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Research Article

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Posted Date: March 3rd, 2022

DOI: <https://doi.org/10.21203/rs.3.rs-1385245/v1>

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A Study on Dynamic Stress of Planetary Gear Transmission in Non-Inertial System

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Abstract: Considering the comprehensive effect of the internal non-inertial system (INIS) of planetary gear transmission (PGT) and external non-inertial system (ENIS) of the body, combined with the modified Heywood formula and Hertz formula, a calculation model for the dynamic bending stress and contact dynamic stress of the PGT in a non-inertial system (NIS) is proposed. The stress non-inertial coefficients (K_{NI}) describing the influence of the additional effects on the dynamic stress in the NIS are defined. Additionally, the variation law of non-inertial coefficients under two typical non-inertial conditions of variable-speed level flight and somersault motion was studied. The effect of gear installation and manufacturing errors on the non-inertial coefficients under two maneuvering conditions was analyzed. The results indicated that non-inertial coefficients increase with an increase in the maneuverability of the body, and the variable-speed horizontal flight and somersault motion have significant effects on the changing trend of non-inertial coefficients with the gear installation and manufacturing errors. Compared with the gear error, the effect of the NIS has a significant influence on the non-inertial coefficients.

Key Words: planetary gear transmission; maneuvering flight; dynamic stress; stress non-inertial coefficient

1. Introduction

In general engineering dynamics, the earth can be approximated as an inertial system, and a reference system that performs variable-speed linear motion or curvilinear motion relative to the inertial reference system of the earth is called a non-inertial system (NIS) [1]. An aircraft changes the flight trajectory by changing the acceleration, realizing several maneuvers such as hovering, climbing, and rolling. It is a typical NIS. Planetary gear transmission (PGT) is widely used in aviation owing to its high transmission ratio, high reliability, and compact structure [2,3]. The planetary gear rotates while revolving with the planet carrier; therefore, the PGT is a NIS, that is, an internal non-inertial system (INIS). When the PGT moves in space following the body, it is affected by the external non-inertial system (ENIS); that is, the components of the PGT are under the action of the dual NIS. For aviation PGTs that work under extreme conditions such as high speed, heavy load, and high vibration, the NIS has a significant impact on its stress, deformation, gear meshing, and other dynamic behaviors. Therefore, it is necessary to study the additional effect of the NIS of maneuvering flight on the PGT, and to ensure good environmental adaptability as well as safe and reliable operation of the PGT in the transient posture and arbitrary motion path.

The bending dynamic stress of the gear tooth root and the contact dynamic stress of the gear pair are the direct reflections of the load-carrying capacity of the gear, as well as the direct cause of fatigue failure, such as gear tooth fracture, tooth surface pitting, and tooth surface spalling. The additional effects of NIS as well as gear installation and manufacturing errors may significantly change them, which poses a major threat to the safety and reliability of an aircraft. Therefore, it is crucial to improve the load-carrying capacity of PGT in a highly maneuverable NIS that analyzes the additional effects of typical NIS environments such as variable-speed level flight and somersault motion on the dynamic stress of the PGT.

In the existing literature, the study of dynamics in the NIS is mainly concentrated in the engine rotor-bearing system. Soni et al. [4] studied the vibration response of a magnetic suspension rotor system during pitch, roll, and yaw, and they proved the superiority of the new control method in the stability control of a magnetic suspension rotor system under moving conditions by comparing the system stability during motion. Dakel et al. [5] established a single-rotor shafting finite element model based on the Timoshenko beam element. By deriving the Lagrangian equation of motion, they established that the basic rotation not only caused changes in the system parameters but also generated additional external force vectors. Basic translational motion only generates additional force vectors. Using rotor stability diagrams, Campbell diagrams, steady-state response, and motion trajectories, they studied the dynamic characteristics of two different rotor configurations: symmetric and asymmetric. Duchemin [6] used a multi-scale method to study the stability of the rotor system when performing sinusoidal rotation and perform experimental verification. Yang et al. [7] studied the nonlinear response of a rigidly supported cracked rotor system under maneuvering flight conditions. Three ways for rotor responses going to chaos were summarized. Han et al. [8] established a dynamic model of an aero-engine single-rotor shafting-ball bearing system based on the Timoshenko beam element, simulated the flight maneuver state as an additional damping matrix, stiffness matrix, and load vector, and they studied the multi-period, irregular, and transient vibrations. The results indicated that both the beginning and end of the maneuvering flight could cause a transient response. As a nonlinear system with self-excitation characteristics, the gear transmission system exhibits significant differences in the dynamic behavior of the engine rotor system. Currently, research on the dynamics of gear transmission systems under NIS is only in the preliminary stage. Parker [9-11] conducted a series of studies on parallel shaft gear transmissions, including modeling methods, internal and external excitation, structural parameters, and their corresponding vibration behaviors. Han et al. [12] deduced the motion equations of the spur gear pair system under the excitation of basic motions such as roll, pitch, and yaw based on the energy theorem and Lagrangian principle, and they discussed the effect of various basic rotations on the amplitude–frequency response of the gear. Qiu et al. [13] deduced the translation-torsion-axial dynamics model of the PGT under the basic pitching motion, and they studied the effect of the additional damping, stiffness, and external force excitation caused by the pitching motion on the dynamic response of the system. The results indicate that the motion of the body introduces more excitation frequencies to increase the system vibration and increase the risk of system resonance. Wei et al. [14-16] considered the comprehensive influence of the INIS and ENIS, and they constructed a kinematic and dynamic model of the gear system in any space motion state of the body in the NIS. They deduced the time-varying influence function of gravity on the carrier in the NIS, derived the additional inertial force and the additional gyro moment, and initially revealed the law of motion change and dynamic response of the transmission system under the condition of the NIS. In addition, multi-excitation and nonlinear factors are being considered more and more comprehensively to capture their coupling effects under NIS conditions [17,18].

For gear dynamic stress, Kahraman et al. [19-21] studied the effect of the thickness of the ring gear flange, the number of planetary gears, and the error of pinholes on the time-domain history of the dynamic stress of the tooth root of the ring gear through experiments. Cornell [22] obtained the modified Heywood formula by simplifying the Heywood formula, and proved the modified formula by comparing different pressure angles, transmission ratios, tooth numbers, and load contact positions corresponding to the tooth root dynamic stress test, finite element, and analytical results. Li et al. [23] analyzed the tooth load, load sharing rate, contact stress, root bending stress, transmission error, and mesh stiffness of spur gears, and they discussed the effect of the contact ratio on the gear strength and basic performance. Zhang et al. [24] established a large-load marine helical gear meshing finite element model based on the explicit dynamics finite element method, simulated the dynamic stress changes of the helical gear under various working

conditions, and analyzed the effect of the working condition changes on the dynamic stress. The results indicate that the speed and center distance errors of the helical gear pair have a significant influence on the dynamic stress of the tooth root. He et al. [25] proposed a numerical model of intersecting bevel gears to evaluate the dynamic stress distribution and dynamic transmission error, and they calculated the dynamic contact characteristics of involute and non-involute bevel gears using the three-dimensional dynamic contact finite element method. Zhou et al. [26] developed a gear test bench that can wirelessly measure the dynamic tooth root stress of a light-load spur gear drive and wirelessly send or receive the tooth root strain, speed, and torque signals of the gear shaft. Tsai and Ye [27] proposed a PGT gear tooth load contact analysis method and studied the effect of the torsional stiffness of the sun gear and planet carrier on the contact stress distribution. Wang et al. [28] used the modal superposition and Newmark beta time integration methods to calculate the dynamic characteristics of the ring gear, and they verified the validity of the calculation method by comparing it with the experimental results.

This study establishes a dynamic stress calculation model of the PGT in a NIS and defines a parameter describing the influence of additional effects on the dynamic stress of gears under a motorized NIS—the stress non-inertial system coefficient (K_{NI}). In addition, we conducted an in-depth study on the influence of gear installation and manufacturing errors on the stress non-inertial coefficients under the two typical non-inertial conditions of the airframe in variable-speed level flight and somersault motion.

2. Dynamic model of planetary gear system in a non-inertial frame

2.1 Coordinate system setting

Considering that the internal meshing position of the gear system rotates with the planet carrier, the dynamic model of the PGT is established in the planet carrier follow-up coordinate system, and the kinematic analysis of each component is performed in the multi-coordinate system. As shown in **Fig. 1(a)**, all the degrees of freedom of the sun gear shaft, ring gear, and translational degrees of freedom of the planet carrier shaft are defined in the planet carrier follow-up coordinate system (INIS), $o_c-x_c y_c z_c$. All degrees of freedom of the planetary gear and pin are defined in the planetary carrier follow-up coordinate system (the origin is at the center of the planetary gear), $o_p-\xi\eta z_p$. The torsional degrees of freedom, θ_c , of the planet carrier and all degrees of freedom of the housing are defined in the coordinate system $o_0-x_0 y_0 z_0$. As shown in **Fig. 1(b)**, body motion is defined in the body follow-up coordinate system, $o_1-x_1 y_1 z_1$. The ground fixed coordinate system, $O-XYZ$, is used to describe the rotation of the carrier around itself and the change in spatial position and posture. Here, o_1 represents the center of gravity of the aircraft; the flying direction of the aircraft is the same as vertical axis $o_1 y_1$; axis $o_1 z_1$ is perpendicular to the vertical axis $o_1 y_1$, and the direction is vertically downward; horizontal axis, $o_1 x_1$, is perpendicular to axis $o_1 y_1$ and axis $o_1 z_1$; each axis of coordinate system $o_0-x_0 y_0 z_0$ is parallel to the corresponding axis of coordinate system $o_1-x_1 y_1 z_1$.

(a) Gear rotor system

(b) Housing

Fig. 1 Coordinate systems of the system

2.2 Kinematics analysis in NISs

2.2.1 Central component

The sun gear, ring gear, and planet carrier are the central components. For the sun gear shaft, the coordinate system is shown in **Fig. 2(a)** when only the INIS conditions are considered. Node M_s represents the real-time position of the sun gear shaft node. The radius vector of moving point M_s in coordinate system $o_c-x_c y_c z_c$ is $\mathbf{r}_{ns1} = \mathbf{r}_s = x_s \cdot \mathbf{k}_c + y_s \cdot \mathbf{j}_c + z_s \cdot \mathbf{i}_c$. x_s, y_s, z_s indicate the real-time dynamic displacement response value of the node and subscript "n" indicates that only INIS is considered.

Fig. 2 Coordinate system of sun shafting under NIS

As shown in **Fig. 1**, when the node finite element method is used to model the shaft system, the nodes of the sun gear shaft are distributed along the direction of its axis. Therefore, the sun shaft axis node M_s in coordinate system $o_c-x_c y_c z_c$ has a radius vector of $\mathbf{r}_{ns2} = \mathbf{r}_{s0} + \mathbf{r}_s$. As shown in **Fig. 2(b)**, $\mathbf{r}_{s0} = z_{s0} \cdot \mathbf{k}_c$, where $\mathbf{k}_c$ represents the initial radius vector and z_{s0} represents the coordinate of node M_s relative to origin o_c .

The second derivative of $\mathbf{r}_{ns1}$ and $\mathbf{r}_{ns2}$ is taken to obtain absolute accelerations $\mathbf{a}_{ns1}$ and $\mathbf{a}_{ns2}$ of node M_s , respectively, according to **Eq. (2)** and **Eq. (3)**. Among them, the derivative of unit vectors $\mathbf{i}_c, \mathbf{j}_c$, and $\mathbf{k}_c$ of coordinate system $o_c-x_c y_c z_c$ with respect to time t is given according to **Eq. (1)**. $\boldsymbol{\omega}$ represents the angular velocity of the planetary carrier follow-up coordinate system $o_c-x_c y_c z_c$ rotating around point o_c , and $\boldsymbol{\omega} = \omega_c \cdot \mathbf{k}_c$. ω_c represents the speed of the planet carrier. $\dot{\mathbf{i}}_c, \dot{\mathbf{j}}_c$, and $\dot{\mathbf{k}}_c$ represent the real-time acceleration of the node. $\mathbf{v}_s = \dot{x}_s \cdot \mathbf{i}_c + \dot{y}_s \cdot \mathbf{j}_c + \dot{z}_s \cdot \mathbf{k}_c$, where $\dot{x}_s, \dot{y}_s$, and $\dot{z}_s$ indicate the real-time speed of the node.

$$\frac{d\mathbf{i}_c}{dt} = \boldsymbol{\omega} \times \mathbf{i}_c, \frac{d\mathbf{j}_c}{dt} = \boldsymbol{\omega} \times \mathbf{j}_c, \frac{d\mathbf{k}_c}{dt} = \boldsymbol{\omega} \times \mathbf{k}_c \quad (1)$$

$$\mathbf{a}_{ns1} = \ddot{x}_s \cdot \mathbf{i}_c + \ddot{y}_s \cdot \mathbf{j}_c + \ddot{z}_s \cdot \mathbf{k}_c + 2\boldsymbol{\omega} \times \mathbf{v}_s + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_s) \quad (2)$$

$$\mathbf{a}_{ns2} = \ddot{x}_s \cdot \mathbf{i}_c + \ddot{y}_s \cdot \mathbf{j}_c + \ddot{z}_s \cdot \mathbf{k}_c + 2\boldsymbol{\omega} \times \mathbf{v}_s + \boldsymbol{\omega} \times (\mathbf{r}_s + \mathbf{r}_{s0}) + \boldsymbol{\omega} \times [\boldsymbol{\omega} \times (\mathbf{r}_s + \mathbf{r}_{s0})] \quad (3)$$

Because $\boldsymbol{\omega} \times \mathbf{k}_c = \mathbf{0}$, **Eq. (3)** can be further expressed as **Eq. (4)**. Comparing **Eq. (2)** and **Eq. (4)**, it can be observed that when only INIS conditions are considered, the axial distribution of nodes will not produce

practical effects.

$$\mathbf{a}_{ns2} = \ddot{\mathbf{x}}_s \cdot \mathbf{i}_c + \ddot{\mathbf{y}}_s \cdot \mathbf{j}_c + \ddot{\mathbf{z}}_s \cdot \mathbf{k}_c + 2\boldsymbol{\omega} \times \mathbf{v}_s + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_s) \quad (4)$$

The planet carrier shaft and the sun gear shaft belong to the same central component, and the node settings are similar; therefore, their absolute acceleration equations are similar. Conversely, the ring gear does not have an initial radius vector (the deformation of the ring gear is not considered in the modeling). Therefore, when only the INIS conditions are considered, the absolute acceleration of the ring gear node is similar to that in **Eq. (4)**.

In contrast, according to **Fig. 2(c)**, considering the additional effects of the ENIS conditions of maneuvering flight, the radius vector of the sun gear axis node M_s in the ground fixed coordinate system $O-XYZ$ is $\mathbf{r}_{Ms} = \mathbf{r}_0 + \mathbf{r}_1 + \mathbf{r}_{s0} + \mathbf{r}_s$. $\mathbf{r}_0$ and $\mathbf{r}_1$ represent the radius vector of o_1 in fixed system $O-XYZ$ and the radius vector of o_c in body coordinate system $o_1-x_1y_1z_1$, respectively.

$$\begin{cases} \mathbf{r}_0 = X \cdot \mathbf{i} + Y \cdot \mathbf{j} + Z \cdot \mathbf{k} \\ \mathbf{r}_1 = x_1 \cdot \mathbf{i}_1 + y_1 \cdot \mathbf{j}_1 + z_1 \cdot \mathbf{k}_1 \end{cases} \quad (5)$$

The derivative of the unit vector of $o_1-x_1y_1z_1$ and planet carrier follow-up coordinate system $o-\xi\eta z_p$ with respect to time t is given according to **Eq. (6)**, where $\boldsymbol{\Omega}$ represents the rotation angular velocity of $o_1-x_1y_1z_1$ around o_1 .

$$\begin{cases} \frac{d\mathbf{i}_1}{dt} = \boldsymbol{\Omega} \times \mathbf{i}_1, \frac{d\mathbf{j}_1}{dt} = \boldsymbol{\Omega} \times \mathbf{j}_1, \frac{d\mathbf{k}_1}{dt} = \boldsymbol{\Omega} \times \mathbf{k}_1 \\ \frac{d\mathbf{i}_c}{dt} = (\boldsymbol{\Omega} + \boldsymbol{\omega}) \times \mathbf{i}_c, \frac{d\mathbf{j}_c}{dt} = (\boldsymbol{\Omega} + \boldsymbol{\omega}) \times \mathbf{j}_c, \frac{d\mathbf{k}_c}{dt} = (\boldsymbol{\Omega} + \boldsymbol{\omega}) \times \mathbf{k}_c \end{cases} \quad (6)$$

Taking the second-order derivation of $\mathbf{r}_{Ms}$, the absolute acceleration of moving point M_s in INIS and ENIS can be obtained. Excluding the terms when only the INIS conditions is considered (the terms in **Eq. (4)**), the absolute acceleration derivative term, $\mathbf{a}_{as}$, in the ENIS is given according to **Eq. (7)**. Subscript "a" represents a derivative item under the ENIS conditions.

$$\begin{aligned} \mathbf{a}_{as} = & \ddot{\mathbf{x}}_s \cdot \mathbf{i} + \ddot{\mathbf{y}}_s \cdot \mathbf{j} + \ddot{\mathbf{z}}_s \cdot \mathbf{k} + \ddot{\mathbf{x}}_1 \cdot \mathbf{i}_1 + \ddot{\mathbf{y}}_1 \cdot \mathbf{j}_1 + \ddot{\mathbf{z}}_1 \cdot \mathbf{k}_1 + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_1) + \boldsymbol{\Omega} \times (\mathbf{r}_s + \mathbf{r}_{s0}) \\ & + \boldsymbol{\Omega} \times [\boldsymbol{\Omega} \times (\mathbf{r}_s + \mathbf{r}_{s0})] + 2\boldsymbol{\Omega} \times \mathbf{v}_s + \boldsymbol{\omega} \times [\boldsymbol{\Omega} \times (\mathbf{r}_s + \mathbf{r}_{s0})] + \boldsymbol{\Omega} \times (\boldsymbol{\omega} \times \mathbf{r}_s) \end{aligned} \quad (7)$$

Similar to the case of considering only the INIS, the coordinate system of the planet carrier shaft node and the absolute acceleration additional items under the ENIS are similar to the sun shaft axis node. In contrast, there is no initial radius vector of the ring gear. The additional items of absolute acceleration of the ring gear and planet carrier shaft nodes are given according to **Eq. (8)** and **Eq. (9)**, respectively, where $\mathbf{r}_{c0}$ represents the initial radius vector of the planet carrier shaft node, similar to $\mathbf{r}_{s0}$.

$$\begin{aligned} \mathbf{a}_{ar} = & \ddot{\mathbf{x}}_r \cdot \mathbf{i} + \ddot{\mathbf{y}}_r \cdot \mathbf{j} + \ddot{\mathbf{z}}_r \cdot \mathbf{k} + \ddot{\mathbf{x}}_1 \cdot \mathbf{i}_1 + \ddot{\mathbf{y}}_1 \cdot \mathbf{j}_1 + \ddot{\mathbf{z}}_1 \cdot \mathbf{k}_1 + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_1) + 2\boldsymbol{\Omega} \times \mathbf{v}_r \\ & + \boldsymbol{\Omega} \times \mathbf{r}_r + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_r) + \boldsymbol{\omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_r) + \boldsymbol{\Omega} \times (\boldsymbol{\omega} \times \mathbf{r}_r) \end{aligned} \quad (8)$$

$$\begin{aligned} \mathbf{a}_{ac} = & \ddot{\mathbf{x}}_c \cdot \mathbf{i} + \ddot{\mathbf{y}}_c \cdot \mathbf{j} + \ddot{\mathbf{z}}_c \cdot \mathbf{k} + \ddot{\mathbf{x}}_1 \cdot \mathbf{i}_1 + \ddot{\mathbf{y}}_1 \cdot \mathbf{j}_1 + \ddot{\mathbf{z}}_1 \cdot \mathbf{k}_1 + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_1) + \boldsymbol{\Omega} \times (\mathbf{r}_c + \mathbf{r}_{c0}) \\ & + \boldsymbol{\Omega} \times [\boldsymbol{\Omega} \times (\mathbf{r}_c + \mathbf{r}_{c0})] + 2\boldsymbol{\Omega} \times \mathbf{v}_c + \boldsymbol{\omega} \times [\boldsymbol{\Omega} \times (\mathbf{r}_c + \mathbf{r}_{c0})] + \boldsymbol{\Omega} \times (\boldsymbol{\omega} \times \mathbf{r}_c) \end{aligned} \quad (9)$$

2.2.2 Planetary gears and pin shaft

In contrast to the central component, the degrees of freedom of the planetary gears and pin shaft nodes are defined in the planetary carrier follow-up coordinate system, $o_p-\xi\eta z_p$. Considering the planetary gear, as shown in **Fig. 3**, planetary gear node M_p is located on the equivalent base circle of the planet carrier. Its

initial radius vector $\mathbf{r}_{bc} = x_{bc} \cdot \mathbf{i}_c + y_{bc} \cdot \mathbf{j}_c$, where x_{bc} and y_{bc} are constants ($\sqrt{x_{bc}^2 + y_{bc}^2} = r_{bc}$).

Fig. 3 Coordinate system of the planet under NIS

When only INIS is considered, the radius vector of the moving point, M_p , in coordinate system $o_p\text{-}\zeta\eta z_p$ is $\mathbf{r}_{np} = \mathbf{r}_{bc} + \mathbf{r}_p$. As shown in **Fig. 3(a)**, similar to the derivation of the absolute acceleration of the central member, the absolute acceleration of M_p is as follows:

$$\mathbf{a}_{np} = \ddot{\theta} \mathbf{i}_p + \ddot{\eta} \mathbf{j}_p + \ddot{\zeta} \mathbf{k}_p + 2\boldsymbol{\omega} \times \mathbf{v}_p + \boldsymbol{\omega} \times (\mathbf{r}_{bc} + \mathbf{r}_p) + \boldsymbol{\omega} \times [\boldsymbol{\omega} \times (\mathbf{r}_{bc} + \mathbf{r}_p)] \quad (10)$$

Compared with the planetary gears, the pin shaft nodes are axially distributed along the $o_p z_p$ direction of their axis; therefore, moving point M_{pp} has an initial radius vector $\mathbf{r}_{pp0}$ similar to $\mathbf{r}_{s0}$, and $\mathbf{r}_{pp0} = z_{p0} \cdot \mathbf{k}_p$. The radius vector of the pin shaft node M_{pp} in coordinate system $o_p\text{-}\zeta\eta z_p$ is $\mathbf{r}_{ppn} = \mathbf{r}_{bc} + \mathbf{r}_{pp0} + \mathbf{r}_{pp}$. When only the INIS conditions are considered, the absolute acceleration equation of the pin shaft moving point M_{pp} is similar to **Eq. (10)**.

According to **Fig. 3(b)**, the radius vector of the planetary gear moving point M_p in fixed system $O\text{-}XYZ$ is $\mathbf{r}_{Mp} = \mathbf{r}_0 + \mathbf{r}_1 + \mathbf{r}_{bc} + \mathbf{r}_p$. Considering the sun gear axis, the additional item of absolute acceleration of the planet gear node under maneuvering flight conditions can be obtained.

$$\begin{aligned} \mathbf{a}_{ap} = & \ddot{X} \mathbf{i} + \ddot{Y} \mathbf{j} + \ddot{Z} \mathbf{k} + \ddot{\Omega} \times \mathbf{r}_1 + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_1) \\ & + \ddot{\Omega} \times (\mathbf{r}_p + \mathbf{r}_{bc}) + \boldsymbol{\Omega} \times [\boldsymbol{\Omega} \times (\mathbf{r}_p + \mathbf{r}_{bc})] + 2\boldsymbol{\Omega} \times \mathbf{v}_p \\ & + \boldsymbol{\omega} \times [\boldsymbol{\Omega} \times (\mathbf{r}_p + \mathbf{r}_{bc})] + \boldsymbol{\Omega} \times [\boldsymbol{\omega} \times (\mathbf{r}_p + \mathbf{r}_{bc})] \end{aligned} \quad (11)$$

Because the pin shaft node M_{pp} has initial radius vectors $\mathbf{r}_{bc}$ and $\mathbf{r}_{pp0}$ at the same time, its radius vector is $\mathbf{r}_{Mpp} = \mathbf{r}_{Mp} + \mathbf{r}_{pp0}$ in fixed system $O\text{-}XYZ$. The additional item of the absolute acceleration of the pin shaft node is shown in **Eq. (12)**.

$$\begin{aligned} \mathbf{a}_{app} = & \ddot{X} \mathbf{i} + \ddot{Y} \mathbf{j} + \ddot{Z} \mathbf{k} + \boldsymbol{\Omega} \times \mathbf{r}_1 + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_1) + 2\boldsymbol{\Omega} \times \mathbf{v}_{pp} \\ & + \ddot{\Omega} \times (\mathbf{r}_{pp} + \mathbf{r}_{bc} + \mathbf{r}_{pp0}) + \boldsymbol{\Omega} \times [\boldsymbol{\Omega} \times (\mathbf{r}_{pp} + \mathbf{r}_{bc} + \mathbf{r}_{pp0})] \\ & + \boldsymbol{\omega} \times [\boldsymbol{\Omega} \times (\mathbf{r}_{pp} + \mathbf{r}_{bc} + \mathbf{r}_{pp0})] + \boldsymbol{\Omega} \times [\boldsymbol{\omega} \times (\mathbf{r}_{pp} + \mathbf{r}_{bc})] \end{aligned} \quad (12)$$

2.3 System-level dynamics model

The author has established a detailed PGT dynamics model [8], using the nodal finite element method and poly condensation technology to model the dynamics of the shafting and housing structures. Thereafter, the shafting and casing structures were coupled to establish a system-level dynamics model. The coupling dynamics equations of the system are as follows:

$$\mathbf{M}\mathbf{A}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = \mathbf{T}(t) + \mathbf{G}(t) \quad (13)$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ represent the overall system mass, damping, and stiffness matrix, respectively; $\mathbf{A}(t)$

represents the absolute acceleration; $\bar{X}(t)$ represents the displacement vector considering nonlinear factors; $T(t)$ represents external excitation; $G(t)$ represents the gravity vector.

When only considering the INIS conditions, $A(t)$ and $G(t)$ are represented according to **Eq. (14)**. When considering the ENIS conditions, the component responds inertially under the internal and external dual NIS, and the gravity component changes, according to **Eq. (15)**.

$$\left\{ \begin{array}{l} A(t) = [\underset{\text{Sun gear shaft node}}{(a_{ps})_{xyz}^{\otimes z}}, L, \underset{\text{Planetary gear node}}{(a_{pp})_{xyz}^{\otimes z}}, L, \underset{\text{Ring gear node}}{(a_{pr})_{xyz}^{\otimes z}}, L, \\ \underset{\text{Pin shaft node}}{(a_{pp})_{xyz}^{\otimes z}}, L, \underset{\text{Carrier shaft node}}{(a_{pc})_{xyz}^{\otimes z}}, L, \underset{\text{Housing node}}{(a_{ph})_{xyz}^{\otimes z}}, L]^T \\ G(t) = [\underset{\text{node 1}}{0, 0, G_1, 0, L}, \underset{\text{node i}}{0, 0, G_i, 0, L}, \underset{\text{node j}}{0, 0, G_j, 0, L}, \underset{\text{Housing node k}}{0, 0, G_k, 0, 0, 0, L}]^T \end{array} \right. \quad (14)$$

$$\left\{ \begin{array}{l} A(t) = [\underset{\text{Sun gear shaft node}}{(a_{ps})_{xyz}^{\otimes z} + (a_{ps})_{xyz}^{\otimes z}}, L, \underset{\text{Planetary gear node}}{(a_{pp})_{xyz}^{\otimes z} + (a_{pp})_{xyz}^{\otimes z}}, L, \\ \underset{\text{Ring gear node}}{(a_{pr})_{xyz}^{\otimes z} + (a_{pr})_{xyz}^{\otimes z}}, L, \underset{\text{Pin shaft node}}{(a_{pp})_{xyz}^{\otimes z} + (a_{pp})_{xyz}^{\otimes z}}, L, \\ \underset{\text{Carrier shaft node}}{(a_{pc})_{xyz}^{\otimes z} + (a_{pc})_{xyz}^{\otimes z}}, L, \underset{\text{Housing node}}{(a_{ph})_{xyz}^{\otimes z} + (a_{ph})_{xyz}^{\otimes z}}, L]^T \\ G(t) = [\underset{\text{node 1}}{(G_1)_{xyz}^{\otimes z}}, 0, L, \underset{\text{node i}}{(G_i)_{xyz}^{\otimes z}}, 0, L, \underset{\text{node j}}{(G_j)_{xyz}^{\otimes z}}, 0, L, \underset{\text{Housing node k}}{(G_k)_{xyz}^{\otimes z}}, 0, 0, 0, L]^T \end{array} \right. \quad (15)$$

2.4 Dynamic stress calculation model

Solving **Eq. (13)** yields the meshing force, F_{mj} , at any meshing position j . According to **Eq. (16)**, the load distribution coefficient, Lsf_{kj} , between the teeth of the tooth pair k ($k = 1, 2$) at meshing position j in the ideal meshing state can be obtained, where K_{kj} represents the meshing stiffness of the tooth pair k at meshing position j . Thereafter, the meshing force of tooth pair k at meshing position j is $F_{kj} = Lsf_{kj} \cdot F_{mj}$. Finally, the modified Heywood formula [14] in **Eq. (17)** and the Hertz contact stress formula in **Eq. (18)** are used to calculate the dynamic root bending stress and dynamic contact stress, respectively.

$$Lsf_{kj} = \frac{K_{kj}}{K_{1j} + K_{2j}} \quad (16)$$

$$\sigma_{Fkj} = \frac{F_{kj} \cos \alpha_j}{W} \left[1 + 0.26 \left(\frac{h_f}{2r_f} \right)^{0.7} \right] \cdot \left[6 \frac{l_f - h_L \cdot \tan \alpha_j / 2}{h_f^2} + \left(\frac{0.72}{h_f l_f} \right)^{0.5} \left(1 - \frac{h_L}{h_f} \cdot \nu \cdot \tan \alpha_j \right) - \frac{\tan \alpha_j}{h_f} \right] \quad (17)$$

$$\sigma_{Hkj} = \sqrt{\frac{F_{kj} \cos \alpha_j}{\pi W' \cos \alpha} \left[\frac{1 / \rho_{kj}}{\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}} \right]} \quad (18)$$

In **Eq. (17)** and **Eq. (18)**, σ_{Fkj} , σ_{Hkj} , and ρ_{kj} represent the maximum dynamic bending stress, dynamic contact stress, and comprehensive curvature of tooth pair k at meshing position j , respectively. $1 / \rho_{kj} = 1 / \rho_{kj1} \pm 1 / \rho_{kj2}$ (the negative sign is used for internal gearing.). ν is 1/4. ρ_{kj1} , ρ_{kj2} , ν_1 , ν_2 , E_1 , and E_2

represent the main curvature radius, Poisson's ratio, and elastic modulus of the contact point of the two gears at meshing position j .

As shown in **Fig. 4**, for an external gear whose basic parameters are known, the base circle radius, R_b , pitch circle radius, R_p , pitch circle pressure angle, α , pitch circle tooth thickness, s_p , pitch circle to line of action, S , tooth root circle radius, R_r , and root fillet radius, r_f , can be obtained. Thus, the load angle α_j , tooth thickness h_L , tooth thickness h_f at the peak position of the tooth root stress, and height l_f can be obtained according to **Eq. (19)–(22)**.

$$\alpha_j = \phi_L - \alpha_L \quad (19)$$

$$h_L = 2R_L \sin \alpha_L \quad (20)$$

$$h_f = h_o + 2r_f(1 - \cos \gamma_f) \quad (21)$$

$$l_f = l_o + r_f \sin \gamma_f + h_L \tan \alpha_j / 2 \quad (22)$$

Fig. 4 Schematic diagram of the modified Heywood formula parameters of the external gear

ϕ_L , α_L , R_L , h_o , and l_o are given from **Eq. (23)** to **Eq. (27)**. The other parameters of **Eq. (23)** to **Eq. (27)** can be obtained according to **Eq. (28)**.

$$\phi_L = \arccos(R_b / R_L) \quad (23)$$

$$\alpha_L = \alpha_b - \tan \phi_L + \phi_L \quad (24)$$

$$R_L = \sqrt{S^2 + 2R_p \sin \alpha \cdot S + R_p^2} \quad (25)$$

$$h_o = \bar{h} - 2r_f(1 - \cos \bar{\gamma}) \quad (26)$$

$$l_o = R_L \cos \alpha_L - h_L \tan \alpha_j / 2 - \bar{R} \cos \bar{\alpha} - r_f \sin \bar{\gamma} \quad (27)$$

$$\left\{ \begin{array}{l} \alpha_b = s_p / 2R_p + \tan \alpha - \alpha \\ \bar{R} = \sqrt{(R_r + r_f)^2 - 2r_f \sqrt{(R_r + r_f)^2 - R_b^2} + r_f^2} \\ \bar{\phi} = \arccos(R_b / \bar{R}) \\ \bar{\alpha} = \alpha_b - \tan \bar{\phi} + \bar{\phi} \\ \bar{h} = 2\bar{R} \sin \bar{\alpha} \\ \bar{\gamma} = \bar{\phi} - \bar{\alpha} \end{array} \right. \quad (28)$$

3. Stress non-inertial system coefficient

The additional effect of the NIS has a significant impact on the dynamic stress of planetary gears. To quantify the influence of the additional effects of NIS on the dynamic stress of train gears to facilitate the design of the PGT, the stress non-inertial system coefficient (K_{NI}) is defined, including the bending dynamic stress non-inertial system coefficient K_{NIFw2} and the contact dynamic stress non-inertial system coefficient K_{NIHu} . In **Eq. (29)**, "NI" represents a non-inertial system, $w2 = s, p, pr, r$, and "pr" represents the planetary gear that participates in internal meshing.

Take K_{NIFs} as an example, which is the non-inertial coefficient of the bending dynamic stress of the sun gear, to illustrate the specific calculation method. First, the minimum value of the root mean square (RMS) value of the bending dynamic stress at the root of the sun gear in INIS, $\sigma_{Fs_n} = \min(\sigma_{Fs_{1n}}, \sigma_{Fs_{2n}}, \sigma_{Fs_{3n}}, \sigma_{Fs_{4n}})$. Thereafter, the ratio of the RMS value of the tooth root bending dynamic stress to σ_{Fs_n} when the sun gear participates in the meshing of different meshing pairs in an ENIS was obtained. Because this model contains four planetary gears, that is, four pairs of s-p gear pairs, four ratios were obtained. Finally, the maximum values of these four ratios were taken to obtain coefficient K_{NIFs} of the dynamic bending stress of the sun gear.

$$\begin{cases} K_{NIFw2} = \max \left[\frac{\sigma_{Fw2i}}{\sigma_{Fw2n}} \right], \sigma_{Fw2n} = \min(\sigma_{Fw2i_n}) \\ K_{NIHu} = \max \left[\frac{\sigma_{Hu_i}}{\sigma_{Hu_n}} \right], \sigma_{Hu_n} = \min(\sigma_{Hu_i_n}) \end{cases} \quad (29)$$

3.1 Non-inertial coefficients of stress during variable-speed level flight and somersault motion

(1) Variable-speed-level flight

Variable-speed-level flight is the most common maneuver. As shown in **Fig. 5**, the airframe flies in a straight line along the OY direction with acceleration a_e . " $a_e < 0$ " and " $a_e > 0$ " indicate decelerated level flight and accelerated level flight, respectively.

Fig. 5 Schematic diagram of plane flight with variable speed

When only the INIS conditions are considered, the absolute acceleration equation of the sun gear shaft node is given according to **Eq. (30)**.

$$\begin{cases} a_{nsx} = \ddot{x}_s - 2\omega_c \cdot \dot{y}_s - \omega_c^2 \cdot x_s - \dot{\omega}_c \cdot y_s \\ a_{nsy} = \ddot{y}_s + 2\omega_c \cdot \dot{x}_s - \omega_c^2 \cdot y_s + \dot{\omega}_c \cdot x_s \\ a_{nsz} = \ddot{z}_s \end{cases} \quad (30)$$

The absolute acceleration equations of the ring gear and planet carrier shaft nodes are the same as those of the sun gear shaft nodes, according to **Eq. (31)** and **Eq. (32)**, respectively.

$$\begin{cases} a_{nrx} = \ddot{x}_r - 2\omega_c \cdot \dot{y}_r - \omega_c^2 \cdot x_r - \dot{\omega}_c \cdot y_r \\ a_{nry} = \ddot{y}_r + 2\omega_c \cdot \dot{x}_r - \omega_c^2 \cdot y_r + \dot{\omega}_c \cdot x_r \\ a_{nrz} = \ddot{z}_r \end{cases} \quad (31)$$

$$\begin{cases} a_{ncx} = \ddot{x}_c - 2\omega_c \cdot \dot{y}_c - \omega_c^2 \cdot x_c - \dot{\omega}_c \cdot y_c \\ a_{ncy} = \ddot{y}_c + 2\omega_c \cdot \dot{x}_c - \omega_c^2 \cdot y_c + \dot{\omega}_c \cdot x_c \\ a_{ncz} = \ddot{z}_c \end{cases} \quad (32)$$

The absolute acceleration of the planetary gear node under INIS conditions is given according to **Eq. (33)**, and the pin node is the same as the planetary gear node, as in **Eq. (34)**.

$$\begin{cases} a_{np\xi} = \ddot{\xi}_p - 2\omega_c \cdot \dot{\eta}_p - \omega_c^2 \cdot (\xi_p + x_{bc}) - \dot{\omega}_c \cdot (\eta_p + y_{bc}) \\ a_{np\eta} = \ddot{\eta}_p + 2\omega_c \cdot \dot{\xi}_p - \omega_c^2 \cdot (\eta_p + y_{bc}) + \dot{\omega}_c \cdot (\xi_p + x_{bc}) \\ a_{npz} = \ddot{z}_p \end{cases} \quad (33)$$

$$\begin{cases} a_{npp\xi} = \ddot{\xi}_{pp} - 2\omega_c \cdot \dot{\eta}_{pp} - \omega_c^2 \cdot (\xi_{pp} + x_{bc}) - \dot{\omega}_c \cdot (\eta_{pp} + y_{bc}) \\ a_{npp\eta} = \ddot{\eta}_{pp} + 2\omega_c \cdot \dot{\xi}_{pp} - \omega_c^2 \cdot (\eta_{pp} + y_{bc}) + \dot{\omega}_c \cdot (\xi_{pp} + x_{bc}) \\ a_{nppz} = \ddot{z}_{pp} \end{cases} \quad (34)$$

The additional term of absolute acceleration of the sun gear shaft during variable-speed level flight is given according to **Eq. (35)**, where $\theta_c = \omega_c \cdot t$.

$$\begin{cases} a_{asx} = a_e \cdot \sin \theta_c \\ a_{asy} = a_e \cdot \cos \theta_c \\ a_{asz} = 0 \end{cases} \quad (35)$$

The additional items of the absolute acceleration of the gear ring and planet carrier shaft nodes are the same as those of the sun gear shaft node. Similarly, the additional terms of the absolute acceleration of the planetary gear and pin shaft node are the same. For the planetary gear node, as in **Eq. (36)**, where $\theta_{cp} = \varphi_{m1} + \theta_c$.

$$\begin{cases} a_{ap\xi} = a_e \cdot \sin \theta_{cp} \\ a_{ap\eta} = a_e \cdot \cos \theta_{cp} \\ a_{apz} = 0 \end{cases} \quad (36)$$

Because the body does not rotate relative to the ground coordinate system, the components of gravity on each coordinate axis do not change. In addition, according to **Eq. (35)–(36)**, the axial distribution of the sun gear shaft, planet carrier shaft, and pin shaft nodes will not add an additional load.

According to **Eq. (29)**, the non-inertial coefficients of the bending stress of each gear and the non-inertial coefficients of the gear pair contact stress under the condition of airframe variable-speed flight are shown in **Fig. 6**.

Fig. 6 K_{NI} of gear stress during variable-speed plane flight

According to **Fig. 6**, with an increase in $|ae|$, the non-inertial coefficient of the bending stress increases nonlinearly, and the non-inertial coefficient of contact stress increases linearly.

(2) Somersault movement

Compared with variable-speed level flight, somersault motion covers maneuvers such as jumping, inverted flight, diving, and it is an important indicator for measuring the maneuverability of an aircraft. As shown in **Fig. 7**, when the body turns somersaults, it turns and flies around the OX axis of the ground fixed coordinate system at the turning angular velocity, Ω_d , and turning radius, R , and it completes a circular trajectory flight in a vertical plane.

Fig. 7 Schematic diagram of somersault movement

The relative motion parameters of somersault movement are given according to **Eq. (37)**, where $\theta_d = \Omega_d \cdot t$.

$$\begin{cases} x_1 = l_1, y_1 = l_2, z_1 = l_3 \\ \Omega = \Omega_d \cdot i_1, \dot{\Omega} = \dot{\Omega}_d \cdot i_1, \ddot{\Omega} = 0 \\ \ddot{x} = -R \cdot \Omega_d^2 \cdot \sin \theta_d + R \cdot \dot{\Omega}_d \cdot \cos \theta_d \\ \ddot{y} = -R \cdot \Omega_d^2 \cdot \cos \theta_d - R \cdot \dot{\Omega}_d \cdot \sin \theta_d \end{cases} \quad (37)$$

The additional term of the absolute acceleration of the sun gear shaft node is given according to **Eq. (38)**, and the additional term of the acceleration of the planet carrier shaft node is the same. The ring gear node has no additional items owing to the axial distribution of the nodes, as shown in **Eq. (39)**.

$$\begin{cases} a_{asx} = -2 \cdot \Omega_d \cdot \sin \theta_c \cdot \dot{x}_s - \Omega_d^2 \cdot \sin^2 \theta_c \cdot x_s - \Omega_d^2 \cdot 0.5 \sin(2\theta_c) \cdot y_s \\ \quad + (\Omega_d \cdot \omega_c \cdot \cos \theta_c - \dot{\Omega}_d \cdot \sin \theta_c) \cdot (z_s + z_{s0}) - l_2 \cdot \Omega_d^2 \cdot \sin \theta_c \\ \quad + R \cdot \sin \theta_c \cdot [\dot{\Omega}_d \cdot \cos(2\theta_d) - \Omega_d^2 \cdot \sin(2\theta_d)] - l_3 \cdot \dot{\Omega}_d \cdot \sin \theta_c \\ a_{asy} = -2 \cdot \Omega_d \cdot \cos \theta_c \cdot \dot{x}_s - \Omega_d^2 \cdot 0.5 \sin(2\theta_c) \cdot x_s - \Omega_d^2 \cdot \cos^2 \theta_c \cdot y_s \\ \quad - (\Omega_d \cdot \omega_c \cdot \sin \theta_c + \dot{\Omega}_d \cdot \cos \theta_c) \cdot (z_s + z_{s0}) - l_2 \cdot \Omega_d^2 \cdot \cos \theta_c \\ \quad + R \cdot \cos \theta_c \cdot [\dot{\Omega}_d \cdot \cos(2\theta_d) - \Omega_d^2 \cdot \sin(2\theta_d)] - l_3 \cdot \dot{\Omega}_d \cdot \cos \theta_c \\ a_{asz} = 2 \cdot \Omega_d \cdot (\sin \theta_c \cdot \dot{x}_s + \cos \theta_c \cdot \dot{y}_s) + (\Omega_d \cdot \omega_c \cdot \cos \theta_c + \dot{\Omega}_d \cdot \sin \theta_c) \cdot x_s \\ \quad - (\Omega_d \cdot \omega_c \cdot \sin \theta_c - \dot{\Omega}_d \cdot \cos \theta_c) \cdot y_s - \Omega_d^2 \cdot (z_s + z_{s0}) \\ \quad - R \cdot [\Omega_d^2 \cdot \cos(2\theta_d) + \dot{\Omega}_d \cdot \sin(2\theta_d)] + l_2 \cdot \dot{\Omega}_d - l_3 \cdot \Omega_d^2 \end{cases} \quad (38)$$

$$\begin{cases}
a_{\text{arx}} = -2 \cdot \Omega_d \cdot \sin \theta_c \cdot \dot{\theta}_c - \Omega_d^2 \cdot \sin^2 \theta_c \cdot x_r - \Omega_d^2 \cdot 0.5 \sin(2\theta_c) \cdot y_r \\
\quad + (\Omega_d \cdot \omega_c \cdot \cos \theta_c - \dot{\Omega}_d \cdot \sin \theta_c) \cdot z_r - l_2 \cdot \Omega_d^2 \cdot \sin \theta_c + R \cdot \sin \theta_c \\
\quad \cdot [\dot{\Omega}_d \cdot \cos(2\theta_d) - \Omega_d^2 \cdot \sin(2\theta_d)] - l_3 \cdot \dot{\Omega}_d \cdot \sin \theta_c \\
a_{\text{ary}} = -2 \cdot \Omega_d \cdot \cos \theta_c \cdot \dot{\theta}_c - \Omega_d^2 \cdot 0.5 \sin(2\theta_c) \cdot x_r - \Omega_d^2 \cdot \cos^2 \theta_c \cdot y_r \\
\quad - (\Omega_d \cdot \omega_c \cdot \sin \theta_c + \dot{\Omega}_d \cdot \sin \theta_c) \cdot z_r - l_2 \cdot \Omega_d^2 \cdot \cos \theta_c + R \cdot \cos \theta_c \\
\quad \cdot [\dot{\Omega}_d \cdot \cos(2\theta_d) - \Omega_d^2 \cdot \sin(2\theta_d)] - l_3 \cdot \dot{\Omega}_d \cdot \cos \theta_c \\
a_{\text{arz}} = 2 \cdot \Omega_d \cdot (\sin \theta_c \cdot \dot{\theta}_c + \cos \theta_c \cdot \ddot{\theta}_c) + (\Omega_d \cdot \omega_c \cdot \cos \theta_c + \dot{\Omega}_d \\
\quad \cdot \sin \theta_c) \cdot x_r - (\Omega_d \cdot \omega_c \cdot \sin \theta_c - \dot{\Omega}_d \cdot \cos \theta_c) \cdot y_r - \Omega_d^2 \cdot z_r \\
\quad - R \cdot [\Omega_d^2 \cdot \cos(2\theta_d) + \dot{\Omega}_d \cdot \sin(2\theta_d)] + l_2 \cdot \dot{\Omega}_d - l_3 \cdot \Omega_d^2
\end{cases} \quad (39)$$

The additional items of the absolute acceleration of the planetary gear and pin shaft nodes are given according to **Eq. (40)** and **Eq. (41)**, respectively.

$$\begin{cases}
a_{\text{np}\xi} = -2 \cdot \Omega_d \cdot \sin \theta_{\text{cp}} \cdot \dot{\theta}_{\text{cp}} - \Omega_d^2 \cdot \sin^2 \theta_{\text{cp}} \cdot (\xi_p + x_{\text{bc}}) - \Omega_d^2 \cdot 0.5 \sin(2\theta_{\text{cp}}) \\
\quad \cdot (\eta_p + y_{\text{bc}}) + (\Omega_d \cdot \omega_c \cdot \cos \theta_{\text{cp}} - \dot{\Omega}_d \cdot \sin \theta_{\text{cp}}) \cdot z_p - l_2 \cdot \Omega_d^2 \cdot \sin \theta_{\text{cp}} \\
\quad + R \cdot \sin \theta_{\text{cp}} \cdot [\dot{\Omega}_d \cdot \cos(2\theta_d) - \Omega_d^2 \cdot \sin(2\theta_d)] - l_3 \cdot \dot{\Omega}_d \cdot \sin \theta_{\text{cp}} \\
a_{\text{ap}\eta} = -2 \cdot \Omega_d \cdot \cos \theta_{\text{cp}} \cdot \dot{\theta}_{\text{cp}} - \Omega_d^2 \cdot 0.5 \sin(2\theta_{\text{cp}}) \cdot (\xi_p + x_{\text{bc}}) - \Omega_d^2 \cdot \cos^2 \theta_{\text{cp}} \\
\quad \cdot (\eta_p + y_{\text{bc}}) - (\Omega_d \cdot \omega_c \cdot \sin \theta_{\text{cp}} + \dot{\Omega}_d \cdot \cos \theta_{\text{cp}}) \cdot z_p - l_2 \cdot \Omega_d^2 \cdot \cos \theta_{\text{cp}} \\
\quad + R \cdot \cos \theta_{\text{cp}} \cdot [\dot{\Omega}_d \cdot \cos(2\theta_d) - \Omega_d^2 \cdot \sin(2\theta_d)] - l_3 \cdot \dot{\Omega}_d \cdot \cos \theta_{\text{cp}} \\
a_{\text{ap}z} = 2 \cdot \Omega_d \cdot (\sin \theta_{\text{cp}} \cdot \dot{\theta}_{\text{cp}} + \cos \theta_{\text{cp}} \cdot \ddot{\theta}_{\text{cp}}) - \Omega_d^2 \cdot z_p + l_2 \cdot \dot{\Omega}_d - l_3 \cdot \Omega_d^2 \\
\quad + (\dot{\Omega}_d \cdot \sin \theta_{\text{cp}} + \Omega_d \cdot \omega_c \cdot \cos \theta_{\text{cp}}) \cdot (\xi_p + x_{\text{bc}}) - R \cdot \Omega_d^2 \cdot \cos(2\theta_d) \\
\quad + (\dot{\Omega}_d \cdot \cos \theta_{\text{cp}} - \Omega_d \cdot \omega_c \cdot \sin \theta_{\text{cp}}) \cdot (\eta_p + y_{\text{bc}}) - R \cdot \dot{\Omega}_d \cdot \sin(2\theta_d)
\end{cases} \quad (40)$$

$$\begin{cases}
a_{\text{app}\xi} = -2 \cdot \Omega_d \cdot \sin \theta_{\text{cp}} \cdot \dot{\theta}_{\text{pp}} - \Omega_d^2 \cdot \sin^2 \theta_{\text{cp}} \cdot (\xi_{\text{pp}} + x_{\text{bc}}) - \Omega_d^2 \cdot 0.5 \sin(2\theta_{\text{cp}}) \\
\quad \cdot (\eta_{\text{pp}} + y_{\text{bc}}) + (\Omega_d \cdot \omega_c \cdot \cos \theta_{\text{cp}} - \dot{\Omega}_d \cdot \sin \theta_{\text{cp}}) \cdot (z_{\text{pp}} + z_{\text{pp}0}) + R \cdot \sin \theta_{\text{cp}} \\
\quad \cdot [\dot{\Omega}_d \cdot \cos(2\theta_d) - \Omega_d^2 \cdot \sin(2\theta_d)] - (l_2 \cdot \Omega_d^2 + l_3 \cdot \dot{\Omega}_d) \cdot \sin \theta_{\text{cp}} \\
a_{\text{app}\eta} = -2 \cdot \Omega_d \cdot \cos \theta_{\text{cp}} \cdot \dot{\theta}_{\text{pp}} - \Omega_d^2 \cdot 0.5 \sin(2\theta_{\text{cp}}) \cdot (\xi_{\text{pp}} + x_{\text{bc}}) - \Omega_d^2 \cdot \cos^2 \theta_{\text{cp}} \\
\quad \cdot (\eta_{\text{pp}} + y_{\text{bc}}) - (\Omega_d \cdot \omega_c \cdot \sin \theta_{\text{cp}} + \dot{\Omega}_d \cdot \cos \theta_{\text{cp}}) \cdot (z_{\text{pp}} + z_{\text{pp}0}) + R \cdot \cos \theta_{\text{cp}} \\
\quad \cdot [\dot{\Omega}_d \cdot \cos(2\theta_d) - \Omega_d^2 \cdot \sin(2\theta_d)] - (l_2 \cdot \Omega_d^2 + l_3 \cdot \dot{\Omega}_d) \cdot \cos \theta_{\text{cp}} \\
a_{\text{app}z} = 2 \cdot \Omega_d \cdot (\sin \theta_{\text{cp}} \cdot \dot{\theta}_{\text{pp}} + \cos \theta_{\text{cp}} \cdot \ddot{\theta}_{\text{pp}}) - \Omega_d^2 \cdot (z_{\text{pp}} + z_{\text{pp}0}) + l_2 \cdot \dot{\Omega}_d - l_3 \cdot \Omega_d^2 \\
\quad + (\dot{\Omega}_d \cdot \sin \theta_{\text{cp}} + \Omega_d \cdot \omega_c \cdot \cos \theta_{\text{cp}}) \cdot (\xi_{\text{pp}} + x_{\text{bc}}) - R \cdot \Omega_d^2 \cdot \cos(2\theta_d) \\
\quad + (\dot{\Omega}_d \cdot \cos \theta_{\text{cp}} - \Omega_d \cdot \omega_c \cdot \sin \theta_{\text{cp}}) \cdot (\eta_{\text{pp}} + y_{\text{bc}}) - R \cdot \dot{\Omega}_d \cdot \sin(2\theta_d)
\end{cases} \quad (41)$$

In addition, the gravity component of each member changes under the condition of the somersault motion, and the time-varying gravity component of each member is as follows:

$$\begin{cases}
F_{\text{gj}1}^m = m_{j1} \cdot g \cdot [-\sin \theta_d \cdot (\mathbf{i}_c \cdot \sin \theta_c + \mathbf{j}_c \cdot \cos \theta_c) + \cos \theta_d \cdot \mathbf{k}_c] \\
F_{\text{gj}2}^m = m_{j2} \cdot g \cdot [-\sin \theta_d \cdot (\mathbf{i}_p \cdot \sin \theta_{\text{cp}} + \mathbf{j}_p \cdot \cos \theta_{\text{cp}}) + \cos \theta_d \cdot \mathbf{k}_p] \\
F_{\text{gx}}^m = m_x \cdot g \cdot (-\sin \theta_d \cdot \mathbf{j}_0 + \cos \theta_d \cdot \mathbf{k}_0)
\end{cases} \quad (42)$$

The change law of the stress non-inertial coefficients of the planetary gear system when the body turns somersaults is shown in **Fig. 8**.

Fig. 8 K_{NI} of gear stress in somersault motion

Figure 8 shows that the change in the non-inertial system coefficient under the influence of the turning radius is relatively simple, and it increases linearly with the increase in R . The change in the non-inertial system coefficients under the influence of the turning angular velocity is more complicated, and K_{NIH} and K_{NIF} increase nonlinearly with the increase in Ω_d . The coefficient of the stress non-inertial system exhibits a small change in $[3, 9]$ rad/s and a large change in $[9, 15]$ rad/s. The K_{NIFp} involved in external and internal gearing changes by 1.2% and 1.0% within $[3, 9]$ rad/s, respectively, and within $[9, 15]$ rad/s, they vary by 12.9% and 12.3%, respectively.

3.2 Effect of installation and manufacturing errors on the coefficient of non-inertial system of stress

Here, we consider the planetary gear tooth root bending stress non-inertial coefficient (K_{NIFp}) and internal meshing contact stress non-inertial coefficient (K_{NIHrp}) for discussion.

3.2.1 Installation and manufacturing errors

(1) Installation error

Installation error is the deviation between the theoretical rotation axis and the actual installation rotation axis of the gear produced during the assembly of the gear. According to the relationship between the installation error and the meshing line in **Fig. 9**, the projection displacement of each gear installation error on the meshing line is given according to **Eq. (43)**, where e_{as} , e_{ap} , and e_{ar} represent the installation error values of the sun, planetary, and ring gears, respectively, and β_s , β_p , and β_r represent the initial phase angle of the installation error of the sun, planet, and ring gears, respectively.

(a) Installation error of sun gear (b) Installation error of planet gear (c) Installation error of ring gear

Fig. 9 Projection diagram of gear installation error to meshing line

$$\begin{cases} E_{as}(t) = -e_{as} \cdot \sin(\omega_c \cdot t + \varphi_{m1} - \beta_s - \alpha_{sp}) \\ E_{asp}(t) = e_{ap} \cdot \sin(\omega_c \cdot t + \varphi_{m1} - \beta_p - \alpha_{sp}) \\ E_{arp}(t) = e_{ap} \cdot \sin(\omega_c \cdot t + \varphi_{m1} - \beta_p + \alpha_{rp}) \\ E_{ar}(t) = -e_{ar} \cdot \sin(\omega_c \cdot t + \varphi_{m1} - \beta_r + \alpha_{rp}) \end{cases} \quad (43)$$

(2) Manufacturing error

Manufacturing error is the deviation of the geometric center of the component from the actual center of rotation produced during the processing of the gear. Similar to the projection derivation process of the installation error on the meshing line, the manufacturing errors of the sun, planetary, and ring gears are projected on the meshing line, according to **Eq. (44)**.

$$\begin{cases} E_{es}(t) = e_{es} \cdot \sin(\omega_{src} \cdot t + \gamma_s + \alpha_{sp} - \varphi_{m1}) \\ E_{esp}(t) = -e_{ep} \cdot \sin(\omega_{prc} \cdot t + \gamma_p + \alpha_{sp}) \\ E_{erp}(t) = -e_{ep} \cdot \sin(\omega_{prc} \cdot t + \gamma_p - \alpha_{rp}) \\ E_{er}(t) = e_{er} \cdot \sin(\omega_{trc} \cdot t + \gamma_r - \alpha_{rp} - \varphi_{m1}) \end{cases} \quad (44)$$

Where e_{es} , e_{ep} , and e_{er} represent the manufacturing error values of the sun, planetary, and ring gears, respectively; γ_s , γ_p , and γ_r represent the initial phase angles of the manufacturing errors of the sun, planetary, and ring gears, respectively; $\omega_{src}(\omega_s - \omega_c)$, $\omega_{prc}(\omega_p - \omega_c)$, and $\omega_{trc}(\omega_r - \omega_c)$ represent the rotational angular velocity of the sun, planetary, and ring gears relative to the planet carrier, respectively.

When only the INIS conditions are considered, the component manufacturing eccentricity error will cause an unbalanced force F_{e1} , according to **Eq. (45)**, where m_{w1} represents the mass of the component, $w1 = s, p, r$, and a_{e1} represents the absolute acceleration, according to **Eq. (46)**.

$$F_{e1} = m_{w1} \cdot a_{e1} \quad (45)$$

$$\begin{aligned} a_{e1} = & \mathcal{E}_{ew1x} \cdot i_c + \mathcal{E}_{ew1y} \cdot j_c + 2 \cdot \omega \times (\mathcal{E}_{ew1x} \cdot i_c + \mathcal{E}_{ew1y} \cdot j_c) \\ & + \mathcal{E}_{ew1x} \cdot i_c + \mathcal{E}_{ew1y} \cdot j_c + \omega \times [\omega \times (\mathcal{E}_{ew1x} \cdot i_c + \mathcal{E}_{ew1y} \cdot j_c)] \end{aligned} \quad (46)$$

$$\begin{cases} e_{ew1x} = e_{ew1} \cdot \cos(\omega_{w1rc} \cdot t + \gamma_w) \\ e_{ew1y} = e_{ew1} \cdot \sin(\omega_{w1rc} \cdot t + \gamma_w) \end{cases} \quad (47)$$

From **Eq. (47)**, **Eq. (46)** can be further expressed as **Eq. (48)**.

$$\begin{aligned} a_{e1} = & i_c \cdot (\omega_{ew1x} - 2 \cdot \omega_c \cdot e_{ew1y} - e_{ew1y} \cdot \omega_c^2 - e_{ew1x} \cdot \omega_c^2) \\ & + j_c \cdot (\omega_{ew1y} + 2 \cdot \omega_c \cdot e_{ew1x} + e_{ew1x} \cdot \omega_c^2 - e_{ew1y} \cdot \omega_c^2) \end{aligned} \quad (48)$$

When considering the ENIS conditions of the maneuver, the manufacturing error will add an unbalanced force F_{e2} , according to Eq. (49).

$$F_{e2} = m_{wl} \cdot a_{e2} \quad (49)$$

$$\begin{aligned} a_{e2} = & 2 \cdot e_{ew1x} \cdot \Omega \times i_c + 2 \cdot e_{ew1y} \cdot \Omega \times j_c \\ & + e_{ew1x} \cdot (\Omega + \omega) \times (\Omega \times i_c) + e_{ew1x} \cdot \Omega \times (\omega \times i_c) + e_{ew1x} \cdot \Omega \times i_c \\ & + e_{ew1y} \cdot (\Omega + \omega) \times (\Omega \times j_c) + e_{ew1y} \cdot \Omega \times (\omega \times j_c) + e_{ew1y} \cdot \Omega \times j_c \end{aligned} \quad (50)$$

3.2.2 Effect of installation and manufacturing errors on the coefficient of non-inertial system under the condition of variable-speed level flight

(1) Installation error

The variation trend of the stress non-inertial coefficient of the airframe with the gear installation error during horizontal flight with variable speed is shown in Fig. 10.

Fig. 10 Effect of gear installation error on K_{NI} of gear stress during variable-speed plane flight

When $a_e = 0$ g, the stress non-inertial coefficients K_{NIFp} and K_{NIHrp} first decrease and then increase with an increase in the installation error of the sun (e_{as}) and ring (e_{ar}) gears, and they increased with an increase in the installation error of the planet gear (e_{ap}). When the level flight is accelerated, K_{NIFp} and K_{NIHrp} decrease linearly with the increase in e_{as} and e_{ar} , and they increase with the increase in e_{ap} . The changing trend during deceleration and level flight is opposite to that during acceleration and level flight.

(2) Manufacturing error

When the airframe is flying at a variable speed, K_{NIFp} and K_{NIHrp} change slowly with the manufacturing error of the sun (e_{es}) and planetary (e_{ep}) gears, as shown in Fig. 11. K_{NIFp} increases nonlinearly with an increase in e_{es} and e_{ep} . K_{NIHrp} decreases as e_{es} and e_{ep} increase when $a_e \geq 0$ g, and it increases as e_{es} and e_{ep} increase when $a_e < 0$ g. The variation of K_{NIFp} and K_{NIHrp} with gear ring manufacturing error (e_{er}) varies with a_e : they decrease first and then increase with the increase in e_{er} during uniform flight, non-linearity decreases with the increase in e_{er} during acceleration flight, and non-linearity increases with the increase in e_{er} during deceleration flight, as shown in Fig. 11(c) and Fig. 11(f).

Fig. 11 Effect of gear manufacturing error on K_{NI} of gear stress during varied-speed plane flight

3.2.3 Effect of installation and manufacturing errors on the coefficient of non-inertial system under somersault motion

(1) Installation error

When the body somersaults, the turning angular velocity of the body changes the trend of K_{NI} of gear stress with gear installation errors, as shown in Fig. 12.

When the body turns somersaults at an angular velocity of [3, 6] rad/s, K_{NIFp} and K_{NIHrp} increase linearly with an increase in the installation error of the sun (e_{as}) and ring (e_{ar}) gears. When $\Omega_d \in [3, 4]$ rad/s, K_{NIFp} and K_{NIHrp} first decrease and then increase with an increase in the installation error of the planetary gear (e_{ap}), and the extreme points of the curve move to the right with the increase in Ω_d . When Ω_d increases to 5 rad/s, K_{NIFp} and K_{NIHrp} decrease with an increase in e_{ap} .

Fig. 12 Effect of gear installation error on K_{NI} of gear stress under somersault motion

(2) Manufacturing error

Figure 13 shows the variation trend of K_{NIFp} and K_{NIHrp} with gear manufacturing errors under different flipping angular velocity conditions. According to Fig. 13(a) and Fig. 13(d), the values of K_{NIFp} and K_{NIHrp}

corresponding to different Ωd are quite different, but the change is slower with the increase in the manufacturing error of the sun gear (e_{es}), and the changing trend is similar. The trend of K_{NIFp} and K_{NIHrp} with e_{ep} does not change under the influence of the flip angular velocity: non-linearly increases with the increase in e_{ep} , as shown in **Fig. 13(b)** and **Fig. 13(e)**. K_{NIFp} increases with e_{er} , and K_{NIHrp} decreases with e_{er} , as shown in **Fig. 13(c)** and **Fig. 13(f)**.

Fig. 13 Effect of gear manufacturing error on K_{NI} of gear stress under somersault motion

It can be observed from **Fig. 10–13** that the increase in gear error will affect the stress non-inertial coefficient, but the NIS effect produced by maneuvering flight is significantly more affected. As shown in **Fig. 12(f)**, when the body is in somersault motion and the ring gear manufacturing error increases from 0 to 120 μm , K_{NIHrp} increases by 0.2%. When the error is constant and the turning angular velocity increased from 3 rad/s to 6 rad/s, K_{NIHrp} increased by 0.8%. According to **Fig. 8**, when the turning angular velocity increases, the stress non-inertial coefficient changes significantly.

4. Conclusion

This work considered the complex combination of INIS and ENIS, and studied the effect of high maneuvering conditions on the dynamic stress of PGT. The specific conclusions are as follows:

(1) The dynamic and dynamic stress calculation models of the PGT under NIS are established, and the dynamic stress of each gear tooth root bending and the contact dynamic stress of each gear pair under NIS conditions were obtained.

(2) The stress non-inertial coefficients are defined, and the influence of the airframe's variable-speed level flight and somersault motion state on the stress non-inertial coefficients is discussed. The results indicate that the stress non-inertial coefficients increase with an increase in the mobility of the random body. The NIS effect has a significant influence on the stress non-inertial system coefficient.

(3) The influence of gear installation and manufacturing errors on the coefficient of the stress NIS under two maneuvering flight states was studied. The maneuvering flight status changes the trend of the stress non-inertial coefficients with gear installation and manufacturing errors. In addition, compared with the gear installation and manufacturing errors, the NIS effect has a greater impact on the stress non-inertial coefficient.

Acknowledgments

This study is part of research project supported by the National Key R&D Program of China (grant number 2020YFB2008101), the National Natural Science Foundation of China (grant number 51775058/52105051), the Innovation Group Science Foundation of Chongqing Natural Science Foundation (No.cstc2019jcyj-cxttX0003) and the Project funded by China Postdoctoral Science Foundation (grant number 2021M700585). The authors would like to express their gratitude for the support of the funding authority.

Author contributions

Conceptualization: J.W. and A.Q.Z.; Methodology: A.Q.Z. and J.W.; Software: H.C.; Validation: A.Q.Z. and M.F.C.; Formal analysis: A.Q.Z. and M.F.C.; Investigation: H.C. and B.P.; Resources: M.F.C.; Data curation: B.P.; Writing—original draft preparation: M.F.C.; Writing—review and editing: M.F.C., J.W. and A.Q.Z.; Visualization: H.C. and B.P.; Supervision: J.W. All authors have read and agreed to the published version of the manuscript.

Competing interests

The authors declare no competing interests.

Availability of data and materials

The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

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