Electromagnetic Wave Focusing with 2-D Gridded Parabolic Reflector Made of Parallel Circular PEC Wires at Oblique Incidence

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Research Article

Keywords: circular wire, gridded parabolic reflector, addition theorem for cylindrical functions, focusing

Posted Date: March 7th, 2022

DOI: <https://doi.org/10.21203/rs.3.rs-1399185/v1>

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Abstract

We consider the two-dimensional (2-D) scattering of the *TE*- and *TM*-polarized plane electromagnetic waves from a discrete grid-like parabolic reflector consisting of parallel circular perfectly electrically conducting (PEC) wires at oblique incidence. Our analysis is performed in the exact formulation, fully taking into account the wave interactions between the wires. We use the method of separation of variables in the local cylindrical coordinate systems in combination with Graf's addition formulas for cylindrical functions. The near field in the vicinity of the reflector is calculated at different angles of incidence. The dependences of the field amplitude at the maximum in the focal area and the position of this maximum on the angle of incidence are plotted. The far field scattered patterns, total scattering cross-sections and backward scattering cross-sections are calculated and discussed.

Introduction

This work is a continuation of our previous studies [1, 2] on the scattering of a plane electromagnetic wave by a discrete parabolic reflector from circular PEC wires for *TE*- and *TM*-polarizations. In these works, scattering problem was solved in the full-wave formulation taking into account the electromagnetic interactions between the wires. In [1] the accent was made on the near-field characteristics of the discrete reflector, namely, the focusing ability, in [2] the far-field characteristics, such as the angular scattering pattern and the total scattering cross-section were calculated and discussed. Continuing this study, here we consider the oblique incidence of a plane electromagnetic wave on a parabolic reflector made of PEC wires. Following [1, 2], full-wave self-consistent calculation taking into account all interwire interactions is performed by the method of separation of variables in the local polar coordinate systems in combination the Graf's addition theorem for the cylindrical functions.

It should be reminded that grid-like reflectors, which have reduced weight and windload, serve as reasonable alternative to solid metal reflectors in harsh atmospheric and precipitation conditions [3]. Their design needs sub-wavelength spacing of wires, otherwise the reflector performance is lost. Crossed-wire grid reflectors are more frequent because of the obvious polarization insensitivity, however, reflectors made of parallel metal wires are also used. They provide a polarization selectivity because, due to the "Hertz effect," only the E- polarization (i.e. the parallel or *TM*-polarization) regime provides sufficiently high reflectivity.

Scattering Problem Formulation

Consider a set of N_c identical infinite parallel circular PEC wires placed in vacuum, the axes of which are located on the parabola, $x = F - (0.25 / F) \cdot y^2$, where F is the focal distance (Fig. 1).). Such a parabola is optimal in the sense that its segment with endpoints at $(0, \pm D/2)$ corresponds to the value of $F/D = 0.25$, which provides the best focusing in the geometric-optics approximation.

The direction of the X axis is chosen to coincide with the symmetry line of such a discrete gridded parabolic reflector cross-section. Suppose that a plane time-harmonic electromagnetic wave of the unit amplitude is incident at this reflector at the angle α to the X axis, so that its wave vector is $\vec{k} = (k_x, k_y, 0)$, where $k_x = k \cos \alpha$, $k_y = k \sin \alpha$.

We denote the radius of the wires as a and introduce a global Cartesian coordinate system (x, y, z) with the z -axis collinear to the wires and N_c local Cartesian (x_q, y_q) and polar (r_q, φ_q) coordinate systems, $q = 1, \dots, N_c$, with the origins in the centers of all wire cross-sections. The time dependence $\exp(-i\omega t)$ is used and will be omitted.

As usual for 2-D problems, we consider two alternative field polarizations, TE and TM . Then, in the global Cartesian coordinate system, the equation of the incident plane is $H_z^i(x, y) = H_a \exp \left[i \left(k_x x + k_y y \right) \right]$ for TE -polarization and $E_z^i(x, y) = E_a \exp \left[i \left(k_x x + k_y y \right) \right]$ for TM -polarization. In the polar coordinate system, these equations are of the form $H_z^i(r, \varphi) = H_a \exp[ikr \cos(\varphi - \alpha)]$ and $E_z^i(r, \varphi) = E_a \exp[ikr \cos(\varphi - \alpha)]$, respectively.

The full-wave numerical solution can be obtained by expanding the field function in the terms of the Fourier series in azimuth exponents in the local polar coordinates associated with each cylinder. Subsequent transformations are performed similarly to those given in [1,2]. Our self-consistent treatment takes into account the wave interactions by using the method of separation of variables in the local polar coordinate systems in combination with Graf's addition theorem for the cylindrical functions. This approach is widely used in electromagnetics since the 1900s [5–14]. However, as stressed in [1,2], to provide the convergence of the numerical solution, the resulting infinite-matrix equation has to be regularized, i.e. cast to the Fredholm second-kind type. This enables reducing the error of computations to the machine precision, if desired, by increasing the matrix truncation order. Note that similar regularized matrix equations were also used in [15–20].

Basic Equations

All the details of mathematical calculations are presented in detail in [1]. Therefore, in this work, we present only the final equations. For the TM -polarization of the incident wave we obtain the following set of linear equations for the field Fourier coefficients, $a_n^{(q)}$, where $n = 0, \pm 1, \pm 2, \dots$ and $q = 1, \dots, N_c$

$$a_n^{(q)} + \left[H_n^{(1)}(ka) \right]^{-1} \sum_{p=1, p \neq q}^{N_c} \sum_{m=-\infty}^{+\infty} A_m^{(p)} J_m(ka) H_{m-n}^{(1)}(kL_{pq}) e^{i(m-n)\varphi_{pq}} = -E_{cq} i^n \left[H_n^{(1)}(ka) \right]^{-1}$$

1

In the case of the TE polarization:

$$a_n^{(q)} + \left[H_n^{(1)'}(ka) \right]^{-1} \sum_{p=1, p \neq q}^{N_c} \sum_{m=-\infty}^{+\infty} a_m^{(p)} J_m'(ka) H_{m-n}^{(1)}(kL_{pq}) e^{i(m-n)\varphi_{pq}} = -H_{cq} i^n \left[H_n^{(1)'}(ka) \right]^{-1}$$

2

The quantities included in the matrix elements can be found in [1]. Note that these systems of equations are the Fredholm equations of the second kind. This guarantees the convergence of the solutions obtained with truncation number N to the exact solution if N is taken greater [1].

Results Of Numerical Modeling

In computations, we take the number of cylinders to be $N_c = 51$, the focal distance of the parabola is $F = 24$ mm, the radii of the wires are $a = 0.5$ mm, the projection of the distance between their centers to the y -axis is $\Delta y = 1.5$ mm, the aperture of the reflector, so that reflector's aperture, i.e. the distance between the most out-standing wire centers, is $D = 76$ mm.

In Figs. 2 and 3, we show the near electric and magnetic fields portraits around explained above 51-wire discrete parabolic reflector at different incident angles for TM and TE - polarizations, respectively. Calculations were carried out at frequency of 77.25 GHz for both polarizations.

First, from a comparison of Fig. 2a and Fig. 3a, it can be seen that the maximum of the absolute value of the total field near the geometrical focus of the parabola is higher for *TM*-polarized plane wave than the *TE*-polarized one and that a much deeper shadow is formed behind discrete reflector than in the case of the *TM*-polarization (see [2]). This is a manifestation of the Hertz effect. Secondly, as can be seen, with an increase in the angle of incidence, the focusing of the field deteriorates, the value of the field amplitude at the focus decreases, and the focus itself is blurred for both polarizations. A more detailed study of the focus position is presented in Fig. 4. Here, we show the true focus (field maximum) position and the value of field amplitude in the true focus versus the incidence angle, for *TM* and *TE* polarizations.

As one can see, with an increase in the angle of incidence, the position of the true focus along the *X* coordinate changes slightly (the focus remains near *X* = 0 mm), but its position along the *Y* coordinate changes larger and in almost linear manner. It is also seen that with an increase in the angle of incidence, the field amplitude at the maximum decreases. This is true for both polarizations.

The plots in Fig. 5 demonstrate the dependence of the focusing ability of the grid-like reflector (the field magnitude in the true focus) on the angle of plane-wave arrival. As visible, it deteriorates within 10% if the angle of incidence lies within 10°. In contrast, the field in the geometrical focus displays oscillations and can drop to some 3% of the maximum value. This comparison explains why a shifted receiver can provide the scanning of the targets in the receiving antenna.

In Fig. 6, we present the value of field amplitude in the geometrical focus versus the incidence angle in wide region, up to 180° for low (77.25 GHz) and high frequencies (500 GHz) for both polarizations. Besides, here we show some critical angle called the spillover angle, $\alpha_{sp} = 180^\circ - \alpha_1$ (see Fig. 1). Angle $\alpha_1 \approx 76^\circ$ can be found from Fig. 1, given the coordinates of the uppermost cylinder (9.35; 37.5) mm. Thus, for the chosen parameters of the parabolic reflector $\alpha_{sp} \approx 104^\circ$. These plots confirm earlier conclusions about that a discrete parabolic reflector focuses the *TM*-waves better than the *TE*-waves and that with an increase in the angle of incidence, the field amplitude at the geometric focus decreases and oscillate. The oscillations become enhanced with increasing the frequency of the incident wave.

In Fig. 7, we show the far-field scattering patterns for the explained above 51-wire discrete parabolic reflector in both polarizations for different incident angles at one low frequency 77.25 GHz (i.e. $\lambda = 3.88$ mm and $ka \approx 0.809$) This means that the electric size of reflector's aperture is $D = 19.33\lambda$.

Figure 8 shows the plots of the normalized total scattering cross-section (TSCS) (see Eq. 12 in [2]),

$$\frac{\sigma_s^{E,H}}{4a} = \frac{1}{2\pi ka} \int_0^{2\pi} |F_{E,H}(\varphi)|^2 d\varphi$$

3

as a function of parameter ka , for the gridded parabolic reflector for the *TM*- and *TE* polarizations, normalized by the high-frequency limit for single wire, $4a$. Here, φ is the polar angle in the global polar coordinates with the origin in the geometrical focus and $F_{E,H}(\varphi)$ is the far-field angular scattering pattern, defined in [1,2]. As one can see, the plots show some oscillations, however, follow the plot of TSCS of single wire times the number of wires, $N_c = 51$. The largest deviations appear at lower frequencies. This proves that at high frequencies the gridded reflector becomes largely transparent in either polarization regime. Here, the *TE* polarization case shows more ripples than the *TM* case. Note also that for the larger angles α the spectra of TSCS show lower values than for $\alpha = 0$ - this can be explained by the overshadowing of distant wires by the forward ones.

Finally, in Fig. 9 we present the plots of the backward scattering cross-section (BSCS) (for $\alpha = 0^\circ, 10^\circ$). Its definition is given in [9] as the ratio of the total power dissipated by a fictitious isotropic scatterer, creating a field in all directions equal to the field from true obstacle towards the source, to the absolute value of the Poynting vector of a plane wave incident on the scatterer. After some algebra and normalization by its high-frequency limit, πa , it reduces to

$$\frac{\sigma_b^{E,H}}{\pi a} = \frac{4}{\pi k a} |F_{E,H}(0)|^2$$

4

In these calculations, the step in parameter ka is taken as 0.1. For both polarizations, BSCS plots show numerous small and sharp oscillations, especially at higher frequencies, in the either polarization regime. This is caused by the wave interactions between the wires.

Conclusions

We have studied in full-wave manner the near- and far-field characteristics of the discrete parabolic reflector made of PEC circular wires at oblique incidence. This analysis has shown that with an increase in the angle of incidence, the focusing ability of the gridded parabolic reflector deteriorates in both polarizations. However, it is possible to minimize such a degradation by adapting the position of receiver to the true focus, which is shifted from the geometrical focus of parabola. This shift almost linearly depends on the incidence angle, counted from the reflector's line of symmetry.

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Figures

Figure 1

Cross-sectional geometry of finite number of circular wires placed on parabola.

Figure 2

E-polarization. Electric field pattern around PEC circular wires arranged in a parabola at a frequency of 77.25 GHz for the incident angles $\alpha = 0^\circ$ (a), $\alpha = 2^\circ$ (b), $\alpha = 10^\circ$ (c), $\alpha = 80^\circ$ (d).

Figure 3

The same as in Fig. 2, however, for the H-polarization.

(a)

(b)

Figure 4

The field maximum position in the focal area (X_{max} , Y_{max}) versus the incidence angle, in the TM (a) and the TE (b) polarization case at 77.25 GHz.

Figure 5

The focusing ability (field amplitude in the geometrical focus) and the field amplitude in the true focus at (X_{max} , Y_{max}) versus the incidence angle, in the TM (a) and the TE (b) polarization case at 77.25 GHz.

Figure 6

The value of field amplitude in the geometrical focus versus the incidence angle in the TM and TE polarization cases, for low (a) and high (b) frequencies. Note the effect of the spillover angle at 104° , better visible in the low-frequency E-polarization case.

Figure 7

Normalized angular far-field scattering patterns (in dB) for 51 cylinders arranged on parabola and illuminated by the normally incident plane wave for incident angles $\alpha=0^\circ$ (a), $\alpha=2^\circ$ (b), $\alpha=10^\circ$ (c), $\alpha=80^\circ$ (d).

Figure 8

Normalized TSCS as a function of parameter ka for different incident angles, for 51 PEC wires arranged on the parabola, for TM- (a) and TE- (b) polarizations.

Figure 9

Normalized BSCS as a function of parameter ka for 51 PEC wires arranged on the parabola, for TM- (a) and TE-(b) polarizations.