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Yaotong Chen

Lei Du

<https://orcid.org/0000-0003-0641-440X>

Lingzhen Guo

<https://orcid.org/0000-0002-5558-3106>

Zhihai Wang

Yan Zhang

Northeast Normal University <https://orcid.org/0000-0003-0371-9953>

Yong Li

Beijing computational science research centre <https://orcid.org/0000-0002-6087-715X>

Jin-Hui Wu (✉ jhwu@nenu.edu.cn)

Northeast normal university

Article

Keywords:

Posted Date: March 11th, 2022

DOI: <https://doi.org/10.21203/rs.3.rs-1407573/v1>

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Version of Record: A version of this preprint was published at Communications Physics on August 22nd, 2022. See the published version at <https://doi.org/10.1038/s42005-022-00991-3>.

Nonreciprocal and chiral single-photon scattering for giant atoms

Yao-Tong Chen,¹ Lei Du,^{2,1} Lingzhen Guo,³ Zhihai Wang,¹ Yan Zhang,^{1,4,*} Yong Li,^{2,†} and Jin-Hui Wu^{1,‡}

¹*School of Physics and Center for Quantum Sciences,
Northeast Normal University, Changchun 130024, China*

²*Beijing Computational Science Research Center, Beijing 100193, China*

³*Max Planck Institute for the Science of Light, Staudtstraße 2, 91058 Erlangen, Germany*

⁴*National Demonstration Center for Experimental Physics Education,
Northeast Normal University, Changchun 130024, China*

(Dated: March 1, 2022)

In this work, we investigate the nontrivial single-photon scattering properties of giant atoms coupled to waveguides that can be an effective platform for realising nonreciprocal and chiral quantum optics. For the two-level giant-atom setup, we identify the condition for nonreciprocal transmission: the external atomic dissipation is further required other than the breaking of time-reversal symmetry by local coupling phases. Especially, in the non-Markovian regime, unconventional revival peaks periodically appear in the reflection spectrum of such a two-level giant-atom system. To explore more interesting scattering behaviours, we further extend the two-level giant-atom system to Δ -type and ∇ -type three-level giant atoms coupled to double waveguides without external atomic dissipation. We analyse the different physical mechanisms for the nonreciprocal and chiral scattering properties of the Δ -type and ∇ -type giant atoms. Our proposed giant-atom structures have potential applications of high-efficient single-photon targeted router and circulator for quantum information processing.

INTRODUCTION

Waveguide quantum electrodynamics (QED) studies the interactions between atoms and one-dimensional waveguide modes, providing an excellent platform for constructing long-range interactions and engineering large-scale quantum networks [1–5]. In experiments, typical candidates of implementing waveguide QED systems include quantum dots coupled to photonic crystal waveguides [6, 7], superconducting qubits coupled to transmission lines [8, 9], ultracold atoms coupled to optical fibers [10, 11], etc. To date, waveguide QED has inspired a number of exotic phenomena, such as atom-like mirrors [12, 13], dynamic Casimir effects [14], single-photon routing [15–17], bound states in the continuum [18].

In general, the atom can be viewed as a point when coupled with the waveguide due to its negligible size compared to the wavelength of waveguide modes. Nevertheless, in a recent experiment, a superconducting transmon qubit was designed to interact with surface acoustic waves (SAWs) via multiple coupling points whose separation distances can be much larger than the wavelength of SAWs [19]. Instead, a generalized theory called giant atom has been developed to describe such situations [20]. Since the first theoretical study in 2014 [21], the giant-atom scheme has been broadly investigated with superconducting qubits [22–26], coupled waveguide arrays [27], and cold atoms [28]. With such nonlocal coupling schemes, a series of tempting quantum phenomena have been demonstrated, including frequency-dependent re-

laxation rate and Lamb shift [21, 25, 29], non-exponential atomic decay [22, 23], decoherence-free interatomic interaction [25, 30, 31], exotic bound states [24, 32], and modified topological effects [33]. Giant atoms have emerged as a new paradigm in quantum optics and require more comprehensive understanding in physics.

On the other hand, controlling the flow of photons, especially realising asymmetric photonic propagations in waveguide QED systems, is crucial for constructing nonreciprocal optical element devices [34–39]. To this end, one could break the time-reversal symmetry of the system such that the interactions between the atoms and the waveguide modes are direction-dependent [16, 40–44]. Such a paradigm, also known as chiral quantum optics [40], can be achieved via several methods, such as the spin-momentum locking effect [45–47], inserting circulators in superconducting circuits [48–50], applying topological waveguides [51, 52], and synthesizing artificial gauge fields [53]. Based on the chiral interaction, targeted photonic routers [17], single-photon circulators [54, 55], cascaded quantum networks [56–58], and enhanced entanglement [59, 60] have been realised. Recently, the concept of giant atom has been introduced to chiral quantum optics, making some advanced functionalities possible, such as chiral bound states [32], dark states without coherent drives [31], and non-Markovicity induced nonreciprocity [61]. These seminal works inspire us to explore more intriguing effects in chiral giant-atom setups, especially with multi-level structure [61–63].

In this paper, we investigate how external atomic dissipations outside the waveguide and local coupling phases affect the single-photon scattering properties of a two-level giant atom with two atom-waveguide coupling points. By taking account of the phase difference between two coupling points, we find that the giant atom behaves

* zhangy345@nenu.edu.cn

† liyong@csrc.ac.cn

‡ jhwu@nenu.edu.cn

86 like a chiral small atom in the Markovian regime but ex-
 87 hibits peculiar giant-atom effects in the non-Markovian
 88 regime. We physically demonstrate that the breaking of
 89 time-reversal symmetry by local coupling phases is not
 90 sufficient for realising nonreciprocal photon scatterings.
 91 In fact, in the absence of the external atomic dissipation,
 92 the scatterings are always reciprocal even if the atomic
 93 spontaneous emission becomes chiral [63, 64]. In order
 94 to realise asymmetric scattering for a giant atom with-
 95 out external dissipation, we propose a ∇ -type giant atom
 96 coupled to two waveguides. In such way, we realise the
 97 nonreciprocal and chiral scatterings with single ∇ -type
 98 atom. Targeted routing and circulation schemes can also
 99 be realised via such scatterings with proper phases. Fi-
 100 nally, we consider a Δ -type giant atom and compare its
 101 properties with that of ∇ -type one. We reveal that, the
 102 nonreciprocal scatterings stem from the quantum inter-
 103 ference effect in the closed-loop atom-level structure for
 104 the Δ -type giant atom, but from the nontrivial coupling
 105 phase difference for the ∇ -type giant atom.

106 RESULTS AND DISCUSSION

107 A. Two-level giant atom coupled to a single 108 waveguide

109 As schematically shown in Fig. 1(a), we consider a two-
 110 level giant atom coupled to a waveguide at two separated
 111 points $x = 0$ and $x = d$. The atom-waveguide coupling
 112 coefficients are $ge^{i\theta_1}$ and $ge^{i\theta_2}$, respectively, with local
 113 coupling phases θ_1 and θ_2 for inducing some intriguing
 114 interference effects to the scattering properties as will
 115 be discussed below. With superconducting quantum de-
 116 vices, the local coupling phases can be introduced with
 117 Josephson loops threaded by external fluxes [64].

118 Under the rotating wave approximation (RWA), the
 119 real-space Hamiltonian of the model can be written as
 120 ($\hbar = 1$ hereafter)

$$\begin{aligned}
 H &= H_w + H_a + H_I, \\
 H_w &= \int_{-\infty}^{+\infty} dx \left[a_L^\dagger(x) \left(\omega_0 + iv_g \frac{\partial}{\partial x} \right) a_L(x) \right. \\
 &\quad \left. + a_R^\dagger(x) \left(\omega_0 - iv_g \frac{\partial}{\partial x} \right) a_R(x) \right], \\
 H_a &= (\omega_e - i\frac{\gamma_e}{2}) |e\rangle\langle e|, \\
 H_I &= \int_{-\infty}^{+\infty} dx \left\{ \delta(x) ge^{i\theta_1} [a_R^\dagger(x) + a_L^\dagger(x)] |g\rangle\langle e| \right. \\
 &\quad \left. + \delta(x-d) ge^{i\theta_2} [a_R^\dagger(x) + a_L^\dagger(x)] |g\rangle\langle e| + \text{H.c.} \right\}. \tag{1}
 \end{aligned}$$

121 Here H_w represents the free Hamiltonian of the wave-
 122 guide modes with v_g being the group velocity of photons
 123 in the waveguide. $a_{R,L}$ ($a_{R,L}^\dagger$) are the bosonic annihila-
 124 tion (creation) operators of the right-going and left-going

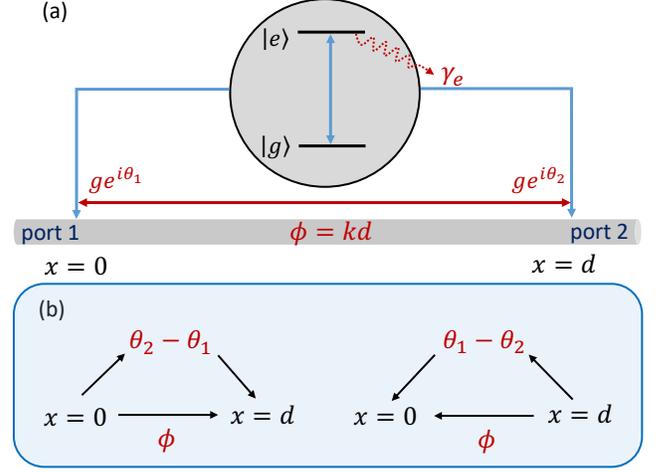


FIG. 1. (a) Schematic configuration of a two-level giant atom coupled to a waveguide at $x = 0$ and $x = d$, respectively, with individual local coupling phases $\theta_{1,2}$. (b) Two paths of a single photon propagating from port 1 to port 2 (left) or from port 2 to port 1 (right).

125 photons in the waveguide, respectively; ω_0 is the frequen-
 126 cy around which the dispersion relation of the waveguide
 127 mode is linearised [1, 65]. H_a is for the atom, where ω_e
 128 describes the transition frequency between the ground s-
 129 tate $|g\rangle$ and the excited state $|e\rangle$; γ_e is the external atomic
 130 dissipation rate due to the non-waveguide modes in the
 131 environment. H_I describes the interactions between the
 132 atom and the waveguide, where the Dirac delta functions
 133 $\delta(x)$ and $\delta(x-d)$ indicate that the atom-waveguide cou-
 134 plings occur at $x = 0$ and $x = d$, respectively. Besides,
 135 there is an accumulated phase $\phi = kd$ of photons be-
 136 tween two coupling points, where k is the renormalized
 137 wave vector that satisfies the linearised dispersion rela-
 138 tion $E = \omega_0 + kv_g$ (E is the eigenenergy as determined
 139 later in Eq. (3)). The relevant physics discussed in this
 140 work is valid for $E \sim \omega_e$, that is, $kv_g \sim \omega_e - \omega_0$.

141 Considering that the total excitation number is con-
 142 served in RWA, the eigenstate of the system can be ex-
 143 pressed in the single-excitation subspace as

$$\begin{aligned}
 |\Psi\rangle &= \int_{-\infty}^{+\infty} dx [\Phi_R(x) a_R^\dagger(x) + \Phi_L(x) a_L^\dagger(x)] |0, g\rangle \\
 &\quad + u_e |0, e\rangle, \tag{2}
 \end{aligned}$$

144 where $\Phi_{R,L}(x)$ are the density of probability amplitudes
 145 of creating the right-going and left-going photons at po-
 146 sition x , respectively; u_e is the excitation amplitude of
 147 the atom; $|0, g\rangle$ denotes the vacuum state of the system.
 148 The probability amplitudes can be determined by solving

149 the eigenequation $H|\Psi\rangle = E|\Psi\rangle$, which leads to

$$\begin{aligned}
 E\Phi_R(x) &= \left(\omega_0 - iv_g \frac{\partial}{\partial x}\right)\Phi_R(x) \\
 &\quad + g\left[e^{i\theta_1}\delta(x) + e^{i\theta_2}\delta(x-d)\right]u_e, \\
 E\Phi_L(x) &= \left(\omega_0 + iv_g \frac{\partial}{\partial x}\right)\Phi_L(x) \\
 &\quad + g\left[e^{i\theta_1}\delta(x) + e^{i\theta_2}\delta(x-d)\right]u_e, \\
 Eu_e &= \left(\omega_e - i\frac{\gamma_e}{2}\right)u_e + ge^{-i\theta_1}[\Phi_R(0) + \Phi_L(0)] \\
 &\quad + ge^{-i\theta_2}[\Phi_R(d) + \Phi_L(d)].
 \end{aligned} \tag{3}$$

150 Assuming that a single photon with wave vector k ($k > 0$)
 151 is incident from port 1 of the waveguide. Then the wave
 152 functions $\Phi_{R,L}(x)$ can be written in the forms of

$$\begin{aligned}
 \Phi_R(x) &= e^{ikx}\{\Theta(-x) + A[\Theta(x) - \Theta(x-d)] \\
 &\quad + t\Theta(x-d)\}, \\
 \Phi_L(x) &= e^{-ikx}\{r\Theta(-x) + B[\Theta(x) - \Theta(x-d)]\},
 \end{aligned} \tag{4}$$

153 where $\Theta(x)$ is the Heaviside step function. Here, t and r
 154 denote the single-photon transmission and reflection am-
 155 plitudes in the regions of $x > d$ and $x < 0$, respectively.
 156 We define A and B as the probability amplitudes for the
 157 right-going and left-going photons between the two cou-
 158 pling points ($0 < x < d$), respectively.

159 Substituting Eq. (4) into Eq. (3), one obtain

$$\begin{aligned}
 0 &= -iv_g(A-1) + ge^{i\theta_1}u_e, \\
 0 &= -iv_g(t-A)e^{i\phi} + ge^{i\theta_2}u_e, \\
 0 &= -iv_g(r-B) + ge^{i\theta_1}u_e, \\
 0 &= -iv_gBe^{-i\phi} + ge^{i\theta_2}u_e, \\
 0 &= \frac{g}{2}e^{-i\theta_1}(A+B+r+1) + \frac{g}{2}e^{-i\theta_2} \\
 &\quad \times (Ae^{i\phi} + Be^{-i\phi} + te^{i\phi}) - (\Delta + i\frac{\gamma_e}{2})u_e
 \end{aligned} \tag{5}$$

160 with $\Delta = E - \omega_e$ being the detuning of frequency be-
 161 tween the incident photon and the atomic transition fre-
 162 quency. From the condition $E \sim \omega_e$, we work in the dis-
 163 persive regime where the detuning is much smaller than
 164 the atomic transition frequency (deducting the offset ω_0
 165 in the linear dispersion) as $|\Delta/(\omega_e - \omega_0)| \ll 1$. Then the
 166 transmission and reflection amplitudes can be obtained
 167 from solving Eq. (5) as

$$t = \frac{\Delta + i\frac{\gamma_e}{2} - 2\Gamma e^{i\theta}\sin\phi}{\Delta + i\frac{\gamma_e}{2} + 2i\Gamma(1 + e^{i\phi}\cos\theta)}, \tag{6a}$$

$$r = \frac{[2i\Gamma(1 + e^{i\phi}\cos\theta) + 2\Gamma e^{i\theta}\sin\phi][1 + e^{i(\theta+\phi)}]}{[\Delta + i\frac{\gamma_e}{2} + 2i\Gamma(1 + e^{i\phi}\cos\theta)][1 + e^{i(\theta-\phi)}]}, \tag{6b}$$

168 where $\theta = \theta_2 - \theta_1$ is the phase difference between the two
 169 atom-waveguide coupling channels and $\Gamma = g^2/v_g$ is the
 170 rate of the atomic emission into the waveguide. Com-
 171 pared with the setup of a two-level small atom coupled

172 locally to a waveguide, such giant atom shows phase-
 173 dependent effective detuning and decay rate given by
 174 $\Delta - 2\Gamma\cos\theta\sin\phi$ and $\gamma_e/2 + 2\Gamma(1 + \cos\theta\cos\phi)$, respectively
 175 [21]. In fact, a left-incident (right-incident) photon can
 176 propagate from $x = 0$ to $x = d$ (from $x = d$ to $x = 0$)
 177 via two different paths: it can either keep on propagating
 178 along the waveguide, or be absorbed at $x = 0$ ($x = d$)
 179 and re-emitted at $x = d$ ($x = 0$) by the atom, as shown
 180 in Fig. 1(b). For the left-incident photon, the two path-
 181 s yield phase accumulations ϕ and θ , respectively, which
 182 determine the phase-dependent interference effect jointly.
 183 For the right-incident photon, the propagation process
 184 is equivalent to that of the left-incident one yet with
 185 exchanged coupling phases, i.e., $\theta_1 \leftrightarrow \theta_2$. Therefore,
 186 the transmission and reflection amplitudes for the right-
 187 incident photon are expressed as

$$t' = \frac{\Delta + i\frac{\gamma_e}{2} - 2\Gamma e^{-i\theta}\sin\phi}{\Delta + i\frac{\gamma_e}{2} + 2i\Gamma(1 + e^{i\phi}\cos\theta)}, \tag{7a}$$

$$r' = \frac{[2i\Gamma(1 + e^{i\phi}\cos\theta) + 2\Gamma e^{-i\theta}\sin\phi][1 + e^{-i(\theta-\phi)}]}{[\Delta + i\frac{\gamma_e}{2} + 2i\Gamma(1 + e^{i\phi}\cos\theta)][1 + e^{-i(\theta+\phi)}]}, \tag{7b}$$

188 which are also consistent with the results obtained by
 189 rewriting the wave functions for the right-incident pho-
 190 ton. For $d = 0$ ($\phi = 0$), the system is reduced to a
 191 two-level small atom coupled to the waveguide at a s-
 192 ingle point; consequently, the phase difference θ is al-
 193 so nonexistent. In addition, note that the accumulat-
 194 ed phase of the propagating photons can be written as
 195 $\phi = kd = \tau(\omega_e - \omega_0 + \Delta) = \phi_0 + \tau\Delta$ with the time delay
 196 $\tau = d/v_g$ and $\phi_0 = \tau(\omega_e - \omega_0)$. Thus, ϕ strongly depends
 197 on the frequency of incident photon in the non-Markovian
 198 regime, where the delay time τ is nonnegligible [61]. As
 199 we discussed above, the linearised dispersion of waveg-
 200 uide is used and we have the condition that $|\tau\Delta/\phi_0| \ll 1$
 201 for the relevant physics around the atomic transition fre-
 202 quency [61].

1. Reciprocal and nonreciprocal transmissions

204 We first focus on the Markovian regime of $\tau \ll 1/(2\Gamma +$
 205 $\gamma_e/2)$, where $\phi \approx \phi_0$ according to the Taylor expansion
 206 because this substitution gives correct Lamb shift and
 207 modified emission rate in the Markovian limit [22, 27].
 208 In Fig. 2, we plot the transmission rates $T_{1 \rightarrow 2} = |t|^2$
 209 and $T_{2 \rightarrow 1} = |t'|^2$ as functions of the detuning Δ and the
 210 phase difference θ with and without external atomic dis-
 211 sipations. Owing to the interference between two photon
 212 paths mentioned above, the scattering behavior changes
 213 periodically with θ . For $\gamma_e = 0$ as shown in Figs. 2(a)
 214 and 2(b), the single-photon scattering is reciprocal, i.e.,
 215 $T_{1 \rightarrow 2} \equiv T_{2 \rightarrow 1}$, although the time-reversal symmetry is
 216 broken due to the nontrivial phase difference θ arising
 217 from the interference.

218 This counterintuitive phenomenon can be explained by
 219 comparing Eqs. (6a) and (7a). On one hand, the trans-

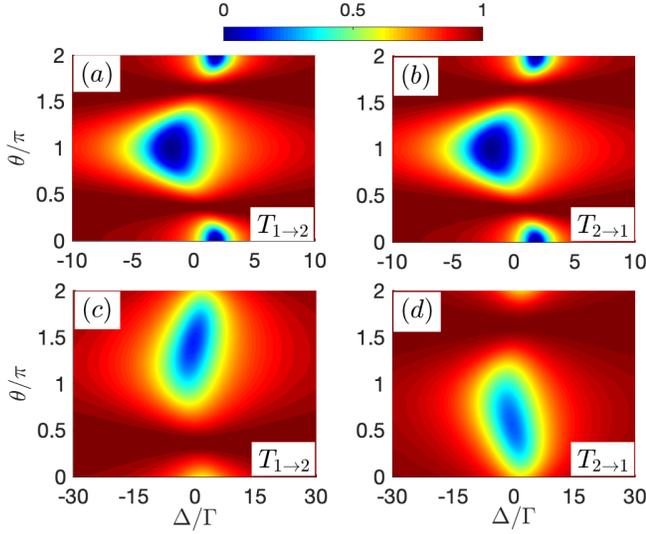


FIG. 2. Transmission rates $T_{1 \rightarrow 2}$ and $T_{2 \rightarrow 1}$ versus the detuning Δ and the phase difference θ with $\gamma_e = 0$ for (a) and (b); $\gamma_e/\Gamma = 10$ for (c) and (d). Other parameters: $\phi_0 = \pi/2$ and $\tau\Gamma = 0.01$.

220 mission amplitudes t and t' share the same denominator
 221 that is an even function of θ . On the other hand, the nu-
 222 merators of t and t' in Eqs. (6a) and (7a) can be rewritten
 223 as

$$\begin{aligned} \Delta - 2\Gamma\sin\phi\cos\theta + i\left(\frac{\gamma_e}{2} - 2\Gamma\sin\phi\sin\theta\right), \\ \Delta - 2\Gamma\sin\phi\cos\theta + i\left(\frac{\gamma_e}{2} + 2\Gamma\sin\phi\sin\theta\right). \end{aligned} \quad (8)$$

224 Equation (8) clearly shows that nonreciprocal single-
 225 photon transmissions ($|t|^2 \neq |t'|^2$) can be achieved only if
 226 a finite external atomic dissipation rate is taken into ac-
 227 count ($\gamma_e > 0$). This can be observed by the transmission
 228 spectra shown in Figs. 2(c) and 2(d).

229 When $\gamma_e = 0$, Fig. 3(a) depicts the transmission rates
 230 $T_{1 \rightarrow 2}$ and $T_{2 \rightarrow 1}$ versus the detuning Δ with various θ .
 231 For $\theta = \pi/2$, we find $T_{1 \rightarrow 2} = T_{2 \rightarrow 1} \equiv 1$ over the w-
 232 hole range of the detuning, implying that reflections are
 233 prevented for both directions. For $\theta = \pi$, however, the
 234 transmission spectrum exhibits the Lorentzian line shape
 235 with phase-dependent Lamb shift and linewidth (decay
 236 rate) [21]. In both cases ($\theta = \pi/2, \pi$), the transmission-
 237 s are reciprocal, yet the atomic excitation probabilities
 238 are different as will be discussed below. When $\gamma_e \neq 0$,
 239 as shown in Figs. 3(b) and 3(c), the scattering becomes
 240 nonreciprocal if $\theta = \pi/2$; however, with $\theta = \pi$, the s-
 241 catterings are still reciprocal even in the presence of the
 242 external dissipation. The yellow dot-dashed, red dotted,
 243 and blue dashed lines in Fig. 3(d) depict the contrast
 244 ratio

$$I = \frac{T_{2 \rightarrow 1} - T_{1 \rightarrow 2}}{T_{2 \rightarrow 1} + T_{1 \rightarrow 2}} \quad (9)$$

245 versus the coupling phase difference θ with different
 246 atomic dissipation rates. It can be seen that this nonre-

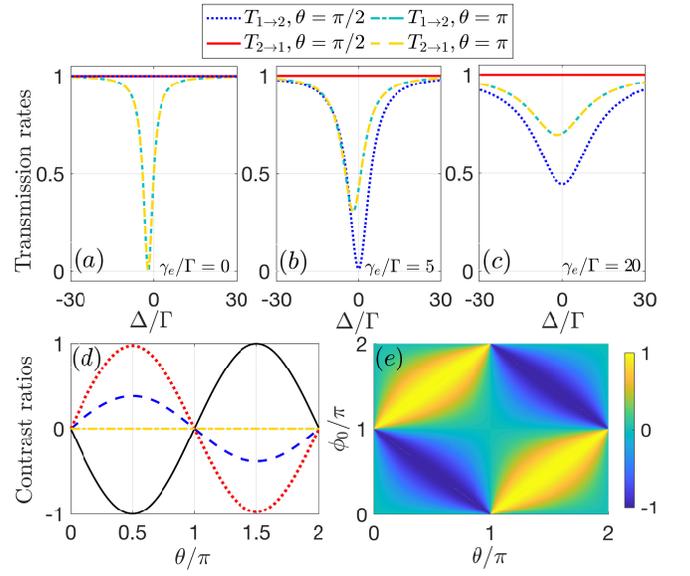


FIG. 3. Transmission rates $T_{1 \rightarrow 2}$ and $T_{2 \rightarrow 1}$ versus the detuning Δ for (a) $\gamma_e/\Gamma = 0$; (b) $\gamma_e/\Gamma = 5$; (c) $\gamma_e/\Gamma = 20$ with $\phi_0 = \pi/2$. (d) Contrast ratios I and D versus the coupling phase difference θ with $\phi_0 = \pi/2$. The yellow dot-dashed, red dotted, and blue dashed lines are I with $\gamma_e/\Gamma = 0$, $\gamma_e/\Gamma = 5$, and $\gamma_e/\Gamma = 20$, respectively, and the black solid one represents D independent of γ_e . (e) Contrast ratio D versus the phase difference θ and the propagating phase ϕ_0 . Other parameters: $\tau\Gamma = 0.01$.

247 ciprocal transmission behaviour is phase-dependent. In-
 248 deed, the above behaviours are easy to be analysed with
 249 Eqs. (6a) and (7a).

250 Furthermore, the underlying physics of the reciprocal
 251 and nonreciprocal scatterings can be understood via ex-
 252 amining the atomic excitation by the single photon. To
 253 this end, we define the contrast ratio D of the atomic
 254 excitation probabilities for two opposite propagating di-
 255 rections as

$$D = \frac{|u_{e2 \rightarrow 1}|^2 - |u_{e1 \rightarrow 2}|^2}{|u_{e2 \rightarrow 1}|^2 + |u_{e1 \rightarrow 2}|^2} \quad (10)$$

$$\begin{aligned} u_{e1 \rightarrow 2} &= \frac{t - 1}{-i\frac{g}{v_g}[e^{i\theta_1} + e^{i(\theta_2 + \phi)}]}, \\ u_{e2 \rightarrow 1} &= \frac{t' - 1}{-i\frac{g}{v_g}[e^{i\theta_2} + e^{i(\theta_1 + \phi)}]}. \end{aligned} \quad (11)$$

257 According to Eqs. (6a) and (7a), parameters $t - 1$ and
 258 $t' - 1$ have the same denominator containing γ_e but dif-
 259 ferent numerators without γ_e . Furthermore, because the
 260 denominator that contains γ_e is eliminated when calcu-
 261 lating Eq. (10), the contrast ratio D is independent of
 262 dissipation rate γ_e . Note that the contrast ratio D can
 263 be used to capture the difference of the atomic excitation
 264 probabilities for opposite directions even if the eigenstate
 265 Eq. (2) is unnormalized.

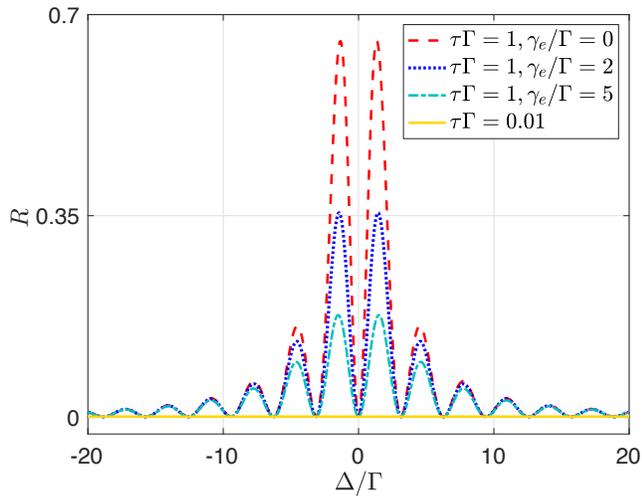


FIG. 4. Reflection rate R versus the detuning Δ with $\theta = \pi/2$ and $\phi_0 = \pi/2$.

We plot in Fig. 3(d) the contrast ratio D (black solid line) as a function of the phase difference θ with $\phi_0 = \pi/2$. For $\theta = \pi/2$, $D = -1$ means that the atom can only be excited by the left-incident photon, and thus the atom-waveguide interaction becomes ideally chiral [64, 66]. In this case, the right-incident photon is guided transparently because it does not interact with the atom. While in the Markovian regime, the reflections are lacking for both directions under the ideal chiral coupling [16, 43], this is in fact not true in the non-Markovian regime as will be discussed in Sec. IIB. For $\theta = 3\pi/2$, $D = 1$ corresponds to the ideal chiral case where the atom can only be excited by the right-incident photon. For other cases of $D = 0$ and $0 < |D| < 1$, the atom-waveguide couplings are nonchiral and nonideal chiral, respectively. In fact, the nonreciprocal scatterings arise from the different dissipations into the environment, i.e., the energy loss into the environment is proportional to the dissipation rate γ_e as well as the atomic population. In addition, we demonstrate in Fig. 3(e) that the contrast ratio D is also sensitive to the propagating phase. This provides an alternative way to tune the chirality of the atom-waveguide interaction and the reciprocal/nonreciprocal scattering on demand.

The results above can also be interpreted from the aspect of Hermitian and non-Hermitian scattering centers [67–69]. In our system with $\gamma_e = 0$ ($\gamma_e \neq 0$), the giant atom can be regarded as a Hermitian (non-Hermitian) scattering center of the Aharonov-Bohm structure supporting two spatial interference paths. For the Hermitian case, the scattering remains reciprocal; however, when introducing an imaginary potential, e.g., the external atomic dissipation, the combination of the non-Hermiticity and the broken time-reversal symmetry gives rise to nonreciprocal scatterings [67, 68]. It is noted that, as discussed in the case in Figs. 3(b) and 3(c) ($\theta = \pi$), not all non-Hermitian scattering centers can

demonstrate nonreciprocal transmissions. The exceptions include, e.g., \mathcal{P} -, \mathcal{T} -, or \mathcal{PT} -symmetric scattering centers [67, 69]. In our model, although the giant atom can exhibit chiral spontaneous emission corresponding to the time-reversal symmetry breaking if $\theta \neq n\pi$ (n is an arbitrary integer) [64], the scatterings are still reciprocal unless the additional non-Hermiticity (such as external dissipations) are introduced.

2. Non-Markovian regime

With nontrivial local coupling phases, as demonstrated above, the current giant-atom model (in the Markovian regime) is able to simulate a chiral atom-waveguide system. However, one important characteristic of the giant atom is the peculiar scattering behaviours arising in the non-Markovian regime, where the propagating phase accumulation $\phi = \phi_0 + \tau\Delta$ is sensitive to the detuning Δ due to the large enough τ that is comparable to or larger than the lifetime of the atom [27]. Such a detuning-dependent phase will undoubtedly result in the non-Markovian features in the transmission and reflection spectra [23, 61]. Here we just consider our system in the non-Markovian regime and demonstrate the reflection with $\phi_0 = \pi/2$ and $\theta = \pi/2$. Note that the reflection is totally prevented in the small-atom case with an ideal chiral coupling, which has been demonstrated in Ref. [16].

We plot in Fig. 4 the reflection rates $R = |r|^2$ for the left-incident photon in the Markovian and non-Markovian regimes. The yellow solid curve shows that the reflection in the Markovian regime disappears completely. Such a reflectionless behavior occurs in the case of $D = \pm 1$, independent of the external atomic dissipation. However, in the non-Markovian regime, due to the Δ -dependent propagating phase ϕ , the reflection revives with multiple peaks aligning periodically in the frequency domain. In addition, the maximums of the reflection peaks decrease gradually with the increasing of γ_e . The underlying physics is that, in the phase accumulation ϕ , the non-Markovian contribution $\tau\Delta$ cannot be ignored relative to ϕ_0 ; thus, $\tau\Delta$ and ϕ_0 determine the scattering behaviors jointly. The reflection disappears at some discrete Δ points satisfying $\tau\Delta = n\pi$.

B. Three-level giant atom coupled to double waveguides

In this section, we extend the giant-atom model to a multi-level version and demonstrate the possibility of realising nonreciprocal scatterings without the additional non-Hermiticity (i.e., external atomic dissipation). Specifically, we introduce here an additional atomic transition coupled to other waveguide modes. As shown in Fig. 5(a), we propose a ∇ -type giant atom coupled to two waveguides via two different atomic transitions, respectively. Each transition is coupled to the correspond-

ing waveguide at two points, and an external microwave field is applied to drive the magnetic dipole transition between two excited states [70]. Such system allows for, without the help of external dissipation, high-efficiency single-photon routing and circulating. Furthermore, at the end of this section, we will also consider a Δ -type scheme and compare the differences between these two three-level structures.

As shown in Fig. 5(a), the atomic transition $|e_1\rangle \leftrightarrow |g\rangle$ of frequency ω_{e_1} is coupled to waveguide W_a with complex coupling coefficient $g_1 e^{i\theta_{1,2}}$ at two separated points $x = 0$ and $x = d_a$, respectively; the transition $|e_2\rangle \leftrightarrow |g\rangle$ of ω_{e_2} is coupled to W_b with $g_2 e^{i\theta_{3,4}}$ at $x = 0$ and $x = d_b$, respectively. The excited states $|e_{1,2}\rangle$ are coupled to an external coherent field of Rabi frequency Ω and initial phase α . The atom is initialized on the ground state $|g\rangle$. The Hamiltonian of the ∇ -type giant atom coupled to two waveguides can be written as

$$\begin{aligned}
H' &= H'_w + H'_a + H'_I, \\
H'_w &= \int_{-\infty}^{+\infty} dx \left[a_L^\dagger(x) \left(\omega_0 + iv_g \frac{\partial}{\partial x} \right) a_L(x) \right. \\
&\quad \left. + a_R^\dagger(x) \left(\omega_0 - iv_g \frac{\partial}{\partial x} \right) a_R(x) \right] \\
&\quad + \int_{-\infty}^{+\infty} dx \left[b_L^\dagger(x) \left(\omega_0 + iv_g \frac{\partial}{\partial x} \right) b_L(x) \right. \\
&\quad \left. + b_R^\dagger(x) \left(\omega_0 - iv_g \frac{\partial}{\partial x} \right) b_R(x) \right], \\
H'_a &= \left(\omega_{e_1} - i \frac{\gamma_{e_1}}{2} \right) |e_1\rangle \langle e_1| + \left(\omega_{e_2} - i \frac{\gamma_{e_2}}{2} \right) |e_2\rangle \langle e_2| \\
&\quad + (\Omega e^{i\alpha} |e_1\rangle \langle e_2| + \text{H.c.}), \\
H'_I &= \int_{-\infty}^{+\infty} dx \left\{ \delta(x) g_1 e^{i\theta_1} [a_R^\dagger(x) + a_L^\dagger(x)] |g\rangle \langle e_1| \right. \\
&\quad + \delta(x-d) g_1 e^{i\theta_2} [a_R^\dagger(x) + a_L^\dagger(x)] |g\rangle \langle e_1| \\
&\quad + \delta(x) g_2 e^{i\theta_3} [b_R^\dagger(x) + b_L^\dagger(x)] |g\rangle \langle e_2| \\
&\quad \left. + \delta(x-d) g_2 e^{i\theta_4} [b_R^\dagger(x) + b_L^\dagger(x)] |g\rangle \langle e_2| + \text{H.c.} \right\},
\end{aligned} \tag{12}$$

where $a_{R,L}/b_{R,L}$ ($a_{R,L}^\dagger/b_{R,L}^\dagger$) annihilates (creates) right-going and left-going photons in the waveguide W_a/W_b , respectively. In the single-excitation subspace, the eigenstate of the system can be expressed as

$$\begin{aligned}
|\Psi\rangle &= \int_{-\infty}^{+\infty} dx \left[\Phi_{aR}(x) a_R^\dagger(x) + \Phi_{aL}(x) a_L^\dagger(x) \right. \\
&\quad \left. + \Phi_{bR}(x) b_R^\dagger(x) + \Phi_{bL}(x) b_L^\dagger(x) \right] |0, g\rangle \\
&\quad + u_{e_1} |0, e_1\rangle + u_{e_2} |0, e_2\rangle,
\end{aligned} \tag{13}$$

where $\Phi_{aR,aL}$ ($\Phi_{bR,bL}$) are the probability amplitudes of creating the right-going and left-going photons in W_a (W_b), respectively.

Assuming that a photon with wave vector k_a is emanated from port 1 of W_a , the probability amplitudes

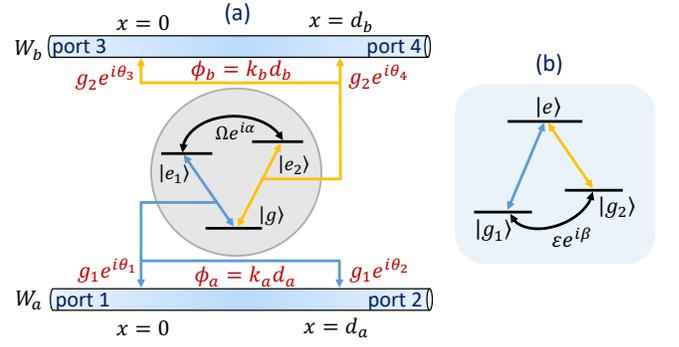


FIG. 5. Schematic configuration of the three-level giant atom. (a) ∇ -type atom: the waveguide W_a (W_b) is coupled to the transition $|g\rangle \leftrightarrow |e_1\rangle$ ($|g\rangle \leftrightarrow |e_2\rangle$) at two separated points. The excited states $|e_1\rangle$ and $|e_2\rangle$ are coupled to an external coherent field of Rabi frequency Ω and initial phase α . (b) Δ -type atom: W_a (W_b) is coupled to $|g_1\rangle \leftrightarrow |e\rangle$ ($|g_2\rangle \leftrightarrow |e\rangle$) at two separated points. Two ground states $|g_1\rangle$ and $|g_2\rangle$ are coupled to an external coherent field $\varepsilon e^{i\beta}$.

can be written as

$$\begin{aligned}
\Phi_{aR}(x) &= e^{ik_a x} \left\{ \Theta(-x) + M [\Theta(x) - \Theta(x-d_a)] \right. \\
&\quad \left. + s_{1 \rightarrow 2} \Theta(x-d_a) \right\}, \\
\Phi_{aL}(x) &= e^{-ik_a x} \left\{ s_{1 \rightarrow 1} \Theta(-x) + N [\Theta(x) - \Theta(x-d_a)] \right\}, \\
\Phi_{bR}(x) &= e^{ik_b x} \left\{ Q [\Theta(x) - \Theta(x-d_b)] + s_{1 \rightarrow 4} \Theta(x-d_b) \right\}, \\
\Phi_{bL}(x) &= e^{-ik_b x} \left\{ s_{1 \rightarrow 3} \Theta(-x) + W [\Theta(x) - \Theta(x-d_b)] \right\},
\end{aligned} \tag{14}$$

where the wave vectors $k_a = (E' - \omega_0)/v_g$ with the eigenenergy E' in W_a and $k_b = k_a + (\omega_{e_2} - \omega_{e_1})/v_g$ in W_b . When excited to state $|e_1\rangle$ by the incident photon from port 1, the atom can either re-emit a photon with the same frequency to W_a via decaying back to state $|g\rangle$ directly, or radiate a photon with frequency ω_{e_2} to W_b via first transferring from state $|e_1\rangle$ to state $|e_2\rangle$ due to the external driving and then decaying to state $|g\rangle$ [62, 71]. If a photon with wave vector k_b is sent from port 4 of W_b , the probability amplitudes can be written as

$$\begin{aligned}
\Phi_{aR}(x) &= e^{ik_a x} \left\{ s_{4 \rightarrow 2} \Theta(x-d_a) + M' [\Theta(x) - \Theta(x-d_a)] \right\}, \\
\Phi_{aL}(x) &= e^{-ik_a x} \left\{ N' [\Theta(x) - \Theta(x-d_a)] + s_{4 \rightarrow 1} \Theta(-x) \right\}, \\
\Phi_{bR}(x) &= e^{ik_b x} \left\{ Q' [\Theta(x) - \Theta(x-d_b)] + s_{4 \rightarrow 4} \Theta(x-d_b) \right\}, \\
\Phi_{bL}(x) &= e^{-ik_b x} \left\{ \Theta(x-d_b) + W' [\Theta(x) - \Theta(x-d_b)] \right. \\
&\quad \left. + s_{4 \rightarrow 3} \Theta(-x) \right\}.
\end{aligned} \tag{15}$$

By solving the stationary Schrödinger equation, one can obtain the scattering amplitudes of ∇ -type giant atom for this case.

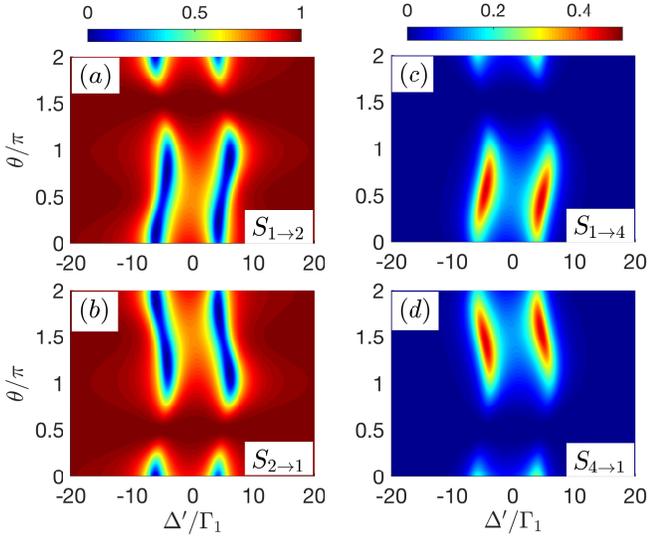


FIG. 6. Scattering probabilities (a) $S_{1\rightarrow 2}$; (b) $S_{2\rightarrow 1}$; (c) $S_{1\rightarrow 4}$; (d) $S_{4\rightarrow 1}$ versus the detuning Δ' and the phase difference θ . Other parameters: $\gamma_{e_1} = \gamma_{e_2} = 0$, $\Omega = 5\Gamma_1$, $\Gamma_2/\Gamma_1 = 1$, $\phi_{a0} = \pi/2$.

1. Nonreciprocal scattering

For simplicity, we start by supposing $\phi_b = \theta_3 = \theta_4 = 0$, i.e., the transition $|g\rangle \leftrightarrow |e_2\rangle$ is coupled to W_b at a single point. Then, the scattering probabilities can be calculated from $S_{1\rightarrow 2} = |s_{1\rightarrow 2}|^2$ and $S_{1\rightarrow 3(4)} = |s_{1\rightarrow 3(4)}|^2$ with the scattering amplitudes given by

$$s_{1\rightarrow 2} = \frac{\Delta' + i\frac{\gamma_{e_1}}{2} - \Omega^2 f - 2\Gamma_1 e^{i\theta} \sin\phi_a}{\Delta' + i\frac{\gamma_{e_1}}{2} - \Omega^2 f + 2i\Gamma_1(1 + e^{i\phi_a} \cos\theta)}, \quad (16)$$

$$s_{1\rightarrow 3(4)} = \frac{g_2 \Omega e^{-i\alpha} (s_{1\rightarrow 2} - 1)}{g_1 (\Delta' + i\gamma_{e_2}/2 + i\Gamma_2) [e^{i\theta_1} + e^{i(\theta_2 - \phi_a)}]}$$

with the detuning $\Delta' = E' - \omega_{e_1}$ and the atomic emission rates $\Gamma_{1,2} = g_{1,2}^2/v_g$. As discussed above, we make the substitution $\phi_a = k_a d_a \simeq \phi_{a0}$ in the Markovian regime. Likewise, one can also obtain $S_{2\rightarrow 1} = |s_{2\rightarrow 1}|^2$ and $S_{2\rightarrow 3(4)} = |s_{2\rightarrow 3(4)}|^2$ with

$$s_{2\rightarrow 1} = \frac{\Delta' + i\frac{\gamma_{e_1}}{2} - \Omega^2 f - 2\Gamma_1 e^{-i\theta} \sin\phi_a}{\Delta' + i\frac{\gamma_{e_1}}{2} - \Omega^2 f + 2i\Gamma_1(1 + e^{i\phi_a} \cos\theta)}, \quad (17)$$

$$s_{2\rightarrow 3(4)} = \frac{g_2 \Omega e^{-i\alpha} (s_{2\rightarrow 1} - 1)}{g_1 (\Delta' + i\gamma_{e_2}/2 + i\Gamma_2) [e^{i\theta_2} + e^{i(\theta_1 - \phi_a)}]}$$

which are achieved via exchanging θ_1 and θ_2 in Eq. (16). It is found that $s_{4\rightarrow 1} = s_{2\rightarrow 3}$ and thus $S_{4\rightarrow 1} = |s_{4\rightarrow 1}|^2 = S_{2\rightarrow 3}$.

Compared with Eq. (6a) and Eq. (7a) of the two-level giant atom, both $s_{1\rightarrow 2}$ and $s_{2\rightarrow 1}$ include an additional coupling term $\Omega^2 f$ with

$$f = \frac{1 - i(\frac{\gamma_{e_2}}{2} + \Gamma_2)}{\Delta'^2 + (\frac{\gamma_{e_2}}{2} + \Gamma_2)^2}, \quad (18)$$

which describes the photon transfer from W_a to W_b . It can be seen from Eqs. (16)-(18) that, in contrast to the two-level giant-atom scheme, the transmission between ports 1 and 2 in W_a is nonreciprocal even if the external dissipations are not considered. In fact, for $\gamma_{e_1} = \gamma_{e_2} = 0$, the imaginary part of f describing the decay of $|e_2\rangle \rightarrow |g\rangle$ into W_b plays the role of an external dissipation for the transition $|e_1\rangle \rightarrow |g\rangle$.

It is worth noting that the scattering probabilities of the ∇ -type system are independent of the phase α of the external coherent field Ω in spite of the closed-loop atom-level structure. This is because the ∇ -type atom cannot provide the inner two-path quantum interference. For instance, when excited to state $|e_1\rangle$ by an incident photon from port 1, the atom may be pumped to state $|e_2\rangle$ by the external field Ω and then return to state $|g\rangle$ after emitting a photon into W_b , which is the only path for the photon transferring from W_a to W_b . This is radically different from the Δ -type structure as will be discussed in Sec. IIC. In fact, the photon cannot be routed from W_a to W_b in the absence of the field Ω , implying that the ∇ -type three-level giant atom reduces to a two-level one. This is also consistent with the fact that $S_{1\rightarrow 3(4)} = S_{2\rightarrow 3(4)} = 0$ when $\Omega = 0$.

Figure 6 shows the single-photon scattering spectra as functions of the detuning Δ' and the phase difference θ . As discussed above, it can be seen from Figs. 6(a) and 6(b) that the nonreciprocal scattering can still be realised in W_a ($S_{1\rightarrow 2} \neq S_{2\rightarrow 1}$) with W_b playing the role of the external thermal reservoir in the two-level scheme as analysed above. According to the conclusion in Sec. II, for $\theta \neq n\pi$ and $\phi_{a0} = \pi/2 + 2n\pi$, the excitation probabilities $|u_{e_1}|^2$ for two opposite directions are unequal, i.e., the effective interaction between the atom and W_a is chiral. Then, as shown in Figs. 6(c) and 6(d), the nonreciprocal scattering between ports 1 and 4 can be led to by the chiral coupling, since the scattering probability $S_{1\rightarrow 4}$ ($S_{4\rightarrow 1}$) is related to the coupling between the atomic transition $|e_1\rangle \leftrightarrow |g\rangle$ and the right-going (left-going) mode in W_a . When $\theta = \pi/2$ ($3\pi/2$), $S_{1\rightarrow 4}$ ($S_{4\rightarrow 1}$) approaches 0.5 and $S_{4\rightarrow 1}$ ($S_{1\rightarrow 4}$) falls to 0. This corresponds to the ideal chiral case where the atom is only coupled to the right-going (left-going) modes effectively in W_a . When $\theta = \pi$, the scatterings between ports 1 and 4 are reciprocal, similar to the results of the non-chiral case in Sec. II.

2. Chiral scattering

Next, we turn to study another kind of asymmetric scattering phenomenon proposed recently called chiral scattering. Specifically, the transmission from port 1 to port 4 and that from port 2 to port 3 are different. Quantitatively, the chiral scattering can be evaluated by the chirality defined as [72]

$$C = \frac{S_{1\rightarrow 4} - S_{2\rightarrow 3}}{S_{1\rightarrow 4} + S_{2\rightarrow 3}}. \quad (19)$$

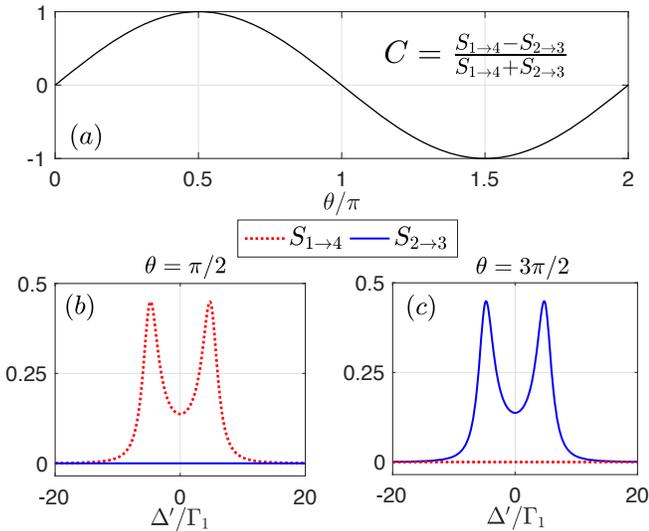


FIG. 7. (a) Chirality C versus the phase difference θ . Scattering probabilities $S_{1 \rightarrow 4}$ and $S_{2 \rightarrow 3}$ versus the detuning Δ' for (b) $\theta = \pi/2$ and (c) $\theta = 3\pi/2$. Other parameters: $\gamma_{e_1} = \gamma_{e_2} = 0$, $\Omega = 5\Gamma_1$, $\Gamma_2/\Gamma_1 = 1$, $\phi_{a0} = \pi/2$.

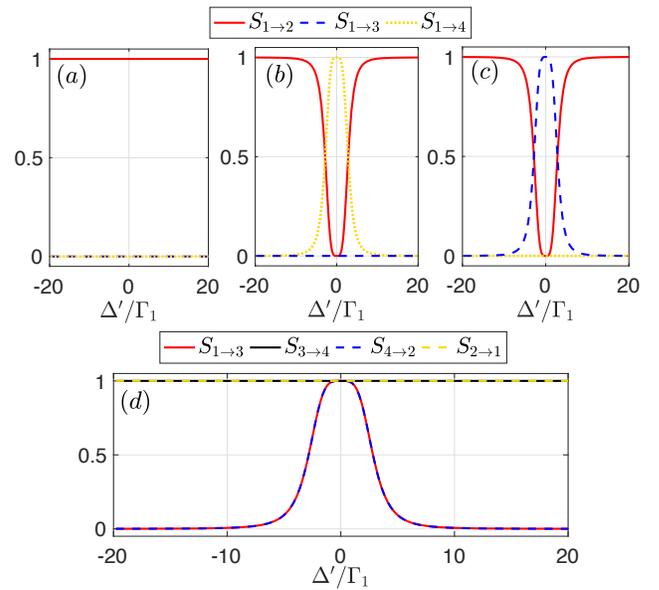


FIG. 8. Scattering probabilities versus the detuning Δ' for (a) $\Omega = 0$, $\theta = \pi/2$; (b) $\Omega = 2\Gamma_1$, $\theta = \pi/2$, $\theta' = \pi/2$; (c) and (d) $\Omega = 2\Gamma_1$, $\theta = \pi/2$, $\theta' = 3\pi/2$. Other parameters: $\gamma_{e_1} = \gamma_{e_2} = 0$, $\Gamma_2/\Gamma_1 = 1$, and $\phi_{a0} = \phi_{b0} = \pi/2$.

466 Figure 7(a) shows the chirality as a sinusoidal function of
 467 the phase difference θ . In view of this, chiral scatterings
 468 can be observed as long as $\theta \neq n\pi$, where the chirality
 469 $C \neq 0$ means $S_{1 \rightarrow 4} \neq S_{2 \rightarrow 3}$. This can be further verified
 470 by the scattering spectra as shown in Figs. 7(b) and 7(c).
 471 Note that $C = 1$ ($C = -1$) corresponding to $\theta = \pi/2$
 472 ($\theta = 3\pi/2$), implies that only the scattering from port
 473 2 (1) to port 3 (4) is prevented, as shown in Fig. 7(b)
 474 [Fig. 7(c)].

475 The underlying physics of the chiral scattering can also
 476 be attributed to the difference between the atomic excitation
 477 probabilities for two incident directions as discussed
 478 above. The excitation probabilities $|u_{e_1}|^2$ by the photon
 479 incident from port 1 and port 2 can be unequal, and
 480 thus the atom is pumped from $|e_1\rangle$ to $|e_2\rangle$ with unequal
 481 probabilities. This leads to different probabilities of routing
 482 photons from W_a to W_b . Furthermore, as shown in
 483 Fig. 7, the chiral scattering scheme here shows the *in-situ*
 484 tunability that the scattering chirality can be controlled
 485 by tuning the phase difference θ .

3. Targeted router and circulator

487 In this subsection, we would like to demonstrate how
 488 to realise a single-photon targeted router and circulator
 489 based on the asymmetric scatterings above. Specifically,
 490 one can send a single photon deterministically from port
 491 1 to one of the other three ports on demand. Note that
 492 the router and circulator can run with very high efficiency
 493 in such a non-loss system. Here we assume the transition
 494 $|e_2\rangle \leftrightarrow |g\rangle$ coupled to W_b at two separated points, i.e.,
 495 $\phi_b \neq 0$, as shown in Fig. 5(a), and define $\theta' = \theta_4 - \theta_3$.
 496 The mechanism of the targeted router can be under-

497 stood from Figs. 8(a)-8(c) showing the scattering proba-
 498 bilities from port 1 to other ports versus the detuning Δ' .
 499 When turning off the external field ($\Omega \equiv 0$), as shown in
 500 Fig. 8(a), the incident photon from port 1 cannot be routed
 501 to W_b ; particularly for $\theta = \pi/2$, the photon is routed
 502 to port 2 totally. Next, we turn on the external field to
 503 enable photon routing to the desired port in W_b with high
 504 efficiency. When setting $\theta' = \pi/2$, as shown in Fig. 8(b),
 505 a photon resonant with the transition $|g\rangle \leftrightarrow |e_1\rangle$ can be
 506 routed from port 1 to port 4 totally. Likewise, when
 507 setting $\theta' = 3\pi/2$ as shown in Fig. 8(c), the resonant
 508 photon can be routed to port 3 totally. In addition, both
 509 the propagating phases ϕ_{a0} and ϕ_{b0} determine the out-
 510 put port of photons in W_b , which is a unique feature of
 511 the giant-atom model.

512 More interestingly, the ∇ -type giant atom is also a
 513 promising candidate of realizing a single-photon circulator.
 514 When turning on the external field and setting
 515 $\theta = \pi/2$ and $\theta' = 3\pi/2$, the two waveguides are coupled
 516 to the atom with ideal chiral couplings in opposite
 517 manners, respectively. That is to say, the atom is only
 518 coupled to the left-incident photons in W_a yet to right-
 519 incident photons in W_b . Then, as shown in Fig. 8(d), one
 520 has $S_{2 \rightarrow 1} = S_{3 \rightarrow 4} \equiv 1$ over the whole frequency range and
 521 $S_{1 \rightarrow 3} = S_{4 \rightarrow 2} = 1$ around the resonance. Consequently,
 522 for a resonant photon, directional scattering along the
 523 direction $1 \rightarrow 3 \rightarrow 4 \rightarrow 2 \rightarrow 1$ can be realised suggesting
 524 a high-performance single-photon circulation scheme for
 525 quantum networks [54, 55].

526

4. Comparison with the Δ -type scheme

527 Finally, we consider a Δ -type giant-atom scheme where
 528 the ∇ -type atom in Fig. 5(a) is replaced by a Δ -type one
 529 in Fig. 5(b) and compare the single-photon scatterings of
 530 these two schemes. The Δ -type structure is construct-
 531 ed with an external coherent field $\epsilon e^{i\beta}$ which couples
 532 the two ground states $|g_{1,2}\rangle$ of a Λ -type atom that has
 533 been broadly studied to demonstrate quantum interfer-
 534 ence phenomena, such as coherent population trapping
 535 [73] and electromagnetically induced transparency [74].

536 For the Δ -type giant-atom system, the Hamiltonians
 537 of the atom and the atom-waveguide interaction become
 538

$$\begin{aligned}
 H'_a &= \left(\omega_{g_2} - i\frac{\gamma_{g_2}}{2}\right)|g_2\rangle\langle g_2| + \left(\omega_e - i\frac{\gamma_e}{2}\right)|e\rangle\langle e| \\
 &\quad + (\epsilon e^{i\beta}|g_1\rangle\langle g_2| + \text{H.c.}), \\
 H'_I &= \int_{-\infty}^{+\infty} dx \left\{ \delta(x)g_1 e^{i\theta_1} [a_R^\dagger(x) + a_L^\dagger(x)]|g_1\rangle\langle e| \right. \\
 &\quad + \delta(x-d)g_1 e^{i\theta_2} [a_R^\dagger(x) + a_L^\dagger(x)]|g_1\rangle\langle e| \\
 &\quad + \delta(x)g_2 e^{i\theta_3} [b_R^\dagger(x) + b_L^\dagger(x)]|g_2\rangle\langle e| \\
 &\quad \left. + \delta(x-d)g_2 e^{i\theta_4} [b_R^\dagger(x) + b_L^\dagger(x)]|g_2\rangle\langle e| + \text{H.c.} \right\}.
 \end{aligned}
 \tag{20}$$

539 The single-excitation eigenstate of the system takes the
 540 form

$$\begin{aligned}
 |\Psi\rangle &= \int_{-\infty}^{+\infty} dx \left\{ [\Phi_{aR}(x)a_R^\dagger(x) + \Phi_{aL}(x)a_L^\dagger(x)]|0, g_1\rangle \right. \\
 &\quad \left. + [\Phi_{bR}(x)b_R^\dagger(x) + \Phi_{bL}(x)b_L^\dagger(x)]|0, g_2\rangle \right\} + u_e|0, e\rangle.
 \end{aligned}
 \tag{21}$$

541 With the same procedure above (see Supplementary for
 542 more details), one can obtain the scattering probabilities
 543 in this case.

544 Setting the atom on the ground state $|g_1\rangle$ initially, we
 545 plot in Fig. 9 the scattering spectra of $\tilde{S}_{1\rightarrow 4}$ and $\tilde{S}_{4\rightarrow 1}$. It
 546 is worth noting that, even in the absence of the local cou-
 547 pling phases, i.e., $\theta = \theta' = 0$, the nonreciprocal scatter-
 548 ings still exist. This is obviously distinct from the ∇ -type
 549 case. The nonreciprocity of the ∇ -type case stems from
 550 the effective chiral couplings owing to the nontrivial cou-
 551 pling phase difference, and is independent of the phase of
 552 the external field. For the Δ -type scheme, however, the
 553 nonreciprocity arises from the typical which-way quan-
 554 tum interference, i.e., the interference between the two
 555 transition paths $|g_1\rangle \rightarrow |g_2\rangle$ and $|g_1\rangle \rightarrow |e\rangle \rightarrow |g_2\rangle$. In
 556 this case, the optical responses are typically sensitive to
 557 the phase of the external field encoded in the closed-loop
 558 level structure [75]. However, the main drawback to the
 559 Δ -type scheme is that one cannot switch on/off the pho-
 560 ton transfer between the two waveguides by tuning the
 561 external field solely.

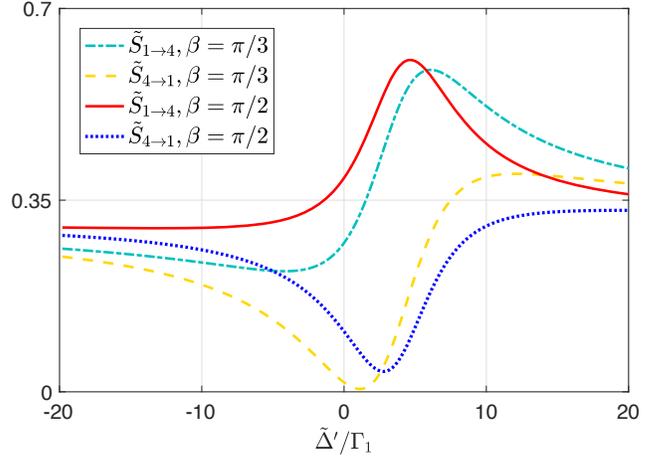


FIG. 9. Scattering probabilities versus the detuning $\tilde{\Delta}'$ with different β . Other parameters: $\gamma_{g_2} = \gamma_e = 0$, $\epsilon = 30\Gamma_1$, $\Gamma_2/\Gamma_1 = 1$, and $\phi_{a0} = \phi_{b0} = \pi/2$.

562

CONCLUSION

563 In summary, we have investigated step-by-step the con-
 564 ditions of single-photon nonreciprocal and chiral scatter-
 565 ings in the two-level and three-level giant-atom struc-
 566 tures with tunable local phase on each atom-waveguide
 567 coupling. We found that the atomic excitation in the
 568 two-level giant-atom structure depends on the propaga-
 569 tion direction of waveguide modes and can be tuned by
 570 the nontrivial coupling phase difference. In such scenar-
 571 io, our two-level giant atom in the Markovian regime is
 572 equivalent to a two-level small atom chirally coupled to
 573 the waveguide mode. However, it is worth noting that the
 574 realisation of nonreciprocal scatterings requires the com-
 575 bination of the time-reversal symmetry breaking induced
 576 by the local coupling phases and the non-Hermiticity in-
 577 duced by the external atomic dissipation due to the sur-
 578 rounding non-waveguide modes. Moreover, in the non-
 579 Markovian regime, the reflection spectra exhibit peculiar
 580 non-Markovian features with multiple reflection peaks
 581 that are absent in the chiral small-atom case.

582 For exploring more interesting asymmetric scatter-
 583 ing properties and applications with such giant-atom
 584 structures, we have extended the two-level structure to
 585 the three-level ∇ -type and Δ -type ones coupled to two
 586 waveguides via different atomic transitions. We found
 587 that, for the atomic transition coupled to one waveguide,
 588 the transition coupled to the other waveguide can serve as
 589 the external dissipation channel. Such three-level giant-
 590 atom structures coupled to double waveguides enable the
 591 nonreciprocal and chiral scatterings without external dis-
 592 sipations. Based on this mechanism, the high-efficiency
 593 single-photon targeted router and circulator can be im-
 594 plemented. Finally, we explained the different physical
 595 mechanisms that lead to the nonreciprocal and chiral s-
 596 catterings for the two phase-sensitive closed-loop three-
 597 level giant-atom structures. We believe that our result-
 598 s have promising applications in designing effective and

599 efficient single-photon optical elements for quantum net- 609
600 work engineering and optical communications.

AUTHOR CONTRIBUTIONS

610 C.Y.T. conceived the study, performed the calculation-
611 s, and wrote the first version of the manuscript. D.L.,
612 G.L.Z., and Z.Y. checked the calculations, analyzed the
613 data, and redrafted the manuscript. W.Z.H., L.Y., and
614 W.J.H. helped in the interpretation of the results and in
615 the writing of the manuscript. All authors discussed the
616 results and reviewed the manuscript.

METHODS

602 In this theoretical work, the methods used are solving 617
603 the stationary Schrödinger equation with the Hamiltoni-
604 an and single-excitation eigenstate (as described in the
605 main text [Eq. (3)] and Supplementary).

COMPETING INTERESTS

618 The authors declare no competing interests.

DATA AVAILABILITY

607 All data are available in the main text or in the sup- 625
608 plementary materials.

ACKNOWLEDGMENTS

619 This work is supported by the National Natural Sci-
620 ence Foundation of China (Grants No. 12074030 and
621 No. U1930402), the Fundamental Research Funds for the
622 Central Universities (Grant No. 2412019FZ045), and the
623 Science Foundation of the Education Department of Jilin
624 Province during the 14th Five-Year Plan Period (Grant
625 No. JJKH20211279KJ).
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