

# Creation and observation of topological magnetic monopoles and their interactions in a ferromagnetic meta-lattice

**Arjun Rana**

University of California Los Angeles

**Chen-Ting Liao**

University of Colorado Boulder <https://orcid.org/0000-0002-3423-277X>

**Ezio Iacocca**

University of Colorado Colorado Springs <https://orcid.org/0000-0002-8870-5106>

**Ji Zou**

University of California Los Angeles

**Minh Pham**

University of California Los Angeles

**Xingyuan Lu**

University of California Los Angeles

**Emma-Elizabeth Cating Subramanian**

University of Colorado

**Yuan Hung Lo**

University of California Los Angeles

**Sinéad Ryan**

University of Colorado

**Charles Bevis**

JILA, University of Colorado

**Robert Karl Jr.**

JILA, University of Colorado

**Andrew Glaid**

Penn State University

**Jeffrey Rable**

Penn State University

**Pratibha Mahale**

Penn State University

**Joel Hirst**

Sheffield Hallam University

**Thomas Ostler**

Sheffield Hallam University

**William Liu**

University of California, Los Angeles

**Colum O'Leary**

University of California, Los Angeles

**Young-Sang Yu**

Lawrence Berkeley National Laboratory

**Karen Bustillo**

Lawrence Berkeley National Lab <https://orcid.org/0000-0002-2096-6078>

**Hendrik Ohldag**

SLAC National Accelerator Laboratory <https://orcid.org/0000-0002-2034-878X>

**David Shapiro**

Lawrence Berkeley National Laboratory <https://orcid.org/0000-0002-4186-6017>

**Sadegh Yazdi**

Univ of colorado <https://orcid.org/0000-0002-3470-9398>

**Thomas Mallouk**

University of Pennsylvania

**Stanley Osher**

University of California, Los Angeles

**Henry Kapteyn**

Joint Institute for Laboratory Astrophysics

**Vincent Crespi**

Pennsylvania State University <https://orcid.org/0000-0003-3846-3193>

**John Badding**

Pennsylvania State University <https://orcid.org/0000-0002-4517-830X>

**Yaroslav Tserkovnyak**

University of California Los Angeles <https://orcid.org/0000-0002-3277-3695>

**Margaret Murnane**

JILA, University of Colorado Boulder

**Jianwei (John) Miao (✉ [miao@physics.ucla.edu](mailto:miao@physics.ucla.edu))**

University of California Los Angeles <https://orcid.org/0000-0003-4033-3945>

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1 **Creation and observation of topological magnetic monopoles and**  
2 **their interactions in a ferromagnetic meta-lattice**

3 Arjun Rana<sup>1,2\*</sup>, Chen-Ting Liao<sup>2,3\*</sup>, Ezio Iacocca<sup>4,5</sup>, Ji Zou<sup>1</sup>, Minh Pham<sup>2,6</sup>, Xingyuan Lu<sup>1,7</sup>,  
4 Emma-Elizabeth Cating Subramanian<sup>2,3</sup>, Yuan Hung Lo<sup>1,2</sup>, Sinéad A. Ryan<sup>2,3</sup>, Charles S.  
5 Bevis<sup>2,3</sup>, Robert M. Karl Jr<sup>2,3</sup>, Andrew J. Glaid<sup>8</sup>, Jeffrey Rable<sup>8</sup>, Pratibha Mahale<sup>8,9</sup>, Joel Hirst<sup>10</sup>,  
6 Thomas Ostler<sup>10,11</sup>, William Liu<sup>1,2</sup>, Colum M. O’Leary<sup>1,2</sup>, Young-Sang Yu<sup>12</sup>, Karen Bustillo<sup>13</sup>,  
7 Hendrik Ohldag<sup>12</sup>, David A. Shapiro<sup>12</sup>, Sadegh Yazdi<sup>14</sup>, Thomas E. Mallouk<sup>8,9</sup>, Stanley J.  
8 Osher<sup>2,6</sup>, Henry C. Kapteyn<sup>2,3</sup>, Vincent H. Crespi<sup>8</sup>, John V. Badding<sup>8</sup>, Yaroslav Tserkovnyak<sup>1</sup>,  
9 Margaret M. Murnane<sup>2,3</sup>, Jianwei Miao<sup>1,2</sup>

10 *<sup>1</sup>Department of Physics & Astronomy and California NanoSystems Institute, University of*  
11 *California, Los Angeles, CA 90095, USA. <sup>2</sup>STROBE Science and Technology Center. <sup>3</sup>JILA*  
12 *and Department of Physics, University of Colorado and NIST, 440 UCB, Boulder, Colorado*  
13 *80309, USA. <sup>4</sup>Department of Mathematics, Physics, and Electrical Engineering, Northumbria*  
14 *University, Newcastle upon Tyne, NE1 8ST, UK. <sup>5</sup>Center for Magnetism and Magnetic*  
15 *Materials, University of Colorado, Colorado Springs, CO 80918, USA. <sup>6</sup>Department of*  
16 *Mathematics, University of California, Los Angeles, CA 90095, USA. <sup>7</sup>School of Physical*  
17 *Science and Technology, Soochow University, Suzhou 215006, China. <sup>8</sup>Departments of*  
18 *Chemistry, Physics, Materials Science and Engineering and Materials Research Institute,*  
19 *Penn State University, University Park, PA 16802, USA. <sup>9</sup>Department of Chemistry, University*  
20 *of Pennsylvania, Philadelphia PA 19104, USA. <sup>10</sup>Materials and Engineering Research*  
21 *Institute, Sheffield Hallam University, Howard Street, Sheffield S1 1WB, UK. <sup>11</sup>Department of*  
22 *Engineering and Mathematics, Sheffield Hallam University, Howard Street, Sheffield S1 1WB,*  
23 *UK. <sup>12</sup>Advanced Light Source, Lawrence Berkeley National Laboratory, Berkeley, CA 94720,*  
24 *USA. <sup>13</sup>National Center for Electron Microscopy, Molecular Foundry, Lawrence Berkeley*

25 *National Laboratory, Berkeley, CA, 94720, USA. <sup>14</sup>Renewable and Sustainable Energy*  
26 *Institute, University of Colorado, Boulder, CO 80309, USA.*

27 *\*These authors contributed equally to this work.*

28 **Topological magnetic monopoles, also known as hedgehogs or Bloch points, are three-**  
29 **dimensional (3D) nonlocal spin textures that are robust to thermal and quantum**  
30 **fluctuations due to their topology<sup>1-4</sup>. Understanding their properties is of both**  
31 **fundamental interest and practical applications<sup>1-9</sup>. However, it has been difficult to**  
32 **experimentally produce topological magnetic monopoles in a controlled manner and**  
33 **directly observe their 3D magnetization vector field and interactions at the nanoscale.**  
34 **Here, we report the creation of 138 stable topological magnetic monopoles at the specific**  
35 **sites of a ferromagnetic meta-lattice at room temperature. We further develop 3D soft x-**  
36 **ray vector ptychography to determine the magnetization vector and emergent magnetic**  
37 **field of the topological monopoles with a 3D spatial resolution of 10 nm. This spatial**  
38 **resolution is comparable to the magnetic exchange length of transition metals<sup>10</sup>, enabling**  
39 **us to probe monopole-monopole interactions. We find that the topological monopole pairs**  
40 **with positive and negative charges are separated by  $18.3 \pm 1.6$  nm, while the positively and**  
41 **negatively charged pairs are stabilized at comparatively longer distances of  $36.1 \pm 2.4$  nm**  
42 **and  $43.1 \pm 2.0$  nm, respectively. We also observe virtual topological monopoles created by**  
43 **magnetic voids in the meta-lattice. This work demonstrates that ferromagnetic meta-**  
44 **lattices could be used as a new platform to create and investigate the interactions and**  
45 **dynamics of topological magnetic monopoles. Furthermore, we expect that soft x-ray**  
46 **vector ptychography can be broadly applied to quantitatively image 3D vector fields in**  
47 **magnetic and anisotropic materials at the nanoscale.**

48       The 3D ferromagnetic meta-lattice was synthesized by self-assembly of a face-centred  
49 cubic template using silica nanospheres of 60 nm in diameter (Methods). The interstitial spaces

50 between the nanospheres of the template were infiltrated with nickel to create a meta-lattice,  
51 comprising octahedral and tetrahedral sites interconnected by thin necks<sup>11,12</sup>. Superconducting  
52 quantum interference device measurements show that the saturation magnetization of the meta-  
53 lattice is consistent with that of the nickel thin film (Extended Data Fig. 1). The complex 3D  
54 curved surfaces of the silica nanospheres in the meta-lattice creates a magnetically frustrated  
55 configuration that harbours topological spin textures. To quantitatively characterize the  
56 topological spin textures, we developed 3D soft x-ray vector ptychography for the simultaneous  
57 determination of the electron density and the magnetization vector field without requiring any  
58 prior knowledge of the sample. This represents a significant advantage over previous methods  
59 that either rely on *a priori* assumptions<sup>13</sup> or use Maxwell's equations as a constraint<sup>14-16</sup> to image  
60 the 3D vector field. Furthermore, by taking advantage of the high differential magnetic contrast  
61 at the  $L_3$ -edge resonance of transition metals<sup>13,17</sup>, we demonstrated soft x-ray vector  
62 ptychography with a 3D spatial resolution of 10 nm, which is close to the magnetic exchange  
63 length of transition metals<sup>10</sup> and an order of magnitude higher than that of hard x-ray vector  
64 tomography<sup>2,9</sup>.

65 The experiment was conducted by focusing circularly polarized soft x-rays onto the  
66 ferromagnetic meta-lattice (Fig. 1). The magnetic contrast of the sample was obtained by using  
67 x-ray magnetic circular dichroism<sup>13,17,18</sup> and tuning the x-ray energy to the  $L_3$ -edge of nickel<sup>19</sup>.  
68 To separate the magnetic contrast from the electron density, two independent measurements  
69 were made with left- and right-circularly polarized soft x-rays. In each measurement, three  
70 independent tilt series were acquired from the sample, corresponding to three in-plane rotation  
71 angles ( $0^\circ$ ,  $120^\circ$  and  $240^\circ$ ) around the  $z$ -axis (Fig. 1 and Extended Data Fig. 2). Each tilt series  
72 was collected by rotating the sample around the  $x$ -axis with a tilt range from  $-62^\circ$  to  $+61^\circ$ . At  
73 each tilt angle, a focused x-ray beam was scanned over the sample with partial overlap between  
74 adjacent scan positions and a far-field diffraction pattern was recorded by a charge-coupled

75 device camera at each scan position (Methods). The full data set consists of six tilt series with  
76 a total of 796,485 diffraction patterns.

77 The diffraction patterns were reconstructed using a regularized ptychographic iterative  
78 engine<sup>20</sup>, where corrupted diffraction patterns were removed and phase unwrapping was  
79 implemented (Methods, Extended Data Fig. 3). Each pair of left- and right-circularly polarized  
80 projections was aligned and converted to the optical density for normalization. The sum of each  
81 pair of the oppositely polarized projections produced three independent tilt series  
82 corresponding to three in-plane rotation angles. The scalar tomographic reconstruction was  
83 performed from the three tilt series of 91 projections using a real space iterative algorithm  
84 (Methods), which can optimize the reconstruction by iteratively refining the spatial and angular  
85 alignment of the projections. Quantitative characterization of the reconstructed 3D electron  
86 density and a scanning transmission electron microscopy image of the sample indicates that,  
87 although there are some imperfections, the meta-lattice has an ordered face-centred cubic  
88 structure (Extended Data Figs. 4 and 5a, b). To determine the magnetization vector field, we  
89 took the difference of the left- and right-circularly polarized projections of the three tilt series  
90 (Extended Data Fig. 6). The 3D vector reconstruction was performed from 91 difference  
91 projections by least-squares optimization with gradient descent (Methods). Supplementary  
92 Video 1 shows the 3D electron density and magnetization vector field of the ferromagnetic  
93 meta-lattice. To validate the 3D vector reconstruction and quantify the spatial resolution, we  
94 divided all the projections into two halves by choosing alternate projections and performed two  
95 independent 3D vector reconstructions. By calculating the Fourier shell correlation from the  
96 two independent reconstructions, we quantified that a spatial resolution of 10 nm was achieved  
97 for the 3D vector reconstruction of the magnetization field (Methods and Extended Data Fig.  
98 7).

99 Next, we applied topological theory<sup>21</sup> to quantitatively analyse the experimental 3D

100 magnetization vector field and identify nonlocal spin textures. In 3D magnetic systems, a  
 101 topological magnetic monopole within a bulk  $\Omega$  follows the bulk-surface relationship<sup>4</sup> (i.e., the  
 102 divergence theorem),

$$103 \quad Q = \int_{\Omega} \rho \, dx dy dz = \int_{\partial\Omega} \mathbf{B}_e \cdot d\mathbf{S}, \quad (1)$$

104 where  $Q$  is the topological charge with the charge density  $\rho = \frac{3}{4\pi} \partial_x \mathbf{n} \cdot (\partial_y \mathbf{n} \times \partial_z \mathbf{n})$ ,  $\partial\Omega$  is the  
 105 bounding surface,  $\mathbf{n}$  is the normalized magnetization vector field,  $B_e^i = \frac{1}{8\pi} \epsilon^{ijk} \mathbf{n} \cdot (\partial_j \mathbf{n} \times$   
 106  $\partial_k \mathbf{n})$  is the emergent magnetic field, and  $\epsilon^{ijk}$  is the Levi-Civita symbol.  $\mathbf{B}_e$  acts on  
 107 (quasi)particles such as electrons and magnons moving through the magnetic texture as long as  
 108 they carry a spin<sup>3</sup>. The right-hand side of Eq. (1) is commonly used to evaluate the skyrmion  
 109 number in a 2D plane<sup>22,23</sup>, but can be generalized to any 3D embedded surface. When the  
 110 magnetization vector on the surface of a sphere enclosing a volume  $\Omega$  covers the orientational  
 111 parameter space exactly once, we have the topological charge  $Q = \pm 1$ , where +1 and -1  
 112 represent positively and negatively charged monopoles, respectively. It is important to note  
 113 that skyrmions and topological magnetic monopoles are fundamentally different spin textures.  
 114 Skyrmions are local textures and can be annihilated by shrinking their cores down to the lattice  
 115 constant without affecting the spin states far away<sup>22,23</sup>. In contrast, topological magnetic  
 116 monopoles are nonlocal spin textures and robust to local fluctuations<sup>1-4</sup>. They are topologically  
 117 conserved, i.e. the bulk-surface relationship of equation (1), even when the system is not well-  
 118 ordered. Topological monopoles can only be removed by the outflow of a topological current  
 119 through the boundary or annihilated in pairs of opposite-charged monopoles.

120 Although we used the normalized magnetization vector field ( $\mathbf{n}$ ) in this study, equation  
 121 (1) holds even when  $\mathbf{n}$  varies in its magnitude<sup>4</sup>. To apply equation (1) to the meta-lattice, we  
 122 computed the local maxima and minima of the topological charge density within the bulk of  
 123 the sample. At each local extremum, we defined an enclosed surface and calculated the  
 124 topological charge (Methods). Figure 2a and Supplementary Video 2 show the 3D spatial

125 distribution of 68 and 70 topological magnetic monopoles with positive (red dots) and negative  
126 (blue dots) charges in the meta-lattice. Although there are twice as many tetrahedral than  
127 octahedral sites in the meta-lattice, we found that more magnetic monopoles exist in the  
128 octahedral than the tetrahedral sites (Extended Data Table 1). Figure 2b and d show two  
129 representative topological magnetic monopoles with a positive and negative charge located in  
130 an octahedral and tetrahedral site, respectively. The 3D magnetization vector field of the  
131 topological magnetic monopoles is shown in Fig. 2c and e. The sign of the charge is not apparent  
132 from the 3D magnetization vector field, but can be unambiguously observed from the emergent  
133 magnetic field (Extended Data Fig. 8a and b).

134         The existence of a large number of topological magnetic monopoles in the  
135 ferromagnetic meta-lattice allowed us to investigate their interactions. According to monopole  
136 confinement theory<sup>4</sup>, the potential energy of a monopole pair with a positive and negative charge  
137 grows linearly with their separation when the exchange energy dominates, with all the emergent  
138 magnetic field lines emanating from the positive charge and ending at the negative charge. A  
139 non-negligible pair separation indicates the existence of other interactions competing with the  
140 exchange energy. Figure 3a shows a representative topological monopole pair with a positive  
141 and negative charge, where the emergent magnetic field lines were computed from the  
142 magnetization vector field using Eq. (1). We observed that only part of the magnetic flux  
143 emanating from the positive charge terminates at the negative charge, indicating that the  
144 emergent magnetic field lines are not completely confined. In comparison, the emergent  
145 magnetic field lines in similarly charged pairs exhibit repulsive interactions (Fig. 3b and c).  
146 The distance of the topological magnetic monopole pairs with positive and negative charges  
147 was fit to be  $18.3 \pm 1.6$  nm (Fig. 3d), while the positively and a negatively charged pairs were  
148 stabilized at longer distances of  $36.1 \pm 2.4$  nm and  $43.1 \pm 2.0$  nm (Fig. 3e and f), respectively.  
149 The statistically significant difference in the nearest-neighbour distance between oppositely and

150 similarly charged magnetic monopoles is consistent with theory<sup>4</sup>. To examine if the  
151 imperfections in the sample affect the interactions of the topological magnetic monopoles, we  
152 chose a more ordered region in the meta-lattice and plotted the histogram of the nearest-  
153 neighbour distance between oppositely and similarly charged magnetic monopoles in the  
154 region (Extended Data Fig. 5), which agrees with that obtained from a larger region including  
155 some imperfections (Fig. 3d-f). The consistency of the two histograms confirms that the  
156 structural imperfections in the meta-lattice do not play a significant role in influencing the  
157 interactions of the topological magnetic monopoles.

158         According to the bulk-surface relationship of equation (1), the topological charge can  
159 be computed on an arbitrary surface. In the meta-lattice, the silica nanospheres are magnetic  
160 voids and create 3D internal surfaces within the ferromagnetic network. To calculate the  
161 topological charge on these 3D surfaces, we performed a non-convex triangulation of the  
162 internal structure of the meta-lattice. The resulting facets were grouped into individual void  
163 surfaces by a community-clustering technique used in network analysis<sup>24</sup>. In accordance with the  
164 literature<sup>25</sup>, we defined any void surface with  $|Q| \geq 0.9$  as a virtual topological magnetic  
165 monopole. Figure 2a and Supplementary Video 2 show the distribution of 8 and 11 virtual  
166 topological monopoles with positive (red surface) and negative charges (blue surface) in the  
167 ferromagnetic meta-lattice. Two representative virtual topological monopoles with  $Q = 1.01$   
168 and  $-1$  are shown in Fig. 4a and b, respectively. The 3D magnetization vector field on the two  
169 magnetic voids was mapped onto a 2D plane to produce two stereographic projections,  
170 exhibiting skyrmion and antiskyrmion configurations (Fig. 4c and d). For the virtual  
171 topological monopole with a positive charge, most spins point down in the centre and up at the  
172 boundary, while for the virtual monopole with a negative charge, most spins point up in the  
173 centre and down at the boundary. The emergent magnetic field of the virtual topological  
174 monopoles (Extended Data Fig. 8c and d) shows features as if real topological monopoles

175 reside at the geometric centres of the magnetic voids, which is a clear manifestation of the bulk-  
176 surface correspondence.

177         Compared to materials systems that usually support topological defects, such as non-  
178 centrosymmetric lattices and magnetic / heavy-metal multilayers<sup>1,22,23</sup>, the ferromagnetic meta-  
179 lattice studied does not possess strong anisotropy or the Dzyaloshinskii-Moriya interaction.  
180 However, surface curvature can stabilize magnetic solitons through the effective  
181 Dzyaloshinskii-Moriya interaction<sup>26,27</sup>. The complex 3D curved surface of the magnetic voids  
182 induces strong magnetostatic frustration in the ferromagnetic meta-lattice, which harbours  
183 topological magnetic monopoles at the octahedral and tetrahedral sites of the lattice and the  
184 associated skyrmion textures on the surface of the magnetic voids. Using our experimental data  
185 as direct input to atomistic simulations, we numerically demonstrated that topological magnetic  
186 monopoles can be stabilized by the boundary conditions (Methods). We extracted four  
187  $15 \times 15 \times 15 \text{ nm}^3$  volumes from the ferromagnetic meta-lattice, containing four topological  
188 magnetic monopoles with two positive and two negative charges. The atomistic spins on the  
189 outer boundary of each volume were fixed, while all the other spins were allowed to relax to  
190 an equilibrium configuration. After 50 ps, a stable magnetic monopole formed in each volume  
191 with a topological charge matching the experimental value (Extended Data Fig. 9a-d). We also  
192 observed that as long as the atomistic spins were fixed on four of the six surfaces of each  
193 volume, the topological magnetic monopole remained stable inside the volume (Extended Data  
194 Fig. 9e-h). These results further confirm that surface constraints can stabilize topological  
195 magnetic monopoles.

196         In conclusion, we have created and directly observed topological magnetic monopoles  
197 and their interactions in a ferromagnetic meta-lattice with a 3D spatial resolution of 10 nm.  
198 This work could open the door to use magnetically frustrated meta-lattices as a new platform  
199 to study the interactions, dynamics, and confinement-deconfinement transition of topological

200 monopoles<sup>4</sup>. Furthermore, as a powerful scanning coherent diffractive imaging method<sup>28-31</sup>, the  
201 3D spatial resolution of soft x-ray vector ptychography can be improved by increasing the  
202 incident coherent flux or the data acquisition time. With the rapid development of advanced  
203 synchrotron radiation, x-ray free electron lasers and high harmonic generation sources  
204 worldwide<sup>29</sup>, we expect that 3D vector ptychography can find broad applications in the  
205 topological spin texture, nanomagnetism and x-ray imaging fields.

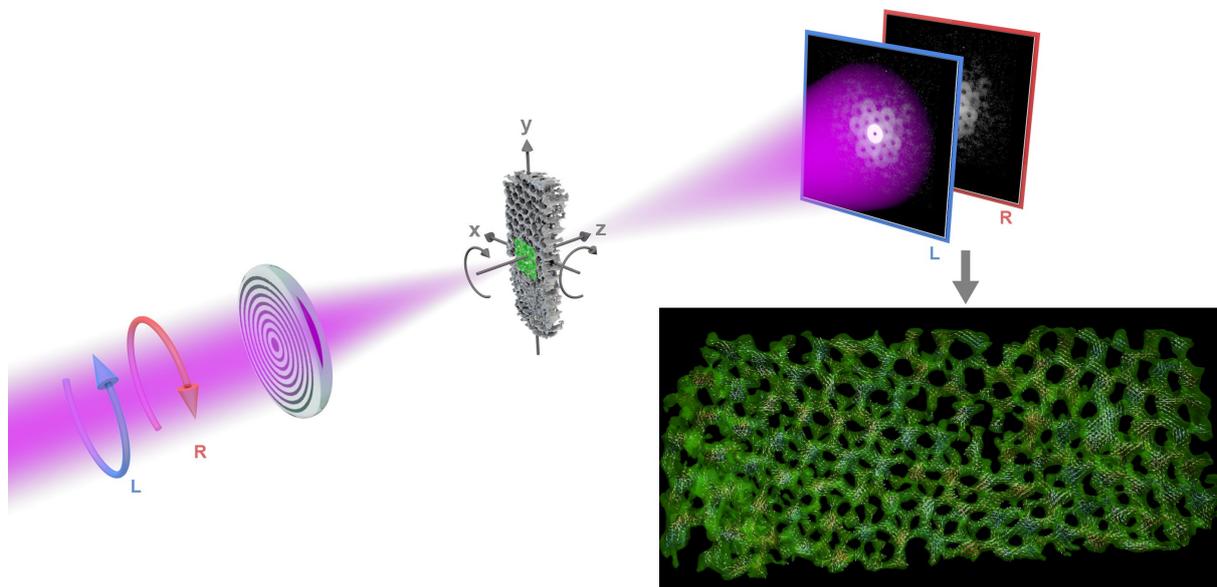
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## 268 Figures and figure legends



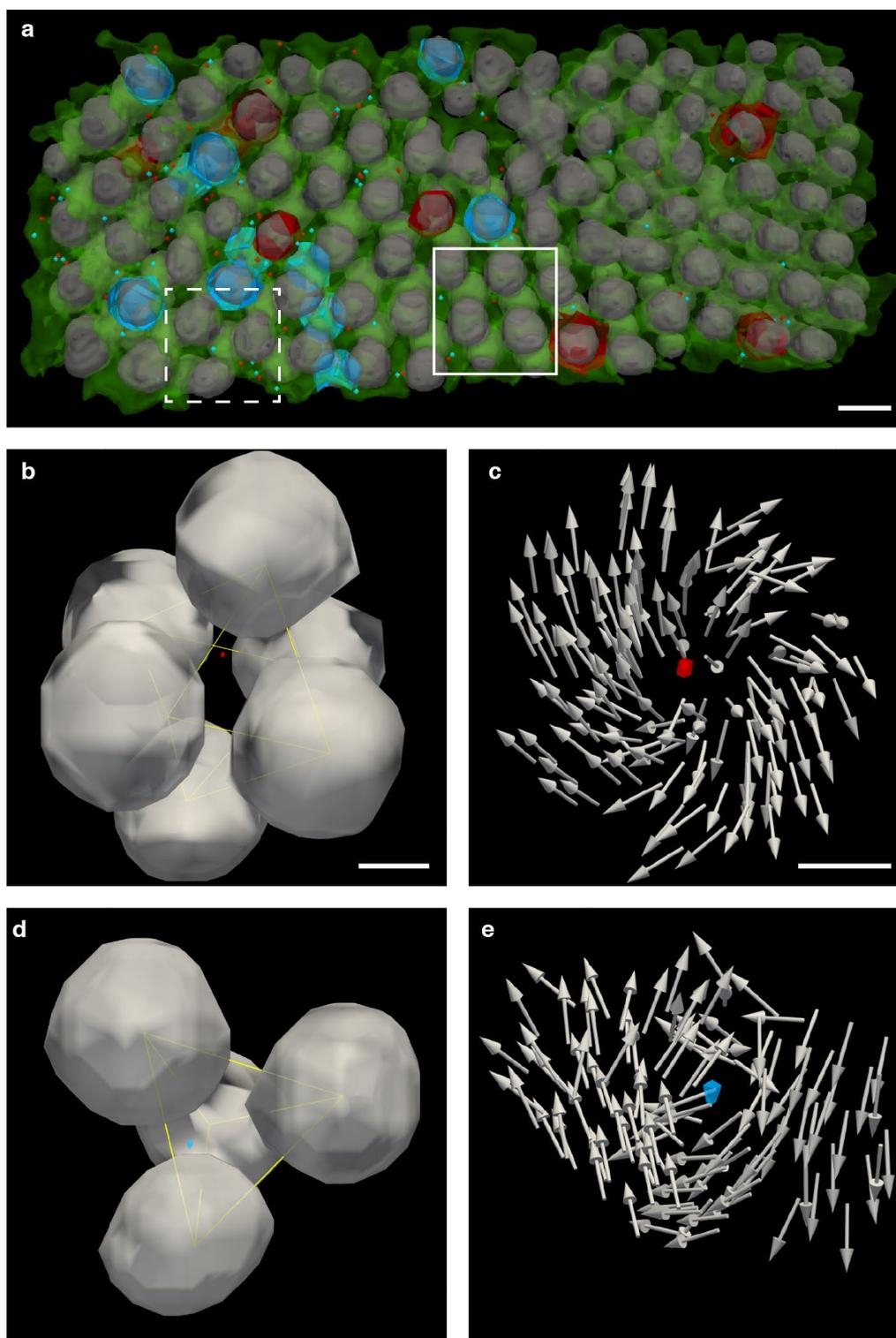
270 **Fig. 1. Experimental schematic of 3D soft x-ray vector ptychography.** Left- and right-

271 circularly polarized x-rays (pink) were focused onto a ferromagnetic meta-lattice sample

272 (centre), on which the green circles indicate the partially overlapped scan positions. The sample

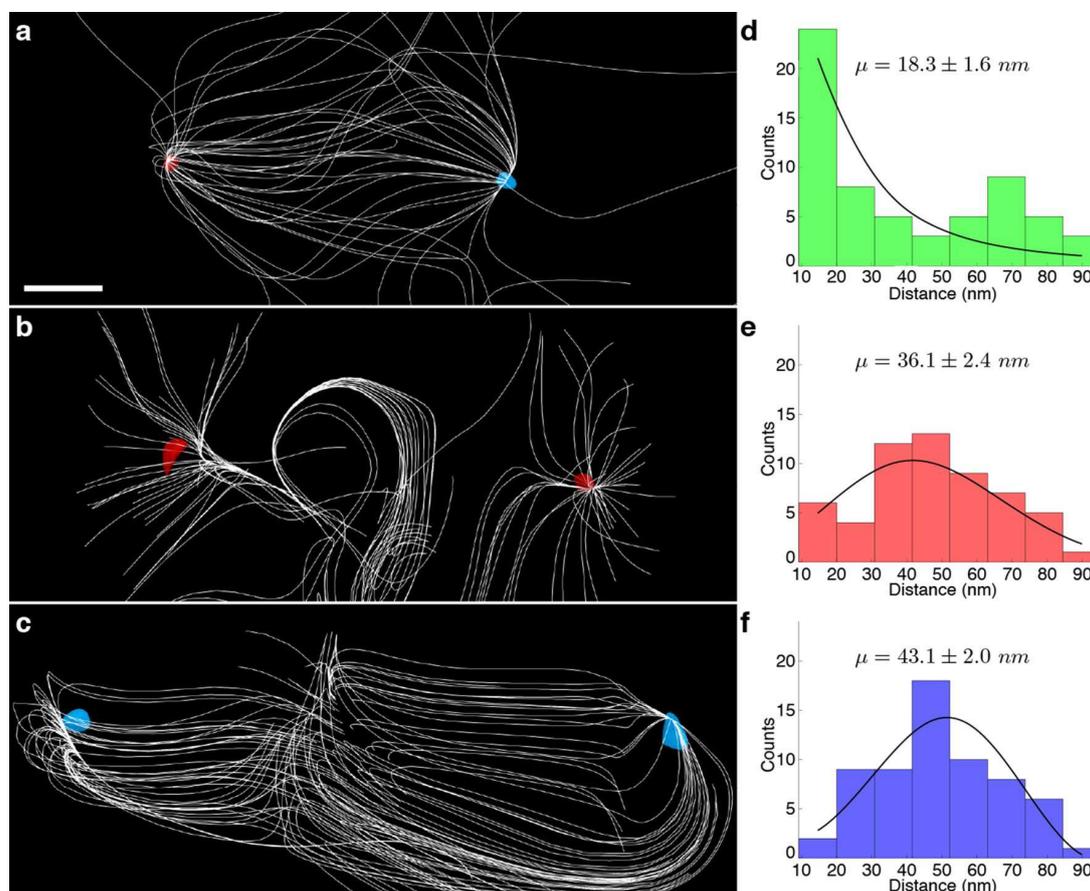
273 was tilted around the x- and z-axis and diffraction patterns were collected by a charge-coupled

274 device camera. The lower right structure shows the 3D electron density (green) and  
275 magnetization vector field (arrows) of the meta-lattice reconstructed from the diffraction  
276 patterns.



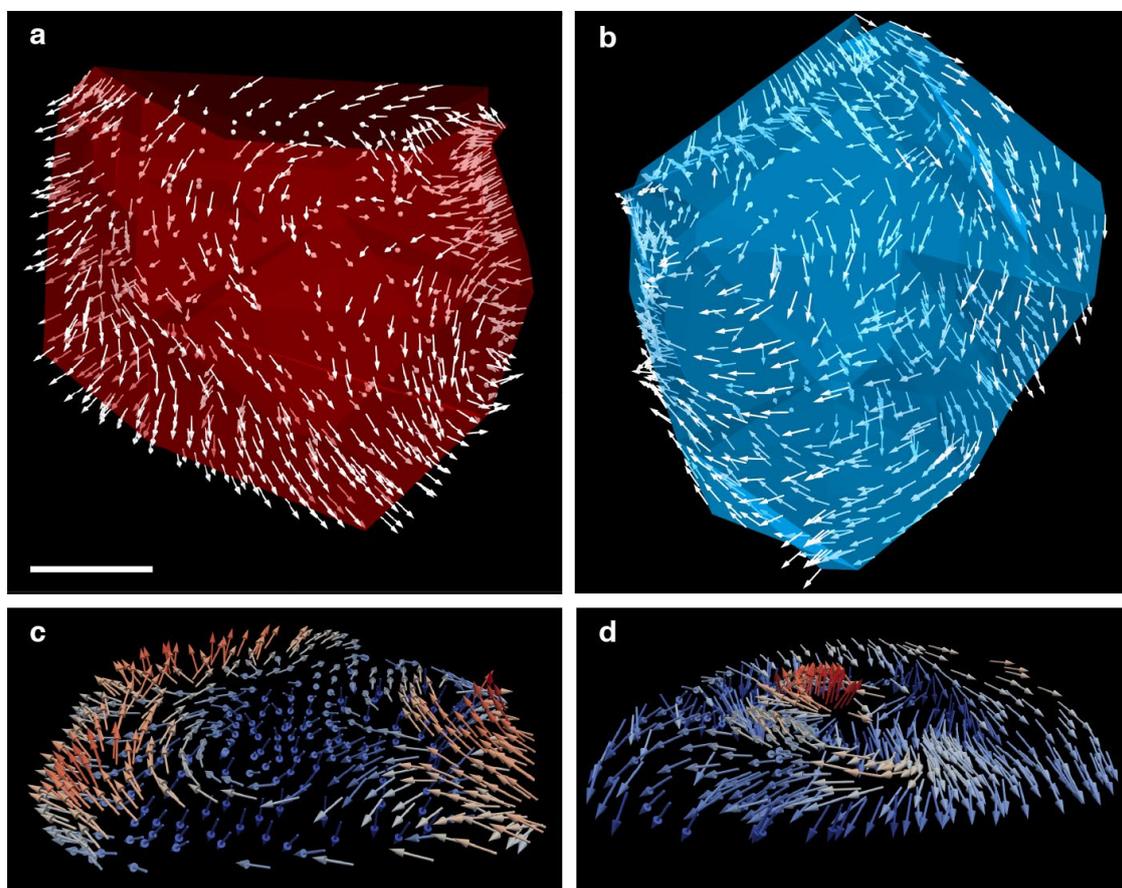
278 **Fig. 2. Quantitative 3D characterization of topological magnetic monopoles in the**

279 **ferromagnetic meta-lattice.** **a**, 3D spatial distribution of 68 and 70 topological monopoles  
 280 with positive (red dots) and negative charges (blue dots) in the meta-lattice, where the surfaces  
 281 of the magnetic voids in red and blue represent virtual topological monopoles with positive and  
 282 negative charges, respectively. The solid and dashed squares mark the region of interest shown  
 283 in **(b)** and **(d)**, respectively. **b, c**, The location and 3D spin textures of a positively charged  
 284 topological monopole within a tetrahedral site of the face-centered cubic meta-lattice. **d, e**, The  
 285 location and 3D spin textures of a negatively charged topological monopole within an  
 286 octahedral site. Scale bars, 60 nm **(a)**; 25 nm **(b)**; and 10 nm **(c)**.



288 **Fig. 3. Interactions of the topological magnetic monopoles in the ferromagnetic meta-**  
 289 **lattice.** **a-c**, Three representative topological monopole pairs with a positive and negative  
 290 charge **(a)**, two positive **(b)** and two negative charges **(c)**, where the white lines represent the  
 291 emergent magnetic field lines. **d-f**, Histograms of the nearest-neighbour distance for the  
 292 topological monopole pairs with a positive and negative charge **(d)**, two positive **(e)** and two

293 negative charges (**f**). The three histograms were fit to a generalized extreme value distribution,  
 294 producing three curves in (**d-f**), where  $\mu$  represents the centre of each fit and the standard error  
 295 was determined from the fit's confidence interval. Scale bar, 5 nm.



297 **Fig. 4. Representative virtual topological monopoles in the ferromagnetic meta-lattice. a,**  
 298 **b,** Two virtual topological monopoles with  $Q = 1.01$  and  $-1$ , respectively, where the arrows  
 299 indicate the 3D magnetization vector field. **c, d,** Stereographic projections of the virtual  
 300 topological monopoles shown in (**a**) and (**b**), respectively, where the colours of the arrows  
 301 represents the z-component of the spin with pointing up (+z) in red and down (-z) in blue. Scale  
 302 bar, 15 nm.

### 303 METHODS

304 **Sample synthesis and preparation.** The 3D ferromagnetic meta-lattice was synthesized by infiltrating  
 305 interconnected voids of a silica nanoparticle template using confined chemical fluid deposition<sup>12</sup>. Monodisperse  
 306 silica nanoparticles of 60 nm in diameter (standard deviation < 5%) were synthesized using a liquid-phase  
 307 method<sup>32</sup>. The evaporation-assisted vertical deposition technique was used to assemble these particles onto silicon

308 substrate<sup>33</sup>. Briefly, 3 cm x 1 cm silicon wafers were placed at a  $\sim 30^\circ$  angle in open plastic vials containing 10x  
309 dilute solution of the as-synthesized particles. The vials were left undisturbed for two weeks in an oven maintained  
310 at  $40^\circ\text{C}$  at 80% relative humidity. The resulting films that were used as the template for nickel infiltration  
311 contained silica particles arranged in a face-centred cubic structure and had thicknesses ranging from 240 nm –  
312 850 nm depending on the vertical position of the silicon substrate<sup>34</sup>.

313 The infiltration of nickel within the template voids was performed using confined chemical fluid  
314 deposition<sup>12</sup>. The template was spatially confined using a 250  $\mu\text{m}$  thick U-shaped titanium spacer and placed  
315 within a custom-built reactor made of parts from High Pressure Equipment Company, McMaster, and Swagelok.  
316 Bis(cyclopentadienyl) nickel (II) was loaded into the reactor in a Vacuum Atmospheres argon glovebox. The  
317 reactor was pressurized with Praxair 4.0 Industrial Grade carbon dioxide using a custom-made manual pump and  
318 heated to  $70^\circ\text{C}$  for 8 hours at a pressure of around 13.8 MPa to dissolve the precursor powder into the supercritical  
319 carbon dioxide. A separate gas reservoir was loaded with Praxair 5.0 ultra-high purity hydrogen using a Newport  
320 Scientific Two Stage 207 MPa Diaphragm Pump and was connected to the reactor. The hydrogen was added to  
321 the reactor to a final reactor pressure of 42.7 MPa and the deposition proceeded at  $100^\circ\text{C}$  for 10 hours. The  
322 interstitial voids between the nanospheres of the template were then infiltrated with nickel, forming a meta-lattice.  
323 An overfilled nickel film over the meta-lattice and template resulting from the deposition process was milled using  
324 a Leica EM TIC 3x Argon ion beam milling system at 3 degrees and 3 kV.

325 To prepare the sample with the correct geometry for the 3D vector ptychography experiment, we lifted  
326 out a portion of the sample from the bulk meta-lattice on a silicon substrate and thinned the sample using a focused  
327 ion beam (FIB, FEI Nova 600 NanoLab DualBeam), which was equipped with a field emission scanning electron  
328 microscope and a scanning gallium ion beam. The FIB prepared sample was mounted on a 3-mm TEM grid  
329 (Omniprobe, 3 posts copper lift-out grid), where the central post was also trimmed by FIB milling to increase the  
330 tilt range. The sample mounted on the TEM grid was examined by the scanning electron microscope and an optical  
331 microscope, and then manually glued on a 3-mm copper ring using a silver paste (Extended Data Fig. 2a-f). The  
332 sample fabricated by this process can be manually rotated in-plane for the 3D vector ptychography experiment.  
333 To examine the surface oxidation of the sample, we conducted an x-ray absorption spectroscopy experiment of  
334 the Ni meta-lattice. By carefully analysing the x-ray absorption spectrum in comparison with that of a pure Ni  
335 film and a NiO film<sup>35</sup> (Extended Data Fig. 2g), we concluded that the surface oxide layer of the sample is very  
336 thin, which is consistent with the previous experimental measurements<sup>36</sup>.

337 **The 3D soft x-ray vector ptychography experiment.** The experiment was conducted at the COSMIC beam line  
338 at the Advanced Light Source, Lawrence Berkeley National Lab<sup>37</sup>. Figure 1 shows the experimental schematic of  
339 3D soft x-ray vector ptychography. An elliptical polarization undulator was used to generate circularly polarized  
340 x-rays of left- and right-helicity and achieve differential contrast enhancement of the magnetic signal. The incident  
341 photon energy was tuned to 856 eV, slightly above the nickel  $L_3$  edge, to obtain the magnetic contrast based on  
342 x-ray magnetic circular dichroism<sup>13,17-19,38</sup>. The polarized beam was focused onto the sample by a Fresnel zone  
343 plate with an outer width of 45 nm. A total of six tilt series with a tilt range from  $-62^\circ$  to  $+61^\circ$  were acquired from  
344 the sample with left- and right-circularly polarized x-rays at three in-plane rotation angles ( $0^\circ$ ,  $120^\circ$  and  $240^\circ$ ). At  
345 each tilt angle, the focused beam was raster-scanned across the sample in 40 nm steps. Diffraction patterns were  
346 collected using both left- and right-circularly polarized x-rays. A charge-coupled device camera was used to record  
347 the diffraction patterns at each scan position. Initial reconstructions were performed on-site in real time using a

348 GPU-based ptychography reconstruction algorithm<sup>39</sup>.

349 **Data processing and ptychographic reconstructions.** A very small number of corrupted diffraction patterns,  
 350 most commonly caused by detector readout malfunction or unstable beam flux, resulted in a global degradation  
 351 of the reconstruction through the coupling of the probe and object. We used the following procedure to  
 352 automatically detect and remove the corrupted diffraction patterns to achieve the high-quality reconstruction. The  
 353 high-angle diffraction intensity at each scan position was integrated to produce a low-resolution map at every  
 354 ptychography scan. Local maxima in the magnitude of the gradient of this map were used to identify and remove  
 355 bad frames (Extended Data Fig. 3a and b). The image reconstructions were performed by using the regularized  
 356 ptychographic iterative engine<sup>20</sup> coupled with phase unwrapping for high tilt angles<sup>40</sup> (Extended Data Fig. 3c and  
 357 d). Specifically, for the first 10 iterations of the ptychographic reconstruction, no phase unwrapping was enforced.  
 358 After that, phase unwrapping was applied to the object in every 3<sup>rd</sup> iteration<sup>40</sup>. The final reconstruction was  
 359 obtained with a total of 500 iterations.

360 From the reconstructed complex-valued exit wave, the absorption component was used as the magnetic  
 361 contrast<sup>17,18</sup> and the two oppositely-polarized projections at each tilt angle were aligned using a feature-based  
 362 image registration package in MATLAB. The projections were converted to optical density<sup>41</sup> to normalize any  
 363 small temporal and polarization-based fluctuations of the beam intensity. In each projection, background  
 364 subtraction was performed by numerically evaluating Laplace's equation,

$$365 \quad \nabla^2 \varphi = 0 \quad , \quad (2)$$

366 where  $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$  is the 2D Laplace operator and  $\varphi$  represents the background of the projection. To determine  
 367  $\varphi$  we solved equation (2) by using the region exterior to the sample as the boundary condition. The value at the  
 368 boundary corresponds to the optical density in vacuum. Mathematically, the calculation of  $\varphi$  is equivalent to the  
 369 determination of the geometry of a soap film from an enclosed boundary. We implemented this procedure by  
 370 using a MATLAB function called 'regionfill'. We found that this method outperforms simple constant background  
 371 subtraction by taking into account the local variation of the background<sup>42</sup>.

372 **The scalar tomography reconstruction.** The relationship between charge and magnetic scattering<sup>17,18,43</sup>,

$$f = f^c \pm i f^m \hat{z} \cdot \mathbf{m} \quad , \quad (3)$$

373 was used to generate a set of scalar and vector projections corresponding to the charge and magnetic scattering,  
 374 where  $f^c$  and  $f^m$  are the charge and magnetic scattering factor, respectively,  $\hat{z}$  is the x-ray propagation direction,  
 375 and  $\mathbf{m}$  is the magnetization vector. The sum of each pair of the oppositely-polarized projections produced three  
 376 independent tilt series corresponding to three in-plane rotation angles. The scalar projections of each tilt series were  
 377 first roughly aligned with cross-correlation, then more accurately aligned using the centre-of-mass and common  
 378 line method<sup>44</sup>. The aligned tilt series was reconstructed by a real space iterative reconstruction (RESIRE)  
 379 algorithm<sup>42,45</sup>, which was able to iteratively perform angular and spatial refinement to adjust any remaining small  
 380 alignment errors. From the three independent reconstructions, transformation matrices were computed to align the  
 381 three tilt series to a global coordinate system. The three aligned tilt series were collectively reconstructed by RESIRE  
 382 using the same angular and spatial refinement procedure, which produced the final scalar tomography  
 383 reconstruction. The transformation matrices obtained from the scalar tomography were used for the vector  
 384 tomography reconstruction.

385 **The vector tomography reconstruction.** The 3D vector magnetization field was reconstructed by taking the  
 386 difference of the left- and right-circularly polarized projections of the six experimental tilt series, producing three  
 387 independent tilt series with the magnetic contrast. The vector tomography algorithm is modelled as a least squares  
 388 optimization problem and solved directly by gradient descent. The least squares problem is given as,

$$\begin{aligned} \min_{O_1, O_2, O_3} f(O_1, O_2, O_3) &= \sum_{i=1}^N \|\alpha_i \Pi_i O_1 + \beta_i \Pi_i O_2 + \gamma_i \Pi_i O_3 - b_i\|^2 \\ &= \sum_{i=1}^N \|\Pi_i(\alpha_i O_1 + \beta_i O_2 + \gamma_i O_3) - b_i\|^2 \quad , \quad (4) \end{aligned}$$

389 where  $O_1, O_2, O_3$  are the three components of the vector field to be reconstructed,  $N$  is the number of the  
 390 projections of the three tilt series,  $\Pi_i$  is the projection operator with respect to the Euler angle set  $\{\phi_i, \theta_i, \psi_i\}$ , and  $b_i$   
 391 is the experimentally measured projection.  $\{\alpha_i, \beta_i, \gamma_i\}$  are the coefficient set with respect to the projection operator  
 392 and are related to the corresponding Euler angle set by,

$$\alpha_i = \sin \theta_i \cos \phi_i, \quad \beta_i = \sin \theta_i \sin \phi_i, \quad \gamma_i = \cos \theta_i \quad . \quad (5)$$

393 The least square problem is solved via gradient descent and the gradients are computed by,

$$\begin{aligned} \nabla_{O_1} f(O_1, O_2, O_3) &= \sum_{i=1}^N \alpha_i \Pi_i^T \Pi_i(\alpha_i O_1 + \beta_i O_2 + \gamma_i O_3) \\ \nabla_{O_2} f(O_1, O_2, O_3) &= \sum_{i=1}^N \beta_i \Pi_i^T \Pi_i(\alpha_i O_1 + \beta_i O_2 + \gamma_i O_3) \quad . \quad (6) \\ \nabla_{O_3} f(O_1, O_2, O_3) &= \sum_{i=1}^N \gamma_i \Pi_i^T \Pi_i(\alpha_i O_1 + \beta_i O_2 + \gamma_i O_3) \end{aligned}$$

394 The  $(j+1)^{\text{th}}$  iteration of the algorithm is updated as,

$$\begin{aligned} O_1^{j+1} &= O_1^j - t \nabla_{O_1} f(O_1, O_2, O_3) = O_1^j - t \sum_{i=1}^N \alpha_i \Pi_i^T \Pi_i(\alpha_i O_1^j + \beta_i O_2^j + \gamma_i O_3^j) \\ O_2^{j+1} &= O_2^j - t \nabla_{O_2} f(O_1, O_2, O_3) = O_2^j - t \sum_{i=1}^N \beta_i \Pi_i^T \Pi_i(\alpha_i O_1^j + \beta_i O_2^j + \gamma_i O_3^j) \quad , \quad (7) \\ O_3^{j+1} &= O_3^j - t \nabla_{O_3} f(O_1, O_2, O_3) = O_3^j - t \sum_{i=1}^N \gamma_i \Pi_i^T \Pi_i(\alpha_i O_1^j + \beta_i O_2^j + \gamma_i O_3^j) \end{aligned}$$

395 where  $t$  is the step size. For a given tilt angle set  $\{\phi_i, \theta_i, \psi_i\}$ , the forward projection of a 3D object is computed  
 396 using the Fourier slice theorem, while the back projection is implemented by linear interpolation.

397 To validate the vector tomography reconstruction algorithm, we simulated 3D topological magnetic  
 398 monopoles and calculated their diffraction patterns based on the experimental parameters. After adding noise to  
 399 the diffraction patterns, we performed ptychographic reconstructions to generate projections. Using the vector  
 400 tomography reconstruction algorithm, we were able to reconstruct the 3D magnetization vector field of the  
 401 topological monopoles from the projections. After validating the vector tomography algorithm using simulated  
 402 data, we applied it to reconstruct the 3D magnetization vector field of the ferromagnetic meta-lattice from the  
 403 experimentally measured tilt series.

404 **Quantification of the 3D spatial resolution.** We quantified the spatial resolution using two independent methods.  
 405 First, we divided the 91 projections of three tilt series into two halves by choosing alternate projections and

406 conducted two independent 3D scalar reconstructions, from which two different supports were generated to  
 407 separate the nickel from the silica region. We then performed two independent vector reconstructions from the  
 408 two halves. After applying the support to exclude the silica region, we calculated the Fourier shell correlation  
 409 (FSC) from the two 3D vector reconstructions. Extended Data Fig. 7a-f shows the FSC for  $|m_x|$ ,  $|m_y|$ ,  $|m_z|$ ,  
 410  $|m_{xy}|$ ,  $|m_{xz}|$  and  $|m_{yz}|$ , respectively, where  $m_x$ ,  $m_y$ , and  $m_z$  are the  $x$ -,  $y$ -, and  $z$ -component of the unnormalized  
 411 magnetization vector field and  $|m_{xy}| = \sqrt{m_x^2 + m_y^2}$ ,  $|m_{xz}| = \sqrt{m_x^2 + m_z^2}$  and  $|m_{yz}| = \sqrt{m_y^2 + m_z^2}$ . As  $m_x$ ,  $m_y$ ,  
 412 and  $m_z$  have both positive and negative values (Supplementary Video 1), their Fourier coefficients in some  
 413 resolution shells have small values. To avoid dividing by small values, we computed the FSC for the magnitude  
 414 of  $m_x$ ,  $m_y$  and  $m_z$ . According to the cut-off of FSC = 0.143, a criterion commonly used in cryo-electron  
 415 microscopy<sup>46</sup>, we characterized the 3D spatial resolution of the vector reconstruction to be 10 nm. We noted that  
 416 the FSC values for  $|m_z|$  are slightly smaller than 0.143 at some high spatial frequency (Extended Data Fig. 7c).  
 417 This was because only a half of the projections were used to perform each 3D vector reconstruction. Compared  
 418 to cryo-electron microscopy that employs a large number of images for a 3D reconstruction<sup>46</sup>, the number of  
 419 projections in our experiment is much smaller. Thus, when only a half of the projections were used for the vector  
 420 reconstruction, the spatial resolution was reduced especially along the beam ( $z$ ) direction. Second, we quantified  
 421 three pairs of topological magnetic monopoles with positive and negative charges distributed along the  $x$ -,  $y$ - and  
 422  $z$ -axis in the 3D vector reconstruction (Extended Data Fig. 7g-o). The net topological charge of each monopole  
 423 pair was calculated to be  $Q = 0$ , while the topological charge of the positive and negative monopole in each pair  
 424 was computed to be  $Q = +1$  (red dot) and  $-1$  (green dot), respectively. The distance between the red and green dot  
 425 in each pair is 2 voxels with a voxel size of 5 nm, further demonstrating that a spatial resolution of 10 nm was  
 426 achieved along the  $x$ -,  $y$ - and  $z$ -axis.

427 **Calculation of the topological magnetic monopole density and charge.** We first calculated the topological  
 428 charge density of every voxel ( $5 \times 5 \times 5 \text{ nm}^3$ ) within the bulk of the meta-lattice by discretizing the expression  $\rho =$   
 429  $\frac{3}{4\pi} \partial_x \mathbf{n} \cdot (\partial_y \mathbf{n} \times \partial_z \mathbf{n})$  on a cubic lattice, producing a 3D map of the local maxima (positive) and minima (negative)  
 430 of the charge density. At each local extremum, we chose  $3 \times 3 \times 3$  vectors surrounding the local extremum. To  
 431 compute the topological charge enclosed by these vectors, we triangulated the surface and calculated the solid  
 432 angle ( $\omega$ ) of each triangle surface subtended by three vectors ( $\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3$ ),

$$433 \quad \tan \frac{\omega}{2} = \frac{\mathbf{n}_1 \cdot (\mathbf{n}_2 \times \mathbf{n}_3)}{1 + \mathbf{n}_1 \cdot \mathbf{n}_2 + \mathbf{n}_1 \cdot \mathbf{n}_3 + \mathbf{n}_2 \cdot \mathbf{n}_3} \quad . \quad (8)$$

434 The topological charge was evaluated by  $Q = \frac{1}{4\pi} \sum_{\text{facets}} \omega$ , which is an integer as the summation of all solid angles  
 435 over an enclosed surface is an integer number of  $4\pi$ . We evaluated the topological charge of the magnetic voids  
 436 using the same approach.

437 **Atomistic simulations using the experimental data as direct input.** Four  $15 \times 15 \times 15 \text{ nm}^3$  volumes of the  
 438 experimentally determined 3D magnetization vector field were extracted from the ferromagnetic meta-lattice as  
 439 direct input to atomistic simulations. The four volumes contain four topological magnetic monopoles with two  
 440 positive and two negative charges and each topological magnetic monopole is located close to the centre of each  
 441 volume. A Ni fcc lattice with a lattice constant of  $3.524 \text{ \AA}$  was constructed for each volume and all atomic sites  
 442 within each  $5 \times 5 \times 5 \text{ nm}^3$  voxel were mapped to the same normalized magnetization vector determined from the

443 experiment, yielding a total of 296,352 atomistic spins in each volume. The dynamics of the individual atomistic  
 444 spins is described by the Landau-Lifshitz-Gilbert equation of motion<sup>47</sup>,

$$445 \quad \frac{\partial \mathbf{S}_i}{\partial t} = -\frac{\gamma}{\mu(1+\lambda^2)} [\mathbf{S}_i \times \mathbf{H}_{\text{eff}}^i + \lambda \mathbf{S}_i \times (\mathbf{S}_i \times \mathbf{H}_{\text{eff}}^i)] , \quad (9)$$

446 where  $\mathbf{S}_i$  is a unit vector at atomistic site  $i$ ,  $\gamma$  is the gyromagnetic ratio,  $\lambda$  is the phenomenological coupling  
 447 constant (damping) and  $\mu$  is the magnetic moment.  $\mathbf{H}_{\text{eff}}^i$ , given by equation (11), is the effective magnetic field at  
 448 site  $i$ . The total energy of the system is represented by the following atomistic spin Hamiltonian,

$$449 \quad \mathcal{H} = -J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - k_u \sum_i (S_i^z)^2 - \mu \mathbf{B} \cdot \sum_i \mathbf{S}_i , \quad (10)$$

450 where the first term on the right hand side is the exchange interaction between spins at site  $i$  and  $j$ , the second is  
 451 the uniaxial anisotropy term, and the third is the Zeeman term. The exchange constant ( $J$ ) and the magnetic  
 452 moment ( $\mu$ ) are  $2.757 \times 10^{-21}$  Joules per link and 0.606 Bohr magnetons, respectively<sup>48</sup>. The anisotropy constant,  
 453  $k_u$ , and the external field  $\mathbf{B}$  were neglected in the simulations. The above Hamiltonian can be represented as an  
 454 effective magnetic field for the spin at site  $i$  by taking the negative first derivative,

$$455 \quad \mathbf{H}_{\text{eff}}^i = -\frac{\partial \mathcal{H}}{\partial \mathbf{S}_i} . \quad (11)$$

456 Based on these equations, we performed atomistic simulations of each volume by fixing the spins on the outer  
 457 boundary of the volume. All the other spins were allowed to relax to an equilibrium configuration. After 50 ps, a  
 458 stable magnetic monopole formed in each volume with a topological charge matching the experimental value  
 459 (Extended Data Fig. 9a-d). Further simulations showed that if the atomistic spins on the outer boundary were  
 460 fixed, any random spin configuration within each volume yielded identical results. We also conducted atomistic  
 461 simulations to determine how much of the boundary can be relaxed before each topological magnetic monopole  
 462 becomes unstable. We found that as long as the atomistic spins were fixed on four of the six surfaces of each  
 463 volume, the topological magnetic monopole remained stable inside the volume (Extended Data Fig. 9e-h). All  
 464 these atomistic simulation results confirm that surface constraints can stabilize topological magnetic monopoles.

#### 465 **Data availability**

466 All the experimental data are available at <https://doi.org/10.5281/zenodo.5450910>.

#### 467 **Code availability**

468 The MATLAB source codes for the scalar and vector tomography reconstruction algorithms and data analysis  
 469 used in this work are available at <https://doi.org/10.5281/zenodo.5450910>.

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517 T.L. and S.Y. synthesized and fabricated the sample; A.R., C.-T.L., Y.H.L., E.E.C.S., S.R., X.L., C.S.B., R.M.K.,  
518 A.J.G., J.R., H.O., Y.S.Y., D.A.S, H.C.K., M.M.M. and J.M. planed and/or performed the experiments; M.P.,

519 A.R., S.J.O. and J.M. developed the scalar and vector tomography algorithms; A.R. and J.M. reconstructed the  
520 3D magnetization vector field; A.R., E.I. J.Z., X.L. and J.M. analysed the data with input from M.M.M., Y.T., C.-  
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522 E.I. and J.Z. wrote the manuscript with input from M.M.M., Y.T., C.-T.L., S.Y., E.E.C.S. and T.E.M.

523 **Competing interests** The authors declare no competing interests.

524 **Correspondence and requests for materials** should be addressed to J.M. (miao@physics.ucla.edu).

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