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BingLin Li (✉ libinglin@swpu.edu.cn)

Southwest Petroleum University

Rui Zhang

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Analytical prediction of cutting forces in cylindrical turning of 304 stainless steel using unequal division shear zone theory

Binglin Li^{a,b,*}, Rui Zhang^a

^aSchool of Mechatronic Engineering, Southwest Petroleum University, Chengdu, China

^bSchool of Mechanical Science and Engineering, Huazhong University of Science and Technology, Wuhan, China

Abstract

In order to establish a more accurate prediction model of turning forces, this paper proposed an analytical model for cylindrical turning with the consideration of the effect of the main cutting edge angle and the nose radius. Meanwhile, the unequal division shear zone theory in orthogonal free cutting is extended and applied to the oblique non-free cutting in the interaction between the chip units. To take into account the real tool nose geometry, the engaged part in the rounded nose is discretized into a set of cutting edge units. The geometrical parameters associated with the cutting edge units are analysed by using the coordinate transformation approach. Then, the improved oblique cutting model is developed for each cutting edge unit to acquire the component forces along the tool rake face. Finally, the resultant cutting forces in the turning process are calculated by the numerical integration method. To verify the effectiveness of the proposed model, the turning force experiment of 304 stainless steel was carried out by changing the cutting conditions, the main edge cutting angle and the nose radius. Through the comparative analysis between the measured results and the calculation values of the proposed model, it was found that the analytical prediction of cutting force is in good agreement with the experiment.

Keyword oblique cutting; non-free cutting; unequal division shear zone theory; cutting edge discretization; cutting force prediction

1. Introduction

Accurate predictions of cutting force are essential for the design and selection of machine tools and cutting tools as well as the effective use of machining operations. And large attention has been paid to oblique cutting by a few researchers all around the world, because the majority of metal machining is non-free oblique cutting. At present, although most of the research is focused on the cutting force prediction of orthogonal free cutting, in which the cutting direction is performed along one straight cutting edge, non-free cutting, in which both primary and minor cutting edges are involved, is most widely used such as cylindrical turning in practice. However, the cutting conditions, the nose radius and the main cutting edge angle also affect the cutting force in turning, therefore, comprehensive consideration of these factors is essential to accurately predicting cutting forces.

Cylindrical turning, one of the commonest processes in manufacturing components, is widely used in automotive, aerospace, and so on. It is a typical non-free cutting operation whose chip deformation process is very complex. The calculation of cutting forces in turning is roughly divided into three methods: empirical, analytical and numerical models. Parakkal et al [1]. developed a relatively simple mechanistic

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*E-mail addresses: libinglin@swpu.edu.cn

Tel numbers: 0086-28-13608183394

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model to predict cutting forces for grooved tool in oblique cutting, which considered the effect of the tool edge radius. This model was used to investigate the effects of cutting conditions and grooved parameters on the cutting forces. Kone et al [2]. identified empirical cutting force equations, from which deduced the chip flow angle when machining 304L austenitic steel with a groove coated tool under dry condition. Colwell [3] proposed the equivalent cutting edge that is used to replace the real cutting edge with an imaginary straight line in the tool rake face, and assumed that the chip flow direction was perpendicular to the equivalent cutting edge. However, the chip flow direction defined above is not suited for cylindrical turning due to the nose radius. Hu et al [4]. extended Colwell's concept of equivalent cutting edge to the case of non-zero inclination angle and normal rake angle. Kushima and Minato [5] divided the cutting edge into many small units and supposed that the component chip flow direction of each unit is perpendicular to the component cutting edge, and then the resultant chip flow direction is obtained by vectorial sum of the component chip flow directions. Young et al [6]. regarded the chip as many independent chip units and assumed that the thickness and direction of undeformed chip part of each unit are variable. The equivalent cutting edge which is a straight line along the perpendicular direction of friction force can be obtained by calculating the resultant force of unit friction. Similarly, Wang and Mathew [7] extended Young's method to the tool nose with non-zero rake and inclination angle, and evaluated cutting force through numerical integration. Endres and Waldorf [8] introduced a nonlinear relationship among the unit friction, cutting force and the chip thickness by considering the size effect of rounded edge on the tool nose. Khelifi et al [9]. applied an equivalent tool geometry method to induce the same cutting force components as the real tool while taking into account the radius of the tool nose and edge. In addition to estimating the tool flank and crater wear, this method can also predict cutting force components in turning.

In the primary shear zone, Merchant [10] proposed a single shear plane analytical model. The shear angle was obtained by the minimum energy theory, many analytical models were then proposed for the modeling of orthogonal cutting process, such as the classical parallel-sided shear zone theory proposed by Oxley [11], which supposed that the primary shear zone is equally divided by the primary shear plane. However, Li et al [12]. established the unequal division shear zone theory by observing the strain rate distribution in the primary shear zone is inconsistent, the analytical results showed that the unequal division shear zone theory has acceptable accuracy. In subsequent research, this model is widely used to predict the cutting force and chip flow direction, and to identify material constitutive equation etc. In addition, the existence of the rounded nose and the main cutting edge angle makes the cutting mechanism more complicated. Simultaneously, they have a significant influence on the geometry of the undeformed chip area, cutting forces, chip geometry, heat generation, tool wear during turning. Redetzky et al [13]. divided the cutting edge into many trapezoidal units that are perpendicular to the component edge and calculated the cutting force of each discrete unit through unit cutting force, unit edge force and unit lateral force. Hagiwara et al [14]. developed the hybrid model by Redetzky et al [13]. to predict the chip side-flow angle for complex grooved tool inserts in contour turning operations by applying the effective cutting conditions and tool geometry. Armarego and Samaranayake [15] put forward to the generalized cutting edge concept to replace the rounded edge of tool nose. Storch and Zawada-Tomkiewicz [16] investigated the unit force distribution on the rounded edge for free cutting, demonstrated that both the tool rounded nose and straight edge affect the chip formation process and the quality of the machined surface. Fang [17] developed an improved oblique cutting model different from traditional model by considering the two cases where the tool main cutting edge angle is not equal to 90° and the influence of the tool feed velocity on the resultant cutting velocity is not ignored. Molinari and Moufki [18] established a new thermo-mechanical model of oblique cutting to take into account the real cutting edge geometry, the rounded nose

is discretized into a set of cutting edge units to obtain the component cutting force and temperature distribution along the rake face, but their oblique model is based on the inertia item, mainly suitable for adiabatic shear process in high-speed machining. Budak and Ozlu [19] proposed a thermo-mechanical model that considered both the sticking and sliding contact on rake face, and applied this model to obtain the elements and the local forces acting on each element by dividing the nose of the turning into small elements. Abdellaoui et al [20]. also analysed the effects of tool nose radius and established a thermo-mechanical model, which allows predicting force cutting components and thermo-mechanical parameters.

In this present work, the oblique cutting theory based on the unequal division shear zone theory investigated by Li et al [11, 21]. and combined with the tool discretization approach of Molinari and Moufki [18], is developed to predict the cutting forces in turning of 304 stainless steel. This paper is organized as follows: Firstly, the analytical modeling of non-free oblique cutting with some geometrical characterization and governing equations is presented. Secondly, the discretization of the tool cutting edge and the component geometry of each cutting edge unit are described. Thirdly, the component and resultant cutting force calculation are detailed. Finally, the theoretically predicted values are found to agree reasonably well with the experimental results under the given experimental conditions. The presented approach can be applied to other cutting processes such as contour turning and face turning.

2. Characteristics of the improved oblique cutting.

To study the turning process, an analytical model of oblique cutting of the unequal division shear zone theory is developed for the case that the tool cutting edge angle relative to the main cutting edge is not equal to 90° , namely, the feed rate f_r is not equal to the undeformed chip thickness t . In view of this, Fang [16] also proposed an improved method for the oblique cutting model and applied it to chip control experiment. Fig. 1 illustrates the process of improved oblique cutting.

In this model, where t , a_w and φ_r are undeformed chip thickness, cutting width and lead angle (complementary angle of the main cutting edge angle κ_r) respectively. The cutting depth a_p , feed rate f_r , cutting velocity V , normal rake angle γ_n and inclination angle λ_s are known, and the geometric relations of these parameters are shown in Fig. 2. Some planes that can be used to determine the location of the cutting edges, rake face and flank face are defined.

- The reference plane P_r
- The working plane P_f
- The back plane P_p
- The cutting plane P_s
- The orthogonal plane P_o
- The normal plane P_n
- The shear plane P_{sh}
- The equivalent plane P_e

As shown in Figs. 1 and 2, to calculate the relationships of geometry, motion and force in the improved oblique cutting model, a series of coordinate systems are defined.

- Frame $(\mathbf{x}_a, \mathbf{y}_a, \mathbf{z}_a)$, $\mathbf{x}_a$ is parallel to the cutting velocity direction, $\mathbf{z}_a$ is parallel to the feed direction, $(\mathbf{x}_a, \mathbf{y}_a)$ is the back plane P_p , $(\mathbf{y}_a, \mathbf{z}_a)$ is the reference plane P_r , $(\mathbf{x}_a, \mathbf{z}_a)$ is the working plane P_f .
- Frame $(\mathbf{x}_o, \mathbf{y}_o, \mathbf{z}_o)$
- Frame $(\mathbf{x}_n, \mathbf{y}_n, \mathbf{z}_n)$
- Frame $(\mathbf{x}_r, \mathbf{y}_r, \mathbf{z}_r)$
- Frame $(\mathbf{x}, \mathbf{y}, \mathbf{z})$
- Frame $(\mathbf{x}_c, \mathbf{y}_c, \mathbf{z}_c)$
- Frame $(\mathbf{x}_s, \mathbf{y}_s, \mathbf{z}_s)$

- Frame ($\mathbf{x}_e, \mathbf{y}_e, \mathbf{z}_e$)

The eight frames can be related by rotation transformation, as shown in Fig. 3. For instance, ($\mathbf{x}_o, \mathbf{y}_o, \mathbf{z}_o$) is obtained by the rotation of angle $-\varphi_r$ of the frame ($\mathbf{x}_a, \mathbf{y}_a, \mathbf{z}_a$) around the $\mathbf{x}_a$ axis, which is expressed as the following form:

$$(\mathbf{x}_a, \mathbf{y}_a, \mathbf{z}_a) \xrightarrow{\mathbf{R}(\mathbf{x}_a, -\varphi_r)} (\mathbf{x}_o, \mathbf{y}_o, \mathbf{z}_o)$$

The rest of frames can also be given by the above method, φ_n is the normal shear angle, η_c is the chip flow angle, η_s is the shear flow angle, η_e is the equivalent plane angle. In order to locate the equivalent plane for discrete edge, of which k is used to represent the number, the equivalent plane angle and shear flow angle are defined by the following expression.

$$\tan \eta_e^k = \frac{\tan \eta_c^k \cos \eta_s^k + \sin(\varphi_n^k - \gamma_n^k) \sin \eta_s^k}{\cos(\varphi_n^k - \gamma_n^k)} \quad (1)$$

$$\tan \eta_s^k = \frac{\tan \lambda_s^k \cos(\varphi_n^k - \gamma_n^k) - \tan \eta_n^k \sin \varphi_n^k}{\cos \gamma_n^k} \quad (2)$$

It should be noted that the chip flow angle η_c formula of free oblique cutting which was proposed by Moufki and Molinari [22] or others no longer applicable in non-free oblique cutting because the chip flow angle is affected by non-free chip in turning. The chip velocity can be calculated as:

$$V_c^k = \frac{V \cos \lambda_s^k \sin \varphi_n^k}{\cos \eta_c^k \cos(\varphi_n^k - \gamma_n^k)} \quad (3)$$

The normal shear angle can be calculated with the Merchants formula.

$$\varphi_n^k = \frac{\pi}{4} - \frac{\beta_k}{2} + \frac{\gamma_n^k}{2} \quad (4)$$

The mean friction angle β_k can be defined by mean friction coefficient, and the mean friction coefficient may be a power function of the chip velocity, as shown in Fig. 4. The friction coefficient is given by:

$$f = f_0 V_c^p \quad (5)$$

Where f_0 is constant, f is mean friction coefficient, while $p < 0$. The coefficients f_0 and p can be identified by the experimental data in orthogonal cutting.

3. Governing equations of the primary shear zone

Johnson-Cook model is used as flow stress model of workpiece material.

$$\tau^k = \frac{1}{\sqrt{3}} [A + B(\frac{\gamma^k}{\sqrt{3}})^n] [1 + C \ln(\frac{\dot{\gamma}^k}{\dot{\gamma}_0})] [1 - (\frac{T^k - T_r}{T_m - T_r})^m] \quad (6)$$

Where τ^k , γ^k , $\dot{\gamma}^k$ and T^k are the shear stress, the shear strain, the shear strain rate and the instantaneous temperature of the workpiece, respectively. $\dot{\gamma}_0$ is reference plastic strain rate, T_m is melting temperature of the workpiece material and T_r is room temperature or initial workpiece temperature. A , B , C , m , n is the material constants which can be identified through split Hopkinson bar (SPHB), the material coefficients of Johnson–Cook model are listed in Table 1.

The unequal division shear zone theory is used to predict the turning forces, as shown in Fig. 5. In this model, the strain, strain rate the temperature of discrete chip are calculated as follows:

$$\dot{\gamma}^k = \begin{cases} \frac{\dot{\gamma}_m^k}{(k_u h)^q} (z_e^k)^q & z_e^k \in [0, k_u^k h] \\ \frac{\dot{\gamma}_m^k}{(1-k_u)^q h^q} (h-z_e^k)^q & z_e^k \in [k_u^k h, h] \end{cases} \quad (7)$$

$$\gamma^k = \begin{cases} \frac{\dot{\gamma}_m^k \cos \eta_e^k}{(q+1)V \cos \lambda_s^k \sin \phi_n^k (k_u^k h)^q} (z_e^k)^{q+1} & z_e^k \in [0, k_u^k h] \\ -\frac{\dot{\gamma}_m^k \cos \eta_e^k}{(q+1)V \cos \lambda_s^k \sin \phi_n^k (1-k_u^k)^q h^q} (h-z_e^k)^{q+1} + \frac{\cos \gamma_n^k \cos \eta_e^k}{\cos(\phi_n^k - \gamma_n^k) \cos \eta_s^k \sin \phi_n^k} z_e^k & z_e^k \in [k_u^k h, h] \end{cases} \quad (8)$$

Where $\dot{\gamma}_m^k$ is the maximum strain rate, k_u^k is unequal division ratio, q is a parameter characterizing the non-uniform distribution of the tangential velocity in the deformation zone, and h is the thickness of the primary shear zone, 0.025mm is an acceptable approximation of h in reference by observing Shaw's research [23]. Besides, the boundary of the primary shear zone can be considered adiabatic, the chip temperature at the entry boundary is equal to the initial workpiece temperature.

$$T \Big|_{z_e^k=0} = T_w \quad (9)$$

The material behavior and thermo-mechanical evolution in the primary shear zone are governed by Eqs. (6), (7) and (8). Using the boundary conditions (9), the shear stress τ_s^k on the main shear plane can be calculated.

4. Cylindrical turning geometry and the cutting edge discretization

Considering the complexity of chip formation in turning, the majority of current research is based on the equivalent cutting edge method, i.e. a fictitious straight cutting edge is used to replace the real cutting edge, as shown in Fig. 6. When some studies need to know the local information about stress and temperature on the rake face, this method is very disadvantageous to the study of tool design and tool wear. Therefore, to obtain the component state of the real cutting edge in the chip formation process, it is necessary to discretize the cutting edge.

In cylindrical turning, the modeling process is more complicated than the traditional oblique cutting with a single cutting edge on account for the existence of the tool rounded nose. In this paper, the discretization method of the cutting edge proposed by Molinnari and Moufki [18] is used to model the cylindrical turning process. The rounded edge is divided into finite straight cutting edge units. For each component edge unit, the expressions of component normal rake angle and inclination angle are derived by using geometric relationships. For the case of cylindrical turning, as shown in Fig. 7. The cutting conditions to be given are cutting velocity V , feed rate f_r , cutting depth a_p and tool geometric parameters (the main cutting edge angle κ_r , inclination angle λ_s , normal rake angle γ_n and nose radius r_n), the tool geometry is defined by six planes (P_r , P_s , P_n , P_o , P_f and P_p) to determine the spatial position of the cutting edge. Fig. 8 illustrates the discrete process of tool nose in cylindrical turning.

On the reference plane P_r , the cutting edge can be discretized into $K+2$ parts of the straight cutting edge section by the index k ($0 \leq k \leq K+1$). The tool nose is discretized into $K+1$ straight cutting edge units, and the chip thickness corresponding to each component edge unit is variable as shown in Fig. 8. In the

turning process, the rounded edge in contact with the workpiece can be divided into two parts by the angles ϕ_s and ϕ_a respectively. ϕ_{st} and ϕ_{ex} are the entry and exit angles in the first part, see Fig. 8. The first part (for the interval angle of ϕ_s) can be subdivided into K cutting edge units which are identified as k ($1 \leq k \leq K$), and $\Delta\phi$ is taken as the increment angle. The second part is also considered as a single unit with the index $k=K+1$. Fig. 8 gives an example of discretization with $K=3$, which corresponds to the projection on the reference plane of the discrete cutting edge. According to the geometric relationship, the following equations of these angles can be obtained:

$$\phi_{st} = \begin{cases} \arcsin((r_n - a_p) / r_n) & a_p < r_n(1 - \cos \kappa_r) \\ \pi / 2 - \kappa_r & a_p \geq r_n(1 - \cos \kappa_r) \end{cases} \quad (10)$$

$$\phi_{ex} = \pi / 2 - \arcsin(0.5f / r_n) \quad (11)$$

$$\phi_k = \phi_{ex} - \phi_{st} \quad (12)$$

$$\Delta\phi = \phi_k / K \quad (13)$$

$$\phi_a = 2 \arcsin(0.5f / r_n) \quad (14)$$

For the rounded edge, the geometric description of the discrete cutting edge is mainly defined by the planes $P_r, P_s^0, P_n^0, P_s^k, P_n^k$, as shown in Fig. 8. P_s^0 and P_n^0 corresponds to the cutting plane and normal plane of the main cutting edge ($k=0$). P_s^k is defined as containing the k th component edge and being perpendicular to P_r . P_n^k is normal to the k th component edge. θ_r^k is the angle between the planes P_s^0 and P_s^k measured in the plane P_r , θ_a^k is the angle between the main cutting edge ($k=0$) and the k th component edge ($1 \leq k \leq K$) measured in the tool rake face, see Fig. 8. As shown in Fig. 9, the component parameters $\phi_k, \kappa_r^k, \varphi_r^k, \theta_r^k, a_w^k$ and t_k in the reference plane P_r can be obtained as below:

$$\phi_k = \phi_{st} + (k-1)\Delta\phi \quad (15)$$

$$\kappa_r^k = \pi/2 - \phi_k - \Delta\phi / 2 \quad (16)$$

$$\varphi_r^k = \pi/2 - \kappa_r^k \quad (17)$$

$$\theta_r^k = \kappa_r - \kappa_r^k = \phi_k - \pi/2 + \kappa_r + \Delta\phi / 2 \quad (18)$$

$$a_w^k = 2r_n \sin(\Delta\phi / 2) \quad (19)$$

$$t_k = a_f \sin(\kappa_r^k) \quad (20)$$

Where $\phi_k, \kappa_r^k, \varphi_r^k, a_w^k$ and t_k represent, respectively, the angular position, the component cutting edge angle, the component lead angle, the component cutting width and the component cutting thickness corresponding to the k th component cutting edge. The following relations can be obtained for the main cutting edge ($k=0$):

$$\begin{cases} \theta_r^0 = 0 \\ a_w^0 = \begin{cases} 0 & a_p < r_n(1 - \cos \kappa_r) \\ (a_p - r_n(1 - \cos \kappa_r^0)) / \sin \kappa_r^0 & a_p \geq r_n(1 - \cos \kappa_r) \end{cases} \\ t_0 = f_r \sin(\kappa_r^0) \end{cases} \quad k=0 \quad (21)$$

The chip load A^k of the k th ($0 \leq k \leq K$) component edge is defined as the undeformed chip area. For the chip load A^{K+1} of the $k=K+1$ component edge, the following relationship can be obtained.

$$A^k = a_w^k t_k \quad 0 \leq k < K \quad (22)$$

$$A^{K+1} = r_n^2 \arcsin(0.5f / r_n) - 0.5f \sqrt{r_n^2 - (f)^2 / 4} \quad (23)$$

The relation between the discrete cutting edge in the rake face and its projection on P_r is treated using the coordinate transformation approach. The component cutting conditions corresponding to each component cutting edge, namely the component cutting angles (cutting edge angle, inclination angle, normal rake angle) and the component undeformed chip area, are also determined. In the rake face, y_r^k is the vector along the k th component edge and is contained in the P_s^k . The projection of y_r^k on the reference plane P_r is oriented by the unit vector y_o^k as shown in Fig. 9. To calculate the angles θ_a^k , λ_s^k and γ_n^k , a set of frames is introduced. These frames can be related by rotation transformation, as shown in Fig. 10. Based on the relationship of the coordinate systems shown in Fig. 10, the following expression can be got:

$$\mathbf{R}(x_r^0, -\theta_a^k) \cdot \mathbf{R}(y_n^0, \gamma_n^0) \cdot \mathbf{R}(z_o^0, \lambda_s^0) = \mathbf{R}(y_n^k, \gamma_n^k) \cdot \mathbf{R}(z_o^k, \lambda_s^k) \cdot \mathbf{R}(x_o^0, -\theta_r^k) \quad (24)$$

Therefore, by considering Eq. (24) and after some algebraic manipulation, θ_r^k corresponding to the projection of θ_a^k in the reference plane P_r is obtained:

$$\tan \theta_a^k = \frac{\cos \lambda_s^0 \tan \theta_r^k}{\cos \gamma_n^0 - \tan \theta_r^k \sin \gamma_n^0 \sin \lambda_s^0} \quad (25)$$

The component inclination angle λ_s^k and the component normal rake angle γ_n^k for the k th component edge are measured in P_s^k and P_n^k , respectively. For the main cutting edge ($k=0$), the cutting angles are given by $\lambda_s^0 = \lambda_s$ and $\gamma_n^0 = \gamma_n$. The angles λ_s^k and γ_n^k , for $1 \leq k \leq K+1$, are determined from the following relationships by Eq. (24).

$$\sin \lambda_s^k = \cos \theta_a^k \sin \lambda_s^0 - \sin \theta_a^k \sin \gamma_n^0 \cos \lambda_s^0 \quad (26)$$

$$\sin \gamma_n^k = \cos \theta_r^k \sin \gamma_n^0 + \sin \theta_r^k \sin \lambda_s^0 \cos \gamma_n^0 \quad (27)$$

The component cutting edge geometry can be characterized by these angles κ_r^k , γ_n^k , λ_s^k , θ_r^k and θ_a^k .

5. Cutting forces model of cylindrical turning

5.1 Component equilibrium of the discrete chip unit

For the improved oblique cutting model, the shear force F_s^k and normal force F_{ns}^k exerted by the workpiece on the k th chip unit can be expressed as:

$$\mathbf{F}_k = -F_s^k \mathbf{x}_s^k + F_{ns}^k \mathbf{z}_s^k \quad (28)$$

The component force balance acting on the k th chip unit can be expressed in the form of a vector:

$$-F_s^k \mathbf{x}_s^k + F_{ns}^k \mathbf{z}_s^k + F_{nc}^k \mathbf{x}_c^k - F_{fc}^k \mathbf{z}_c^k + F_{mc}^k \mathbf{y}_c^k = 0 \quad (29)$$

Eq. (29) can be transformed into the component equations in the coordinate system $(\mathbf{x}^k, \mathbf{y}^k, \mathbf{z}^k)$. The following expressions were deduced by Moufki and Molinari [22].

$$\begin{cases} F_{nc}^k (\cos(\phi_n^k - \gamma_n^k) - \tan \beta_k \sin(\phi_n^k - \gamma_n^k) \cos \eta_c^k) - F_{mc}^k \sin(\phi_n^k - \gamma_n^k) \sin \eta_c^k - F_s^k \cos \eta_s^k = 0 \\ F_{nc}^k \tan \beta_k \sin \eta_c^k - F_{mc}^k \cos \eta_c^k - F_s^k \sin \eta_s^k = 0 \\ F_{nc}^k (\sin(\phi_n^k - \gamma_n^k) + \tan \beta_k \cos(\phi_n^k - \gamma_n^k) \cos \eta_c^k) + F_{mc}^k \cos(\phi_n^k - \gamma_n^k) \sin \eta_c^k - F_{ns}^k = 0 \end{cases} \quad (30)$$

Eq. (30) is algebraic equation which can be solved by combining with F_s^k . Meanwhile, the unknown forces F_{nc}^k , F_{mc}^k , and F_{ns}^k can be determined. The shearing force F_s^k is given by:

$$F_s^k = \frac{\tau_s^k}{\cos \lambda_s^k \sin \phi_n^k} A_k \quad (31)$$

Where τ_s^k is the shear stress on the main shear plane of the primary shear zone, it can be obtained from the unequal division shear zone theory.

5.2 Resultant chip equilibrium and cutting forces.

In order to determine the value of the main chip flow angle η_c^0 , the resultant force balance equation acting on the entire chip can be simplified as:

$$\sum_{k=0}^{K+1} (-F_s^k \mathbf{x}_s^k + F_{ns}^k \mathbf{z}_s^k + F_{nc}^k \mathbf{x}_c - F_{fc}^k \mathbf{z}_c) = 0 \quad (32)$$

Through combining Eqs. (29) and (32), the implicit Eq. (33) can be obtained, and the main chip flow angle can be obtained by Eqs. (33) and (29). In the resultant chip balance, the sum of F_{mc}^k is equal to zero for the interaction force F_{mc}^k between discrete chip units.

$$\sum_{k=0}^{K+1} F_{mc}^k = 0 \quad (33)$$

Because the variables η_e^k , η_s^k , ϕ_n^k , β_k , and F_{mc}^k all depend on the main chip flow angle η_c^0 , numerical iteration is applied to obtain their values. The main chip flow angle η_c^0 is determined by Eq. (33), it considers the influence of material properties of workpiece, tool geometric parameters and cutting parameters. For traditionally defined equivalent cutting edge, the effects of tool nose radius and cutting angle on chip flow direction are considered separately when the cutting angles are unequal to zero by Adibi-Sedeh et al [24].

For the improved oblique cutting model of the k th component edge, in Fig. 3, the resultant force $\mathbf{F}_k$ is expressed as the $(\mathbf{x}_o^k, \mathbf{y}_o^k, \mathbf{z}_o^k)$ components of F_c^k (cutting force), F_d^k (back force), and F_f^k (feed force).

$$\mathbf{F}_k = -F_c^k \mathbf{x}_o^k - F_d^k \mathbf{y}_o^k + F_f^k \mathbf{z}_o^k \quad (34)$$

$$\begin{pmatrix} F_c^k \\ F_d^k \\ F_f^k \end{pmatrix} = \begin{pmatrix} \cos \eta_s^k \cos \phi_n^k \cos \lambda_s^k + \sin \eta_s^k \sin \lambda_s^k & \sin \phi_n^k \cos \lambda_s^k \\ \cos \eta_s^k \cos \phi_n^k \sin \lambda_s^k - \sin \eta_s^k \cos \lambda_s^k & \sin \phi_n^k \sin \lambda_s^k \\ -\cos \eta_s^k \sin \phi_n^k & \cos \phi_n^k \end{pmatrix} \begin{pmatrix} F_s^k \\ F_{ns}^k \end{pmatrix} \quad (35)$$

These force components can then be transformed to the frame $(\mathbf{x}_a, \mathbf{y}_a, \mathbf{z}_a)$ by:

$$\begin{pmatrix} F_x^k \\ F_y^k \\ F_z^k \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\cos \phi_r^k & \sin \phi_r^k \\ 0 & \sin \phi_r^k & \cos \phi_r^k \end{pmatrix} \begin{pmatrix} F_c^k \\ F_d^k \\ F_f^k \end{pmatrix} \quad (36)$$

Where F_x^k is the tangential force, F_y^k is radial force, and F_z^k is the longitudinal force as shown in Fig. 2. The algorithm to calculate cutting forces for cylindrical turning is as follows in Fig. 11.

At the beginning, the component cutting angles can be calculated by giving an initial value of η_c^0 . These output parameters such as strain, strain rate, stress, temperature distributions in the primary shear zone for each chip unit can be computed by numerical iterative method. If $\sum_{k=0}^{K+1} F_{mc}^k$ is not equal to zero, a new estimated value of η_c^0 will be obtained by adding an increment $\Delta \eta$. The computation will stop until the iteration requirement is satisfied. Conclusively, the component cutting force (F_x^k, F_y^k, F_z^k) of the k th

cutting edge unit is deduced by Eqs. (35) and (36), F_{mc} can be determined by Eq. (33). All steps of calculations were performed programmatically by MATLAB software.

6. Experimental validation and analysis

In order to validate the above proposed method above, a series of turning cutting experiments for 304 stainless steel were conducted without cutting fluid in Mazak lathe. The cutting forces were measured by Kisler Dynamometer (6275A), the fore signals that pass through the charge amplifier (5070A) was addressed by the Dynoware software, as shown in Fig. 12. The experiment was carried out in three cases:

- The influence of cutting depth a_p , feed rate f_r and cutting velocity V on turning forces was analysed with different cutting parameters.
- The influence of the tool nose radius r_n on turning forces was analysed with different tool insert.
- The influence of the main cutting edge angle κ_r on turning forces was analysed with different toolholder.

6.1 Effects of cutting conditions.

The experiment used SNMG120408-MR2025 insert and DSBNR2525M12 holder of Sandvik to study the influence of cutting parameters on turning forces of 304 stainless steel. The geometry parameters of the insert and the cutting conditions are shown in Tables 2 and 3, respectively. Specifically, there were three procedures in the experiment to obtain the reliable results, namely: the experiment with the same cutting parameters was repeated twice; the measurement interval time of the cutting process is more than ten seconds, and the insert was replaced by a new one after each cutting test with the same cutting parameters.

In Fig. 13, it can be seen that the cutting force increases approximately linearly with the cutting depth increase. However, the variation of cutting force shows obvious nonlinearity when the cutting depth is very small. The reason is that the straight main cutting edge plays a leading role in the cutting process when the cutting width a_w ($a_w = a_p / \sin(\kappa_r)$) increases in a positive proportion with the cutting depth increases. Furthermore, the influence of tool nose radius on the cutting force gradually decreases. If the cutting depth is much greater than the feed rate and the tool nose radius, the cutting edge involved in cutting can be regarded as a straight line. However, when the cutting depth is close to the nose radius, the influence of the nose radius on the cutting force must be considered. At this time, the nonlinear variation can only be measured in a more stable environment with precise equipment.

The cutting force also presents an increase linearly trend with the feed rate increases as show in Fig. 14, which is different from the effect of cutting depth on cutting force. This is because the chip thickness t ($t = f_r \sin(\kappa_r)$) has been increasing in a positive proportion with the increase of feed rate. However, the temperature increases with the increase of feed rate will result in the friction coefficient and cutting force decrease in turning. Comprehensively considering the positive and negative effects, the effect of feed rate on cutting force may be nonlinear. At the same time, it can be found from Figs. 14 (a) and (b) that the cutting force increases is slow with the increase of feed rate when the cutting velocity is very high.

For the 304 stainless steels, the nonlinear effect of cutting velocity on turning force is shown in Fig. 15. It can be seen that the cutting force decreases with the increase of cutting velocity in Fig. 15, the variation trend of cutting forces is consistent with the experimental results of Young et al [6]. and Arsecularate et al [25]. This is due to the fact that the deformation coefficient and friction coefficient decrease with the cutting velocity increases. It should be noted that the tool-chip friction is mainly caused

by high temperature, high pressure and high chip velocity. In cylindrical turning, the friction coefficient of each discrete chip unit will change, the average tool-chip temperature decreases with the cutting velocity increases, which will lead to the decrease of average tool-chip friction coefficient and cutting force. Meanwhile, it can be seen in Fig. 15 that the changing ratio of cutting force is obviously slow when cutting velocity is very low.

6.2 Effects of the main cutting edge angle

The cutting experiments of 304 stainless steel with three kinds of the main cutting edge angle were carried out. Table 4 illustrates the geometric parameters of the tool, and normal rake angle γ_n is approximately regarded as 0° , inclination angle λ_s is -6° , nose radius r_n is 0.8mm. The predicted values of the proposed model and the experimental results are shown in Fig. 16, the main cutting edge angle κ_r is investigated in the range of 30° to 90° . It can be found from Fig. 16 that the influence of the main cutting edge angle is approximately linear. As the main cutting edge angle increases, F_x and F_z is increasing, F_y is decreasing, and this trend is consistent with the experimental results of Young et al [6]. However, F_x basically remains constant with the increase of the main cutting edge angle. Although the incensement of the main cutting edge angle will result in a decrease in the engaged length in cutting, the area A ($A=f_r a_p$) of the undeformed chip which mainly determines the cutting force not depend on the tool main cutting edge angle. It can be seen in Fig. 16 that F_x is increased significantly when the main cutting edge angle increases to the range between 60° and 90° . It can generally be considered that the main cutting edge angle mainly affects the cutting force through the cutting thickness. With the increase of κ_r , the contact length of the tool rounded edge will also increase (at this time, the non-free cutting effect will be more significant). When the cutting area is constant, the cutting thickness increases with the increase of the main cutting edge angle. As a result, the deformation of chip formation will become large, which will lead to the increase of F_x . Nevertheless, the cutting thicknesses of the discrete chip units for tool nose part decreases in order as shown in Fig. 16, and they are smaller than the cutting thickness of the main cutting edge. When the cutting area is constant and the main cutting edge angle is increased, the cutting thickness will increase. The influence of the main cutting edge angle on F_z and F_y is mainly through the projection components of the resultant cutting force in the Z and Y directions. Therefore, F_z will decrease and F_y will increase as the main cutting edge angle increases.

6.3 Effects of the tool nose radius

The effect of four kinds of the tool nose radius on turning force of 304 stainless steel was studied. The rounded edge parameters are presented in Table 5. Fig. 17 illustrates the predicted variation of turning force with the nose radius and the experimental results. In the experiments, the main cutting edge is not involved in cutting process ($a_p < r_n(1-\cos(\kappa_r))$). It can be seen in Fig. 17 (a) that F_x is basically a constant, which may be because the total undeformed chip area A is nearly independent of the nose radius and only related to feed rate and cutting depth. The predicted curve in Fig. 17 shows an increase of F_y and a decrease of F_z with the increase of the nose radius. The experimental results of 304 stainless steel are also basically consistent with the predicted law, which is also consistent with the experimental results of Young [6] and Arsecularatne et al [25]. According to the previous discussion, F_z will increase with the increase of the main cutting edge angle and decrease with the increase of η_c^0 , the change law of F_y is opposite.

However, when the main cutting edge is involved in cutting ($a_p > r_n(1 - \cos(\kappa_r))$), as shown in Figs. 17 (b) and (c). In the case that the cutting depth a_p and the feed rate f_r are constant, the nose radius length and cutting width will increase with the increase of the main cutting edge angle. Adversely, the cutting thickness, the tool component cutting edge angle of the discrete edge units of the tool rounded edge will decrease. These lead to the increase of the cutting deformation and cutting force. It has a greater influence on F_y than on F_x as shown in Fig. 17.

7. Conclusion

This paper extended the previous work for the traditional oblique cutting model to non-free turning with a rounded edge. The main conclusions can be summarized as follows:

- The improved oblique cutting model considers the condition that the main cutting edge angle is unequal to be 90° and the tool nose has a rounded edge.
- Through applying the coordinate transformation method, the geometry relations for the non-free turning are analysed. The unequal division shear zone theory in orthogonal cutting is used in oblique cutting modeling.
- The tool rounded edge was discretized into small oblique cutting edges characterized by component cutting angles. For every engaged chip unit, the differential cutting forces are obtained from the oblique cutting process. The effects of the tool nose radius, the cutting angles, and workpiece material properties on the cutting force are considered as their coupling interaction.

A MATLAB-based program has been given to facilitate the application of the analytical model for cutting forces. The experiments with three types of cutting conditions have been conducted to validate the prediction model and the deviation between the predicted results and experiment data is within an acceptable range. The proposed models have also been used to investigate the effects of cutting conditions and tool angles on the cutting forces. This quantitative information is helpful to selection of appropriate cutting parameters and optimization of cutting edge geometry. It should be noted that the above description law of the influence of the machining parameters on the turning forces has appeared in many literatures, but most are obtained by curve fitting from the experimental results. The significance of this paper lies in proposing a new analytical model to predict the trend, and these methods are mutually verified.

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References

- [1] Parakkal G, Zhu RX, Kappor SG, DeVor RE (2002) Modeling of turning process cutting forces for grooved tools. *International Journal of Machine Tools and Manufacture* 1:179-191
- [2] Kone F, Czarnotab C, Haddaga B, Nouari M (2013) Modeling of velocity-dependent chip flow angle and experimental analysis when machining 304L austenitic stainless steel with groove coated-carbide tool. *Journal of Materials Processing Technology* 213:1166-1178
- [3] Colwell LV (1954) Predicting the angle of chip flow for single-point cutting tools. *Transactions of the ASME* 76:199-204
- [4] Hu RS, Mathew P, Oxley PLB, Young HT (1986) Allowing for end cutting edge effects in predicting forces in bar turning with oblique machining conditions. *Proceedings of the Institution of Mechanical Engineers* 200: 89-99
- [5] Kushima KO, Minato K (1987) On the behaviors of chip in steel cutting. *JSME International Journal* 2:58-64
- [6] Young HT, Mathew P, Oxley PLB (1987) Allowing for nose radius effects in predicting the chip flow direction and cutting forces in bar turning. *Proceedings of the Institution of Mechanical Engineers* 201:213-226
- [7] Wang J, Mathew P (1995) Development of a general tool model for turning operations based on a variable flow stress theory. *International Journal of Machine Tools and Manufacture* 1:71-90
- [8] Endres WJ, Waldorf DJ (1994) The importance of considering size effect along the cutting edge in predicting the effective lead angle for turning. *North American Manufacturing Research Conference* 65–72.
- [9] Khlifi H, Abdellaoui L, Bouzid SW (2019) An equivalent geometry model for turning tool with nose and edge radii. *The International Journal of Advanced Manufacturing Technology* 103:4233-4251
- [10] Merchant E (1945) Mechanics of the metal cutting process. *Journal of Applied Physics* 16:267-275
- [11] Oxley PLB, Young H. *The mechanics of machining: an analytical approach to assessing machinability*. Ellis Horwood Publisher.
- [12] Li BL, Wang XL, Hu YJ, Li, CG (2011) Analytical prediction of cutting forces in orthogonal cutting using unequal division shear zone model. *The International Journal of Advanced Manufacturing Technology* 54:431-443
- [13] Redetzky M, Balaji AK, Jawahir IS (1999) Predictive modeling of cutting forces and chip flow angle

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- in machining with nose radius tools. The second CIRP international workshop on modeling of machining operations 160-180
- [14] Hagiwara M, Chen S, Jawahir IS (2009) A hybrid predictive model and validation for chip flow incontour finish turning operations with coated grooved tools. *Journal of Materials Processing Technology* 209:1417-1427
- [15] Armarego EJA, Samaranayake P (1999) Performance prediction models for turning with rounded corner plane faced lathe tool theoretical development. *Journal of Mechanical Science and Technology* 3143–172
- [16] Storch B, Zawada-Tomkiewicz A (2012) Distribution of unit forces on the tool nose rounding in the case of constrained turning. *International Journal of Machine Tools and Manufacture* 57:1-9
- [17] Fang N (1998) An improved model for oblique cutting and its application to chip-control research. *Journal of Materials Processing Technology* 79:79-85
- [18] Molinari A, Moufki A (2005) A new thermomechanical model of cutting applied to turning operations. Part I. Theory. *International Journal of Machine Tools and Manufacture* 45:166-180
- [19] Budak A, Ozlu E (2008) Development of a thermomechanical cutting process model for machining process simulations. *CIRP Annals-Manufacturing Technology* 57:97-100
- [20] Abdellaoui L, Khlifi H, Bouzid SW, Hamdi H (2020) Tool nose radius effects in turning process. *Journal of Mechanical Science and Technology* 25:1-30.
- [21] Li BL, Hu YJ, Wang XL, Li CG, Li XX (2011) An analytical model of oblique cutting with application to end milling. *Journal of Mechanical Science and Technology* 15:453-484
- [22] Moufki A, Molinari A (2005) A new thermomechanical model of cutting applied to turning operations. Part II. Parametric study. *International Journal of Machine Tools and Manufacture* 45:181-193
- [23] Shaw MC (2005) *Metal cutting principles*, Second published. Oxford University Press, Oxford, UK.
- [24] Adibi-Sedeh AH, Madhavan V, Bahr B (2003) Upper bound analysis of oblique cutting: Improved method of calculating the friction area. *International Journal of Machine Tools and Manufacture* 43:485-492
- [25] Arsecularatne JA, Mathew P, Oxley PLB (1995) Prediction of chip flow direction and cutting forces in oblique machining with nose radius tools. *Proceedings of the Institution of Mechanical Engineers* 209:305–315

Table 1

Johnson-Cook parameters of 304 stainless steel.

$A(\text{MPa})$	$B(\text{MPa})$	C	n	$\dot{\epsilon}_0 (\text{s}^{-1})$	m
310	1000	0.07	0.65	1.00	1.00

Table 2

Geometry parameters of tool insert.

The main cutting edge angle (κ_r)	Inclination angle (λ_s)	Normal Rake angle (γ_n)	Radius of tool nose (r_n)
75°	-6°	0°	0.8mm

Table 3

Cylindrical turning conditions of 304 stainless steel.

Feed rate $f_r(\text{mm/r})$	Cutting depth $a_p(\text{mm})$	Cutting velocity $V(\text{m/min})$
0.075	0.5	62
0.1	1	90
0.15	1.5	132
0.2	2.0	188

Table 4

The tool geometric parameters.

Tool insert	Tool holder	The main cutting edge angle κ_r
SNMG 12 04 08-MM 2025	DSDNN 2525 M12	45°
SNMG 12 04 08-MM 2025	DSBNR 2525 M12	75°
SNMG 12 04 08-MM 2025	PTGNN 2525 M16	90°

Table 5

The tool rounded edge parameters.

Tool insert	Tool holder	Tool nose radius
SNMG 12 04 04-MF 2015	DSBNR 2525 M12	0.4mm
SNMG 12 04 08-MM 2025	DSBNR 2525 M12	0.8mm
SNMG 12 04 12-MM 2025	DSBNR 2525 M12	1.2mm
SNMG 12 04 16-MM 2025	DSBNR 2525 M12	1.6mm

Fig. 1. The improved oblique cutting process.

Fig. 2. The geometrical relationship of oblique cutting

Fig. 3. The rotation transformation relationships between the coordinate systems.

Fig. 4. The fitting curve of mean friction coefficient f with cutting velocity V .

Fig. 5. The unequal division shear zone model.

Fig. 6. The equivalent cutting edge method.

Fig. 7. The Geometric model of cylindrical turning.

Fig. 8. The cutting edge discretization of the engaged part in the rake face.

Fig. 9. Projection of discrete cutting edge on base reference plane.

Fig. 10. Coordinate transformation relationship in the discrete process of turning tool.

Fig. 11. Flow chart for the improved oblique cutting model applied to cylindrical turning.

Fig. 12. The experiments of cutting forces in cylindrical turning of 304 stainless steel

Fig. 13. The effect of cutting depth a_p on turning forces.

Fig. 14. The effect of feed rate f_r on turning force.

Fig. 15. The effect of cutting speed V on turning force.

Fig. 16. The effect of the main cutting edge angle κ_r on turning force.

Fig. 17. The effect of tool nose radius r_n on turning force.