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Keywords:

Posted Date: February 24th, 2022

DOI: <https://doi.org/10.21203/rs.3.rs-1389516/v1>

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Extracting high-order cosmological information in galaxy surveys with power spectra

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The “reconstruction” method¹ was proposed more than a decade ago to boost the signal of baryonic acoustic oscillations² measured in galaxy redshift surveys, which is one of key probes for dark energy^{3,4}. After moving observed galaxies in galaxy surveys back to their initial position, the reconstructed density field is closer to a linear Gaussian field, with higher-order information moved back into the power spectrum. By combining power spectra measured from the pre- and post-reconstructed galaxy samples, higher-order information beyond the 2-point power spectrum can be efficiently extracted, as power spectra can be measured more quickly than direct measurements of the bispectrum and other higher order functions. This opens a new window to easily use higher-order information when constraining cosmological models.

The science driver for massive galaxy spectroscopic surveys is to extract cosmological information from the clustering of galaxies in the past lightcone. Baryon acoustic oscillations (BAO)², formed in the early Universe due to interactions between photons and baryons under pressure and gravity, yield a special clustering pattern of galaxies around a characteristic comoving scale around 150 Mpc. The increasing size of galaxy redshift surveys over the decade 2000-2010 led ultimately to

a 5σ detection of BAO by the Baryon Oscillation Spectroscopic Survey (BOSS)⁵. This enabled the BAO to be used as an accurate *standard ruler* to measure the geometry of the Universe and constrain the cosmic expansion history. A compilation of results from the Sloan Digital Sky Survey (SDSS) galaxy survey recently demonstrated the power of this technique⁶.

The BAO feature is generally blurred by the nonlinear evolution of the Universe reducing its strength as a standard ruler, and various reconstruction methods have been developed to sharpen the BAO peak by undoing the nonlinear evolution of the density field. The commonly used Lagrangian reconstruction, for example, linearises the density field by shifting the galaxies using the displacement field^{1,7,8}, while for the Eulerian reconstruction, manipulation is performed at the field level without moving the galaxies⁹.

Although designed to boost the BAO signal originally, the reconstruction method can, in principle, better extract the general cosmological information from the clustering. For example, the redshift space distortions (RSD)¹⁰, which lead to clustering pattern of galaxies probing the structure growth of the Universe, can also be better constrained using the reconstructed sample¹¹.

The standard method for BAO-reconstruction reverses over-densities back to those of the seed perturbations from which they grew. It achieves this by inferring the bulk-flows using the observed galaxy field and then removing these displacements from both the galaxy positions and the map of expected density setting the baseline from which the over-densities are found. The power spectrum of the post-reconstructed sample (P_{post}) provides additional information for cosmology compared with the pre-reconstructed sample (P_{pre}) because the reconstruction restores

the linear signal reduced by the non-linear evolution.

The higher order correlation functions such as B , the bispectrum, induced by the non-linear evolution are, in turn, reduced. Thus we can extract more cosmological information encoded in the linear density field from P_{post} than P_{pre} . On the other hand, the non-linear field contains information on small-scale clustering such as galaxy biases, which provide better constraints on cosmological parameters by breaking the degeneracy between them. For the pre-reconstructed sample, this information can be extracted by combining the power spectrum with higher-order statistics. However, the power spectrum and higher order statistics such as the bispectrum are correlated, reducing our ability to estimate cosmological parameters. If we instead consider the post-reconstructed power spectrum, the covariance between the power spectrum and the bispectrum is reduced, and we can extract the information more efficiently. In this work, we show that the same improvement can be achieved by a joint analysis of P_{pre} , P_{post} and P_{cross} (the cross-power spectrum between the pre- and post-reconstructed density fields). Due to the restored linear signal in the reconstructed density field, P_{post} is decorrelated with P_{pre} on small scales, which are dominated by the non-linear field. On these scales, the combination of P_{pre} , P_{post} and P_{cross} has a similar ability to extract cosmological information as the combination of P_{post} or P_{pre} with the bispectrum because we are able to use the linear information in P_{post} and higher-order information in P_{pre} separately.

Let us rewrite the non-linear over-density field as $\delta = R + \Delta$, where R is the density field after reconstruction, which is closer to the linear field. It is then straightforward to express P_{pre} , P_{cross}

in terms of $P_{RR}(= P_{\text{post}})$, $P_{\Delta\Delta}$ (the power spectrum of Δ) and $P_{\Delta R}$ (the cross-power spectrum between Δ and R). Using perturbation theory¹², we can show that, at the leading order, $P_{\Delta\Delta}$ contains the integrated contribution from the bispectrum of squeezed-limit triangles while $P_{R\Delta}$ contains the integrated contribution from the trispectrum of folded/squeeze-limit quadrilaterals (those quadrilaterals with zero area and two-by-two sides equal). In this fashion when we add to the post-reconstructed power spectrum the information from P_{pre} and P_{cross} , we are not doing anything but including part of the high-order signal, and naturally gaining information. Note that in order to match the information obtained by adding these two extra statistics, it is not enough to consider the bispectrum signal of the pre-reconstructed field, but both bispectrum and trispectrum signals. For this reason the information content of $P_{\text{pre}} + B_{\text{pre}}$ is different from that contained by $P_{\text{pre}} + P_{\text{cross}} + P_{\text{post}}$. However, it is important to note that higher-order information that reconstruction brings is only a part from the total contained by the full bispectrum and trispectrum data-vectors. This is why a full analysis using $P + B + T$ will always provide more information. However, such an analysis is not very practical because of the size of the full data-vector and the computational time typically required to measure B and especially T directly. In this paper we show that $P_{\text{pre}} + P_{\text{cross}} + P_{\text{post}}$ is a practical alternative way of compressing much of the data that captures the most relevant information of these higher-order moments for cosmology inference purposes.

To demonstrate, we perform an *anisotropic* Lagrangian reconstruction (*i.e.*, the RSD effect is kept during the reconstruction process¹, see Method for details) on each realisation of the MOLINO galaxy mocks¹³, which is a large suite of realistic galaxy mocks produced from the Quijote simulations¹⁵ at $z = 0$. We then use this to calculate the derivatives required for a Fisher

matrix analysis¹⁴ using the measured multipoles (up to $\ell = 4$) of P_{pre} , P_{post} and P_{cross} on the parameter set $\Theta \equiv \{\Omega_{\text{m}}, \Omega_{\text{b}}, h, n_s, \sigma_8, M_{\nu}, \mathbf{H}\}$ where $\mathbf{H}$ denotes the Halo Occupation Distribution (HOD) parameters, *i.e.*, $\mathbf{H} \equiv \{\log M_{\text{min}}, \sigma_{\log M}, \log M_0, \alpha, \log M_1\}$ ¹⁶. The covariance matrix and the derivatives with respect to Θ are evaluated numerically, as in the joint power spectrum-bispectrum analysis¹³ (see Method for details).

Panel **a** in Figure 1 shows the measured power spectra monopole, and we see that P_{cross} decreases dramatically with scale compared to P_{pre} and P_{post} . This indicates a decorrelation between P_{pre} and P_{post} below quasi-nonlinear scales ($k \gtrsim 0.1 h \text{ Mpc}^{-1}$), which is largely due to the difference in levels of nonlinearity in P_{pre} and P_{post} .

From the original data vector $\{P_{\text{pre}}, P_{\text{post}}, P_{\text{cross}}\}$, we can construct their linear combinations, $P_{\text{R}\Delta}$, $P_{\Delta\Delta}$, $P_{\delta\Delta}$ defined as

$$P_{\text{R}\Delta} = P_{\text{cross}} - P_{\text{post}}, \quad P_{\Delta\Delta} = P_{\text{pre}} + P_{\text{post}} - 2P_{\text{cross}}, \quad P_{\delta\Delta} = P_{\text{pre}} - P_{\text{cross}}. \quad (1)$$

Panel **b** in Figure 1 show these power spectra. As discussed above, these power spectra involving Δ contain the information on part of the high-order statistics such as bispectrum and trispectrum.

The correlation matrix for the power spectrum and bispectrum monopoles (only the correlation with the squeezed-limit of B_0 is visualised for brevity) is shown in Figure 2. It is seen that P_0^{pre} highly correlates with B_0 , confirming that the bispectrum is induced by nonlinearities. In contrast, P_0^{post} is almost uncorrelated with B_0 , or with P_0^{pre} and P_0^{cross} on nonlinear scales (*e.g.*, at $k \gtrsim 0.2 h \text{ Mpc}^{-1}$). This implies that the information content in P_{post} combined with B_0 may

be similar to that in P_{post} combined with P_{pre} and P_{cross} , which is confirmed to be the case by the results from the Fisher analysis presented below.

Contour plots for $(\log M_0, \sigma_8)$ derived from different datasets with two choices of k_{max} (the maximal k for the observables used in the analysis) are shown in Figure 3 (the full contour plots for all parameters are shown in Supplement Figures 3, 4). The smoothing scale is set to be $S = 10 h^{-1} \text{ Mpc}$ when performing the reconstruction. The degeneracies between parameters using P_{pre} , P_{post} and P_{cross} are generally different, because P_{pre} , P_{post} and P_{cross} differ to a large extent in terms of nonlinearity on small scales. This is easier to see in Supplement Figure 2, in which contours for the same parameters are shown for observables used in several k intervals. The contours derived from P_{pre} and P_{post} generally rotate as k increases because of the kick-in of nonlinear effects, which affects P_{pre} and P_{post} at different levels on the same scale. This significantly improves the constraint when these power spectra are combined, labelled as P_{all} , which is tighter than that from the traditional joint power spectrum-bispectrum analysis ($P_{\text{pre}} + B_0$). Interestingly, P_{all} can even win against $P_{\text{post}} + B_0$ in some cases, demonstrating the robustness of this new method. The contour plots with 1D posterior distributions for all parameters with $S = 10$ and $20 h^{-1} \text{ Mpc}$ and $k_{\text{max}} = 0.2$ and $0.5 h \text{ Mpc}^{-1}$ are shown in Supplement Figures 3-6, respectively. In all cases, P_{all} offers competitive constraints on all parameters, even compared to the joint $P_{\text{post}} + B_0$ analysis.

To further quantify our results, in Figure 4 we compare the square root of the Fisher matrix element for each parameter, with and without marginalising over others, derived from P_{all} and

$P_{\text{pre}} + B_0$, respectively, with two choices of k_{max} .

For $k_{\text{max}} = 0.2 \, h \, \text{Mpc}^{-1}$, we see that the Fisher information for each parameter (panel **a**: without marginalising over others) in the model for P_{all} is identical or even greater than that in $P_{\text{pre}} + B_0$. In other words, combining all power spectra we can exhaust the information in $P_{\text{pre}} + B_0$. After marginalising over other parameters, panel **b** shows that the uncertainty on each parameter gets redistributed due to the degeneracy. The ratios for the HOD parameters are all greater than unity especially for $\log M_0$ and $\sigma_{\log M}$, demonstrating the power of our new method on constraining HOD parameters. The information content for cosmological parameters in $P_{\text{pre}} + B_0$ is well recovered by using P_{all} , although the recovery for M_ν is relatively worse. The overall trend for the case of $k_{\text{max}} = 0.5 \, h \, \text{Mpc}^{-1}$ is similar, although the advantage of using P_{all} over $P_{\text{pre}} + B_0$ gets degraded to some extent. However, P_{all} is still competitive: it almost fully recovers the information for the HOD parameters in $P_{\text{pre}} + B_0$ with or without marginalisation, and largely wins against $P_{\text{pre}} + B_0$ after marginalisation. Regarding the cosmological parameters, P_{all} recovers all information in $P_{\text{pre}} + B_0$ before the marginalisation, although the recovery is slightly worse for M_ν . After marginalisation when the uncertainties are redistributed, the constraint from P_{all} is generally worse than $P_{\text{pre}} + B_0$, especially for M_ν .

The 68% CL constraints on each parameter expected fitting to different sets of data are shown in Table 1. To quantify the information gain, we evaluate the Figure-of-Merit (FoM) defined as $[\det(F)]^{1/(2N_p)}$, where F denotes the Fisher matrix and N_p is the total number of free parameters. For the ease of comparison, for cases with different k_{max} , we normalise the FoM using the corresponding

$(\text{FoM})_{P_{\text{pre}}}$. As shown, for $k_{\text{max}} = 0.2 \, h \, \text{Mpc}^{-1}$, $(\text{FoM})_{P_{\text{all}}}$ is greater than all others, namely, it is larger than $(\text{FoM})_{P_{\text{pre}}}$ and $(\text{FoM})_{P_{\text{post}}}$ by a factor of 2.7 and 1.6, respectively, and it is even greater than $(\text{FoM})_{P_{\text{post}}+B_0}$ by $\sim 13\%$. For $k_{\text{max}} = 0.5 \, h \, \text{Mpc}^{-1}$, $(\text{FoM})_{P_{\text{all}}}$ is also more informative than $(\text{FoM})_{P_{\text{pre}}}$ and $(\text{FoM})_{P_{\text{post}}}$ by a factor of 2.1 and 1.5, respectively, and is the same as $(\text{FoM})_{P_{\text{pre}}+B_0}$, but is less than $(\text{FoM})_{P_{\text{post}}+B_0}$ by $\sim 10\%$ in this case.

To highlight the constraining power on cosmological parameters, we also list FoM_{cos} , which is the FoM with all HOD parameters fixed. It shows a similar trend as FoM_{Θ} : P_{all} is the most informative data combination for $k_{\text{max}} = 0.2 \, h \, \text{Mpc}^{-1}$, but it is outnumbered by $P_{\text{pre}} + B_0$ and $P_{\text{post}} + B_0$ by 13% and 35%, respectively, for the case of $k_{\text{max}} = 0.5 \, h \, \text{Mpc}^{-1}$.

In this example, the k -binning for P and B are different, namely, $\Delta k(B) = 3k_f \sim 1.9 \, \Delta k(P)$ where k_f denotes the fundamental k mode given the box size of the simulation. We have checked that using a much finer k -binning for B does not change significantly the constraints¹⁷. The FoM can only be raised by $\sim 10\%$ when $\Delta k(B)$ is reduced from $3k_f$ to k_f , which is largely due to the strong mode-coupling effect in B shown in Figure 2. Such a binning is not practical anyway as, for example, using $\Delta k(B) = k_f$ up to $k = 0.5 \, h \, \text{Mpc}^{-1}$, we end up with more than 50,000 data points to measure for B_0 .

Another concern might be that the MOLINO mock was investigated at $z = 0$, where the nonlinear effect are the strongest. At higher redshifts where the density fields are more linear and more Gaussian, we may expect less gain from the new method. This is also the case for the joint $P+B$ analysis, compared to analyses of only P . To quantify, we also perform an independent analysis

on a suite of 4000 high-resolution N -body mocks produced at $z = 1.02$, and confirm the gain in cosmological information in terms of BAO and RSD parameters (see Supplement information). Therefore, for redshift ranges covered by ongoing and future galaxy surveys (with median $z \sim 1$), our method manifests itself as a highly efficient and powerful way of transferring higher-order information back to the 2-point statistics.

As demonstrated, a joint analysis using P_{pre} , P_{post} and P_{cross} is an efficient way to extract high-order information from galaxy catalogues, and in some cases, the compression of information into P_{all} is more informative even than to $P_{\text{post}} + B_0$, which is more computationally expensive. This may be attributed to the fact that considering the full multipole expansion of P_{all} in terms of Legendre polynomials includes information from high-multipoles of the bispectrum and trispectrum, which are challenging to measure and use in traditional approaches.

Stage-IV redshift surveys including the Dark Energy Spectroscopic Instrument (DESI)¹⁸, Euclid¹⁹ and the Prime Focus Spectrograph (PFS)²⁰ will release galaxy maps over a wide range of redshift with an exquisite precision. Given that the number density of these galaxies is expected to be sufficiently high to allow for a reconstruction, the novel method presented in this work can be directly applied to yield a much more precise measurement of cosmological parameters, compared to the traditional 2-point, or even the 2-point + 3-point analyses. Since the reconstruction will be performed anyway for most ongoing and forthcoming galaxy surveys to improve the BAO signal, our proposed analysis can be performed at almost no additional computational cost.

Additional work is required to link the cosmological parameters to the observables, especially

the reconstructed power spectrum^{12,21} and the cross power spectrum²² on (quasi-) nonlinear scales, and this is challenging using traditional approaches, *e.g.*, methods based on perturbation theories. However, new model-free approaches including the simulation-based emulators, will be ideal for implementing our analysis on actual galaxy surveys.

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Acknowledgements We thank Florian Beutler, Shi-Fan Chen, Yipeng Jing, Baojiu Li, Levon Pogossian, Uroš Seljak, Shun Saito, Atsushi Taruya, Francisco Villaescusa-Navarro, Martin White, Hans Winther, Hanyu Zhang and Pengjie Zhang for discussions. YW is supported by NSFC Grant 11890691, by the Youth Innovation Promotion Association CAS, and by the Nebula Talents Program of NAOC. GBZ is

supported by the National Key Basic Research and Development Program of China (No. 2018YFA0404503), NSFC Grants 11925303, 11720101004, and a grant of CAS Interdisciplinary Innovation Team. Research at Perimeter Institute is supported in part by the Government of Canada through the Department of Innovation, Science and Economic Development Canada and by the Province of Ontario through the Ministry of Colleges and Universities. RT is supported by MEXT/JSPS KAKENHI Grant Numbers 20H05855 and 20H04723. Numeric work was performed on the UK Sciama High Performance Computing cluster supported by the ICG, University of Portsmouth.

Author contributions YW contributed to the idea and the development of the pipeline, performed the analysis on the Molino mocks, produced the results and contributed to the draft. GBZ proposed the idea, developed the pipeline, performed the analysis on the N -body mocks and wrote the draft. KK contributed to the idea and pipeline, led the effort on the theoretical interpretation of the result and co-wrote the draft. WJP contributed to the theoretical interpretation and observational implications of this work, and co-wrote the draft. RT and CH provided the N -body mocks, performed the reconstruction and measured the power spectra. HGM and CHH contributed to the bispectrum analysis, and to the draft. WZ, RZ and XM contributed to building an EFT-based theoretical model for the cross-power, which helped with interpretation of the result. YY and HZ provided another set of N -body mock for a cross validation, and helped with interpreting the de-correlation effect. FG performed a test on the eBOSS mocks to confirm the de-correlation effect between P_{pre} and P_{post} .

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Competing Interests The authors declare that they have no competing financial interests.

Figure 1. The measured power spectrum monopoles.

Panels **a**: the monopole (multiplied by k) of three types of power spectra indicated in the legend, measured from the MOLINO galaxy mocks; **b**: the rotated power spectra monopoles as defined in Eq. 1. In both panels, the lines in the center denote the mean of the mocks, and the shades represent the 68% CL uncertainty.

Figure 2. Part of the correlation matrix between the power spectra and bispectrum.

The correlation matrix for the monopoles of three types of power spectra (P_0^{pre} , P_0^{post} , P_0^{cross}), and of the bispectrum in the squeezed limit (B_0^{SL}), *i.e.* $k_1 = k_2 \gg k_3$, derived from the MOLINO galaxy mocks. The horizontal and vertical lines separate each block for visualisation. For all blocks, the associated k or k_1 increases from 0.01 to $0.5 h \text{ Mpc}^{-1}$, from left to right, and from bottom to top.

Figure 3. The 68% CL contour plots on σ_8 and $\log M_0$ derived from various data combinations.

Panels **a** and **b** show the constraint with $k_{\text{max}} = 0.2$ and $0.5 \, h \, \text{Mpc}^{-1}$, respectively.

Figure 4. The Figure of Merit (FoM) of each individual parameter derived from all power spectrum combined, rescaled by those derived from $P_{\text{pre}} + B_0$.

The quantities X and Y are defined as the FoM of each individual parameter with or without all other parameters fixed. Specifically, $X \equiv \sqrt{F_{ii}}$ and $Y \equiv 1/\sqrt{C_{ii}}$ where F is the Fisher matrix and $C \equiv F^{-1}$. D_0 and D_1 denote P_{all} and $P_{\text{pre}} + B_0$ respectively. The upper light gray and lower

dark gray bars show the cases with $k = 0.2$ and $0.5 \ h \text{ Mpc}^{-1}$, respectively. The vertical dashed lines shows a full recovery of information from dataset $P_{\text{pre}} + B_0$. The smoothing scale is set to be $10 \ h^{-1} \text{ Mpc}$ when performing the reconstruction.

	$k_{\text{max}} = 0.2 \, h \, \text{Mpc}^{-1}$					$k_{\text{max}} = 0.5 \, h \, \text{Mpc}^{-1}$				
	P_{pre}	P_{post}	P_{all}	$P_{\text{pre}} + B_0$	$P_{\text{post}} + B_0$	P_{pre}	P_{post}	P_{all}	$P_{\text{pre}} + B_0$	$P_{\text{post}} + B_0$
Ω_{m}	0.054	0.042	0.027	0.030	0.027	0.029	0.026	0.015	0.019	0.012
Ω_{b}	0.025	0.018	0.011	0.013	0.011	0.014	0.010	0.007	0.007	0.004
h	0.329	0.235	0.141	0.154	0.142	0.155	0.132	0.075	0.068	0.043
n_{s}	0.395	0.267	0.152	0.165	0.154	0.171	0.158	0.077	0.068	0.047
σ_8	0.183	0.107	0.043	0.045	0.045	0.057	0.041	0.019	0.022	0.014
$M_{\nu} \, [\text{eV}]$	0.865	0.655	0.307	0.283	0.274	0.355	0.283	0.160	0.102	0.070
FoM _{cos}	1.0	1.6	2.5	2.0	2.4	1.0	1.6	2.3	2.6	3.1
$\log M_{\text{min}}$	0.678	0.455	0.143	0.254	0.267	0.149	0.081	0.028	0.075	0.047
$\sigma_{\log M}$	1.333	0.848	0.250	0.610	0.545	0.464	0.238	0.078	0.202	0.102
$\log M_0$	1.170	0.389	0.118	0.362	0.162	0.102	0.056	0.029	0.040	0.025
$\log M_1$	0.679	0.625	0.186	0.213	0.250	0.116	0.115	0.051	0.066	0.057
α	0.601	0.330	0.084	0.162	0.119	0.080	0.076	0.033	0.032	0.027
FoM _Θ	1.0	1.7	2.7	1.9	2.4	1.0	1.4	2.1	2.1	2.3

Table 1: **A quantification of the information content extracted from various observables measured from the Molino mocks.** The 68% CL uncertainty of parameters derived from different datasets, with two different choices of the maximal wavenumber k_{max} . The last row, FoM_Θ shows the Figure-of-Merit (FoM), defined as $[\det(F)]^{1/(2N_{\text{p}})}$, where F denotes the Fisher matrix and N_{p} is the total number of free parameters. The FoM is normalised by the corresponding $(\text{FoM})_{P_{\text{pre}}}$ for two cases with different k_{max} . The row of FoM_{cos} shows the same FoM as defined above, but with all HOD parameters fixed.

Methods

1 The MOLINO galaxy mocks

The MOLINO catalogs¹³ are suite of publicly available galaxy mock catalogs that were constructed to quantify the total cosmological information content of different galaxy clustering observables using Fisher matrix forecasting¹. They are constructed from the QUIJOTE suite of N -body simulations¹⁵ using the halo occupation distribution (HOD) framework. HOD provides a statistical prescription for populating dark matter halos with central and satellite galaxies and has been successful in reproducing a wide range of observed galaxy clustering statistics. In particular, the MOLINO catalogs use the standard¹⁶ HOD model, which has five free parameters: $\{ \log M_{\min}, \sigma_{\log M}, \log M_0, \alpha, \log M_1 \}$. MOLINO includes 15,000 galaxy catalogs that are constructed at a fiducial set of cosmological parameters ($\Omega_m = 0.3175, \Omega_b = 0.049, h = 0.6711, n_s = 0.9624, \sigma_8 = 0.834, M_\nu = 0$) and HOD parameters ($\log M_{\min} = 13.65, \sigma_{\log M} = 0.2, \log M_0 = 14.0, \alpha = 1.1, \log M_1 = 14.0$), which are based on the best-fit HOD parameters for the SDSS $M_r < 21.5$ and -22 samples¹⁶. They are designed for accurately estimating the covariance matrices of high dimensional galaxy clustering observables. It also includes 60,000 galaxy catalogs that are constructed at 24 different parameter values and designed for accurately estimating the derivatives with respect to each of the 11 parameters. The parameter values are determined by varying each parameter of the fiducial values one step above and below. In total, the MOLINO suite includes 75,000 galaxy catalogs.

¹changhoonhahn.github.io/molino

2 The reconstruction on the MOLINO mocks

An anisotropic reconstruction ²¹, which means that the RSD effect is kept during the process of reconstruction, is performed on each realisation of the MOLINO galaxy mocks with two choices of the smoothing scale, $S = 10$ and $20 \ h^{-1} \text{ Mpc}$ (All results presented in the main text are for the $10 \ h^{-1} \text{ Mpc}$ case, while results for $20 \ h^{-1} \text{ Mpc}$ are shown in the Supplement information). Specifically, a smoothing is performed by convolving density field with the kernel $K(k) = \exp [-(k \ S)^2/2]$ in Fourier space. Then the displacement vector is solved using the Zeldovich approximation, *i.e.*, $\tilde{s}(\mathbf{k}) = -\frac{i\mathbf{k}}{k^2} \frac{\delta(\mathbf{k})}{b+f\mu^2} K(k)$, where δ denotes the nonlinear redshift-space overdensity, b is the linear bias measured from the mocks, and f is the fiducial logarithmic growth rate for the MOLINO mocks. An inverse Fourier transformation on $\tilde{s}$ returns the configuration-space displacement field $s(\mathbf{x})$, which is used to move both the galaxies and randoms by $s(\mathbf{x}) + f(s \cdot \hat{\mathbf{z}})\hat{\mathbf{z}}$.

3 Measurement of the power spectrum multipoles

The multipoles of both the pre- and post-reconstructed density fields are measured using an FFT-based estimator ¹ that is implemented in `N-body kit` ². The shot-noise, which reflects the discreteness of the density field, is removed as a constant for the monopole of the auto-power.

Care needs to be taken when measuring the cross-power spectrum between the pre- and post-reconstructed density fields, since the raw measurement using the FFT-based estimator is contaminated by a scale-dependent shot-noise: on large scales, the post-reconstructed field resembles the unreconstructed one, making the cross-power spectrum essentially an auto-power, thus it is subject to a shot-noise component. On small scales, however, the shot-noise largely drops because

the two fields effectively decorrelate.

To obtain a shot-noise-free cross-power spectrum, we adopt the “half-sum (HS) half-difference (HD)” approach, motivated by that used for measuring the shot-noise-free auto-power spectrum³. We start by randomly dividing the catalog into two halves, dubbed δ_1 and δ_2 , and the corresponding reconstructed density fields are R_1 and R_2 , respectively.

Let

$$\text{HS} \equiv \frac{\delta_1 + \delta_2}{2}, \quad \text{HD} \equiv \frac{\delta_1 - \delta_2}{2}, \quad (2)$$

and

$$\text{HS}^R \equiv \frac{R_1 + R_2}{2}, \quad \text{HD}^R \equiv \frac{R_1 - R_2}{2}. \quad (3)$$

Then $\text{HS}^{(R)}$ contains both the signal and noise, but $\text{HD}^{(R)}$ only contains noise. Hence the noise-free cross-power spectrum can be estimated by,

$$P_{\text{cross}} = \langle \text{HS}, \text{HS}^R \rangle - \langle \text{HD}, \text{HD}^R \rangle = \frac{\langle \delta_1, R_2 \rangle + \langle \delta_2, R_1 \rangle}{2}. \quad (4)$$

This allows for an estimation of the k -dependent shot noise of the cross power spectrum, as a by-product. As shown in Supplement Figures 7 and 8 (for cases with $S = 10$ and $20 \ h^{-1} \text{ Mpc}$, respectively), the noise is anisotropic thus it exists even for multipoles with $\ell \neq 0$, which could be understood from statistics and perturbation theory⁴.

4 Measurement of the bispectrum monopole

We measure the galaxy bispectrum monopole, B_0 , for all of the MOLINO catalogs using the publicly available pySpectrum package². Galaxy positions are first interpolated onto a grid with $N_{\text{grid}} = 360$ using a fourth-order interpolation scheme and then Fourier transformed to obtain $\delta(k)$. Afterwards B_0 is estimated using

$$B_0(k_1, k_2, k_3) = \frac{1}{V_B} \int_{k_1} d^3 q_1 \int_{k_2} d^3 q_2 \int_{k_3} d^3 q_3 \delta_D(\mathbf{q}_{123}) \delta(\mathbf{q}_1) \delta(\mathbf{q}_2) \delta(\mathbf{q}_3) - B_0^{\text{SN}} \quad (5)$$

where δ_D is the Dirac delta function, V_B is the normalization factor proportional to the number of triplets that can be found in the k_1, k_2, k_3 triangle bin, and B_0^{SN} is the Poisson shot noise correction term. Triangle configurations are defined by k_1, k_2, k_3 bins of width $\Delta k = 3k_f$, where $k_f = 2\pi/(1000 h^{-1}\text{Mpc})$.

Data Availability The data that support the plots within this paper and other findings of this study are available from the corresponding author upon reasonable request.

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3. Ando, S., Benoit-Lévy, A. & Komatsu, E. Angular power spectrum of galaxies in the 2MASS Redshift Survey. Mon. Not. Roy. Astron. Soc. **473**, 4318–4325 (2018). 1706.05422.

²<https://github.com/changhoonhahn/pySpectrum>

4. Zhao, R. et al. (To appear).

Supplementary Information

An independent mock test for demonstration

To support the main text, here we present an independent mock test using a suite of high-resolution dark matter-only mocks, to forecast the sensitivity of the BAO and RSD parameters. The mock is a suite of 4000 N -body simulations with 512^3 dark matter particles in a $L = 500 h^{-1}\text{Mpc}$ box at $z = 1.02$. We perform the anisotropic Lagrangian reconstruction¹ on each realisation of the mocks with a smoothing scale $S = 10 h^{-1}\text{Mpc}$, before measuring the multipoles (up to $\ell = 4$) of P_{pre} , P_{post} and P_{cross} , respectively. To investigate the sensitivity on cosmological parameters, we perform a Fisher matrix analysis¹⁴ on BAO parameters ($\alpha_{\perp}, \alpha_{\parallel}$) and a parameter describing the change of velocities along the line of sight (Δv) by computing the derivatives with respect to each parameter numerically. The parameter Δv mimics the change of the linear growth rate on large scales, but it also changes the velocity of particles coherently on small scales.

The result is shown in Supplement Figure 1. From panel **a** we see that the amplitude of P_{cross} decreases dramatically with scales, indicating a decorrelation effect between P_{pre} and P_{post} below quasi-nonlinear scales, which is confirmed by the correlation coefficient (the normalised covariance) plotted in panel **b**. This decorrelation, which is not caused by the shot noise given the noise level in the mocks, is a clear evidence of the complementarity among the power spectra.

The projection onto BAO parameters and Δv , shown in panels **c**₁ – **c**₈, well demonstrates the advantage for performing a joint analysis using P_{pre} , P_{post} and P_{cross} . On large scales, P_{pre} and

P_{post} are both determined by the linear density field, making the power spectra highly correlated, as shown in panel **b**. As shown in panels **c**₅, the contours derived from P_{pre} and P_{post} have similar orientations and we do not gain by combining them. For $k > 0.15 \, h \, \text{Mpc}^{-1}$, the correlation between P_{pre} and P_{post} decreases as the pre-reconstructed density field is dominated by the non-linear field while P_{post} still retains the correlation with the linear density field. The contours shown in lines in panel **c**₇, which are derived from power spectra in the k range of $[0.2, 0.25] \, h \, \text{Mpc}^{-1}$, are almost orthogonal to each other, making the constraint from the combined spectra, as illustrated in the shaded region, significantly tightened. On smaller scales, the post-reconstructed density field is also dominated by the non-linear field and the orientations of the contours are again aligned and the complementarity on smaller scales weakens. This shows that the level of nonlinearity in the power spectrum determines the degeneracies between parameters. Since P_{pre} and P_{post} are affected by different levels of nonlinearities on a given scale, which gives rise to different degeneracies, a joint analysis using both P_{pre} and P_{post} (and P_{cross}) can yield a better constraint by breaking the degeneracies.

Supplement Figure 1. An illustration of the complementarity between P_{pre} , P_{post} and P_{cross} (with the smoothing scale $S = 10 h^{-1} \text{ Mpc}$) when performing the reconstruction on the 4000 N-body mocks.

Panel **a**: the mean (lines) and 68% uncertainty (shades) of the measured three types of power spectrum monopole (multiplied by the wavenumber k); Panel **b**: the measured correlation coefficient between various types of power spectrum monopoles. The dashed horizontal line shows a perfect correlation (*i.e.*, $\text{corr} = 1$) for a reference; Panels **c**: the 68% CL contour plots for BAO and RSD parameters including $\alpha_{\parallel}$, $\alpha_{\perp}$ and Δv derived from three types of power spectrum multipoles ($\ell = 0, 2, 4$) separately (lines) and jointly (filled). Power spectra used in different k ranges are shown in different columns, as illustrated in the legend.

Supplementary figures for analyses using the Molino mocks

Supplement Figure 2. The 68% CL contour plots on $(\sigma_8, \log M_0)$ and $(\sigma_8, \sigma_{\log M})$ for different combination of summary statistics in several k ranges.

Supplement Figure 3. The 1D posterior distribution and 68% CL contour plots for cosmological and HOD parameters up to $k_{\text{max}} = 0.2 \, h \, \text{Mpc}^{-1}$. The smoothing scale for the reconstruction is $S = 10 \, h^{-1} \, \text{Mpc}$.

Supplement Figure 4. The 1D posterior distribution and 68% CL contour plots for cosmological and HOD parameters up to $k_{\max} = 0.5 \, h \, \text{Mpc}^{-1}$. The smoothing scale for the reconstruction is $S = 10 \, h^{-1} \, \text{Mpc}$.

Supplement Figure 5. The 1D posterior distribution and 68% CL contour plots for cosmological and HOD parameters up to $k_{\text{max}} = 0.2 \, h \, \text{Mpc}^{-1}$. The smoothing scale for the reconstruction is $S = 20 \, h^{-1} \, \text{Mpc}$.

Supplement Figure 6. The 1D posterior distribution and 68% CL contour plots for cosmological and HOD parameters up to $k_{\text{max}} = 0.5 \, h \, \text{Mpc}^{-1}$. The smoothing scale for the reconstruction is $S = 20 \, h^{-1} \, \text{Mpc}$.

Supplement Figure 7. The measured shot-noise for the cross power spectrum multipoles. The smoothing scale $S = 10 \ h^{-1} \text{ Mpc}$ is used for the reconstruction.

Supplement Figure 8. The measured shot-noise for the cross power spectrum multipoles. The smoothing scale $S = 20 \, h^{-1} \text{ Mpc}$ is used for the reconstruction.

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