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Research Article

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Posted Date: February 28th, 2022

DOI: https://doi.org/10.21203/rs.3.rs-1395185/v1

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Entanglement Dynamics and Quantum Dense Encoding of Heisenberg Spin Chain in Non-Markovian Environment

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Abstract: Quantum entanglement is a special nonlocal association between quantum systems. It is a fundamental feature of the difference between quantum and classical. At the same time, it acts as a very important resource in quantum informatics. Therefore, entanglement plays a noble role in quantum information and quantum computing. Quantum dense coding, as an application of quantum entanglement in practice, is a quantum communication protocol characterized by security and secrecy, which plays a very important role in quantum informatics and cryptography. This paper is mainly in the Non-Markovian environment of Heisenberg Spin chain system, using the quantum dense coding scheme proposed by Bennett et al. to study The dynamics of quantum dense coding channel capacity evolution with time and the entanglement dynamics of the system are discussed in detail.Based on the theoretical knowledge of open quantum systems and the maximum entangled state $|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}(|11\rangle + |00\rangle)$ as the initial state, the density matrix of the system evolution over time is obtained by quantum state diffusion method,and the evolution of channel capacity and entanglement over time of Heisenberg spin chains is simulated accurately. The results show that: the capacity and entanglement of dense coded channel oscillate with time and reach different stable values. Environmental correlation coefficient γ and coupling strength η and a play an important role in dense coding and quantum entanglement of quantum systems.Proper selection of parameters can keep the system good entanglement characteristics and increase the channel capacity χ of the system, thus ensuring the effective transmission and processing of quantum information in practice. It lays a solid foundation for quantum computing

Keywords: Quantum information and quantum computing, Quantum dense coding, Quantum entanglement, Non-Markovian quantum state diffusion

PACS: 03.67.Pp, 03.65.Yz

1. Introduction

Quantum informatics has achieved rapid development in both theory and technology, which is able to achieve calculations difficult to complete with classical information, thus demonstrating its superiority. The core resource in quantum information processing is quantum entanglement. The prototype of entanglement was proposed by Einstein [1] et al. and then Schrodinger formally named entanglement[2]. First gave mathematically expression of entanglement by Werner[3] until 1989.It was then widely used by researchers in the fields of quantum dense coding, quantum entangle and quantum teleportation,so it is seen that quantum entanglement is very significant[4] in the kingdom of quantum information science. Quantum dense encoding[5] is a quantum communication scheme pioneered by Bennett and Wiesner[6] in 1992, with a two-level quantum system as the research object and entangled states as the channel.The main purpose of the scheme is to act as a quantum channel with two entangled particles (EPR pairs) in which each transmitted particle with two states can transmit more than one bit of classical information. Dense coding has attracted wide attention from countless physics and information scientists since its introduction and has inspired their research[7,8] in this field. Ekert,[9] et al. theoretically proposes a standard dense coding scheme for partially entangled states and demonstrates maximizing sending and receiving information using Bell measurements. Hao,[10] et al.

studied controllable dense coding schemes in three-particle entangled states and generalize to controllable dense coding of continuous variable. Schaetz,[11] et al. studied quantum dense coding using an atom, and found that two trapped ions achieve quantum dense coding beyond the capacity in one without entanglement. Nowadays, dense coding is widely used in solid-state quantum systems such as photon qubits [12], optical mode [13], MRI[14], and atomic system[15].

Considering that no physical system existing in real life can be completely closed, so that the quantum system is inevitably affected by the environment and lead to the quantum correlation properties of a quantum system are partially or completely destroyed. Therefore, it is very meaningful to study the evolution relationship between the system's channel capacity and the dynamics of system entanglement in open quantum systems. According to the extent of external environment on open quantum system, the environment of quantum system can be attributed to Markovian environment and Non-Markovian environment[16].In the Markovian environment, the environmental correlation time is very short, which is usually expressed by the delta function, so there is no environmental memory effect[17]. At the time, the energy and information in the system can only be flowing into the environment and soon disappear in the environment, so the environment can not in turn affect the quantum system; The Non-Markovian environment has an environmental memory effect and the information, energy exchange[18] between the system and the environment, so the current state of the quantum system is now affected by its historical state, so the feedback effect of the environment on the quantum systems must be taken into account in the Non-Markovian environment.

The Non-Markovian quantum state diffusion (NMQSD) approach proposed by Diosi and Gisin[19] and solved the quantum correlation properties in Non-Markovian environments in later, successful applications to the solid-state quantum system[20,21,22]. JingJun[23] studies many-body quantum trajectories of Non-Markovian open quantum systems,verifies the applicability of the method in multi-quantum systems, and finds that the Non-Markovian environment has a positive effect on the system in many-body quantum systems. Yu Ting[24],et al. studied the Fermi bath and successfully derive the NMQSD master equation, finding that environmental memory effects have very significant modulation effects on the generation of entanglement. Zhao Xinyu,[25] et al. used this method to demonstrate the applicability of system quantum correlation dynamics in Fermi environments, and to discuss the universality and applicability of Grassmann stochastic processes. Mu qingxia,[26] et al. used the method to study entanglement transfer in coupled superconducting resonators, demonstrating that the memory effects of the environment can lead to higher entanglement resurrection and make the entanglement last longer, where a Non-Markovian environment can effectively perform enhanced entanglement transfer.Nothing has been reported, discussing the dynamics evolution vs time of its quantum dense encoded channel capacity and its entanglement dynamics with the Heisenberg spin chain as an open quantum system.

2 Heisenberg spin chain model with accurate quantum state diffusion equation

The Heisenberg spin chain model in a solid-state quantum system is a relatively simple model[27,28], proving to be a very useful model[29] in quantum information processing due to its very rich entanglement properties.It is, therefore, widely adopted in theoretical and experimental studies with quantum associations. The Heisenberg model describes the nearest-neighbor spin-exchange interaction, and therefore it is a magnetic quantum system[30].The J_1-J_2 spin-chain model[31] is a non-trivial generalization of the in the Heisenberg model, which involves the spin interactions of the nearest and lower neighbors[32-33]. Although it is very difficult to accurately solve the J_1-J_2 spin chains, the model is solvable with some additional terms[34-38].For example: Popkov and Zvyagin propose double and multi-chain quantum spin models that can be computed[39-42]; Ikhlef, Jacobsen and Saleur construct the Z_2 staggered vertex model[43,44] using derivatives of the product of two transfer matrices with different spectral parameters;Tavares and Ribeiro discuss the thermodynamic properties of the system using the quantum transfer matrix method[45,46]. The Hamiltonian of the Heisenberg spin chain model discussed here comes from literature [31] and considers only 2 spins, coupling the Heisenberg spin chain to a Bose environment, the total Hamiltonian of quantum systems in interacting picture are (using natural units in ours context, where all physical quantities are dimensionless physics, η=1):

$$\boldsymbol{H}_{tot} = \boldsymbol{H}_{sys} + \boldsymbol{H}_{env} + \boldsymbol{H}_{int} \tag{1}$$

Among them,

$$\boldsymbol{H}_{sys} = 2J\cosh(2a)\left[\left(\boldsymbol{\sigma}_1^x\boldsymbol{\sigma}_2^x + \boldsymbol{\sigma}_1^y\boldsymbol{\sigma}_2^y\right)\right] + 2\cosh(\eta)\boldsymbol{\sigma}_1^Z\boldsymbol{\sigma}_2^Z - \frac{\sinh^2(2a)\cosh(\eta)}{2\sinh^2(\eta)} \tag{2}$$

$$\boldsymbol{H}_{env} = \sum_j \omega_j \boldsymbol{b}_j^{+}\boldsymbol{b}_j \tag{3}$$

$$\boldsymbol{H}_{int} = \sum\left(g_j \boldsymbol{b}_j^{+} L + h.c.\right) \tag{4}$$

In the formula $\boldsymbol{\sigma}_i^j(i=1,2; j=x,y,z)$ is the Pauli matrix and a, η is the coefficient used to describe the coupling strength of the systems. $\boldsymbol{H}_{sys}$ represents the Hamiltonian of systems in a Bose environment,$\boldsymbol{H}_{int}$ represents the Hamiltonian interacting the system is coupled to the Bose environment and $\boldsymbol{H}_{env}$ as the Hamiltonian of the Bose environment. L is Lindblad operator describing the system when coupled to the environment and defined as $L = \kappa^A\boldsymbol{\sigma}_1^z + \kappa^B\boldsymbol{\sigma}_2^z$. $\boldsymbol{\kappa}^A$、 $\boldsymbol{\kappa}^B$ is constant, take the $\boldsymbol{\kappa}^A = \boldsymbol{\kappa}^B = 1$ in this article.

The stochastic Schrodinger differential equation[24] of the Heisenberg spin chain under interaction picture can be represented as:

$$\frac{\partial}{\partial t}\boldsymbol{\psi}_t = -i\boldsymbol{H}_{sys}\psi_t + \boldsymbol{L}z_t^*\boldsymbol{\psi}_t - \boldsymbol{L}^{+}\int_0^t ds\,\alpha(t,s)\frac{\delta\boldsymbol{\psi}_t}{\delta z_s^*} \tag{5}$$

$\alpha(t,s)$ is environment association function in the formula,Determined by the noise operator $B(t) = \sum_k g_k a_k e^{-i\omega_k t}$ at zero temperature (T=0) and its mathematical expression is: $\alpha(t,s) = \langle 0|\left[B(t)+B^\dagger(t)\right]\left[B(s)+B^\dagger(s)\right]|0\rangle = \sum_j |g_j|^2 e^{-i\omega_j(t-s)}$. z_t^* is the noise function representing some complex Gaussian processes and its mathematical expression is $z_t^* = -i\sum_k g_k^* z_k^* e^{i\omega_k t}$. $M[\bullet]$ represent Set average amount on classical noise by z_t and satisfied $M[z_t] = M[z_t z_s] = 0$, $M[z_t^* z_s] = \alpha(t,s)$. Average density operator for quantum trajectories $|\psi_t(z^*)\rangle$ is $\boldsymbol{\rho}(t) = M\left[|\psi_t(z^*)\rangle\langle\psi_t(z^*)|\right] = \int\frac{dz^2}{\pi}|\psi_t(z^*)\rangle\langle\psi_t(z^*)|$.It can be seen in equation (5) that contains a time non-local term, and practically an approximation method has to be used for handling the procedure.The time-dependent part in equation (5) take the place of $\boldsymbol{O}(t,s,z^*)$ is:

$$\frac{\delta\boldsymbol{\psi}_t}{\delta z_s^*} = \boldsymbol{O}\left(t,s,z^*\right)\boldsymbol{\psi}_t\left(z^*\right) \tag{6}$$

It is a polynomial function acting on the system operators in the Hilbert space, using the consistency condition[25] in mathematics,

$$\frac{\delta}{\delta z_s^*}\frac{\partial \psi_t}{\partial t}=\frac{\partial}{\partial t}\frac{\delta \psi_t}{\delta z_s^*} \tag{7}$$

Get the time evolution equation for the operator $\boldsymbol{O(t,s,z^*)}$,

$$\frac{\partial}{\partial t}\boldsymbol{O}=\left[-i\boldsymbol{H}_{sys}+\boldsymbol{L}z_t^*-\boldsymbol{L}^{+}\bar{\boldsymbol{O}},\boldsymbol{O}\right]-\boldsymbol{L}\frac{\delta}{\delta z_s^*}\bar{\boldsymbol{O}} \tag{8}$$

The description in the formula $\bar{\boldsymbol{O}}$ equal to $\bar{\boldsymbol{O}}=\int_0^t ds\alpha(t-s)\boldsymbol{O}(t,s,z^*)$.Initial conditions for the equation of motion of the operator $\boldsymbol{O}(\boldsymbol{t,s,z^*})$ are given by $\boldsymbol{O}(t,t,z^*)=\boldsymbol{L}$.For any given model, the equation (8) enables an exact QSD equation for a time domain.

Since the exact solution of the $\boldsymbol{O}(\boldsymbol{t,s,z^*})$ operator is very difficult or even unachievable. In this paper, the $\boldsymbol{O}(\boldsymbol{t,s,z^*})$ operator is extended and ignores the higher order terms as follows:

$$\boldsymbol{O}(\boldsymbol{t,s,z^*})=f_1(t,s)\boldsymbol{O_1}+f_2(t,s)\boldsymbol{O_2}+f_3(t,s)\boldsymbol{O_3}+f_4(t,s)\boldsymbol{O_4} \tag{9}$$

Above equation of $\boldsymbol{O_1}=\boldsymbol{\sigma}_1^z$, $\boldsymbol{O_2}=\boldsymbol{\sigma}_2^z$, $\boldsymbol{O_3}=\boldsymbol{\sigma}_1^y\boldsymbol{\sigma}_2^x$, $\boldsymbol{O_4}=\boldsymbol{\sigma}_1^x\boldsymbol{\sigma}_2^y$,and f_i (i=1,2,3,4) is some coefficients that change vs time.Puts expansion of operator $\boldsymbol{O}(\boldsymbol{t,s,z^*})$,equation (9), into the partial differential equation (8), as follows:

$$\frac{\partial}{\partial t}f_1=4J\cosh(2a)f_7-4J\cosh(2a)f_4-2F_3f_3-2F_4f_4-2F_3f_4-2F_4f_4;$$

$$\frac{\partial}{\partial t}f_2=4J\cosh(2a)f_4-4J\cosh(2a)f_3-2F_3f_4-2F_4f_3-2F_3f_3-2F_4f_4;$$

$$\frac{\partial}{\partial t}f_3=-4J\cosh(2a)f_1+4J\cosh(2a)f_2+2F_3f_1+2F_4f_2+2F_3f_2+2F_4f_1;$$

$$\frac{\partial}{\partial t}f_4=-4J\cosh(2a)f_2+4J\cosh(2a)f_1+2F_3f_2+2F_4f_1+2F_3f_1+2F_4f_2。$$

In the above formula: $F_i(t)=\int_0^t\alpha(t,s)f_i(t,s)$ (i=1,2,3,4).Entry initial conditions,

$$f_1(t,t=s)=1,\qquad f_2(t,t=s)=1,$$
$$f_3(t,t=s)=0,\qquad f_4(t,t=s)=0.$$

The stochastic Schrodinger differential equation of the Heisenberg spin chain can be solved numerically, and the equation (5) can be reduced to the local equation of time, as follows:

$$\frac{\partial}{\partial t}\psi_t=-i\boldsymbol{H}_{sys}\psi_t+\boldsymbol{L}z_t^*\psi_t-\boldsymbol{L}^{+}\bar{\boldsymbol{O}}(\boldsymbol{t,z^*})\psi_t \tag{10}$$

Among them, $\bar{\boldsymbol{O}}(\boldsymbol{t,z^*})=\sum_{i=1}^{8}F_8(t)\boldsymbol{O}_i$ .Because the definition of $\bar{\boldsymbol{O}}(\boldsymbol{t,z^*})$ is related to the environmental correlation function $\alpha(t,s)$, it is necessary to select the type of the environmental correlation function.The noise mode selected is Ornstein-Uhlenbeck mode, when the environment correlation function can be written as $\alpha(t,s)=\frac{\gamma}{2}e^{-\gamma|t-s|}$. γ As the environmental noise correlation coefficient, γ value size can be used to distinguish the system from being in Markovian environment or Non-Markovian environment. In practice, the system can be implemented in a Non-Markovian environment or in a Markovian environment by changing the parameter fetch size by controlling the memory effect of the reservoir.When $\gamma>2$,system is very close to the Markovian environment.Otherwise, if <2 can usually be considered as the system is close to a Non-Markovian environment. Note that at this time, the evolution of the system reduction density matrix vs time is obtained by taking the statistical mean matrix of the noise $\boldsymbol{\rho}_t = \boldsymbol{M}\left[\left|\boldsymbol{\psi}_t\left(z^*\right)\right\rangle\left\langle\boldsymbol{\psi}_t\left(z^*\right)\right|\right]$,

$$\frac{\partial}{\partial t}\rho_s(t) = -i\left[\boldsymbol{H}_{sys},\rho_s\right] + \left[\boldsymbol{L},\rho_t\bar{O}^{(0)}(t)\right] - \left[\boldsymbol{L}^+{}_{,t}\bar{O}^{(0)}(t)\rho\right] \quad (11).$$

3. Quantum entanglement

3.1 Quantum entanglement theory

The simpler and very valuable associations in quantum correlations are quantum entanglement. To quantify the degree of entanglement in the system, one names the amount of entanglement that the entanglement state carries information. In 988, Wootters[47] et al. studied two-level atoms proposed Symsymbiotic entanglement , whose mathematical expression is:

$$C(\boldsymbol{\rho}_t) = \max\left\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\right\} \quad (12).$$

$\boldsymbol{\rho}_t$ is density matrix of the two-mass subsystems in the formula and reduced density matrix equation for the system calculated above (11). λ_1, λ_2, λ_3, λ_4 is the eigenvalue of the matrix $\boldsymbol{R}_{AB} = \boldsymbol{\rho}_{AB}\left(\boldsymbol{\sigma}_1^y \otimes \boldsymbol{\sigma}_2^y\right)\boldsymbol{\rho}^*{}_{AB}\left(\boldsymbol{\sigma}_1^y \otimes \boldsymbol{\sigma}_2^y\right)$ and has $\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4$. asterisk means common together.The main motivation behind the introduction of this entanglement measure is based on its faithful quantization of quantum entanglement.If C=0, then the quantum state corresponds to the separable state, a Non-Entangled state. If not equal to zero, it will quantify how many states are separable.

3.2 Results and analysis of the entanglement dynamics

The following analysis of the quantum entanglement dynamics of the system when changing various parameters.First, in order to investigate the entanglement dynamics of how the environmental memory time can shadow the systems. in Figure (1), it depicts the influence of the environmental correlation coefficient on the entanglement decay rate of the Heisenberg spin chain system, and the entanglement evolution diagram of different parameters is drawn. It can be found from the graph that the entanglement dynamics has certain fluctuations and obvious decay characteristics in the process of time evolution, and they all tend to a certain stable value. Maximum entangled state $|\boldsymbol{\varphi}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$ when the system starts and Memory time of the environment is $\frac{1}{\gamma}$ .When γ =0.1, memory effect of the environment is very obvious, namely that the correlation time is relatively long.At this time, the Non-Markovianian property is very strong, the system entanglement in the Non-Markovianian regime exhibits a strong fluctuating regime and remains highly entangled for long periods of the state[48].This is because the information and energy flowing into the environment return to the system to the original state because the environment has a memory effect, so it very significantly increases the entanglement enabling the system to obtain more robustness. When γ =1,the system is in a relatively mild Non-Markovian environment. As the size of entanglement becomes smaller over time, the decay of entanglement is very pronounced.When γ =2, The entanglement dynamics of the system at this time is thought to be very close to the Markovian limit.Thus, the reservoir is able to destroy the coherence of the initial state very quickly and rapidly evolve into a separable state. It is shown that in strong Non-Markovian properties, the local maximima and expectation can be increased, the computational efficiency[49,50] can be effectively improved in quantum computation, and also shows that the system carries more information under Non-Markovian conditions.

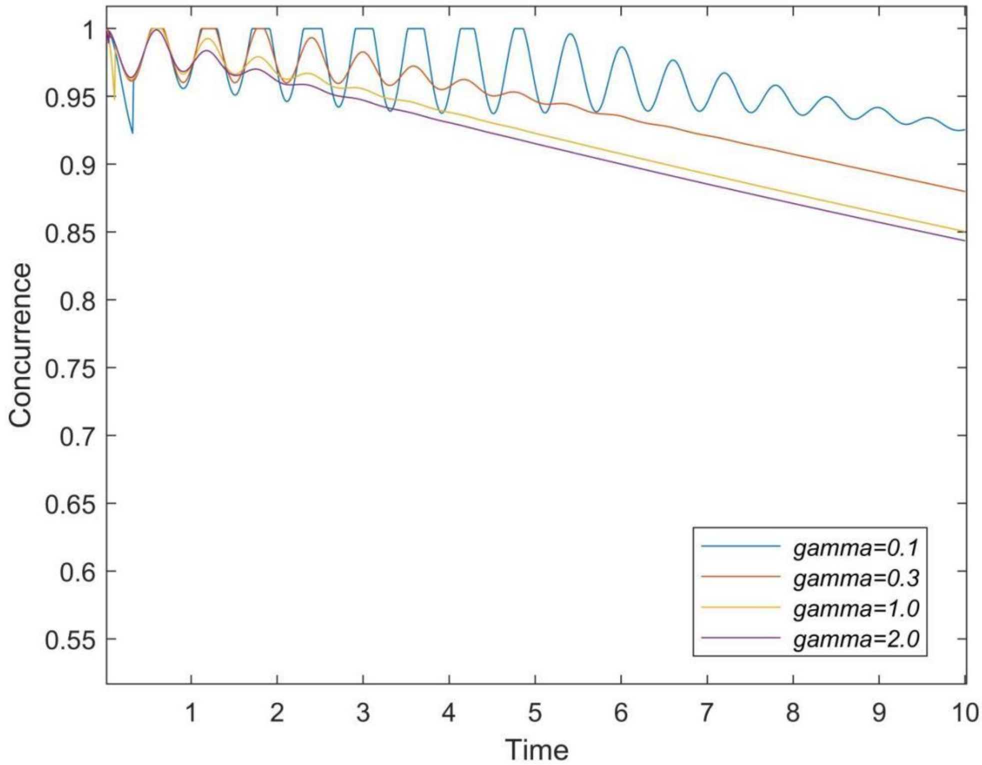

Figure 1. Time evolution of quantum entanglement dynamics vs environmental correlation coefficient γ .The parameters are set as: γ takes different values, J =0.8， a =0.6, η =0.5.

In Figure (2), it depicts changed parameters vs the quantum entanglement dynamics of the system. It is very clear from the graph that the entanglement dynamics exhibits the onset of periodic oscillations over time.However, when the parameters J increase, the entanglement characteristic of the system is significantly enhanced, and the entanglement duration increases significantly. When J =0.1, The system soon loses partial entanglement, the effective information carried significantly reduced and therefore cannot be applied to the transmission of quantum information.When J =0.8, The entanglement of the system is very high and the duration is very long and the fluctuation amplitude is relatively small, so the probability of distortion in the information transmission is relatively small, which effectively can be applied to[51] quantum information science.We show that increasing the parameters J can improve the system entanglement under the maximum entangled state $|\varphi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$ as initial state.

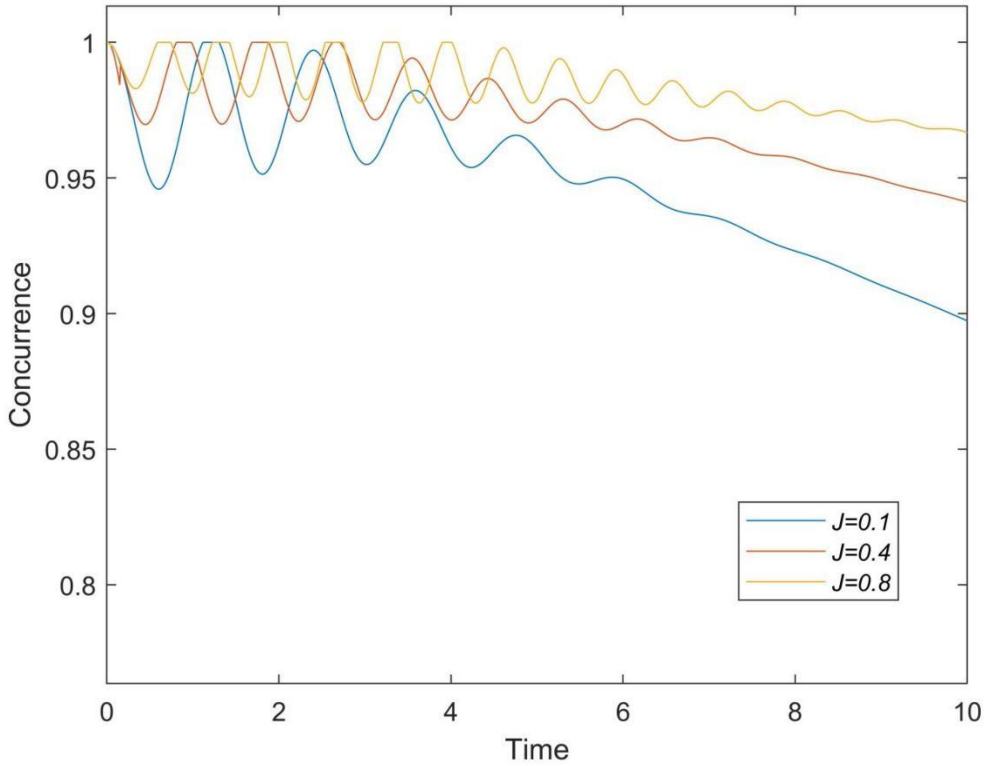

Figure 2. Time evolution of quantum entanglement dynamics vs different coefficients J .The parameters are set as: J takes different values, γ =0.1， a =0.5, η =0.5.

In Figure (3), the relation of the changed parameters a and the system quantum entanglement dynamics evolution vs time is clearly presented. It is very clear to find that the quantum entanglement of the system is characterized by oscillatory decay over time, and eventually they all tend to a certain stable value. When a =0.3,the entanglement is very good during time evolution, and the system maintains a high entanglement of almost 1 and the fluctuation range is very small during the evolution, so a straight line is presented in the graph. When a =0.5, from the graph, a relatively slow decay occurs after a period of periodic fluctuations during the evolution. After a period of time, although the system entanglement has decreased, but also maintains a high degree of entanglement. When a =0.7, from the graph, intense fluctuations occur during the evolution and a relatively rapid decay occurs. It is shown that when the maximum entanglement state $|\varphi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$ as the initial state, the parameters a can affect the entanglement properties of the system, but the influence is not too large.

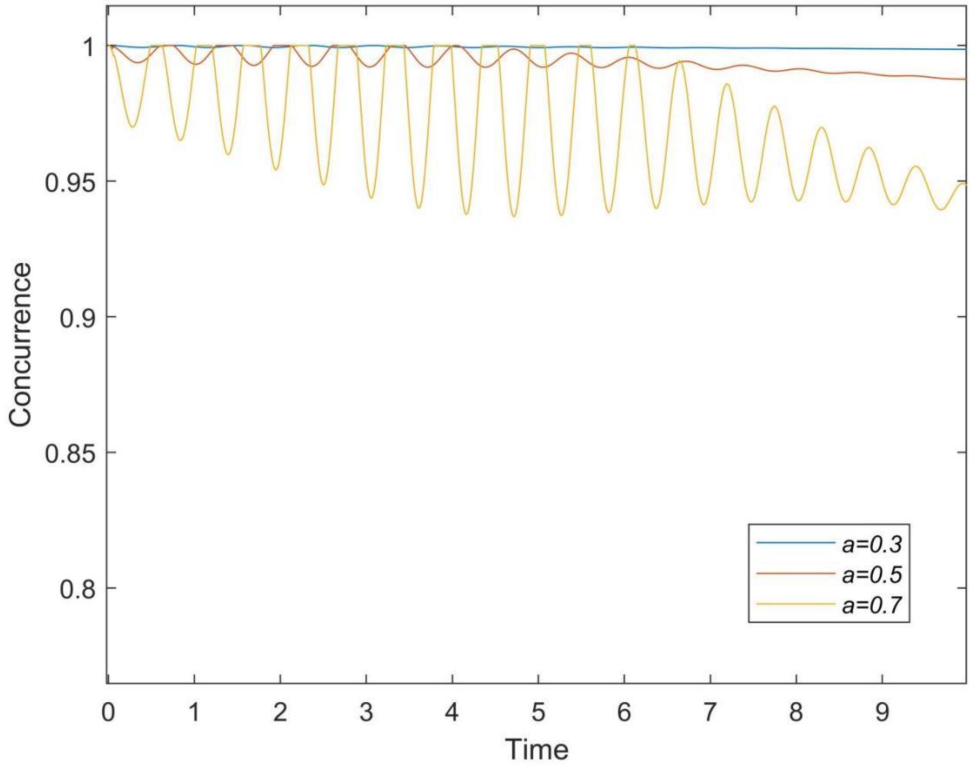

Figure 3. Time evolution of quantum entanglement dynamics vs different coefficients a .The parameters are set as: a takes different values, γ =0.1， J =0.8, η =0.4.

In Figure (4), the relationship between the parameters η and the system entanglement dynamics evolution vs time is clearly presented. It is very clear to find that the quantum entanglement of the system is characterized by oscillatory decay over time.When η =0.3,It is seen from the graph that the very strong entanglement characteristics of the system remain very high after a long evolution. However, when the entanglement fluctuates strongly, the fluctuation amplitude increases significantly, and the entanglement obviously shows a trend of decay, especially at η =0.5. It is shown that under the maximum entangled state $|\varphi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ as the initial state, the smaller parameters η can increase the entanglement of the system, and the entangled state generated by the Heisenberg spin chain can act as a good quantum channel to effectively conduct the quantum information transmission [52].

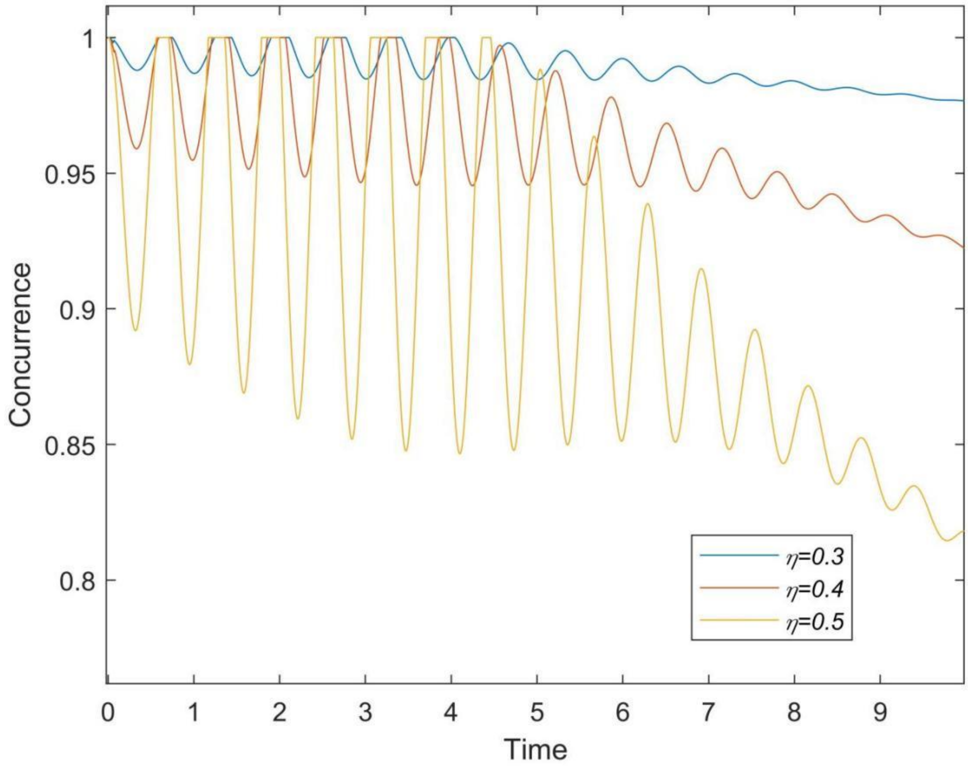

Figure 4. Time evolution of quantum entanglement dynamics vs different coefficients η .The parameters are set as: η takes different values, γ =0.1， J =0.6, a =0.7.

4. Quantum dense encoding

4.1 Quantum dense encoding scheme

In the standard quantum dense encoded protocol, the sender (Alice) receives an encoded qubit with the maximum entangled state[53] $|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}(|11\rangle + |00\rangle)$ as the channel send to Bob.Bob is able to get more than one classical information[54]. Alice and Bob prepare maximum entanglement and mark the corresponding particles as AB, in the A particles in the Alice side and B in the Bob side. The specific requirements of the quantum dense encoding scheme are as follows:First, the Alice encodes the classical information and marks it with 0,1,2,3. Alice then takes the A particles he owns as four local unitary transformations, namely:unit operation using matrix I for number 0, Bit reversal operation using matrix σ_x for number 1, Phase-reversal operation using matrix σ_z for number 2 and Bit-phase-reversal manipulation using matrix $-i\sigma_y$ for number 3. Second, the Alice sends the already prepared A particles to the Bob,Bob to perform controlled Non-transform on the two particles. Again, the Bob is able to perform measurement operations on the B particles without affecting the A particle state. If the result of the Bob measurement is 0, the coded number is 0 or 3; if the measurement result is 1, the coded number is 1 or 2. Next, the Bob performs the Hadamard transform of the A particles, thus fully affirming the encoding serial number that the Alice wants to transmit, so that the correct decoding result can be obtained according to the above operation. When the quantum system codes the maximum entangled state as the channel, the unitary transform to:

$$\begin{aligned} U_{00}|j\rangle &= |j\rangle,\ U_{01}|j\rangle = |j+1(mod2)\rangle \\ U_{10}|j\rangle &= e^{\sqrt{-1}\pi j}|j\rangle,\ U_{11}|j\rangle = e^{\sqrt{-1}\pi j}|j+1(mod2)\rangle \end{aligned} \quad (13).$$

In here, $|j\rangle$ is the orthogonal base vector of the ($|j\rangle=|0\rangle, |1\rangle$). After the above operation, the signal ensemble starts the input information into:

$$\bar{\rho}=\frac{1}{4}\sum_{i=0}^{3}(U_i\otimes I_2)\rho(t)\left(U_i^{+}\otimes I_2\right) \quad (14).$$

$\rho(t)$ is a density matrix corresponding to the equation (12) in above calculation.In a real-world environment, the maximum amount of information passed in the channel capable is described as dense coded channel capacity, often defined by the Holevo quantity[55].Depending on the Holevo capacity, the densely encoded channel capacity is defined as:

$$\chi=S(\bar{\rho})-S(\rho) \quad (15).$$

$S(\rho)$ in formula (15) is the Von-Neumann entropy at the initial moment , it represents the mean density matrix of the signal ensemble and defined as: $S(\rho)=-Tr(\rho\log_2\rho)$.Encoded Von-Neumann entropy is $S(\bar{\rho})=-Tr(\bar{\rho}\log_2\bar{\rho})$.

4.2 Dense coding results and analysis

The variation of the quantum dense coded channel capacity χ is discussed below. First, in Figure (5), the relationship between the different environmental correlation coefficients γ and the channel capacity χ is clearly presented.From the graph, the channel capacity χ appears periodic oscillations vs time and shows a trend of obvious decay, and they all tend to some stable value after a while.When γ =0.1,The system is in a Non-Markovian environment, there are χ always $\chi>1$ knowing from Figure (5), indicating that the dense coded transmission capacity of the system in the entangled state is always superior to the limit transmission capacity[56] of classical communication.When γ =0.3,the system is in a relatively mild Non-Markovian environment, which can be learned from Figure (5) that the capacity of the system attenuated rapidly in the time evolution, soon below the limit transmission capacity[57] of classical communication, the channel capacity of the system attenuated faster in the Markovian environment, and below the limit value of the transmission capacity in a very short time of classical communication.It is shown that with the maximum entangled state $|\varphi\rangle=\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$ as the channel, the system has the largest channel capacity χ in the Non-Markovian environment, so when the environmental correlation coefficient is relatively small, it is beneficial to transmit more information and obtain efficient quantum dense coding.The γ smaller the more oscillations, the longer the optimal coding capacity takes to decay to the stable value[58,59].The positive role of Non-Markovian properties in quantum dense coding is important for high-quality quantum communication[60].

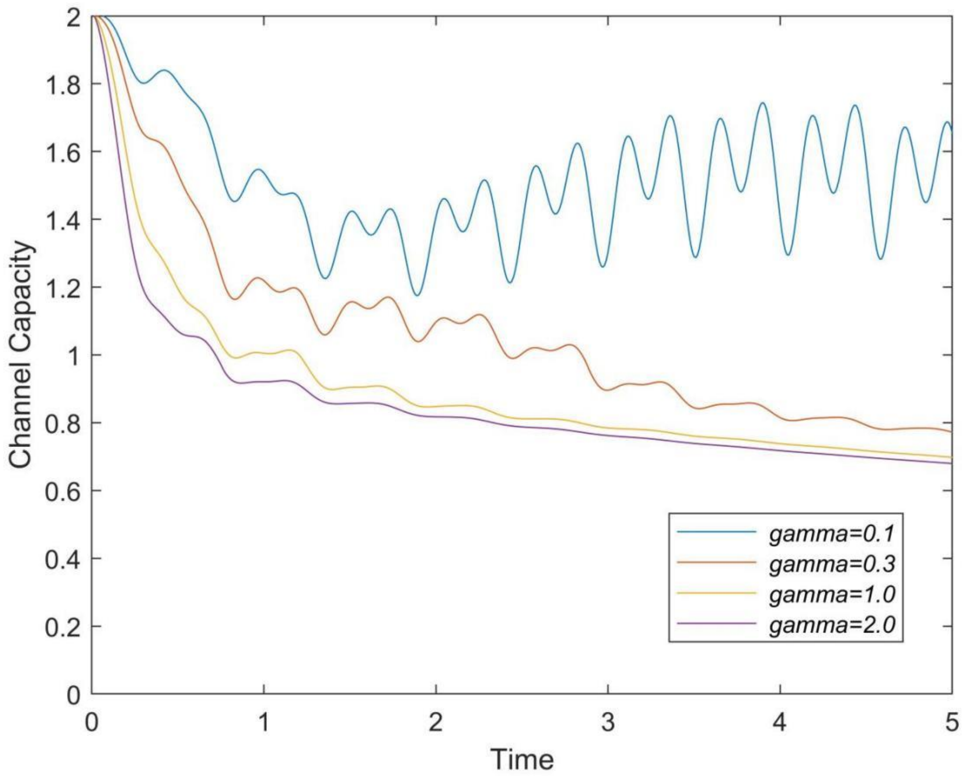

Figure 5. Time evolution of channel capacity vs environmental correlation coefficient γ .The parameters are set as: γ takes different values, J =0.8, a =0.7, η =0.5.

To discuss the influence of the parameters J on the system quantum dense coding, the evolution relation of the different parameters J and the quantum dense coding channel capacity χ is depicted in Figure (6).When parameter J =1.1,quantum dense encoding has the largest channel capacity and consistently exceeds the limit of the classical channel during evolution. Moreover, it is known from the figure that the fluctuation amplitude of the channel capacity is significantly larger, but the oscillation period is minimal and the largest number of shocks occurs, which represents that the optimal quantum dense encoding capacity takes a long time to decay to the stable value, which is conducive to improving the efficiency of information transmission.When the parameter J taking value decreases, channel capacity of the system is soon below the limit of the classical channel and thus does not guarantee the efficient transmission of quantum information.It is shown that when using the maximum entangled state $|\varphi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$ as a quantum channel, choosing large parameters J to obtain superior channel capacity χ over classical transmission so the parameters play a positive role in quantum dense coding.

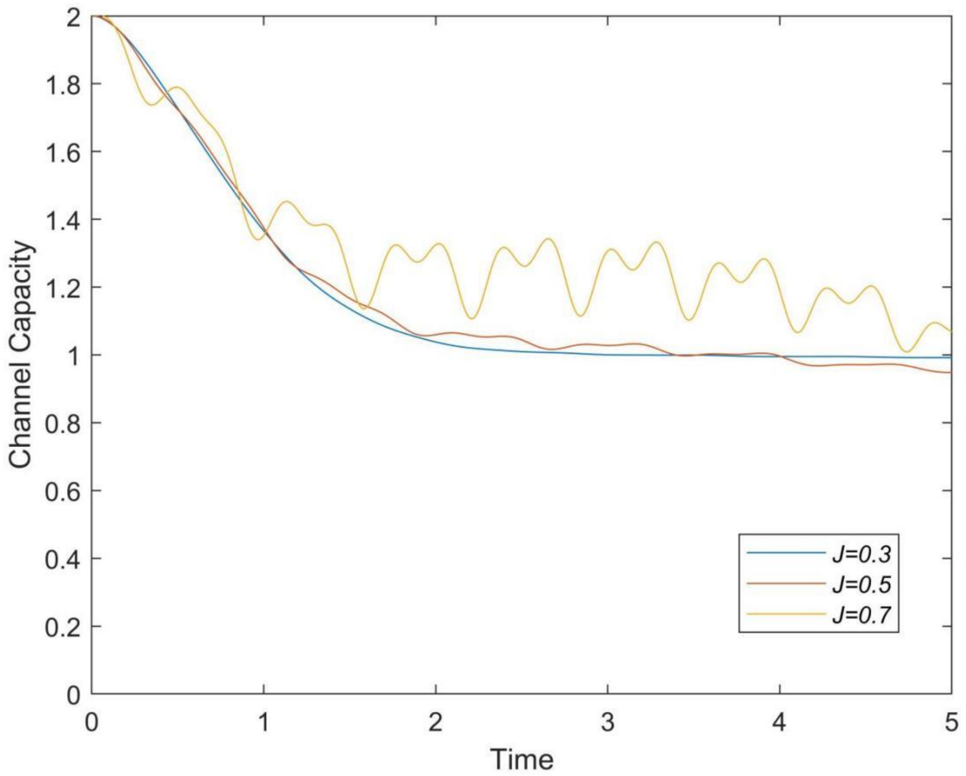

Figure 6. Time evolution of channel capacity vs different coefficients J .The parameters are set as: J takes different values, γ =0.1， a =0.6, η =0.5.

To discuss the role of parameters a in quantum dense coding, the evolution relation of quantum dense coding of the Heisenberg spin chain under different parameters a is depicted in Figure (7).When a =0.7,capacity of the system is always for 1, so the use of quantum dense coding does improve the channel capacity compared to classical communication.When a <0.5, as time the evolving channel capacity is quickly below the classical limit and cannot achieve the transmission of more information. Thus in the case of a <0.5, the study of dense coding is meaningless.It is shown that when using the maximum entangled state $|\varphi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$ as a quantum channel, the value size of the parameter a has an important influence on the quantum dense coding of the system. Improved value of a is able to obtain the optimal channel capacity.

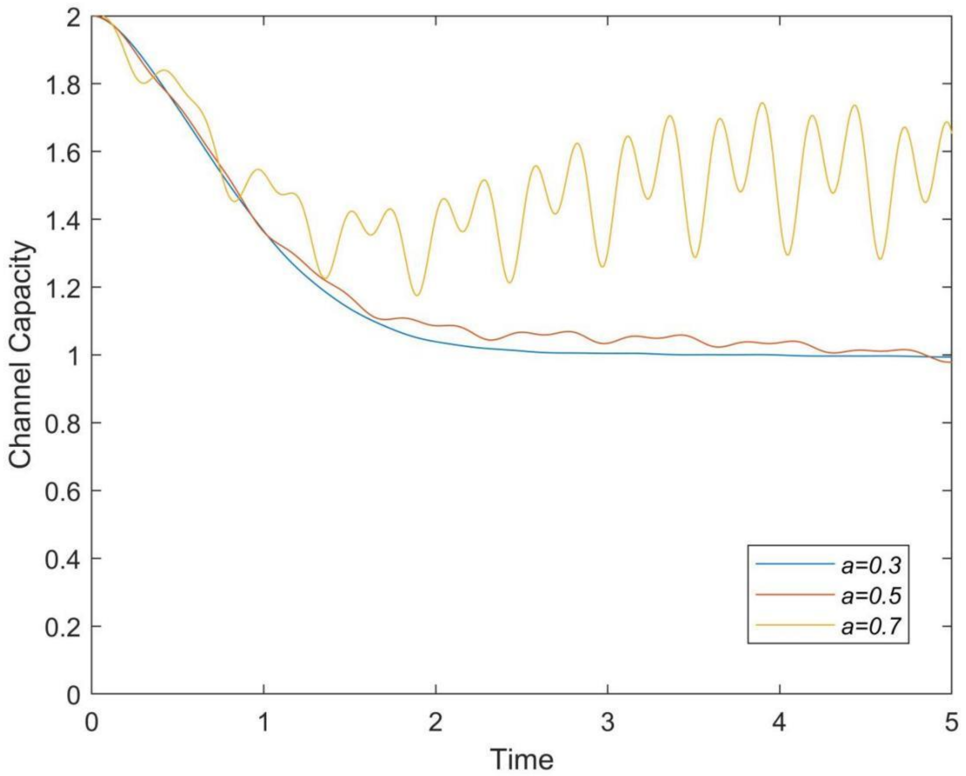

Figure 7. Time evolution of channel capacity vs different coefficients a .The parameters are set as: a takes different values, γ =0.1， J =0.8, η =0.5.

To discuss the role of parameters η in quantum dense encoding, Figure (8) depicts the evolution diagram of the quantum dense encoded channel capacity χ under the action of different parameters.When η =0.1,channel capacity quickly fades to the limit value of the classical information and remains at that value.Therefore, channel capacity cannot be increased in smaller value of η, so more information cannot be transmitted.When η =0.3, channel capacity is always greater than the capacity value of classical information, enabling information transmission that classical information cannot be completed.When η =0.5, during the evolution, the channel capacity is significantly improved and the oscillation period increases, indicating the longer time for the coding capacity to decay to the stable value. This shows that when using the maximum entangled state $|\varphi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$ as a quantum channel, the larger parameter η yields a better than the classical transmitted channel capacity χ .

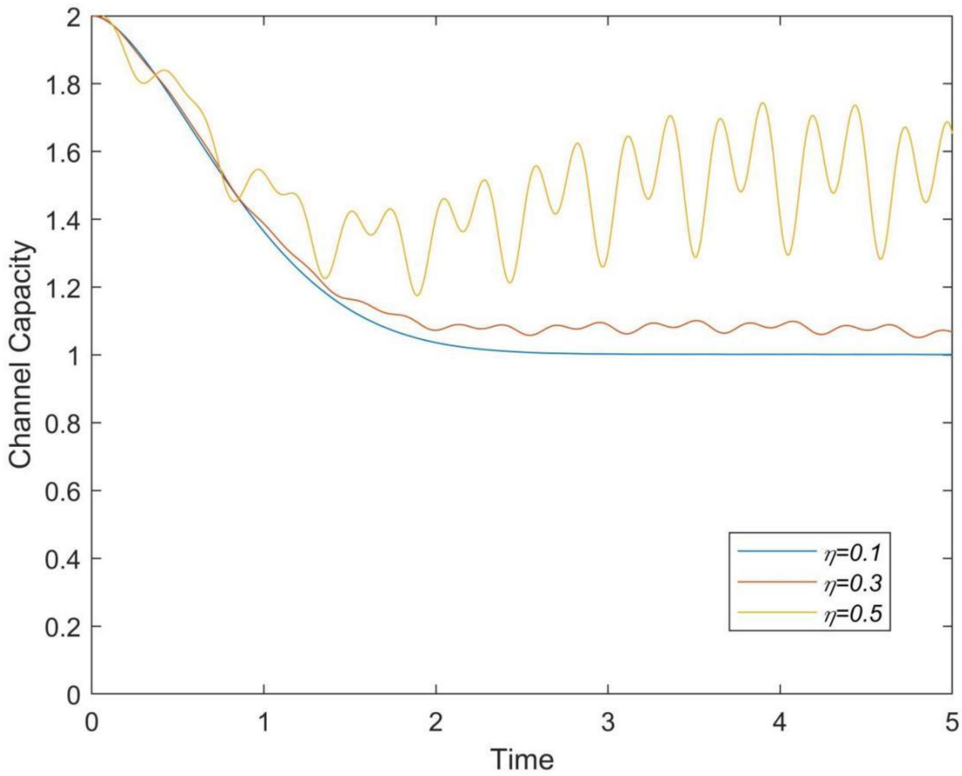

Figure 8. Time evolution of channel capacity vs different coefficients η .The parameters are set as: η takes different values, γ =0.1， J =0.8, a =0.7.

5. Conclusion

In this paper, we study the entanglement dynamics of the Heisenberg spin chain in a Non-Markovian environment with quantum dense coded channel capacity using quantum state diffusion methods. Since dissipation is inevitable,since we using the Ornstein-Uhlenbek type of noise in this paper. evolution of the channel capacity and entanglement of the system vs time is simulated by numerical calculations with the maximum entangled state $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ as the initial state Discuss the noise correlation coefficients in different environments γ , parameters J , a and η effect on the capacity of a quantum dense encoding channel and the entanglement dynamics. The results show that different types of parameters can not affect the dense channel capacity and entanglement size in this quantum system. In quantum entanglement dynamics of the system, smaller environmental correlation coefficients are capable of achieving the maximum entanglement and longest duration of entanglement, where quantum systems carry more information in a Non-Markovian environment.The parameters J show a positive role on the quantum entanglement dynamics of the system and have great significance for improving the entanglement of the system.Although the entanglement dynamics of the system all play a negative role for larger parameters η and a , during the increase η and a , the entanglement dynamics of the system is not too large. We expect that these results will help researchers to use entanglement indicators as an important resource for quantum information transmission with quantum computation.

In terms of dense coding, the coding capability of dense coding is modeled through numerical calculations.Stable and effective dense coding capability controlled by the system parameters can always be realized under this model.Small environmental correlation coefficients can increase the size of the channel capacity, highlighting the superiority of the Non-Markovian. Larger parameters J , a and η ,that is important for improving the channel capacity.Therefore, in the Heisenberg spin chain system,improving the selection of appropriate parameters can improve the entanglement dynamics and the channel capacity of the system, which has a certain reference significance and guiding role in improving the quantum correlation.

Acknowledgment

The scientific contributions from Abuniz Ahmati and our's group are acknowledged here.

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