Novel Ring Compression Test Method to Determine the Stress-strain Relations and Mechanical Properties of Metallic Materials

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Title page

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ORIGINAL ARTICLE

Novel ring compression test method to determine the stress–strain relations and mechanical properties of metallic materials

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Abstract: Although there are methods for testing the stress–strain relation and strength, which are the most fundamental and important properties of metallic materials, their application to small size specimens is limited. In this study, a new dimensionless elastoplastic load–displacement (EPLD-Ring) model for compressed metal rings with isotropy and constitutive power law is proposed to describe the relation between the geometric dimensions, Hollomon law parameters, load, and displacement based on energy density equivalence. Furthermore, a novel test method for the rings is developed to obtain the elastic modulus, stress–strain relation, yield strength, and tensile strength. The universality and accuracy of the model are verified within a wide range of imaginary materials via finite element analysis (FEA), and the results show that the stress–strain relations obtained with the model are more consistent with those inputted in the FEA software. Additionally, for seven metallic materials, a series of ring compression tests with various dimensions were performed. It was found that the stress–strain relations and mechanical properties predicted by the model are in agreement with the normal tensile test results. It is believed that the new method is reliable and effective for testing the mechanical properties of small size materials and tube components.

Keywords: Ring compression • Energy density equivalence • Power law material • Stress–strain relation • metallic material

1 Introduction

As the most fundamental and important properties of metallic materials, stress–strain (σ – ε) relations and strengths are usually obtained via normal tests upon centimeter-grade tensile specimens; however, it is difficult or impossible to

perform tensile tests for small size materials and tube components. Accordingly, a new testing method for nontraditional configuration specimens, such as rings with small sizes, is needed.

Millimeter-grade ring specimens are easy to machine and test, testing the mechanical properties of ring materials has been the focus of research works over the last forty years. Arsene and Bai [1, 2] proposed a ring hoop tension test (RHTT) method to acquire the hoop σ – ε relations of nuclear fuel cladding tubes. In this test, a dog-bone specimen was machined on a ring extracted from the tube. Next, the ring was placed over two loading D-blocks that were parted by a test machine. However, the RHTT is not suitable for thin-wall rings, and the results are affected by the friction between the test specimen and the loading D-blocks. For linear hardening materials, Reddy and Reid [3–5] introduced a simple theory to obtain the yield strength via a ring compression test (RCT). The load–displacement (P – h) test curve is obtained by applying a unidirectional load to the ring specimen through two rigid platens, and then the yield stress is determined according to the linear relation between the yield load and stress. However, for most metallic materials with constitutive power law, this method cannot be universally applied. With the development of simulation technology, Nemat-Alla [6] developed an inverse method for compressed rings based on finite element analysis (FEA). In this method, an imaginary σ – ε relation is inputted in the FEA to predict the RTC P – h curve. Then, the test and analysis curves are compared to adjust the inputted σ – ε relation until they are in agreement. This method is not convenient for wide applications owing to the complicated iterative calculations involved. Chen and Cai [7, 8] proposed a theoretical model for compressed rings under plane-stress conditions based on energy density equivalence. This model can predict the elastoplastic mechanical properties of materials with constitutive power law; however, it does not give details regarding the elastic modulus testing method for ring materials and the thickness effect of the ring specimen

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is not considered.

For three-dimensional rings, the main purpose of this investigation is to develop a method that can acquire the basic mechanical properties of metallic materials with constitutive power law through RCT. To achieve this, a dimensionless elastoplastic load–displacement (EPLD-Ring) model for compressed rings is proposed based on energy density equivalence. By solving this model through the information contained in the P – h curves, the elastic modulus, σ – ε relation, and strength of the testing material can be determined. Finally, the model is verified via FEA with a wide range of imaginary materials and through experiments with seven metallic materials.

2 Theoretical model for ring compression

2.1 Energy density equivalence method

For unidirectional loaded rings, as shown in Figure 1, assuming that a geometric point M exists in the effective deformation region Ω , and that the energy density of the representative volume element (RVE) at M is equal to the average energy density of all RVEs in Ω , then:

$$u_M = \int_0^{\varepsilon_{ij-M}} \sigma_{ij} d\varepsilon_{ij} = U / V_{\text{eff}}, \quad (1)$$

where u_M , σ_{ij} and ε_{ij} are the energy density, stress tensor, and strain tensor of the RVE at M, respectively; ε_{ij-M} is the strain tensor of the RVE at M in a deformation state; V_{eff} is the effective volume of Ω ; and U is the total deformation energy of Ω , which is given by:

$$U = \iiint_{\Omega} u(x, y, z) dx dy dz, \quad (2)$$

Figure 1 Schematic diagram of the ring compression test

According to the von Mises energy equivalence principle, the energy density of the RVE at M in a complex stress state is equivalent to that in a uniaxial stress state:

$$u_M = \int_0^{\varepsilon_{ij-M}} \sigma_{ij} d\varepsilon_{ij} = \int_0^{\varepsilon_{eq-M}} \sigma_{eq} d\varepsilon_{eq}, \quad (3)$$

where ε_{eq-M} is the equivalent strain of the RVE at M in a deformation state; σ_{eq} and ε_{eq} are the equivalent stress and strain, respectively.

Combining Eqs. (1) and (3), the total deformation energy is expressed as:

$$U = V_{\text{eff}} \int_0^{\varepsilon_{eq-M}} \sigma_{eq} d\varepsilon_{eq}, \quad (4)$$

2.2 Elastoplastic load–displacement model

For most metallic materials with homogeneity and isotropy, the uniaxial equivalent σ – ε relations are almost consistent with the Hollomon law model:

$$\sigma_{eq} = \begin{cases} E\varepsilon_{eq} & \varepsilon_{eq} \leq \varepsilon_y \\ K\varepsilon_{eq}^n & \varepsilon_{eq} > \varepsilon_y \end{cases}, \quad (5)$$

where K is the strain hardening coefficient ($K = E^n \sigma_y^{1-n}$); E and σ_y are the elastic modulus and nominal yield stress, respectively; n is the strain hardening exponent; and ε_y is the nominal yield strain ($\varepsilon_y = \sigma_y/E$).

By substituting Eq. (5) into Eq. (4), the total deformation energy U is derived as follows:

$$U = \begin{cases} \frac{EV_{\text{eff}}}{2} \varepsilon_{eq-M}^2 & \varepsilon_{eq-M} \leq \varepsilon_y \\ \frac{KV_{\text{eff}}}{n+1} \left(\varepsilon_{eq-M}^{n+1} - \frac{1-n}{2} \varepsilon_y^{n+1} \right) & \varepsilon_{eq-M} > \varepsilon_y \end{cases}, \quad (6)$$

When ε_{eq-M} is considerably larger than ε_y , $(1-n)\varepsilon_y^{n+1}/2$ is a small quantity compared to ε_{eq-M}^{n+1} and can be ignored. Next, Eq. (6) can be simplified as follows:

$$U = \begin{cases} \frac{EV_{\text{eff}}}{2} \varepsilon_{eq-M}^2 & \varepsilon_{eq-M} \leq \varepsilon_y \\ \frac{KV_{\text{eff}}}{n+1} \varepsilon_{eq-M}^{n+1} & \varepsilon_{eq-M} > \varepsilon_y \end{cases}, \quad (7)$$

Under linear-elastic and elastoplastic conditions, the following assumptions can be made:

$$\begin{cases} \frac{V_{\text{eff}}}{V^*} \varepsilon_{eq-M}^2 = k_0 \left(\frac{h}{h^*} \right)^2 & \text{Linear-elastic} \\ \frac{V_{\text{eff}}}{V^*} = k_1 \left(\frac{h}{h^*} \right)^{k_2} & \\ \varepsilon_{eq-M} = k_3 \left(\frac{h}{h^*} \right)^{k_4} & \text{Elastoplastic} \end{cases}, \quad (8)$$

Where V^* is the characteristic volume ($V^* = A^* h^*$); and A^* and h^* are the characteristic area and length, respectively.

For ring specimens, $A^* = \beta BD(1-\rho^2)$, $h^* = D$, and $V^* = \beta BD^2(1-\rho^2)$, where B and D are the thickness and diameter of the ring specimens, respectively; ρ is the diameter ratio of the ring; β is the thickness effect coefficient; k_0 is the elastic deformation coefficient; k_1 and k_2 are the effective volume coefficient and exponent, respectively; and k_3 and k_4 are the equivalent strain coefficient and exponent, respectively. By substituting Eq. (8) into Eq. (7), U can be derived as follows:

$$U = \begin{cases} \frac{EV^*}{2} k_0 \left(\frac{h}{h^*} \right)^2 & \text{Linear-elastic} \\ \frac{KV^*}{n+1} k_1 k_3^{n+1} \left(\frac{h}{h^*} \right)^{k_4(n+1)+k_2} & \text{Elastoplastic} \end{cases}, \quad (9)$$

According to the work–energy principle ($W=U$), by taking the derivative of the displacement with respect to both sides of Eq. (9), the P – h relations can be deduced as follows:

$$P = \begin{cases} k_0 EA^* h & \text{Linear-elastic} \\ KA^* \frac{k_1 k_3^{n+1} [k_4(n+1) + k_2 - 1]}{(n+1) h^{*k_4(n+1)+k_2-1}} h^{k_4(n+1)+k_2-1} & \text{Elastoplastic} \end{cases}, \quad (10)$$

The dimensionless arrangement of Eq. (10) is expressed as follows:

$$\left\{ \begin{aligned} \frac{P}{P^*} &= \begin{cases} \xi_e \frac{h}{h^*} & \text{Linear-elastic} \\ \xi_{ep} \left(\frac{h}{h^*} \right)^m & \text{Elastoplastic} \end{cases} \\ \left\{ \begin{aligned} P^* &= EA^* \\ A^* &= \beta BD(1-\rho^2) \\ h^* &= D \end{aligned} \right. \\ \left\{ \begin{aligned} \xi_e &= k_0 \\ \xi_{ep} &= \frac{K k_1 k_3^{n+1} (m+1)}{E(n+1)} \\ m &= k_4(n+1) + k_2 - 1 \\ K &= E^n \sigma_y^{1-n} \end{aligned} \right. \end{aligned} \right., \quad (11)$$

where P^* is the characteristic load; ξ_e is the dimensionless P – h deformation coefficient under linear-elastic deformation; and ξ_{ep} and m are the dimensionless P – h deformation coefficient and exponent under elastoplastic deformation, respectively. Eq. (11) is defined as a dimensionless elastoplastic load–displacement (EPLD-Ring) model for

compressed rings.

Moreover, according to Eq. (11), for compressed rings under ideal linear-elastic loading conditions, the loading stiffness $S_L (= P/h)$ is expressed as follows:

$$S_L = \beta k_0 B (1-\rho^2) E \quad \text{Linear-elastic}, \quad (12)$$

3 Parameter determination

The EPLD-Ring model is a dimensionless equation that relates the geometric dimensions of rings (D , ρ , B), Hollomon law parameters (E , σ_y , n), load, and displacement. The six constants contained in the model, including the thickness effect coefficient β , elastic deformation coefficient k_0 and elastoplastic deformation coefficients k_1 – k_4 , can be determined by FEA directly towards ring specimens with various geometric dimensions and Hollomon law parameters.

3.1 Elastoplastic load–displacement model

FEA for ring compression was conducted through the commercial FEA software ANSYS14.5. Considering the symmetry of the ring specimens, a 1/4 three-dimensional FEA model was established, as shown in Figure 2. The imaginary materials inputted in the FEA were assumed to meet the homogenous, isotropic hardening, and von Mises principles, and only linear-elastic deformation occurred in the clamps with elastic modulus E_p . Target172 and Contact173 elements were used to establish a contact pair between the lower surface of the clamps and the outer surface of the ring, and a Solid185 element was used for the main part of both the ring and the clamp. The left and lower cross-section of the ring and the left cross-section of the clamp were applied with a symmetry constraint, and the upper surface of the clamp was applied with a displacement load. The influence of mesh size on the FEA model results was analyzed using various geometric rings, and the mesh size that stabilized the calculation results of the P – h curves was selected.

(a) Front view (b) Isometric view
Figure 2 Finite element analysis model for ring compression

For rings with various diameters, according to the EPLD-Ring model, the P – h curves can be normalized by the normalized load $P/BD(1-\rho^2)$ and the dimensionless displacement h/D , as shown in Figure 3. Thus, the diameter of the rings was fixed to 10 mm during the subsequent parameter determination.

Figure 3 Normalized load–dimensionless displacement curves for various dimeters

3.2 Thickness effect coefficient β

The thickness effect coefficient β is closely related to the characteristic thickness B/D of the ring specimens, and the loading capacity per unit thickness of the ring increases with the increase in B/D . For various B/D of the rings, $P/B \sim h$ curves can be obtained via FEA, as shown in Figure 4. Table 1 lists the FEA conditions.

Figure 4 Per unit load–displacement curves for various B/D

Table 1 Finite element analysis conditions for β

Clamp	Geometry size of rings			Power-law parameters		
E_p/GPa	D/mm	d/mm	B/mm	E/GPa	σ_y/MPa	n
400	10	6	1 to 4	200	400	0.1

In the dimensionless displacement h/D range from 0.05 to

0.15, by fitting the $P/B \sim h/D$ curves of various B/D with the power law, the data array $\{B/D, C_{B/D}\}$ was obtained, where $C_{B/D}$ is the fitting coefficient and $C = \beta \xi_{ep} E D (1-\rho^2)$. Defined as $\beta = 1$ when $B/D = 0.2$, β for other B/D can be obtained with the equation $\beta_{B/D} = C_{B/D}/C_{0.2}$. The β versus B/D curve shown in Figure 5 and the relation between β and B/D can be expressed as an arctangent equation:

$$\beta = b_1 \arctan \left[b_2 \left(\frac{B}{D} \right) + b_3 \right] + b_4, \quad (13)$$

where b_1 to b_4 are the fitting constants (see Table 2).

Figure 5 β versus B/D relation

3.3 Elastic deformation coefficient k_0

The elastic deformation coefficient k_0 is related to the clamp elastic modulus E_p and the diameter ratio of the ring specimen and can be obtained by fitting the $P/P^* \sim h/D$ curves with a linear equation under linear-elastic deformation. The k_0 versus $1-\rho$ curve obtained by FEA, as shown in Figure 6(a) with fixed $E_p = 400$ GPa, and the relation between k_0 and $1-\rho$ can be described by the following power-law equation: $k_0 = a_1(1-\rho)^{a_2}$, where a_1 and a_2 are the fitting coefficient and exponent, respectively. We defined $E_p^* = 400$ GPa as the characteristic elastic modulus and $k_0^* = a_1(1-\rho)^{a_2}$ ($E_p = 400$ GPa) as the characteristic elastic deformation coefficient. Subsequently, E_p/E_p^* versus k_0/k_0^* obtained by FEA, as shown in Figure 6(b) with fixed $\rho = 0.6$, and the relation between E_p/E_p^* and k_0/k_0^* can be described by the following power-law equation: $k_0/k_0^* = a_3(E_p/E_p^*)^{a_4}$, where a_3 and a_4 are the fitting coefficient and exponent, respectively. Thus, the relation between k_0 , E_p , and ρ is expressed as follows:

$$k_0 = c_{01} (1-\rho)^{c_{02}} \left(E_p / E_p^* \right)^{c_{03}}, \quad (14)$$

where c_{01} , c_{02} , and c_{03} are the fitting constants, and $c_{01} = a_1 a_3$, $c_{01} = a_2$, and $c_{03} = a_4$ (see Table 2).

Figure 6 Fitting relations between (E_p, ρ) and k_0

3.4 Elastoplastic deformation coefficient k_1 – k_4

Because the influence of E_p on the elastoplastic P – h curves is slight, the elastoplastic deformation coefficients k_1 – k_4 are only related to the diameter ratio ρ of the ring specimens. For the ring specimens with fixed ρ , they can be determined by fitting the elastoplastic P – h curves with various n . Table 3 shows the FEA conditions to determine k_1 – k_4 of the ring specimen with a diameter ratio of 0.6.

Table 3 FEA conditions for k_1 to k_4 ($\rho=0.6$)

Clamp	Geometry size of rings			Power-law parameters		
E_p/GPa	D/mm	d/mm	B/mm	E/GPa	σ_y/MPa	n
400	10	6	1	200	400	0.1 to 0.5

In the dimensionless displacement range from 0.05 to 0.15, by fitting the $P/P^* \sim h/D$ curves of various n with the power law, the data arrays $\{n, \xi_{ep}\}$ and $\{n, m\}$ were obtained. The data arrays $\{n+1, \xi_{ep}E(n+1)/K(m+1)\}$ and $\{n+1, m+1\}$ were acquired by combining the FEA conditions and Eq. (11). By fitting $n+1 \sim \xi_{ep}E(n+1)/K(m+1)$ with an exponent equation, k_1 and k_3 were determined. By fitting $n+1 \sim m+1$ with a linear equation, k_2 and k_4 were determined.

Similarly, k_1 – k_4 directly towards various ρ values (0.625–0.7) can be determined, as shown in Figure 7. The relations between k_1 – k_4 and ρ are as follows:

$$\begin{cases} k_1 = c_{11}\rho^{c_{12}} \\ k_2 = c_{21}\rho^{c_{22}} \\ k_3 = c_{31} + c_{32}\rho \\ k_4 = c_{41} + c_{42}\rho + c_{43}\rho^2 \end{cases}, \quad (15)$$

where c_{11} 、 c_{12} 、 c_{21} 、 c_{22} 、 c_{31} 、 c_{32} 、 c_{41} – c_{43} are the fitting constants (see Table 2).

(a) k_1 and k_2 versus ρ

(b) k_3 and k_4 versus ρ

Figure 7 Fitting relations between k_1 – k_4 and ρ

Table 2 Fitting constants of the EPLD-Ring model parameters

b_1	b_2	b_3	b_4	c_{01}	c_{02}	c_{03}	c_{11}
0.0473	9.25	-2.13	1.01	0.557	2.21	0.040	0.0639
c_{12}	c_{21}	c_{22}	c_{31}	c_{32}	c_{41}	c_{42}	c_{43}
-3.32	0.0316	-3.15	0.829	-0.737	0.183	2.05	-1.46

In summary, the fitting constants of the EPLD-Ring model parameters are valid for rings with thickness B within [0.1D, 0.4D] and diameter ratio ρ within [0.6, 0.7]. If the rings

geometric dimensions (D , ρ , B) are acquired, then β and k_0 – k_4 can be calculated by combining Eqs. (13)–(15) and Table 2. In fact, by re-calibrating the model parameters β and k_0 – k_4 , the EPLD-Ring model is appropriate for ring specimens whose sizes is not within the aforementioned range.

4 Ring compression test method based on the EPLD-Ring model

4.1 Elastic modulus testing method

During the ring compression test, because of the interference of the initial contact nonlinearity, geometric dimensions error, and other factors, the linear-elastic condition in the initial loading stage of the P – h test curves is difficult to satisfy; therefore, the elastic modulus E obtained through the initial loading stiffness S_L of the P – h test curve is difficult to realize. Using the initial unloading stiffness S_u of the loading–unloading test to calculate E is a common method in elastoplastic mechanical tests, especially, in the elastoplastic indentation problem [9]. Accordingly, this method was applied in this investigation.

In the loading–unloading test for ring compression, the power-law equation $P_u = ah_u^t$ was used to fit the data points of the unloading-stage P_u – h_u curve. The S_u is expressed as follows:

$$S_u = \left. \frac{dP_u}{dh_u} \right|_{h_u=h_{u\max}} = at h_{u\max}^{t-1}, \quad (16)$$

where a and t are the fitting coefficient and exponent of the P_u – h_u curve, respectively, and $h_{u\max}$ is the displacement of the P_u – h_u curve initial unloading point. The research shows a proportional relation between S_L and S_u :

$$S_L = \alpha S_u, \quad (17)$$

where α is the stiffness ratio, which can be determined via a multilevel loading–unloading test for rings with known E .

Multilevel loading–unloading tests were performed on 30Cr2Ni4MoV with $E = 202$ GPa and 7075Al with $E = 72$ GPa. The P – h test curves and α for various h/D are shown in Figure 8.

According to Figure 8, α is approximately constant in the dimensionless displacement h/D range of 0.01–0.05, and the average value α_m is 0.967, with a coefficient of variation of 0.75%. Thus, according to Eqs. (12) and (17), E can be derived as follows:

$$E = \frac{\alpha_m S_u}{\beta k_0 B (1 - \rho^2)}, \quad (18)$$

where β and k_0 can be calculated with Eq.(13), Eq.(14), and Table 2.

(a) Multilevel loading–unloading P – h curves

(b) α for various h/D

Figure 8 Multilevel loading–unloading test to determine α

4.2 Hollomon law parameters testing method

According to Eq. (11), σ_y and n are derived as follows:

$$\begin{cases} \sigma_y = \left[\frac{\xi_{ep} E^{1-n} (n+1)}{k_1 k_3^{n+1} (m+1)} \right]^{1/(1-n)} \\ n = \frac{(m+1) - k_2 - k_4}{k_4} \end{cases}, \quad (19)$$

where k_1 – k_4 can be calculated with Eq.(15) and Table 2.

For steels, using the power-law equation to fit the P – h obtained by RCT within the dimensionless displacement h/D range of 0.05–0.15, ξ_{ep} and m are obtained, for light alloy, the fitting range is 0.07–0.15. Next, by combining Eq. (19), the Hollomon law parameters σ_y and n are acquired. Additionally, the strain hardening coefficients K and σ – ϵ curve can be obtained according to Eq. (5). The yield strength $R_{p0.2}$ can be determined by referring to the recommended method of ISO 12135 standard, and the tensile strength [7, 10] can be calculated from the Hollomon parameters as follows:

$$R_m = K \left(\frac{n}{e} \right)^n, \quad (20)$$

where e is the natural constant, $e = 2.718$.

5 Results and discussion

5.1 Verification by finite element analysis

To verify the universality and accuracy of the EPLD-Ring model, a wide range of imaginary materials with given constitutive relations are inputted in the FEA for rings with various geometric dimensions. Through the method proposed in Section 4.2, by fitting the P - h curves obtained by FEA, the Hollomon law parameters and σ - ε can be acquired. The inputted σ - ε relations and the predicted σ - ε relations are shown in Figure 9.

According to Figure 9, for rings with various geometric dimensions (D , ρ , B) and various Hollomon parameters (ε_y , n), the σ - ε relations obtained by the model correspond to the constitutive relations inputted in the FEA. Thus, the EPLD-Ring model is universal and accurate for predicting the σ - ε relations for rings with various geometric dimensions and various Hollomon law parameters.

(a) $\rho = 0.6$

(b) $\rho = 0.65$

(c) $\rho = 0.7$

Figure 9 Comparison between the σ - ε relations predicted by the EPLD-Ring model and those inputted into the finite element analysis

5.2 Experimental verification

5.2.1 Experimental conditions

The normal tensile test and the RCT were carried out on seven metallic materials, including five steels, one titanium alloy, and one aluminum alloy. Both tests were performed on an MTS 809 testing machine; the extensometer model is MTS 634.11F-24 uniaxial strain extender with an accuracy level of 0.5. The basic mechanical properties of the seven metallic materials obtained by the normal tensile test and the fitting Hollomon law parameters are listed in Table 4.

Four types of ring specimens with various D , B/D , and ρ were machined for the RCT of seven metallic materials. The nominal sizes of the specimens were $\{D, d, B\}$ (unit: mm): $\{6, 4, 1\}$, $\{8, 5, 1\}$, $\{10, 6, 1\}$, and $\{10, 6, 2\}$. The clamps material was 40Cr with $E_p = 210$ GPa. During the tests, the two elastic arms of the extensometer were fixed with the upper and lower clamps, respectively, to investigate the deformation of the ring specimens, as shown in Figure 10.

Table 4 Basic mechanical properties and Hollomon law parameters of seven metallic materials

Materials	Basic mechanical properties			Hollomon law parameters	
	E/GPa	$R_{p0.2}/\text{MPa}$	R_m/MPa	σ_y	n
Q345E	210	467	585	348	0.147
16MnR	210	300	515	254	0.172
A508-III	210	511	656	468	0.118
P91	208	597	692	562	0.0825
30Cr	202	798	886	751	0.0798
TC4-DT	120	804	875	800	0.0661
7075Al	72	437	519	431	0.102

Figure 10 Ring compression test set-up

5.2.2 Experimental results and discussion

Before the tests, the geometric sizes of the specimens were accurately measured with a micrometer. All RCTs were controlled by displacement with a loading rate of 3 mm/s under quasi-static conditions. Four specimens were tested for each geometric size of each material: one for testing E with the ring compression loading–unloading test and three for testing the Hollomon law parameters (σ_y , n) with the RCT (without unloading). The P – h test curves of both tests are shown in Figures 11 and 12, respectively.

(a) $D = 6 \text{ mm}$, $B = 1 \text{ mm}$

(b) $D = 8 \text{ mm}$, $B = 1 \text{ mm}$

(c) $D = 10 \text{ mm}$, $B = 1 \text{ mm}$

(d) $D = 10 \text{ mm}$, $B = 2 \text{ mm}$

Figure 11 P – h curves of the loading–unloading test for seven materials

(a) $D = 6 \text{ mm}$, $B = 1 \text{ mm}$

Figure 12 P - h curves of the ring compression test without unloading for seven materials

Through the method proposed in Section 4.1, the elastic moduli E of the seven materials were determined, as shown in Table 5. The results show that the measured elastic moduli for the seven metallic rings are all close to those given by the normal tensile tests, and that the relative error between the two results is less than 3% for most cases, with the maximum error being 5.6%.

Table 5 Comparison between the elastic moduli predicted by the ring compression test and those obtained by the normal tensile test

Materials	Specimen 6-4-1		Specimen 8-5-1		Specimen 10-6-1		Specimen 10-6-2	
	E_R /GPa	Error /%	E_R /GPa	Error /%	E_R /GPa	Error /%	E_R /GPa	Error /%
Q345E	209	-0.50	213	1.4	209	-0.50	206	-1.9
16MnR	207	-1.4	209	-0.50	214	1.9	206	-1.9
A508-III	208	-1.0	202	-3.8	212	1.0	207	-1.4
P91	211	1.4	216	3.8	213	2.4	211	1.4
30Cr	198	-2.0	206	2.0	205	1.5	198	-2.0
TC4-DT	119	-0.80	123	2.5	124	3.3	116	-3.3
7075Al	73	1.3	74	2.8	74	2.8	76	5.6

According to Figure 12, the P - h curves of three parallel specimens with the same geometric dimensions and material are consistent; thus, according to Section 4.2, the average value of the predicted results of the three parallel specimens is taken as the final test result, as shown in Figure 13. It can be seen that the predicted σ - ε relations of the seven metallic materials are in good agreement with the normal tensile test results.

Figure 13 Comparison between the σ - ε relations predicted by the EPLD-Ring model and those obtained by the normal tensile tests

Based on the method proposed in Section 4.2, the yield and tensile strengths of the seven metallic materials were obtained, as shown in Figure 14. It can be seen that the yield strength $R_{p0.2}$ errors between the RCT results and the normal tensile tests were mostly within 3% and individually within 8% for six metallic materials, except for Q345E. Because a large yield platform exists in Q345E, the $R_{p0.2}$ error obtained through the RCT method is approximately 20% compared with the normal tensile test results. The tensile strength R_m error predicted by the RCT method was within 3% of the normal tensile test results.

Figure 14 Comparison between the yield and tensile strengths $R_{p0.2}$ and R_m predicted by the RCTs and those obtained by the normal tensile tests

6 Conclusions

- (1) In the present study, based on energy density equivalence, a dimensionless elastoplastic load-displacement (EPLD-Ring) model for compressed rings with three-dimensional sizes is proposed. The proposed model describes the relation between the geometric dimensions, Hollomon law parameters, load, and displacement of the tested specimens. The six constants contained in the model can be determined via FEA.
- (2) The test investigation for rings made of two metallic materials shows that the initial theoretical ring stiffness is proportional to the unloading stiffness at the unloading point of the P - h curve in the h/D range of 0.01–0.05. Subsequently, a novel method is developed to obtain the elastic modulus and the stress-strain relations and strengths of metallic materials. For rings of imaginary materials with various geometric dimensions, the σ - ε relations predicted by the proposed method due to the P - h curves given by FEA are consistent with the imaginary constitutive relations inputted in the FEA.
- (3) For seven metallic materials, a series of compression tests for rings with four types of geometric dimensions for each material were performed, and the elastic moduli and σ - ε relations and strengths of the materials predicted by the proposed test method are in agreement with the results given by the normal tensile tests.
- (4) Because of their low material consumption, millimeter-grade ring specimens have high potential to test the mechanical properties for small size materials and tube components.

7 Declaration

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Availability of data and materials

The datasets supporting the conclusions of this article are included within the article.

Authors' contributions

GZH and CB were in charge of the whole trial; GZH wrote the manuscript; LXC was in charge of guidance; MBH, BL, LY and XKL assisted with sampling and data analysis; All the authors read and approved the final manuscript.

Competing interests

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References

- [1] Petersen, D. et al., A new approach to measuring transverse properties of structural tubing by a ring test. *Journal of Testing and Evaluation*, 1996. 24(6): 386-391.
- [2] Petersen, D. et al., A new approach to measuring transverse properties of structural tubing by a ring test—Experimental investigation. *Journal of Testing and Evaluation*, 1998. 26(1): 26-30.
- [3] Yella Reddy, T.Y. and S.R. Reid, On obtaining material properties from the ring compression test. *Nuclear Engineering and Design*, 1979. 52(2): 257-263.
- [4] Reddy, T.Y., S.R. Reid and J.H. Ahn, Phenomena associated with the crushing of metal tubes between rigid plates. *International Journal of Solids and Structures*, 1980. 16(6): 545-562.
- [5] Reddy, T.Y., Tube inversion. An experiment in plasticity. *International Journal of Mechanical Engineering Education*, 1989. 17(4): 277-291.
- [6] Nemat-Alla, M., Reproducing hoop stress-strain behavior for tubular material using lateral compression test. *International Journal of Mechanical Sciences*, 2003. 45(4): 605-621.
- [7] Chen, H. and L.X. Cai, Unified ring-compression model for determining tensile properties of tubular materials. *Materials Today Communications*, 2017. 13: 210-220.
- [8] Chen, H. and L.X. Cai, A universal elastic-plastic model correlating load-displacement relation and constitutive parameters for typical testing components. *Results in Physics*, 2019. 13: 102230.
- [9] Z.J. Zhang, et al., Spherical indentation method to determine stress-strain relations and tensile strength of metallic materials. *Chinese Journal of Theoretical and Applied Mechanics*, 2019. 51(01): 159-169.(in Chinese)
- [10] Byun, T.S., et al., Measurement of through-the-thickness variations of mechanical properties in SA508 Gr.3 pressure vessel steels using ball indentation test technique. *International Journal of Pressure Vessels and Piping*, 1997. 74(3): 231-238.

Figures

Figure 1

Schematic diagram of the ring compression test

Figure 2

Finite element analysis model for ring compression

Figure 3

Normalized load–dimensionless displacement curves for various dimeters

Figure 4

Per unit load–displacement curves for various B/D

Figure 5

β versus B/D relation

Figure 6

Fitting relations between (E_p, ρ) and k_0

(a) k_1 and k_2 versus ρ

(b) k_3 and k_4 versus ρ

Figure 7

Fitting relations between k_1 – k_4 and ρ

(a) Multilevel loading-unloading P – h curves

(b) α for various h/D

Figure 8

Multilevel loading-unloading test to determine α

Figure 9

Comparison between the σ - ϵ relations predicted by the EPLD-Ring model and those inputted into the finite element analysis

Figure 10

Ring compression test set-up

Figure 11

P-h curves of the loading-unloading test for seven materials

Figure 12

P-h curves of the ring compression test without unloading for seven materials

Figure 13

Comparison between the σ - ϵ relations predicted by the EPLD-Ring model and those obtained by the normal tensile tests

Figure 14

Comparison between the yield and tensile strengths $R_{p0.2}$ and R_m predicted by the RCTs and those obtained by the normal tensile tests