Nuclear Excitation by Muon Capture

Simone Gargiulo (✉ simone.gargiulo@epfl.ch)

Ecole Polytechnique Federale de Lausanne <https://orcid.org/0000-0002-7820-3372>

Ming Gu

University of California, Berkeley

Fabrizio Carbone

École Polytechnique Fédérale de Lausanne <https://orcid.org/0000-0001-7854-6167>

Ivan Madan

École Polytechnique Fédérale de Lausanne

Article

Keywords: Electro-nucleus interactions, isomer, nuclear excitation, muon

Posted Date: February 24th, 2022

DOI: <https://doi.org/10.21203/rs.3.rs-1386584/v1>

License: This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Nuclear Excitation by Muon Capture

Simone Gargiulo^{1*}, Ming Feng Gu², Fabrizio Carbone¹ and Ivan Madan^{1*}

¹Institute of Physics (IPhys), Laboratory for Ultrafast Microscopy and Electron Scattering (LUMES), École Polytechnique Fédérale de Lausanne (EPFL), Lausanne 1015 CH, Switzerland.

²Space Science Laboratory, University of California, Berkeley, CA 94720, USA.

*Corresponding author(s). E-mail(s): simone.gargiulo@epfl.ch; ivanmadan@gmail.com;

22/02/2022

Abstract

Efficient excitation of nuclei via exchange of a real or virtual photon has a fundamental importance for nuclear science and technology development. Here, we present a new mechanism of nuclear excitation based on the capture of a free muon into the atomic orbits (NE μ C). The cross section of such a new process is evaluated using the Feshbach projection operator formalism and compared to other known excitation phenomena, i.e. photo-excitation and nuclear excitation by electron capture (NEEC), showing up to ten orders of magnitude increase in cross section. NE μ C is particularly interesting for MeV excitations that become accessible thanks to the stronger binding of muons to the nucleus. The binding energies of muonic atoms have been calculated introducing a state of the art modification to the Flexible Atomic Code (FAC). An analysis of an experimental scenarios in the context of modern muon production facilities shows that the effect can be detectable for selected isotopes. The total probability of NE μ C is predicted to be $P \approx 1 \times 10^{-6}$ per incident muon in a beam-based scenario. Given the high transition energy provided by muons, NE μ C can have important consequences for isomer feeding and particle-induced fission.

Keywords: Electro-nucleus interactions, isomer, nuclear excitation, muon

Manipulating nuclear transitions is a highly desirable goal due to its implications in the energy sector [1–8]. Long-lived nuclear excitations, formally

2 Nuclear Excitation by Muon Capture

isomers, have lifetimes that are sometimes comparable to the age of the universe and have a potential to release hundreds of megajoules of energy stored in few cubic centimeter. The former aspect is crucial in designing new energy storage solutions: long-duration has been suggested to be the key driver towards decarbonized future [9]. Unfortunately, an efficient process to excite and control the lifetime of isomers is currently lacking.

Nuclear levels, in general, are not easily accessible: they often have high spin with respect to the ground state, the resonances are very narrow and predominantly in the MeV range, in which no high-intensity monochromatic light sources exist yet. Alternatively to direct excitation via photon absorption, few other secondary electro-magnetic processes exist - such as Coulomb excitation [10], nuclear excitation upon electron capture (NEEC) [11, 12] or transition (NEET) [13, 14] and excitation upon muon cascade [15, 16]. Here we discuss a new electro-nucleus excitation mechanism that presents one of the highest excitation cross sections: the nuclear excitation by muon capture (NE μ C). The high energy transferred to the nucleus expands the range of isotopes suitable for the process and makes NE μ C a relevant process for muon-induced fission [17–19] and for feeding of long-lived isomers [1], as shown in Fig. 1.

Fig. 1 Nuclear excitation by muon capture (NE μ C): the capture of a free muon leads to a resonant excitation of the nucleus. The excited nucleus can subsequently decay towards lower levels reaching a long-lived state, i.e. isomer feeding. Another possibility is present if the excitation of the nucleus is in resonance with the giant dipole (GDR) or quadrupole resonances (GQR), and the latter is above the fission barrier: the nucleus can undergo a prompt fission induced by NE μ C.

It is instructive to compare electronic and muonic electro-nucleus processes. Excitation upon muon cascade [20, 21] and NEET occur as a result of transition between bound muonic and electronic orbitals respectively. The two processes have been experimentally observed [15, 16] and are considered to be well established. The NEEC process is believed to be observed in a

single experiment [12, 22, 23] and its cross section differs from a theoretical estimate by nine orders of magnitude [24]. To our knowledge, a muonic analogue of NEEC has never been proposed and in this article we investigate this possibility theoretically, underlining key differences with the known processes.

Similarly to NEEC and contrary to NEET and muon cascade excitation, NE μ C describes capture of a *free* lepton in a corresponding atomic orbital, and is thus not constrained by the restriction of matching the transition energies between *bound* atomic and nuclear levels. Since both NEEC and NE μ C depend on the interaction between nuclear and atomic environment, tight muon orbits are expected to provide higher nuclear excitation cross section than their electronic counter parts. In particular, here we report findings of NE μ C integrated cross section up to 1.82×10^5 b eV, that is five orders of magnitude higher than any corresponding NEEC cross section reported so far [25–29].

To evaluate the NE μ C cross section we used the advanced theory based on the Feshbach projection operator formalism developed by A. Pálffy for the NEEC process and presented in Refs. [25, 30]. In this context, the NE μ C rate for an electric transition can be written in muonic atomic units as:

$$Y_n^{(e)L} = \frac{1}{4\pi\alpha} \frac{4\pi^2\rho_i}{(2L+1)^2} B\uparrow(EL)(2j_b+1) \times \sum_k |\tilde{R}_{L,k_b,k}|^2 C(j_b \ L \ j; \ 1/2 \ 0 \ 1/2)^2, \quad (1)$$

where $B\uparrow(EL)$ is the reduced transition probability of the L^{th} multipolar transition, $C(j_1 \ j_2 \ j; \ m_1 \ m_2 \ m)$ is the Clebsch-Gordan coefficient, j_b and j , are the total angular momentum, while k_b and k are the Dirac angular momentum of the bound and free muon respectively. $\tilde{R}_{L,k_b,k}$ is the radial integral that depends on the muon bound and free wavefunctions, which are obtained as solutions of the Dirac equations for a specific atomic configuration using the modified version of FAC [31].

The NE μ C cross section can be expressed as:

$$\sigma_{\text{NE}\mu\text{C}} = 2\pi^2 \lambda_\mu^2 Y_n \frac{\Gamma_r/2\pi}{(E - E_r)^2 + \Gamma_r^2/4}, \quad (2)$$

where λ_μ is the free muon wavelength and E_r the resonance energy. The integration of the cross section over the continuum energies, considering that $\Gamma_r \ll E_r$, gives the so-called resonance strength:

$$S_{\text{NE}\mu\text{C}} = \int \sigma_{\text{NE}\mu\text{C}}(E) \, dE = 2\pi^2 \lambda_\mu^2 Y_n. \quad (3)$$

For NE μ C to be possible the nuclear transition energy (E_n) has to be larger than the muon binding energy (E_b), and the free muon energy (E_r) has to match their difference (i.e., $E_r = E_n - E_b$). This condition defines the search for the nuclear transitions that can be excited by the NE μ C mechanism. In Fig.

4 Nuclear Excitation by Muon Capture

2, we plot the muonic binding energies for K and L shells, calculated with the Flexible Atomic Code (FAC) [31] modified for muonic atoms (see Methods), and nuclear excited levels that satisfy the above criteria for E_r up to 0.4 MeV above the corresponding muonic levels. For the sake of the presentation we only show the E2 transition, which are generally the strongest. The table 1 reports the NE μ C rates and resonance strengths for selected isotopes together with nuclear transition energy and required energy of the free muon, including several E1 transitions.

Fig. 2 Isotopes matching the search criteria in case of E2 transition. Only the nuclear transitions with a $B \downarrow (E2) > 10$ W.u. have been included in the plot. The color of the markers indicate the closest shell allowing NE μ C. Vertical black lines group several isotopes of the same element.

In the table 2 we compare the NE μ C and NEEC resonance strengths of few of the strongest transitions. For all considered cases the NE μ C is substantially stronger than NEEC. The enhancement found ranges between 5 to 10 order of magnitude. Table 2 offers also a comparison with the direct process of photoexcitation. Results show that in the case of an E1 transition, as for ^{138}Ba and ^{207}Pb , S_γ and $S_{\text{NE}\mu\text{C}}$ are comparable, while for quadrupolar excitations $S_{\text{NE}\mu\text{C}}$ is substantially larger than S_γ .

Similarly to the NEEC case, the NE μ C cross section is greatly enhanced if the resonance is met at low kinetic energy, given the λ^2 prefactor in Eq. 2. In this respect it is important to inspect the precision of atomic orbital calculations. Considering that FAC has never been used before to compute muonic binding energies, we compare in Fig. 3 the values obtained using FAC with the state-of-the-art theoretical calculations for muonic atoms presented in Ref. [32], in the case of ^{40}Zr , ^{147}Sm and ^{209}Bi .

Table 1 Resonance strengths for the isotopes highlighted by the searching criteria for E1 and E2 transitions and capture in the K and L shells. Isotopes are ordered with respect to the mass number.

Isotope	L th	E_n (keV)	nl_j	E_r (keV)	$Y_{\text{NE}\mu\text{C}}$ (1/s)	$S_{\text{NE}\mu\text{C}}$ (b eV)
¹¹ Be	E1	320.04	1s _{1/2}	275.54	1.39×10^{14}	12.09
¹⁹ Ne	E2	238.27	2p _{3/2}	168.21	1.16×10^{14}	16.58
⁴³ Ca	E1	1394.47	1s _{1/2}	329.02	4.95×10^{10}	3.62×10^{-3}
⁴⁴ Ca	E2	1157.02	1s _{1/2}	92.20	4.14×10^{12}	1.08
⁴⁵ Sc	E2	1236.70	1s _{1/2}	69.08	2.84×10^{12}	0.99
⁴⁸ Cr	E2	752.19	2s _{1/2}	360.43	4.69×10^{13}	3.13
⁴⁸ Cr	E2	752.19	2p _{1/2}	342.84	2.63×10^{16}	1.84×10^3
⁴⁸ Cr	E2	752.19	2p _{3/2}	345.88	5.15×10^{16}	3.58×10^3
⁵² Mn	E2	731.66	2p _{1/2}	287.05	9.28×10^{15}	771.14
⁶⁸ Se	E2	853.75	2s _{1/2}	92.94	3.52×10^{14}	90.87
⁶⁸ Se	E2	853.75	2p _{1/2}	24.54	1.42×10^{17}	1.38×10^5
⁶⁸ Se	E2	853.75	2p _{3/2}	36.18	2.75×10^{17}	1.82×10^5
⁷³ Ge	E2	825.8	2p _{1/2}	92.66	3.81×10^{16}	9.85×10^3
⁷³ Ge	E2	825.8	2p _{3/2}	101.84	7.41×10^{16}	1.75×10^4
⁸¹ Br	E2	836.8	2s _{1/2}	37.22	1.43×10^{14}	91.82
⁸⁶ Sr	E2	1076.7	2p _{3/2}	54.77	9.52×10^{15}	4.17×10^3
⁹¹ Zr	E2	1204.8	2p _{1/2}	51.18	1.32×10^{16}	6.19×10^3
⁹¹ Zr	E2	1204.8	2p _{3/2}	72.26	2.56×10^{16}	8.49×10^3
⁹³ Mo	E2	1477.2	2p _{1/2}	203.4	6.25×10^{16}	7.38×10^3
⁹³ Mo	E2	1477.2	2p _{3/2}	228.53	1.21×10^{17}	1.27×10^4
¹³⁸ Ba	E1	6244.8	1s _{1/2}	44.19	2.69×10^{14}	146
²⁰² Hg	E1	4922	2p _{1/2}	329.41	6.12×10^{14}	44.66
²⁰⁷ Pb	E1	4980.5	2p _{1/2}	165.51	3.09×10^{15}	447.50
²⁰⁷ Pb	E1	4980.5	2p _{3/2}	350.64	5.67×10^{15}	388.75

Table 2 Comparison between NE μ C, NEEC and direct photo-excitation for the same nuclear transition for several isotopes. The apex i indicates a bare nucleus configuration while n the one for a neutral atom. Integrated cross sections are expressed in b-eV, while E_n in keV.

Isotope	E_n	nl_j	$S_{\text{NE}\mu\text{C}}^i$	S_{NEEC}^i	S_γ	$S_{\text{NE}\mu\text{C}}^n$
⁵² Mn	731.66	2p _{1/2}	771.14	6.83×10^{-7}	0.58	764.73
⁶⁸ Se	853.75	2p _{3/2}	1.82×10^5	1.44×10^{-5}	4.29	1.62×10^5
⁷³ Ge	825.8	2p _{1/2}	9.85×10^3	4.35×10^{-6}	1.34	9.45×10^3
⁹³ Mo	1477.2	2p _{3/2}	1.27×10^4	8.91×10^{-6}	5.00	1.24×10^4
¹³⁸ Ba	6244.8	1s _{1/2}	146	1.57×10^{-2}	164.74	120.66
²⁰⁷ Pb	4980.5	2p _{1/2}	447.5	6.52×10^{-2}	713.96	432.6

Fig. 3 Accuracy assessment of the Flexible Atomic Code. Muonic binding energies computed with FAC (E_{FAC}) are compared with those of Ref. [32] (E_{ref}). The color of the marker indicates the muonic state.

The overall standard deviations between the differences in the binding energies range from 0.36 keV for ^{40}Zr to 0.87 keV for ^{209}Bi , assessing FAC as a valuable tool for predicting binding energies in muonic atoms (see Methods for further details).

Another important difference with respect to NEEC is the absence of the high ionization state requirement. In case of muons, the muonic inner-shells are always available for capture and cannot be filled with electrons even for neutral atoms. The presence of electrons in the atomic environment will screen the muonic levels, making the muons less bound by up to few tens of keV, depending on the number of electrons in the shells [32–34]. This means that the resonance strengths evaluated for bare nuclei in Table 1 will be only slightly affected by the electronic charge state of the capturing ion. Thus, NE μ C allows for a capture in the 1s shell of an entirely filled atom. For this reason, we evaluated the NE μ C resonance strengths for the isotopes with the highest $S_{\text{NE}\mu\text{C}}$ of Table 1 also in case of a neutral electronic configuration (see Methods for further details). Results are shown in Table 2. Here we notice, as expected, that the $S_{\text{NE}\mu\text{C}}^i$ and $S_{\text{NE}\mu\text{C}}^n$ are very close to each other, with a slight difference due to the different resonance energy of the neutral case induced by the electron screening.

From the experimental point of view, the possibility of capture in neutral atoms can be tremendously useful and could offer an interesting perspective

lifting the stringent experimental requirements for NEEC. Indeed, as NEEC simultaneously requires high ionization state and high density of resonant electrons, the simultaneous realization of both poses experimental challenges. Lifting the ionization requirement for NE μ C simplifies the experimental scenario. For example, in a beam-based setup NE μ C can be observed by sending a muon beam into a solid target. Analogously to NEEC, The NE μ C probability can be written as [24, 35]:

$$P = \sum_{\alpha_r} n_i S_{\text{NE}\mu\text{C}}^{\alpha_r} \frac{1}{-(dE_\mu/dx)|_{E_r}} \quad (4)$$

where α_r represent the available capture channels, n_i is the density of atoms and $-(dE_\mu/dx)|_{E_r}$ is the muon stopping power at the resonance energy. The number of excited nuclei per second, assuming a continuous muon beam with a flux ϕ_μ (1/s) is given by:

$$N_{\text{NE}\mu\text{C}}^{\text{exc}} = P\phi_\mu. \quad (5)$$

If we limit ourselves to solid targets with stable or long-lived ground states, essential for practical experiments, ^{73}Ge and ^{93}Mo are the most promising isotopes. The stopping power calculated with GEANT4 [36] is of $dE_\mu/dx \simeq -501 \text{ MeV cm}^{-1}$ and -607 MeV cm^{-1} at the resonant energies of 101.84 keV and 228.53 keV respectively. Considering the capture only in the $2p_{3/2}$ channel the resulting probabilities are $P = 1.54 \times 10^{-6}$ and $P = 1.39 \times 10^{-6}$ respectively. Remarkably, these theoretical probabilities are five orders of magnitude larger than those theoretically estimated for the ^{93m}Mo isomer depletion through NEEC [24, 37]. If we expand the calculations to short lived isotopes, e.g. ^{68}Se , the single channel excitation probability reaches $P = 1.02 \times 10^{-5}$.

Currently, the brightest μ^- beam facilities at PSI (Villigen, Switzerland) and MuSIC (Osaka, Japan) are able to deliver a continuous flux of 10^7 muons per second [38]. Planned upgrades would make it feasible in the next years to have fluxes up to 10^8 and 10^9 muons per second, resulting approximately in ten to one thousand nuclear excitation per second. Furthermore, an increase in the excitation cross section is expected if the wavefunction of the muon is engineered [39], i.e. considering muon vortex beams [40], as recently suggested for NEEC [8, 29, 41]. This modification of the wavefunction could make unfavourable transitions with higher multipolarity more likely to happen.

Given the high energy of nuclear transitions involved in NE μ C and its increased efficiency compared to direct photoexcitation at higher multiplicities, NE μ C can be the most suitable process for isomer feeding. In this case the feeding, as shown in Fig. 1 will not happen directly to the isomer state, but arriving to it through subsequent decays upon the initial excitation from the ground state. This is, for example, the case of the energy level schemes of ^{113}In and ^{87}Sr .

Typically, at energies tens of MeV above the ground state, the density of the excitation states is so high that they overlap in a broad energy range, giving rise to the so called giant resonances. Excitation of these resonances, independently from the particular excitation mechanism, can lead to fission if

the resonance is above the fission barrier. Prompt fission of the nucleus has been achieved under muon excitation and attributed to the muon cascade in ^{238}U [19]. Yet, the possibility of the NE μ C has not been considered despite it could provide substantially better energy overlap given that one has an additional degree of freedom, that is the energy of the free lepton. Indeed, fission induced by muonic transitions is governed by the energy difference between two muonic bound states, while in the case of NE μ C the resonance condition is satisfied throughout the whole width of the giant resonance, that can be several MeV wide. To estimate the contribution of the NE μ C process into the muon induced fission we calculate the fission cross section induced by NE μ C for ^{238}U presented in the Extended Data Figure 1 (see Methods for further details). Integrating the cross section with the energy dependent stopping power (further details are available in the Methods), provides us with final fission probability of $\sim 4.30 \times 10^{-5}$ per incident muon. This probability is still small if compared with prompt fission induced by muon cascade ($\sim 10^{-3}$) and delayed fission induced by muon capture ($\sim 10^{-2} - 10^{-1}$) [42]. Nevertheless, for lighter isotopes the muon cascade eventually becomes non-resonant with the giant resonances, while NE μ C is theoretically always possible.

Most remarkably, the NE μ C has the highest chance to be observed than the NEEC process in which disagreement between experiment and theory is of nine orders of magnitude. Measuring the NE μ C rates and comparing to the estimates provided in presented paper will hopefully help to resolve the contradiction and establish the origins of the extremely high experimentally measured NEEC cross sections.

Acknowledgments. S.G., F.C. and I.M. acknowledge support from Google Inc. The authors would like to thank Giovanni dal Maso for the useful insights about muonic beam facilities.

Methods

Binding energy calculations in muonic atoms using the Flexible Atomic Code

Binding energies for muonic atoms are obtained by numerically solving the Dirac equation including the effect of the finite size of the nucleus using the Fermi distribution function with parameters adjusted to reproduce the rms charge radii of Ref. [43]. Vacuum polarization is taken into account using the standard Uehling potential, while self-energy correction is included using the method of Ref. [44]. The nucleus recoil effect is approximated with an effective Hamiltonian term proposed in Ref. [45]. As we can see from Fig. 3, maximum discrepancy is obtained for the 1s shell with differences ranging between 1.08 keV for ^{40}Zr and 2.81 keV for ^{209}Bi . In the case of the 2s subshell, discrepancies drops down to a minimum of 0.07 keV for ^{40}Zr to a maximum of 0.94 keV for ^{147}Sm . For the 2p $_{1/2}$ the maximum disagreement, of about 1.19 keV, results from ^{209}Bi . Binding energies for the 2p $_{3/2}$ subshell similarly

differ at most by 1.24 keV in the case of ^{209}Bi , while the discrepancies are smaller for ^{40}Zr and ^{147}Sm . Much of these discrepancies can be attributed to the self-energy correction included in the present work, and omitted in Ref. [32]. The agreement between FAC and Ref. [32] improves significantly for the M-shell, as the self-energy term becomes negligible. In this context we used the rms nuclear charge radii reported in Ref. [43] and $t = 2.30$ fm as the parameter of the Fermi-type charge distribution. Screening by an arbitrary electronic configuration has been included in FAC by solving the Dirac equations of both the muon and electrons self consistently via iteration.

Radial integral calculation

The radial integral $\tilde{R}_{L,k_b,k}$ has been calculated using its full expression, reported in Ref. [30], that is:

$$\tilde{R}_{L,k_b,k} = \frac{1}{R_0^{L-1}} \int_0^{R_0} dr r^{L+2} [f_{k_b}(r)f_{Ek}(r) + g_{k_b}(r)g_{Ek}(r)] + R_0^{L+2} \int_{R_0}^{+\infty} dr r^{-L+1} [f_{k_b}(r)f_{Ek}(r) + g_{k_b}(r)g_{Ek}(r)] , \quad (6)$$

where $f(r)$ and $g(r)$ are the large and small radial components of the bound and free muon wavefunctions. R_0 is the nuclear radius for which we used the values reported in Ref. [43] for the available isotopes. For those not available we used the value reported by FAC, that is 3.56208 fm for ^{48}Cr , 3.96475 fm for ^{68}Se and 4.36807 fm for ^{93}Mo .

NE μ C-induced fission cross section

^{238}U presents the giant dipole resonance peak at $E_{\text{GDR}} = 12.8$ MeV, that is slightly above the muonic binding energy of the K-shell, $E_b^K = 12.12$ MeV. As similarly shown in Ref. [18], the NE μ C-induced fission has been related to the value of the photofission cross section through the matrix element of the dipole transition, thus to the photoexcitation cross section. Here, we use Eq. (XII, 7.27) in Ref. [46] as an expression for the direct photo-excitation cross section in case of high-energy nuclear transitions. Since, under Bohr assumption, the decay mode of the excited nucleus is independent from its formation, we express the relation between NE μ C-induced fission cross section and the photo-fission cross section as:

$$\sigma_{\text{NE}\mu\text{C}}(\mu, F) = \frac{1}{4\pi\alpha} \frac{4\pi^2\rho_i}{(2L+1)^2} (2j_b+1) \times \sum_k |\tilde{R}_{L,k_b,k}|^2 C(j_b \ L \ j; \ 1/2 \ 0 \ 1/2)^2 \times 2\pi^2 \lambda_\mu^2 \frac{3}{8\pi^3} \frac{1}{E} \sigma_\gamma(\gamma, F). \quad (7)$$

where E is the energy of the transition. Photo-fission cross section has been retrieved from Fig. 4 of Ref. [47] and is also shown for comparison in the Extended Data Figure 1. The radial integral $\tilde{R}_{L,k_b,k}$ has been calculated over the range 100 eV to 8 MeV for the free muon energy, considering a neutral electronic configuration for ^{238}U . The probability of fission induced by the nuclear excitation by muon capture is then evaluated as:

$$P = n_i \int dE \frac{\sigma_{\text{NEPC}}(\mu, F)}{-(dE_\mu/dx)} \quad (8)$$

considering the only capture in the 1s shell. The stopping power is taken by Ref. [48], neglecting the small difference between positive and negative muons. The integral in Eq. 8 has been performed choosing $E = 100$ eV as lower limit for the integration, that is the minimum energy for which the stopping power is available.

Extended Data Figures

Extended Data Figure 1 NE μ C-induced fission cross section as function of the free muon kinetic energy E_μ (blue axes and curve) and photo-fission cross section (red axes and curve) retrieved from Ref. [47] for ^{238}U . X-axes are aligned considering the relation between the energy of the free muon and the nuclear transtion, i.e. $E_\mu = E_n - E_b^K$. In this case, $E_\gamma = E_n$ and $E_b^K = 12.12\text{ MeV}$.

References

- [1] Aprahamian, A., Sun, Y.: Long live isomer research. *Nature Physics* **1**(2), 81–82 (2005)
- [2] Walker, P., Dracoulis, G.: Energy traps in atomic nuclei. *Nature* **399**(6731), 35–40 (1999). <https://doi.org/10.1038/19911>
- [3] Collins, C., Davanloo, F., Iosif, M., Dussart, R., Hicks, J., Karamian, S., Ur, C., Popescu, I., Kirischuk, V., Carroll, J., *et al.*: Accelerated emission of gamma rays from the 31-yr isomer of 178 hf induced by x-ray irradiation. *Physical review letters* **82**(4), 695 (1999)
- [4] Litz, M., Merkel, G.: Controlled extraction of energy from nuclear isomers. Technical report, Army Research Lab Adelphi MD (2004)
- [5] Collins, C., Zoita, N., Davanloo, F., Emura, S., Yoda, Y., Uruga, T., Patterson, B., Schmitt, B., Pouvesle, J., Popescu, I., *et al.*: Accelerated γ -emission from isomeric nuclei. *Radiation Physics and Chemistry* **71**(3-4), 619–625 (2004)
- [6] Carroll, J., Litz, M., Netherton, K., Henriquez, S., Pereira, N., Burns, D., Karamian, S.: Nuclear structure and depletion of nuclear isomers using electron linacs. In: *AIP Conference Proceedings*, vol. 1525, pp. 586–594 (2013). American Institute of Physics
- [7] Vanacore, G.M., Madan, I., Berruto, G., Wang, K., Pomarico, E., Lamb, R., McGrouther, D., Kaminer, I., Barwick, B., de Abajo, F.J.G., *et al.*: Attosecond coherent control of free-electron wave functions using semi-infinite light fields. *Nature communications* **9**(1), 1–11 (2018)
- [8] Madan, I., Vanacore, G.M., Gargiulo, S., Lagrange, T., Carbone, F.: The quantum future of microscopy: Wave function engineering of electrons, ions, and nuclei. *Appl. Phys. Lett* **116**, 230502 (2020). <https://doi.org/10.1063/1.5143008>
- [9] Sepulveda, N.A., Jenkins, J.D., Edington, A., Mallapragada, D.S., Lester, R.K.: The design space for long-duration energy storage in decarbonized power systems. *Nature Energy* **6**(5), 506–516 (2021)
- [10] Pieper, G., Heydenburg, N.: Coulomb excitation of iron-57. *Physical Review* **107**(5), 1300 (1957)
- [11] Goldanskii, V., Namiot, V.: On the excitation of isomeric nuclear levels by laser radiation through inverse internal electron conversion. *Physics Letters B* **62**(4), 393–394 (1976)

- [12] Chiara, C.J., Carroll, J.J., Carpenter, M.P., Greene, J.P., Hartley, D.J., Janssens, R.V.F., Lane, G.J., Marsh, J.C., Matters, D.A., Polasik, M., Rzadkiewicz, J., Seweryniak, D., Zhu, S., Bottoni, S., Hayes, A.B.: Isomer depletion as experimental evidence of nuclear excitation by electron capture. *Nature* **554**(7691), 216–218 (2018). <https://doi.org/10.1038/nature25483>
- [13] Morel, P., Daugas, J., Gosselin, G., Méot, V., Gogny, D.: Nuclear excitation by electronic processes: Neec and neet effects. *Nuclear Physics A* **746**, 608–612 (2004)
- [14] Morel, P., Méot, V., Gosselin, G., Faussurier, G., Blancard, C.: Calculations of nuclear excitation by electron capture (NEET) in nonlocal thermodynamic equilibrium plasmas. *Physical Review C* **81**, 34609 (2010). <https://doi.org/10.1103/PhysRevC.81.034609>
- [15] Backe, H.: Intensity measurements of x-ray and nuclear gamma-ray transitions in some heavy muonic atoms. *Z. Phys.* **241**: 435-58(1971). (1971)
- [16] Hoehn, M., Shera, E.: Muonic resonance excitation of os 188 and yb 172. *Physical Review C* **20**(5), 1934 (1979)
- [17] Wheeler, J.A.: Some consequences of the electromagnetic interaction between μ -mesons and nuclei. *Reviews of Modern Physics* **21**(1), 133 (1949)
- [18] Zaretski, D., Novikov, V.: Theory of nuclear excitations by muons in heavy mesic atoms. *Nuclear Physics* **28**(1), 177–191 (1961)
- [19] Oberacker, V., Umar, A., Wells, J., Bottcher, C., Strayer, M., Maruhn, J.: Muon-induced fission: A probe for nuclear dissipation and fission dynamics. *Physical Review C* **48**(3), 1297 (1993)
- [20] Jacobsohn, B.A.: Nuclear fine structure in the μ -mesonic atom. *Physical Review* **96**(6), 1637 (1954)
- [21] Wilet, L.: Excitation of nuclear rotational states in μ -mesonic atoms. *Dan: Mat. Fys . Medd.* **29**(3) (1954)
- [22] Guo, S., Fang, Y., Zhou, X., Petrache, C.: Possible overestimation of isomer depletion due to contamination. *Nature* **594**(7861), 1–2 (2021)
- [23] Chiara, C., Carroll, J., Carpenter, M., Greene, J., Hartley, D., Janssens, R., Lane, G., Marsh, J., Matters, D., Polasik, M., *et al.*: Reply to: Possible overestimation of isomer depletion due to contamination. *Nature* **594**(7861), 3–4 (2021)

- [24] Wu, Y., Keitel, C.H., Pálffy, A.: ^{93m}Mo isomer depletion via beam-based nuclear excitation by electron capture. *Phys. Rev. Lett.* **122**, 212501 (2019). <https://doi.org/10.1103/PhysRevLett.122.212501>
- [25] Pálffy, A., Scheid, W., Harman, Z.: Theory of nuclear excitation by electron capture for heavy ions. *Physical Review A* **73**(1), 012715 (2006) [arXiv:0701206](https://doi.org/10.1103/PhysRevA.73.012715) [physics]. <https://doi.org/10.1103/PhysRevA.73.012715>
- [26] Pálffy, A., Harman, Z., Scheid, W.: Quantum interference between nuclear excitation by electron capture and radiative recombination. *Physical Review A* **75**(1), 012709 (2007)
- [27] Rządkiwicz, J., Polasik, M., Ślabkowska, K., Syrocki, L., Węder, E., Carroll, J.J., Chiara, C.J.: Beam-based scenario for ^{242m}Am isomer depletion via nuclear excitation by electron capture. *Phys. Rev. C* **99**, 044309 (2019). <https://doi.org/10.1103/PhysRevC.99.044309>
- [28] Wu, Y., Keitel, C.H., Pálffy, A.: X-ray-assisted nuclear excitation by electron capture in optical laser-generated plasmas. *Physical Review A* **100**(6), 063420 (2019)
- [29] Gargiulo, S., Madan, I., Carbone, F.: Nuclear excitation by electron capture in excited ions. *arXiv preprint arXiv:2102.05718* (2021)
- [30] Pálffy, A.: Theory of nuclear excitation by electron capture for heavy ions. phdthesis, Justus-Liebig-Universität (2006). <https://doi.org/10.1103/PhysRevA.73.012715>. <https://core.ac.uk/download/pdf/56343138.pdf>
- [31] Gu, M.F.: The flexible atomic code. *Canadian Journal of Physics* **86**(5), 675–689 (2008)
- [32] Michel, N., Oreshkina, N.S., Keitel, C.H.: Theoretical prediction of the fine and hyperfine structure of heavy muonic atoms. *Physical Review A* **96**(3), 032510 (2017)
- [33] Vogel, P.: Electron screening in muonic atoms. *Physical Review A* **7**(1), 63 (1973)
- [34] Fricke, B.: On the screening of muons by electrons in muonic atoms. *Lettere al Nuovo Cimento* (1969-1970) **2**(18), 859–862 (1969)
- [35] Aikawa, M., Ebata, S., Imai, S.: Thick-target yields of radioactive targets deduced from inverse kinematics. *Nuclear Instruments and Methods in Physics Research Section B: Beam Interactions with Materials and Atoms* **353**, 1–3 (2015)

- [36] Agostinelli, S., Allison, J., Amako, K.a., Apostolakis, J., Araujo, H., Arce, P., Asai, M., Axen, D., Banerjee, S., Barrand, G., *et al.*: Geant4—a simulation toolkit. Nuclear instruments and methods in physics research section A: Accelerators, Spectrometers, Detectors and Associated Equipment **506**(3), 250–303 (2003)
- [37] Rzedkiewicz, J., Polasik, M., Słabkowska, K., Carroll, J., Chiara, C., *et al.*: Novel approach to $^{93}\text{m}\text{Mo}$ isomer depletion: Nuclear excitation by electron capture in resonant transfer process. Physical Review Letters **127**(4), 042501 (2021)
- [38] Cook, S., D’arcy, R., Edmonds, A., Fukuda, M., Hatanaka, K., Hino, Y., Kuno, Y., Lancaster, M., Mori, Y., Ogitsu, T., *et al.*: Delivering the world’s most intense muon beam. Physical Review Accelerators and Beams **20**(3), 030101 (2017)
- [39] Vanacore, G.M., Madan, I., Carbone, F.: Spatio-temporal shaping of a free-electron wave function via coherent light–electron interaction. La Rivista del Nuovo Cimento, 1–31 (2020)
- [40] Zhao, P., Ivanov, I.P., Zhang, P.: Decay of vortex muon. arXiv preprint arXiv:2106.00345 (2021)
- [41] Wu, Y., Gargiulo, S., Carbone, F., Keitel, C.H., Pálffy, A.: Dynamical control of nuclear isomer depletion via electron vortex beams. arXiv preprint arXiv:2107.12448 (2021)
- [42] Johansson, T., Polikanov, S., Tibell, G., Krogulski, T., Konijn, J., Reist, H.: Muon induced quadrupole photofission. Phys. Lett. B **97**(CERN-EP-80-123), 29–32 (1980)
- [43] Angeli, I., Marinova, K.P.: Table of experimental nuclear ground state charge radii: An update. Atomic Data and Nuclear Data Tables **99**(1), 69–95 (2013)
- [44] Barrett, R.C.: Radiative corrections in muonic atoms. Physics Letters B **28**(2), 93–95 (1968)
- [45] Cheng, K.T., Johnson, W.R., Sapirstein, J.: Screened Lamb shift calculations for lithiumlike uranium, sodiumlike platinum, and copperlike gold. Phys. Rev. Lett. **66**(23), 2960–2963 (1991)
- [46] Blatt, J.M., Weisskopf, V.F.: Theoretical Nuclear Physics, p. 653. Springer, New York (1991)
- [47] Gindler, J., Huizenga, J., Schmitt, R.: Photofission and photoneutron emission in thorium and uranium. Physical Review **104**(2), 425 (1956)

- 387 [48] Groom, D.E., Mokhov, N.V., Striganov, S.I.: Muon stopping power and
388 range tables 10 mev–100 tev. Atomic Data and Nuclear Data Tables
389 **78**(2), 183–356 (2001)