Dynamic equilibrium of the sustainable ecosystem variables

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Abstract

Strategies for achieving sustainability equilibrium between human activity and nature are essential to the global and local scenario. Our work highlights the importance of restoring the stability between economic activity and nature. Developing indicators to monitor this equilibrium is necessary to reduce uncertainty in formulating strategies, decisions, or actions. The Matrix decomposition analysis (MDA) adapts the Leontief input-output equations for the disaggregated structural decomposition of key performance indicators (KPI). At the farm level, three experiments denominated "marginal exponentiation" are proposed to compare the MDA with the Data envelopment analysis (DEA) and Stochastic frontier analysis (SFA). RMarkdown was used for methodological operationalization. Data science steps are coded in specific chunks, applying a layered process with modeling and successive analyses such as descriptive statistics, correlation, cluster, and Linear discriminant analysis (LDA). Given the results, we may argue that the MDA is a Leontief partial equilibrium model that produces indicators with dual interpretation, enabling measurement of the dynamic equilibrium of the sustainable ecosystem variables. The method offers a new ranking system that detects relative changes in the use of resources correlated with efficiency analysis. MDA might provide a new robust ranking system capable of detecting relative changes in the use of resources that can be applied to input-output relationships of many organizations. We found that MDA can identify if a given ecosystem is in equilibrium and that the instability can be caused by the excessive use of resources or abnormal productivity.

Keywords: benchmarking, data envelopment analysis, input-output model, matrix decomposition analysis, stochastic frontier analysis; Sustainability equilibrium

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1. Introduction

Strategies for achieving sustainability equilibrium are important to the global and local scenario (Fang, 2022). One example is the plan for sustainable economy of the EU "*The European Green Deal*", which is a growth strategy to protect biodiversity, stimulating the circular economy, sustainable food and eliminating pollution (EC, 2019-2020). According to Frans Timmermans[1], executive vice-president of the European Commission, the COVID-19 crisis revealed the vulnerability of the system and consequently the importance of restoring the equilibrium between human activity and nature.

[1] https://ec.europa.eu/info/strategy/priorities-2019-2024/european-green-deal/actions-being-taken-eu/farm-fork_en

Our work highlights the importance of restoring the equilibrium between economic activity and nature. Developing indicators to monitor the equilibrium is necessary to reduce the level of uncertainty in the formulation of strategies, decisions or actions (Warhurst, 2002; Joung et al., 2012; Hassini et al., 2012; Hák et al., 2016; Machado et al., 2017; Schmidt-Traub et al., 2017; Perroni et al., 2018; Qiu et al., 2019). The study of Perroni et al. (2020) developed a methodology derived from Leontief input-output model that may be able to monitor the evolution of sustainability through the analysis of indicators. For simplicity, we identify this methodology by the acronym MDA (Matrix Decomposition Analysis). The MDA method was originally proposed with the possibility of benchmarking analysis in various instances of sustainability science.

The literature addresses two other models that are more often used: Data envelopment analysis (DEA) and Stochastic frontier analysis (SFA). The origin of the DEA is in operational research using mathematical programming and the SFA originates from econometrics that uses regression techniques. The MDA is descendent from general economic equilibrium models that use matrix techniques (Leontief 1966; Aigner et al., 1977; Charnes et al., 1978). The main research question that guides our work is: Does the MDA offer a robust ranking system capable of complementing the DEA and SFA approaches?

The objective of this work is to systematically explore the MDA methodology proposed by Perroni et al. (2020). As complementary objectives we establish: (i) Operationalize the Matrix decomposition analysis (MDA) for multiple inputs using MS-Excel®; (ii) Develop a systematic comparison procedure with the Data envelopment analysis (DEA) and Stochastic frontier analysis (SFA); (iii) Discuss the application of the MDA methodology for the analysis of dual performance in sustainable ecosystems.

The main justification for the work is related to the fact that the Leontief approach is not fully explored in the literature for the purpose of benchmarking. In Perroni et al. (2020) the method was compared with a simple example of stochastic frontier using one input. In this work, we developed a robust experiment (marginal exponentiation), using multiple inputs. The systematic exploration of the method makes it possible to identify its advantages and disadvantages in relation to the DEA and SFA. In addition, it allows evidence of its empirical utility for monitoring performance in sustainable ecosystems.

Based on the results of the experiments, we are convinced that the Leontief partial equilibrium model or (MDA) might offer a new robust ranking system capable of detecting relative changes in the use of

resources which can be applied to input-output relationships of many organizations. Its stability was tested both by correlations with the DEA and SFA and by its discriminating power.

The remainder of this paper is divided as follows: Section 2 presents the brief literature review, Section 3 presents the experiment design, Section 4 presents the results, and Sections 5 and 6 discuss and conclude the paper, respectively.

2. Ecosystems concept and techniques for benchmarking

2.1 Ecosystem and Indicators

The indicators are part of a system, "*A system defines a set of bounded interrelated elements with emergent properties and represents it within the context of a paradigm*" (Shehabuddeen et al., 1999, p. 4). The elements might be the indicators developed within the paradigm of transdisciplinary sustainability science (TSS) (Brandt et al., 2013). Indicators are implemented in the processes: "*A process is an approach to achieving a managerial objective, through the transformation of inputs into outputs*" (Shehabuddeen et al., 1999, p. 4). Examples of methodologies that produce indicators from the input-output relationship are the traditional methods of benchmarking: Data envelopment analysis (DEA) and Stochastic frontier analysis (SFA). Matrix decomposition analysis (MDA) was recently developed in Perroni et al. (2020).

A broader perspective is about the ecosystems where the indicators live. Tsujimoto et al. (2018) provide a robust review of the concept of ecosystems from the point of view of management: "*To provide a product/service system, an historically self-organized or managerially designed multilayer social network consists of actors that have different attributes, decision principles, and beliefs" (Tsujimoto et al., 2018, p. 55).* The authors also establish the boundaries of the ecosystem identifying four hierarchical classes: Industrial ecology (natural resource management by industry), business ecosystem (private firms), platform management (extended and virtual enterprise), multi-actor network (government, private firms, universities, consumers, entrepreneurs, investors) (Tsujimoto et al., 2018, p. 52).

In an ecosystem the decision-making might cause unintended results because the actors have different priorities, attributes, beliefs and experiences. In this ecosystem, coexist both positive characteristics (collaboration, symbiosis and improvement) like negatives (predation, parasitism, and destruction of the whole system) (Tsujimoto et al., 2018, p. 52). These characteristics influence the way that the indicators are interpreted. Although the indicators enable to reduce uncertainty in decision-making

for the ecosystem, sometimes there is resistance in adoption (McAfee and Brynjolfsson, 2012; Assis et al. 2019).

2.2 Techniques for benchmarking

The MDA adapts the Leontief input-output matrix equations for the simultaneous decomposition of multiple key performance indicators (KPI - Input/Output) (Leontief, 1936; Leontief, 1966; Lin and Polenske, 1998; Albino and Kühtz, 2004; Albino et al., 2002; Albino et al., 2003a; Albino et al., 2003b; Albino et al., 2004; Miller and Blair, 2009; Kühtz et al., 2010; Perroni et al., 2018; Perroni et al., 2020). The contribution of each series is measured by the Effects (I and II) and aggregated by π^{I} and π^{II}. Briefly explained the effects measure the contribution of variables to the formation of the indicator. The contribution of the numerator is captured by Effect I and the denominator by Effect II (content and activity effects respectively). The π^{I} and π^{II} may be used for both internally (comparing different resources within the organization) as externally (comparing the same resources between organizations), enabling benchmarking between different measurement units e.g., cubic meter, joule, ton, square meter.

DEA and SFA are the main methods for benchmarking, most frequently applied in the areas: finance, transport, agriculture, utilities and environmental analysis (Battese and Coeli, 1992; Coeli, 1995; Fried et al., 2002; Hoang and Nguyen, 2013; Daraio et al., 2019). DEA is a mathematical programming technique developed by Charnes et al. (1978) and SFA an econometric technique developed simultaneously by Aigner et al. (1977) and Meeusen and van Den Broeck (1977). The literature presents advantages for the DEA: works with a small sample, having no assumption of efficiency distribution, does not require a functional form for the data and enables modelling with multiple outputs. Regarding limitations, the assumption of the absence of errors and sensitivity to outliers can be mentioned (Coelli et al., 2005; Fethi and Pasiouras, 2010). The SFA can measure efficiency while considering the presence of statistical noise. Its main limitations are the assumption of the functional form, needing two error terms and using only one output as the dependent variable (Coelli et al., 2005; Andor et al., 2019).

Since the development of both techniques in the 1970s, there has been a debate in the literature to know which technique is best suitable to measure performance/efficiency. Several surveys use the techniques together (DEA and SFA) and it is possible to compare the results. They can be separated into two groups: (i) those whose the main objective is to compare the two methodologies (Cullinane et al., 2006; Krüger, 2012; Kuosmanen et al., 2013; Oh and Shin, 2015; Andor et al., 2019). Within this group the

comparison techniques are: Monte Carlo simulation (MCS), Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Spearman rank correlation and bias. (ii) There are also works in which comparison is not the main objective, but use both approaches to obtain robustness in the analysis (Reinhard et al., 2000; Odeck, 2007; Iglesias et al. 2010; Zhou et al., 2012; Honma and Hu, 2014; Nguyen et al., 2015). In this group, the main comparison technique is the correlation (Pearson and Spearman correlations).

Based on the results of the surveys of the two groups, the answer to which method (DEA or SFA) is more appropriate for estimating performance is not conclusive. Dong et al. (2014) finds a moderate consistency between the two approaches, while Oh end Shin (2015) establishes that SFA outperforms DEA for small measurement errors, but DEA wins SFA as the measurement errors increase. Andor et al. (2019) identifies that combination approaches, such as taking the maximum or the mean over DEA and SFA efficiency scores, might offer a useful alternative to strict reliance on a singular method. The correlation between the two approaches also depends on the model chosen, varying between 0.23 and 1.0, normally most of the correlations are in the range (0.70-1.0). (Odeck, 2007; Iglesias et al., 2010; Honma and Hu, 2014).

3. Methodology: experiment design

The methodological process is guided by the general theory of data analysis by Grolemund and Wickham (2014), in which data analysis consists of an investigative process used to extract knowledge, information and insights from reality, in other words, a sensemaking task that updates a given schema using data to fit a model (Grolemund and Wickham, 2014). Our main objective is the exploration, starting from a data set in order to analyze and compare the MDA with the DEA and SFA methods (Charnes et al., 1978; Aigner et al., 1977; Perroni et al., 2020). Inspired by a "toy example" that used fictitious data, we developed a procedure named "marginal exponentiation", which consists of raising the inputs and outputs to the power of marginal exponent and observing what happens with the results. Marginal exponentiation is applied in DEA, MDA and SFA modeling.

Figure 1 describes the methodological process coded in the RMarkdown analytic framework using several steps of the data science (import, tidy, transform, view, model, program and communicate) implemented through chunks[2] (Wickham and Grolemundand, 2017). In stage 1, the indicators for the MDA,

[2] https://rmarkdown.rstudio.com/lesson-3.html

DEA and SFA are computed. For MDA, the values π^{I} and π^{II} are calculated, for the SFA, the functional forms log (SFA-LOG) and translog (SFA-TRANSLOG) are estimated and for DEA the models input-oriented CRS (DEA-IN) and output-oriented CRS (DEA-OUT) are computed. The experiments were modeled mathematically according to Equations 1 to 3. The indicator matrices generated by the experiments are analyzed using correlation (2), cluster (3), Linear discriminant analysis (LDA) (4) and verification of asymptotic convergence (5). The analysis proposition is similar to the layered grammar of graphics, where the parts of a graph are added layer after layer according to the needs of the analysis (Wickham, 2011; Grolemund and Wickham, 2014-2015).

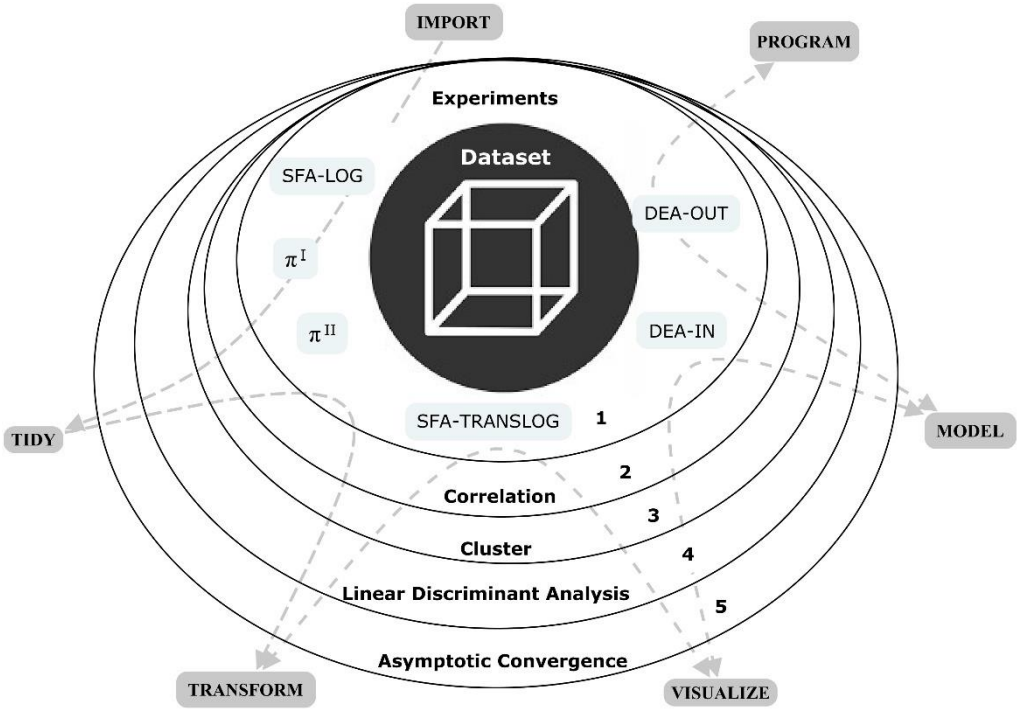


Fig. 1 Methodological layers of data science
Note: (a) Implanted in RMarkdown

Experiment 1 contains the MDA equations, considering that lines 3, 4 and 5 are the same equations presented in Perroni et al. (2020). Marginal exponentiation occurs because the inputs (X) and outputs (I) are raised to the power of $\delta/_{k}$ so that the higher the value of k, the lower the exponent. The k constant is necessary so that the inputs and outputs are not explosive and the value of δ varies in the range [1, N]. The objective of the experiment is to obtain new values for π^{I} and π^{II} for each value of δ.

$$
\begin{aligned}
&1 \ggg \; For\ \delta = 1 \ldots N \\
&2 \ggg \; X^{\delta}/_{k},\ I^{\delta}/_{k} \\
&3 \ggg \; A = Z_t\,\hat{X}_t^{-1};\ \ P = I_{tj}\hat{X}_t^{-1};\ \ L^i = (A)^{-1};\ \ PF = P_{tj}L^i \\
&4 \ggg \; EFFECT\ I = \nabla PF_t = (PF_{tj} - PF_{(t-1)j});\ EFFECT\ II = P_{tj} - EFFECT\ I_{tj} \\
&5 \ggg \; \pi^I = \frac{\sum_1^t EFFECT\ I}{\sum_1^t P};\ \pi^{II} = \frac{\sum_1^t EFFECT\ II}{\sum_1^t P};\ \pi^I + \pi^{II} = 1 \\
&6 \ggg \; Next\ \delta
\end{aligned}
\tag{1}
$$

Experiment 1 – marginal exponentiation for Matrix decomposition analysis (MDA)
Note: Proportional coefficients; I_{tj} - Input matrix; j - Type of input used; L^i - Equivalent to Leontief Inverse; P - Performance indicators; PF - Performance flow indicators; t – Time; π^I - Aggregate dimensionless effect for numerator; π^{II} - Aggregate dimensionless effect for denominator; $\hat{X}_t^{-1}$ - Diagonal matrix of inverse output; Z_t - Special output matrix (upper bidiagonal matrix); ∇ - Difference

Experiment 2 was operationalized in the same way as Experiment 1, but for the Cobb–Douglas log-linear model (SFA-LOG) on line 3 and the translog model on line 4. More details for these models can be verified in other works of literature (Coelli et al., 2005, Bogetoft and Otto, 2010; O'Donnell, 2018; Henningsen, 2019).

$$
\begin{aligned}
&1 \ggg \; For\ \delta = 1 \ldots N \\
&2 \ggg \; X^{\delta}/_{k},\ Y^{\delta}/_{k} \\
&3 \ggg \; \ln x_{ij} = \beta_0 + \sum_{r=1}^{s} \beta_r \ln y_{ijr} \\
&4 \ggg \; \ln x_{ij} = \beta_0 + \sum_{r=1}^{s} \beta_r \ln y_{ijr} + \frac{1}{2} \sum_{r=1}^{s} \beta_r \left(\ln y_{ijr}\right)^2 + \sum_{r=1}^{s}\sum_{l=1}^{s} \ln y_{ijr} \ln y_{ijl} \\
&5 \ggg \; Next\ \delta
\end{aligned}
\tag{2}
$$

Experiment 2 – marginal exponentiation for Stochastic Frontier Analysis (SFA)
Note: x_{ij} – panel dependent variable; β - intercept/coefficients; $y_{ijr} - y_{ijl}$ - panel independent variables

Experiment 3 follows the same rationality as Experiment 1 and 2 but applied to DEA (DEA-IN-DEA-OUT). More details about the DEA can also be found in the literature (Coelli et al., 2005, Bogetoft and Otto, 2011; Zhu, 2014; O'Donnell, 2018).

$$
\begin{aligned}
&1 \ggg \; For\ \delta = 1 \ldots N \\
&2 \ggg \; X^{\delta}/_{k},\ Y^{\delta}/_{k} \\
&\qquad Input\ oriented \qquad\qquad\qquad Ouput\ oriented \\
&3 \ggg \; \theta^* = \min \theta\,, subject\ to \qquad \theta^* = \max \theta\,, subject\ to \\
&4 \ggg \; \sum_{j=1}^{n} \lambda_j\, x_{ij} \le \theta x_{i0} \quad i=1,2,\ldots,m; \qquad \sum_{j=1}^{n} \lambda_j\, x_{ij} \le x_{i0} \quad i=1,2,\ldots,m; \\
&5 \ggg \; \sum_{j=1}^{n} \lambda_j\, y_{rj} \ge y_{r0} \quad r=1,2,\ldots,s; \qquad \sum_{j=1}^{n} \lambda_j\, y_{rj} \ge \theta y_{r0} \quad r=1,2,\ldots,s; \\
&6 \ggg \; \lambda_j \ge 0 \qquad\qquad\qquad\qquad \lambda_j \ge 0 \\
&7 \ggg \; Next\ \delta
\end{aligned}
\tag{3}
$$

Experiment 3 - marginal exponentiation for Data envelopment analysis (DEA)
Note: x_{ij} –x_{i0} inputs/input under analysis - y_{rj} - y_{r0} - outputs/output under analysis; θ – efficiency; λ_j – coefficients.

The layers 2 to 5 of Figure 1 are proposed to analyze the results of the experiments, first through the correlation (Pearson and Spearman), then the cluster analysis is applied to confirm the result of the

correlation (Equation 4). For the cluster, we use the sum of the squared Euclidean distances between each data point x_i and the centroid m_k of the subset C_k which contains x_i(Likas et al., 2003).

$$E(m_{1,} \dots, m_M) = \sum_{i=1}^{N} \sum_{k=1}^{M} I(x_i \in C_k) \|x_i - m_k\|^2 \tag{4}$$

The discriminant analysis (LDA) was applied to check the possibility of forecasting the indicators generated by the experiments. In practice, the LDA will compare the discriminating potential of each indicator. The prediction errors in a supervised approach are used as a metric to check the randomness of the results of the experiments. Linear discriminant analysis (LDA) is a technique for dimensionality reduction problems as a preprocessing step for machine learning and pattern classification applications. The Algorithm for LDA Class-Dependent is presented by Tharwat et al (2017) in Equation 5-8.

$$\mu_i = \frac{1}{n_j} \sum_{x_i \in \omega_j} x_i \tag{5}$$

$$\mu = \frac{1}{N} \sum_{i=1}^{N} x_i = \sum_{i=1}^{c} \frac{n_i}{N} \mu_i \tag{6}$$

$$S_B = \sum_{j=1}^{c} n_i \ (\mu_i - \mu)(\mu_i - \mu)^T \tag{7}$$

$$S_w = \sum_{j=1}^{c} \sum_{i=1}^{n_j} (x_{ij} - \mu_j)(x_{ij} - \mu_j)^T \tag{8}$$

First is calculated the mean of each class and the total mean as Equation 5 and 6 respectively (considering a set of N samples $[x_i]_{i=1}^{N}$). After calculating the between-class matrix $S_B(M\ x\ M)$ (Equation 7) and finally compute the within-class matrix $S_w(M\ x\ M)$ (Equation 8) (Tharwat et al., 2017).

Finally, to test the limits of the π^I and π^{II}, an asymptotic test will be performed to verify their convergence. This test consists of generating values of N in Equation 1 increasingly larger so that the asymptotic behaviour of π^I and π^{II}can be observed.

4. Results of the systematic experiments

4.1 Experiment – toy example

The study by Perroni et al. (2020) assigned the Fibonacci sequence to the energy input, identifying that the MDA model works appropriately with extreme values. In this study, six hypothetical situations are considered in Table 1 (A-F).

In situation A the value of the input is equal to the output raised by power (2,3,4), in this case, the greater potentiation in the input causes an increase of π^I and decrease of π^{II}, with both remaining in the range [0,1]. Case D is the same experiment, but for outputs in reverse order, the changes now are that the values are clearly out of range [0,1]. In situation B, the inputs are multiplied by the values (1,2,3). For this case, firstly, if the values of the inputs and outputs are equal, π^I and π^{II} will be in equilibrium with the value of 0.50, second, the multiplication of the inputs by higher values does not change the indicators, thereby, only relative changes are able to change the indicator. Case C is similar to case B, with outputs in reverse order and values of π^I and π^{II} also outside the range [0,1]. For case E the inputs are considered constant and for F the outputs are constant. In these situations, for any value of the constant, in both cases, the values of π^I and π^{II} do not change, but being [0 and 1] for case E and [1 and 0] for case F.

[Table 1 – toy example for π^I and π^{II} (MDA)]

4.2 Real Example for MDA model in Excel

The toy example describes hypothetical situations enabling the understanding of the behavior of indicators π^I and π^{II}. For the next implementations, we use data from the Philippine rice production ecosystem. According to Andor et al. (2017) these agribusiness data has become a benchmark example in applied efficiency analysis. In the literature, several studies have used these data to test models (Coelli et al., 2005; Rho and Schmidt, 2015; Andor et al., 2017; O'Donnell, 2018; Henningsen, 2019). We used the same methodological implementation as Henningsen (2019): tonnes of freshly threshed rice (PROD), area planted (AREA), labor used (LABOR) and fertilizer used (NPK)[3].

In Perroni et al. (2020) a web application was developed using the R-Shiny script language. In this work, we implemented MDA *via* RMarkdown and MS-Excel®. Figure 2 shows the calculation flow and Excel® formulas needed to implement MDA equations in 10 steps.

[3] *See Coelli et al. (2005, Appendix 2) for a complete description of the data.*

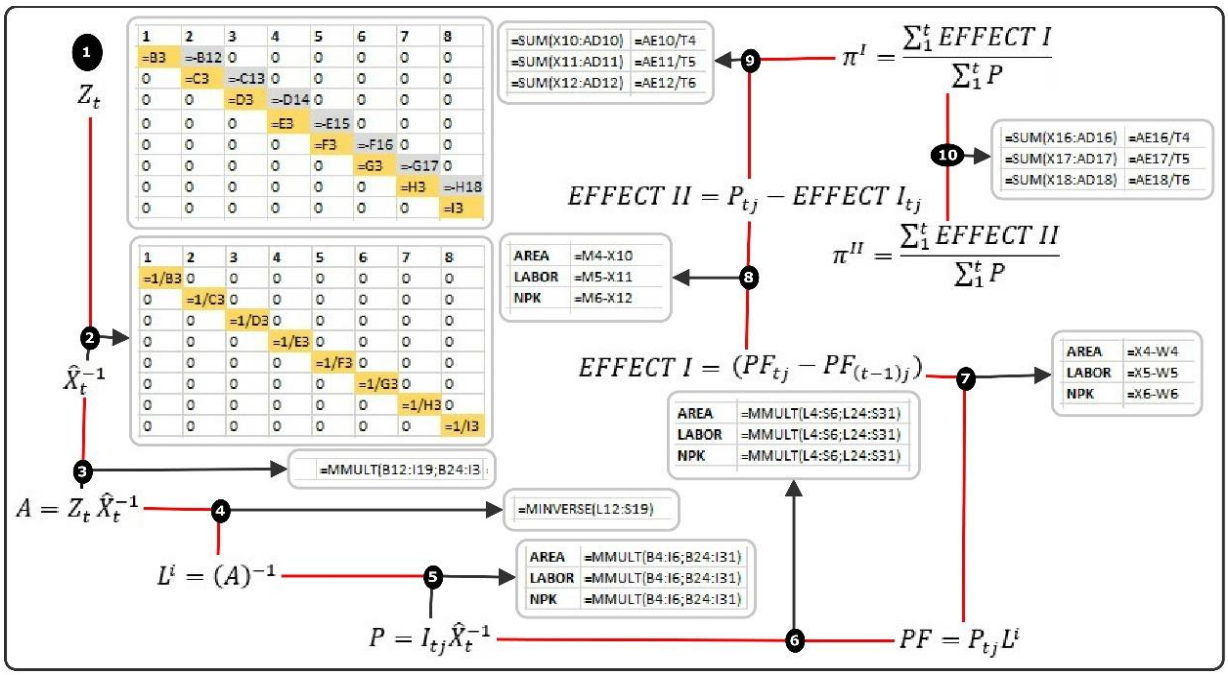


Fig. 2 – Calculation flow in Excel for MDA
Note: See note in Equation 1

Figure 3 presents the results and the information can be found in the supplementary material (RMarkdown-Analytics). In quadrant A of the graph the value of π^{II} is greater than π^{I}, in other words, for six farms the activity effect (contribution of production) surpassed the content effect (contribution of inputs). In quadrant B the π^{I} becomes greater than π^{II}, in this case, the content effect is greater than the activity effect for 26 farms. In quadrant C the content effect is enough to make the activity negative.

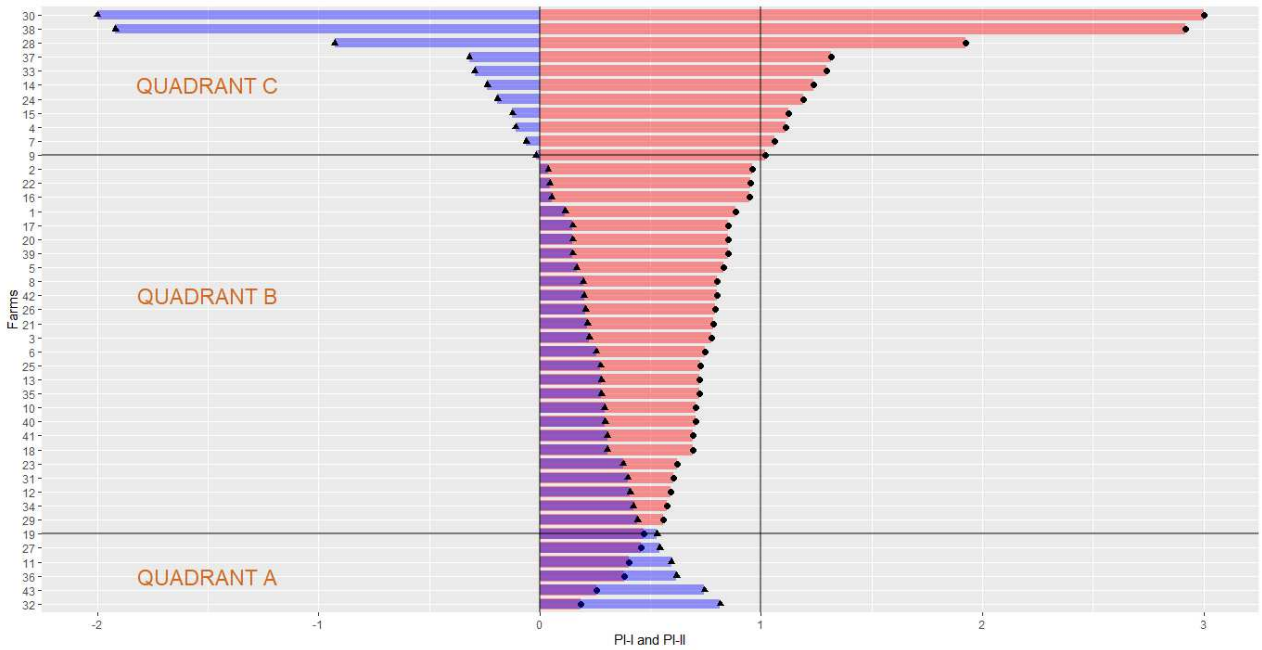


Fig. 3 – π^{I}(dot), π^{II}(triangle) for rice production in 43 Philippine farms
Note: (a) Average of Area, Labor and NPK; (b) Incomplete visualization farm 38 π^{I} = 2,92; farm 30 π^{I} = 11,35; (c) π^{II}=π^{I}-1, (b) Code in chunk 06

The MDA does not generate only the aggregate effects (π^{I} and π^{II}), it is possible to view the effects for each period, shown in Figure 4 (for more details about the effects see Perroni et al. 2020).

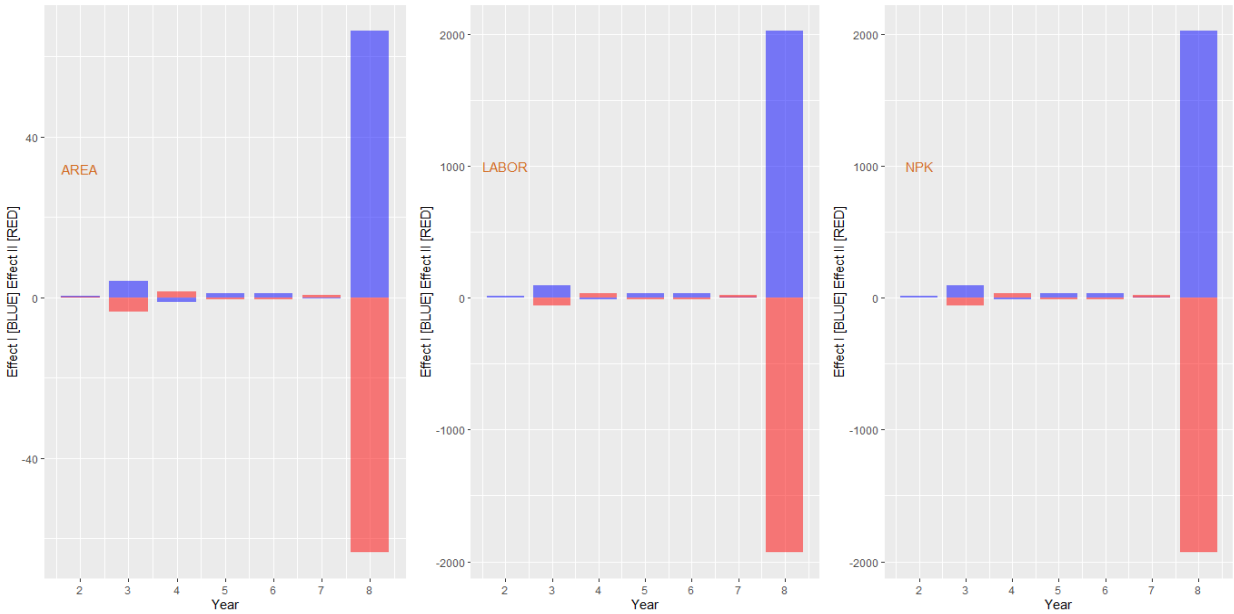


Fig. 4 – Effect I and Effect II for farm 30
Note: (a) AREA $\pi^{I} = 13.45$; $\pi^{II} = -12.45$ *// NPK* $\pi^{I} = 10.57$; $\pi^{II} = -9.57$*// LABOR* $\pi^{I} = 10.02$; $\pi^{II} = -9.02$*; (b) Code in chunk 07*

According to the graph in Figure 4, Effect I can be considered an outlier in period 8 for all inputs (AREA, LABOR and NPK). The outliers may be justified because the production of farm 30 fell from 1.06 tonnes in the year 7 to 0.09 in year 8, in other words, a reduction of more than 1,000%. To fix the analysis the reduction in inputs was less than proportional for this period (AREA = 60%; NPK = 174% and LABOR = 238%). It is interesting noting that the values of π^{I} and π^{II} carry the entire change history, therefore, extreme values should be investigated. This same procedure could be applied to other farms.

4.3 Marginal exponentiation experiment

The marginal experiments were presented in level 1 of Figure 1 and mathematically described in Equations 1 to 3. The analytical process was implemented *via* RMarkdown using 19 chunks [see supplementary materials in RMarkdown-Analytics *(.rmd; .pdf)*]. The experiments for the DEA and MDA were also modelled in Excel-VBA, generating the same results as in RMarkdown (chunk 01). For comparison feasibility, farm averages for the DEA and SFA in the period were calculated. After several tests that considered the computational capacity of the packages, the parameter values N = 40 and k = 10 were chosen for the three experiments. This means that both inputs (Area, Labor, NPK) and output (PROD) are raised to the fourth power with a step of 0.10. Was possible to calculate 40 observations for each farm, totaling 40X43 = 1720 data points for each indicator (SFA-LOG, SFA-TRANSLOG, DEA-IN, DEA-OUT,

π^{I} and π^{II}). The experiments enabled the computation of 80 models for SFA, 1,720 models for MDA and 3,440 DEA models. For SFA there are only 80 because the panel structure was used (40x2) (log, translog), in the case of MDA the computation was realized for each farm separately (40X43), in the same way as MDA, the DEA calculation is made individually for input/output-oriented models (40X43X2). The results of the experiments are found in the supplementary materials, totaling 5,240 models that generate 1,720X6 = 10,320 performance measurement points (see RMarkdown-Analytics).

Figure 5 shows the stacked histogram for the real performance of the 43 farms. In the experiments the real indicators are obtained when the value of δ is equal to 10 in Equation 1 ($X^{\delta/k}$, $I^{\delta/k}$), since k = 10, we have : $X^{10/10}$, $I^{10/10}$, obtaining the dataset without potentiation. Can be noted that most models have performance values in the range [-1.2]. An exception is farm 30 which is in the range [11,12] for π^{I} and [-11, -10] for π^{II}.

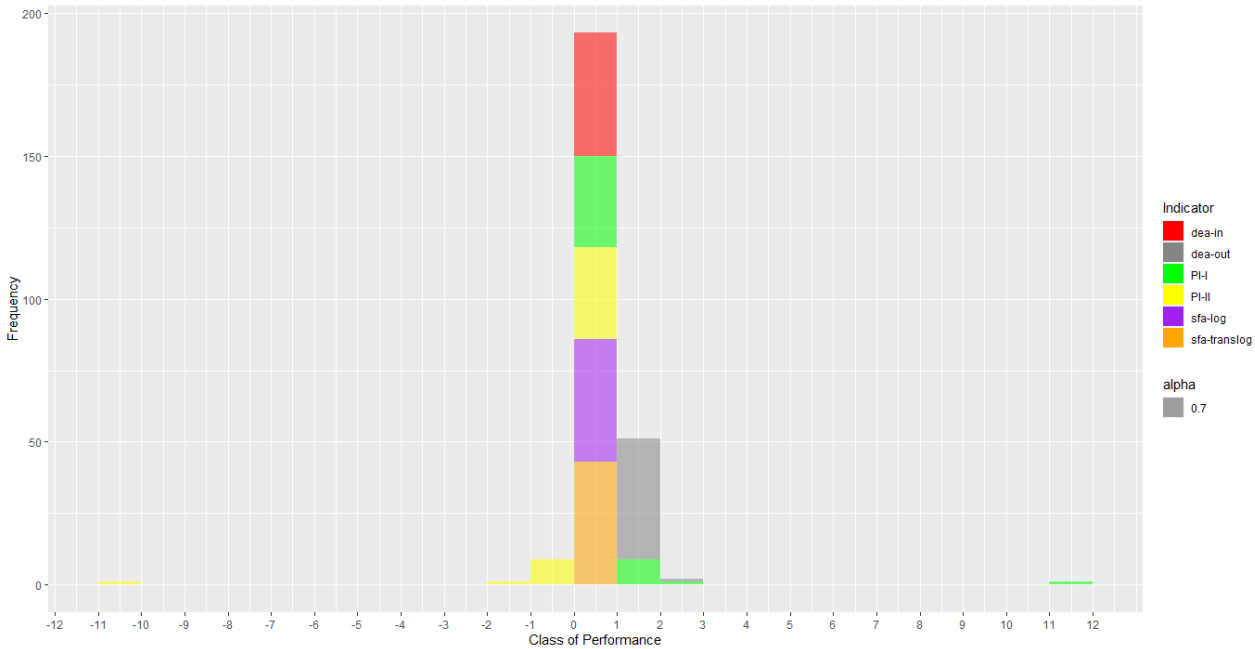


Fig. 5 – Histogram SFA-TRANSLOG , SFA-LOG, DEA-OUT, DEA-IN, π^{II} and π^{I}
Note: Code in chunk 08

Figure 6 shows the distribution of each indicator through the boxplot, aggregating 43 farms by average. The mean is affected by outliers, appearing in π^{I}, π^{II} and DEA-OUT. Figures 7 and 8 show that the distortions are being caused by farms 28, 30 and 38.

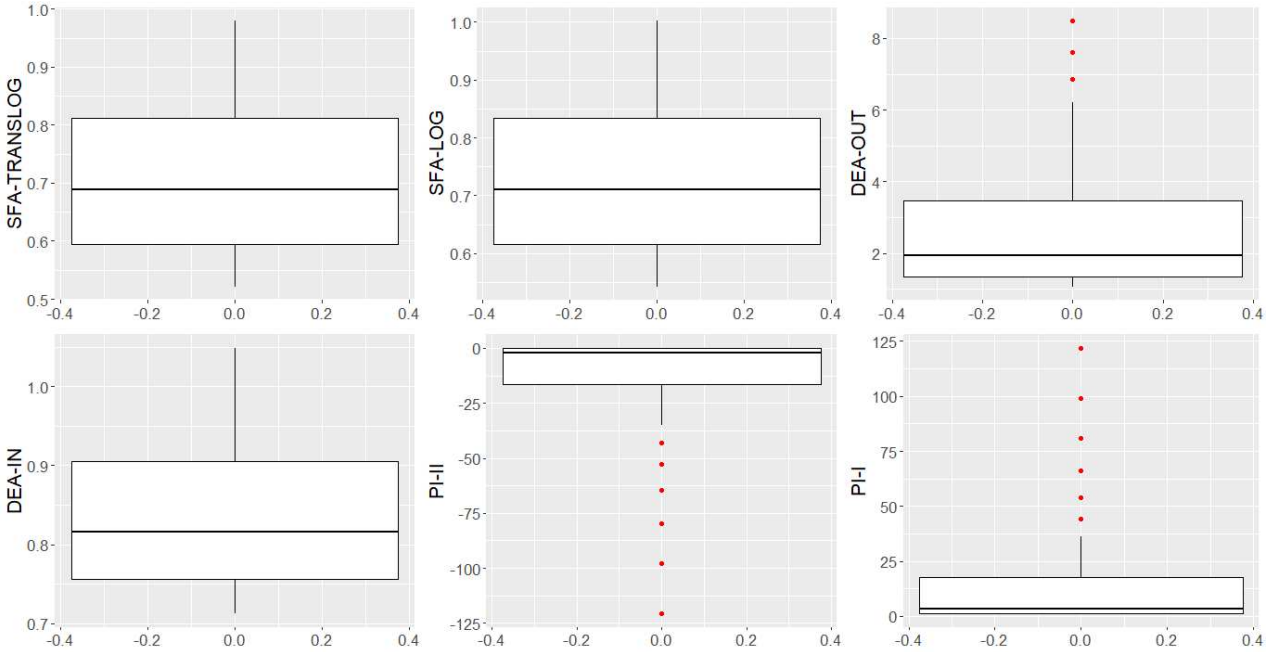
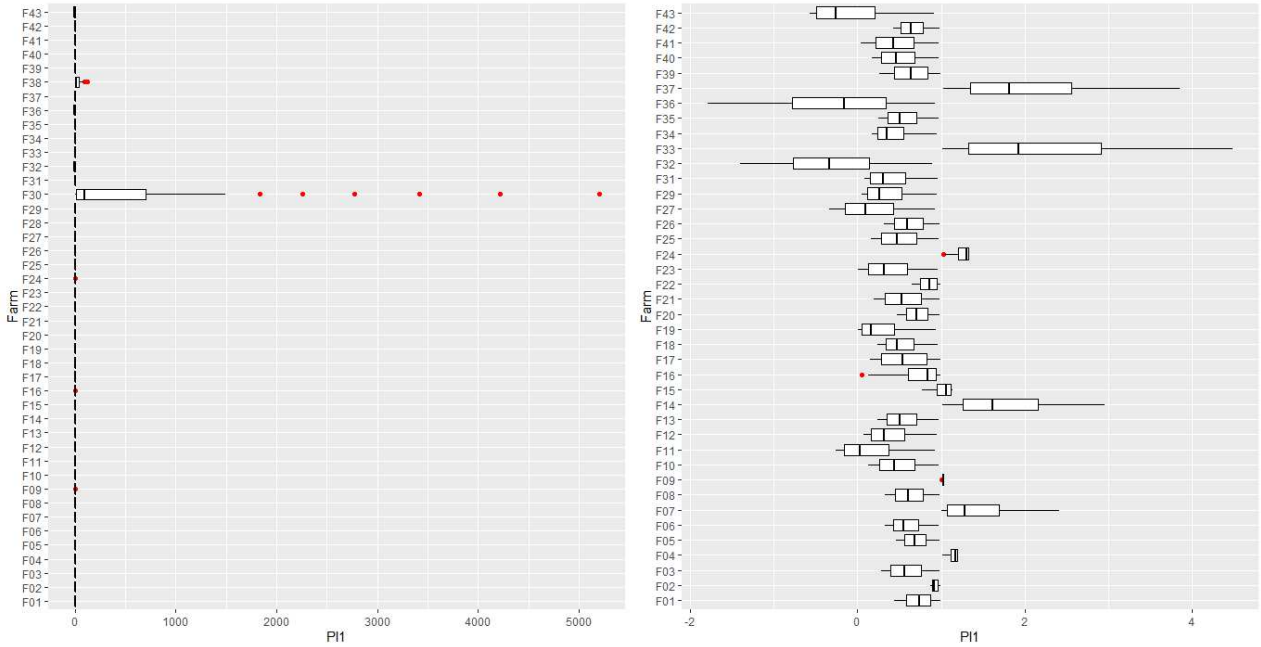


Fig. 6 - Boxplot SFA-TRANSLOG ,SFA-LOG, DEA-OUT, DEA-IN, π^{II} and π^{I}
Note: (a) Code in chunk 09



(a) 43 farms *(b) 40 farms (without considering farms 28, 30 and 38)*

Fig. 7 - Box Plot for π^{I}
Note: Code in chunk 10

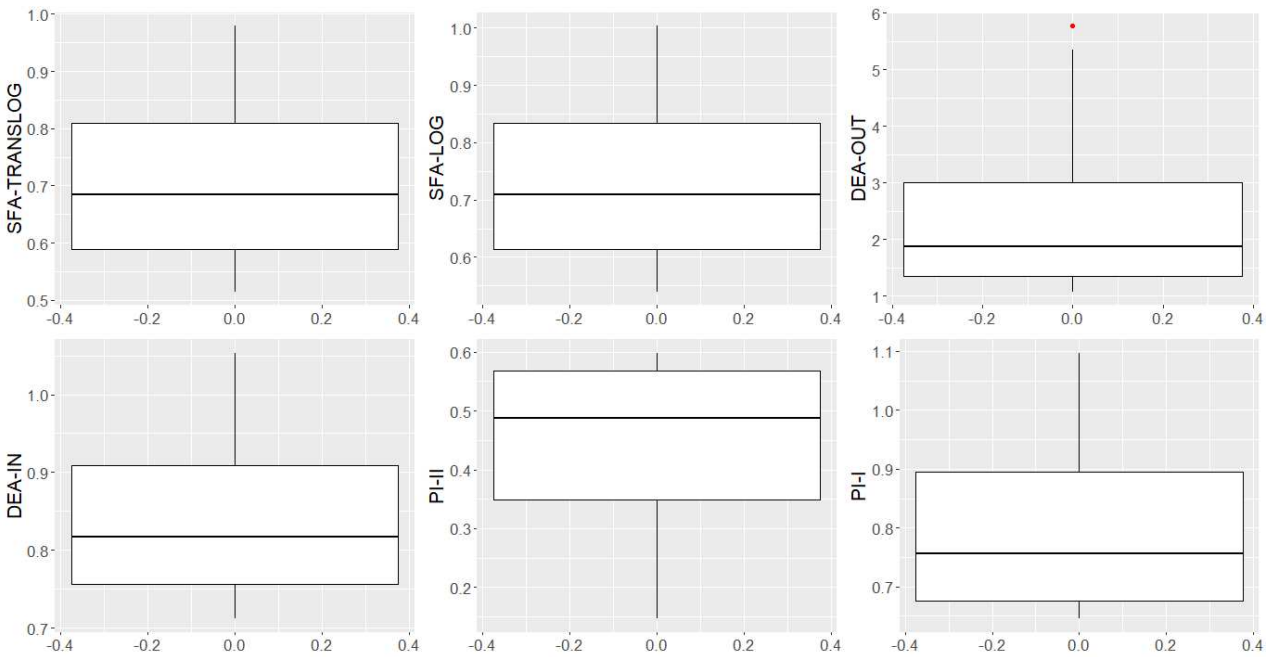


Fig. 8 - Box Plot SFA-TRANSLOG, SFA-LOG, DEA-OUT, DEA-IN, π^{II} and π^{I}
Note: (a) 40 farms (without considering farms 28, 30 and 38); (b) Code in chunk 11

For check the correlation between the six indicators matrices (40x43) of the results of the experiments (Equation 1, 2 and 3), both Spearman's rank correlation and Pearson's product-moment correlation with pairs of the matrices were calculated[4]. The complete results are in the supplementary material (chunk 18 and 19). Interestingly, there are positive and negative correlations. For the indicators pairs [$(\pi^{I}, DEAIN)$, $(\pi^{I}, SFALOG)$, $(\pi^{I}, SFATRANSLOG)$ and $(\pi^{II}, DEAOUT)$] the correlation is negative for ten farms and positive for the others, being the opposite for the pairs [$(\pi^{II}, DEAIN)$, $(\pi^{II}, SFALOG)$, $(\pi^{II}, SFATRANSLOG)$, $(\pi^{I}, DEAOUT)$].

Figure 9 adds the correlation $(\pi^{I}, DEAIN)$ captured from the diagonal of the correlation matrix (same farm between indicators) (for more details see chunk 18 and 19). One issue to explain is the positive and negative differences in correlations. The graph in Figure 9 shows that when the value of π^{I} is greater than one the correlation is negative, the opposite occurs when the value is less than one, an exception is farm 15. Based on Perroni et al. (2020) the π^{I} and π^{II} measures the aggregate effects associated with the total changes in inputs and outputs, respectively (The toy example, clarifies this assertion). Since π^{I} measures the content effect and π^{II} the activity effect, for the farms in quadrant C of Figure 9, the content effect was more than proportionally greater than the activity effect. This evidence suggests that marginal

[4] *Using Spearman's method, most individual correlations have values equal to 1.*

exponentiation amplified the effects on π^I, in other words, it amplified the stochastic structure already existing in the data of this ecosystem.

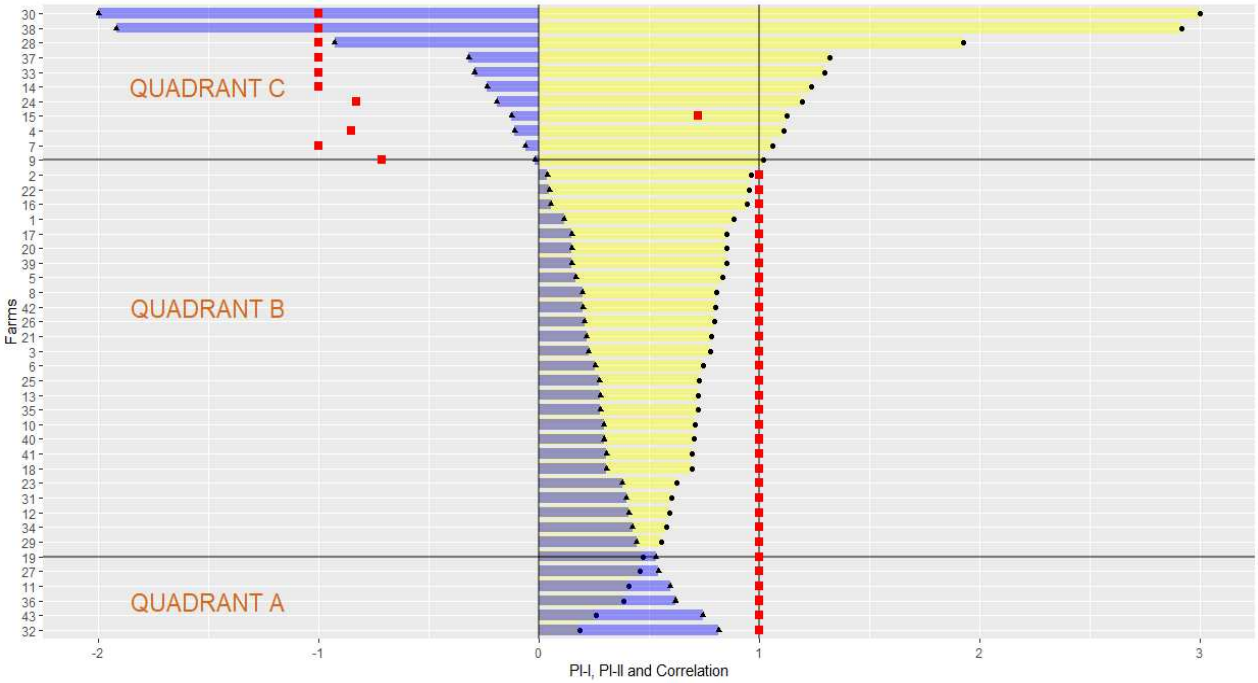


Fig. 9 – π^I(dot), π^{II} (triangle) and Spearman's rank correlation (π^I,DEAIN) (square)
Note: (a) p-value =2.2e-16, (b) The same pattern occurs with Pearson's correlations, but with most correlations less than 1 (c) Code in chunk 12.

Basically, what we want to highlight is that the graph in Figure 9 shows that the correlation is negative for farms that have a greater aggregate content effect (π^I in quadrant C), in other words, used proportionally more resources (area, labor and npk) to produce the same amount of rice. This interpretation is compatible with the efficient use of resources. Interestingly, the threshold occurs exactly where the value of π^{II} becomes negative. An exception is the farm 15, but with a lower positive correlation than other farms.

To distinguish the 43 farms in relation to the indicator π^I and π^{II} we apply the kmeans clustering technique (Kassambara and Mundt, 2020). In the clusters of Figure 10, a clear distinction of eight farms can be identified (7, 14, 24, 28, 30, 33, 37, 38), all belonging to quadrant C of Figures 3 or 9.

To complete the analysis of the correlations, Figures 11 and 12 present the visualization matrix with the correlations (upper), density function (diag) and the scatterplot of the indicators (lower). Can be noted that all correlations are statistically significant, but with an inversion of signs when the value of π^{II} becomes negative, in other words, farms that are in quadrant C, except the farm 15. This means that the result in Figure 9 for ($\pi^I, DEAIN$) can be generalized for all indicators.

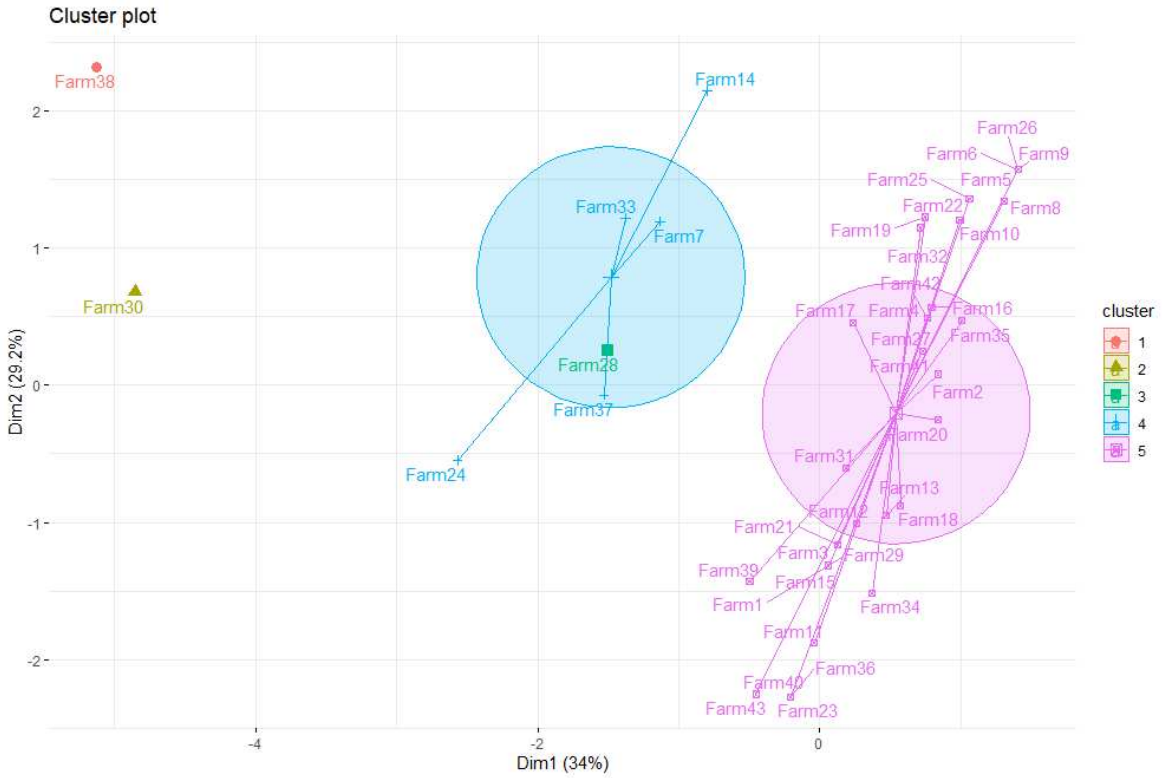


Fig. 10 – Farms clusters for the π^1 indicator
Note: Code in chunk 13

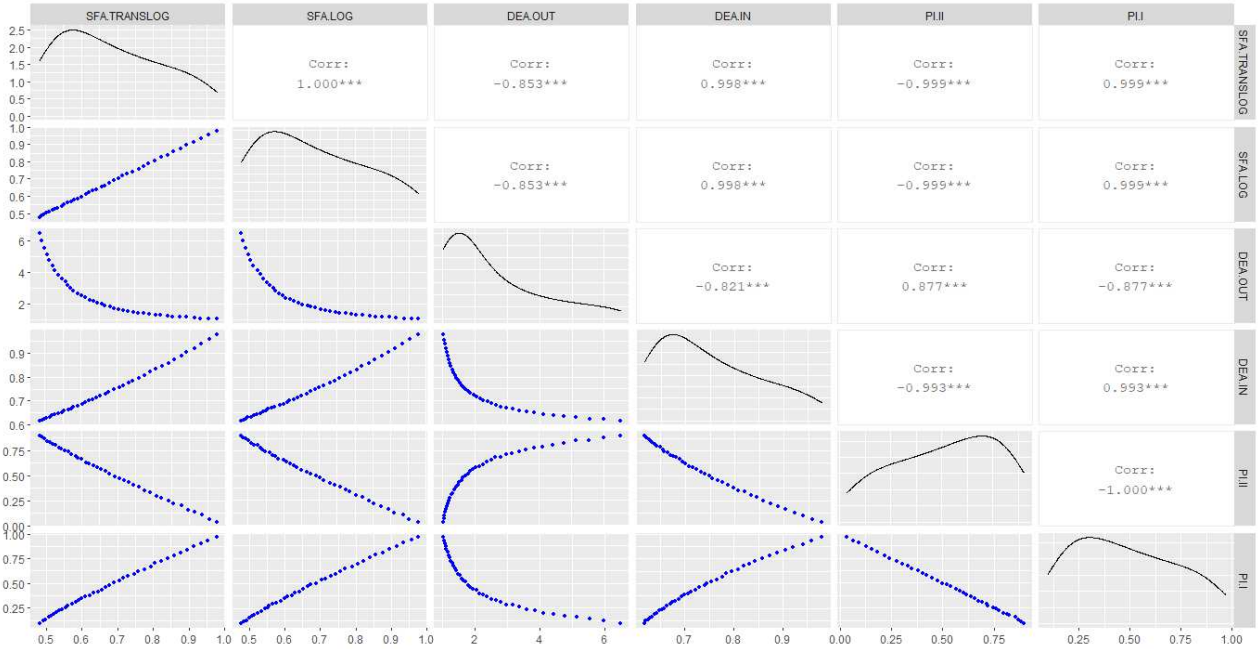


Fig. 11 – Correlation matrix of indicators
*Note: (a) "***" p-value is < 0.001; ; (b) Considering the average of the farms in quadrant A and B of Figure 9 (plus the farm 15); (c) Code in chunk 14*

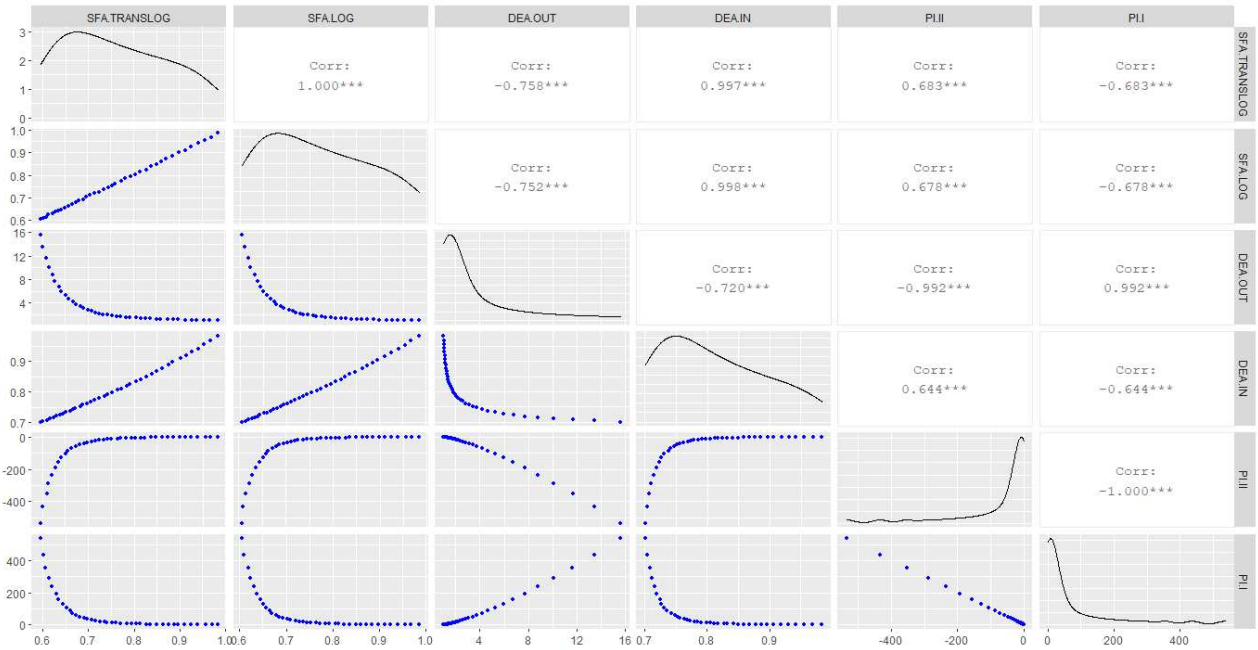


Fig. 12 – Correlation matrix of indicators
*Note: (a) "***" p-value is < 0.001; (b) Considering the average of the farms in quadrant C of Figure 9 (minus the farm 15). (c) Code in chunk 15.*

An additional question is whether the π^{I} and π^{II} of the MDA method has the same discriminating potential as the indicators generated in the DEA and SFA approaches and can be predicted with similar accuracy. This may indicate that the results of the MDA are not random. Our work uses Linear Discriminant Analysis (LDA) because it is a technique often used in the literature that operates with the non-metric dependent variable (Tharwat et al., 2017). Table 2 shows the LDA confusion matrix implemented in the R caret package and coded in chunk 16 (Kuhn, 2020).

[Table 2 – LDA Confusion Matrix]

The operationalization of the LDA would have multicollinearity problems if we used all 43 farms as dependent variables, therefore, we adopted the procedure: an RMarkdown script was developed that randomly selects 5 farms where the indicator is the categorical dependent variable (06 categories) and the farms are the independent variables. Table 2 shows the average of 30 LDA models to reduce the presentation of the results. The overall accuracy of the models was 91%. In Table 2 the values generated by the MDA model (π^{I} and π^{II}) are at least as stable as those generated in the DEA or SFA (the complete result is in chunk 16 of RMarkdown).

A final analysis identified in Figure 1 refers to the asymptoticity of π^{I} and π^{II}. It is not possible to observe in the previous examples because the comparison occurred in the range [1,40]. For checking asymptoticity, among the packages used, the best performance was the matrix inversion of Excel, combined

with VBA, allowing us to test the behavior of π^{I} and π^{II} with a N = 500 for the 43 farms. With the reduction factor $\delta/_{k}$, the maximum power was 50, being enough to verify the convergence. The graphs in Figure 13 show the behavior of the indicators for farms 1, 4, 10 and 24.

The visualization of farms 1 and 10 show that when the parameter δ increases, π^{I} tends to 0 and π^{II} tends to 1. This result can be generalized to 29 farms, understood as a long-term stochastic behavior. For these farms the structure of the data indicates that in the long run the activity effect has a dominance over the content effect. The farm 4 graph indicates a dominance of the content effect that can be generalized to 9 farms. The behavior of farm 24 represents a group of 5 farms in which the effects are divided into the interval [0,1].

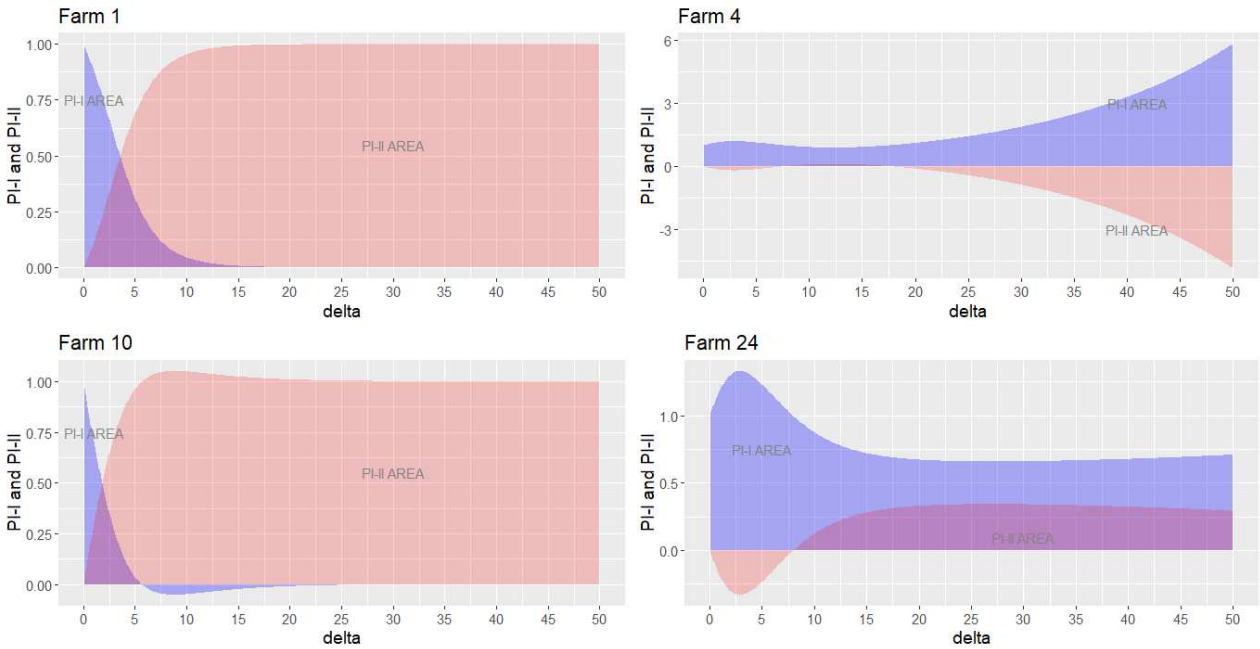


Fig. 13 - Asymptotic convergence for farms (1, 10, 4 and 24)
Note: Code in chunk 17

5. Discussion

In this research we systematically explore the MDA method, where the main objective is to verify its robustness when compared to the DEA and the SFA. Firstly, a toy example was built to demonstrate the behavior of π^{I} and π^{II} given a certain class of hypothetical variation in input and output. Second, the model was operationalized in MS-Excel® using data from an agribusiness ecosystem. Subsequently, three

experiments were developed to compare the MDA with DEA and SFA (marginal exponentiation). The implementation occurred through the development of chunks in RMarkdown framework. As described in Perroni et al. (2020) the indicator π^I measures the contribution of input (content effect) and π^{II} the contribution of output (activity effect) over time. To understand these effects, we can imagine that $\pi^I$is on one side of a seesaw and π^{II} on the other side, but with the difference that the seesaw will only oscillate with proportionally different weights. According to the toy example, only the relative changes in input and output can change the indicators (situation A and D in Table 1). Multiplying the output by any scalar does not change the relative distance of the series, therefore, does not change the indicator (situation B and C in Table 1). In situation C and D there is a disequilibrium due to the decrease in output. The case E and F are specific situations of equilibrium of our hypothetical seesaw, where for case E all weight of change accumulates on the output [$\pi^I = 0$ and $\pi^{II} = 1$] and the reverse for the input in the situation F [π^I =1 and $\pi^{II} = 0$]. By toy example, we can argue that the maximum equilibrium difference will always be one (Perroni et al., 2020).

Was proposed in Perroni et al. (2020) that π^I and π^{II} might be used for internal and external benchmarking in sustainability science. It is possible to compare which decision-making unit (DMU) had the greatest aggregate content effect (π^I). For example, if we want to analyze the use of energy, water or emissions, a higher π^I reflects a greater aggregation of content in the production of goods or services.

Figure 3 shows the benchmarking for 43 rice-producing farms using the resources (Area, Labor and NPK). Quadrant A revealed farms with greater activity by means of π^{II}. The farms in quadrant B are those with the lowest activity and those in quadrant C are those with negative activity (related to the evolution of rice production). Another perspective is the utilization of resources (π^I). Figure 3 also ranks farms in ascending order of resource utilization. The graphs in 3 reveal that extreme cases can be inspected by analyzing Effects I and II. An extreme change can also mean an error in the data (for more information about the effects see Perroni et al., 2020).

In Perroni et al. (2020) the MDA approach was compared with the SFA identifying a negative correlation of -0.80 and -0.40 between $\pi^I$and the SFA. But these are just two isolated examples needing a more robust experiment for detailed comparison. The inspiration for the development of the "marginal exponentiation" experiment was derived from the observations of the toy example, initiated in Perroni et

al. (2020), considering that exponentiation changes the relative distance of the series (inputs, outputs), therefore, it is possible to use multiple databases to compare the behavior of indicators of different models in a controlled manner. The MDA model was compared to DEA and SFA efficiency analyzes.

From the distributions shown in the boxplots of Figure 6, can be noted that there was less variability for the DEA-IN, SFA-LOG and SFA-TRANSLOG models when compared to the models (π^{I} , π^{II}, DEA-OUT). In Figures 7 and 8 can be noted that the asymmetries are caused by farms 28, 30 and 38.

The main finding of the correlations is shown in Figure 9, considering (DEA-IN, π^{I}) to exemplify, most farms have a positive correlation with the efficiency indicator (quadrant A and B), but some have a negative correlation (quadrant C). This result momentarily differentiates the MDA approach from DEA and SFA since 10 farms have a positive correlation and 33 have a negative correlation with efficiency. Since the application of the three experiments followed identical rules, there is evidence to suspect of the stochastic pattern in which the data was generated in this ecosystem, captured by the MDA method. The stochastic pattern in the data was potentialized by the experiment (marginal exponentiation). The most plausible explanation for this difference is the fact that farms with the highest content effect (π^{I}) are negatively correlated with efficiency and farms with the least content effect are positively correlated. The threshold is shown in Figure 9, when the value of π^{II}becomes negative, therefore, the content effect dominates the activity effect.

The cluster analysis identifies that the majority of farms belonging to quadrant C in Figure 9 form distinct clusters from farms that are in quadrants A and B. Correlations are also inverse for $\pi^{I}$and π^{II} and between DEA-IN and DEA-OUT, as expected. The application of the LDA confirms similar forecasting chances, statistically significant between the three models (DEA, MDA and SFA). The asymptotic test shows that there is a long-term pattern for the behavior of π^{I} and π^{II}. Was not possible to verify the asymptotic behavior for DEA and SFA due to model errors caused by computational complexity. It seems probable from these results that MDA can work consistently with more extreme conditions than DEA and SFA.

We must emphasize that the MDA does not have assumptions of optimization (maximum-likelihood estimation or mathematical programming). The estimation of π^{I} and π^{II} is based on indicators decomposition approach derived from Leontief's input-output model (Leontief 1966, Albino and Kühtz,

2004; Bogetoft and Otto, 2011). Part of the decomposition is attributed to the inputs (π^I) and part to the outputs (π^{II}), therefore we think it is reasonable to assume that the MDA is a Leontief partial equilibrium model that produces dual indicators. The duality of the indicators π^I and π^{II} is related to the content and activity effects discussed in Perroni et al. (2020). Interpreted from another point of view, the content effect is related to efficiency (INPUT/OUTPUT) and the activity effect on productivity (OUTPUT/INPUT). In the case of efficiency, we want to reduce the use of resources (Area, Labor and NPK) per unit of output (rice) and in the second case, we seek to increase the output (rice) per unit of input. The indicators π^I and π^{II} seeks to measure this duality.

Both in the toy example as in rice production analysis, there are situations of disequilibrium concerning the productivity and efficiency, in other words, content and activity effects. From the point of view of sustainable management, this disequilibrium is related to resource management *versus* output management. In practical terms, duality unveils the condition of producing with a minimum impact on the environment and at the same time meeting the need for financial results with the achievement of productivity. The MDA method reveals the minimization/maximization effort over a period.

The MDA methodology has the potential to quantify the duality existing in an ecosystem for a given period. Figure 14 highlights the MDA contribution to the analysis of performance in sustainable ecosystems.

In Perroni et al. (2020) the MDA enabled the benchmarking across indicators with different units of measurement, e.g., cubic meter, joule, ton, square meter. The main application discipline of the MDA is the transdisciplinary sustainability science (TSS) (Brandt, et al., 2013). In this work, we introduce the notion of ecosystems in which the indicators live (industrial ecology, private firms, platform management and multi-actor network). The indicators are created in systems with dual processes (increase, decrease) belonging to a given ecosystem, providing learning (benchmark), coordination (allocation of tasks) and motivation (reduction of uncertainties) (Shehabuddeen et al., 1999, p. 4; Bogetoft and Otto, 2011; Tsujimoto et al., 2018; Perroni et al., 2020).

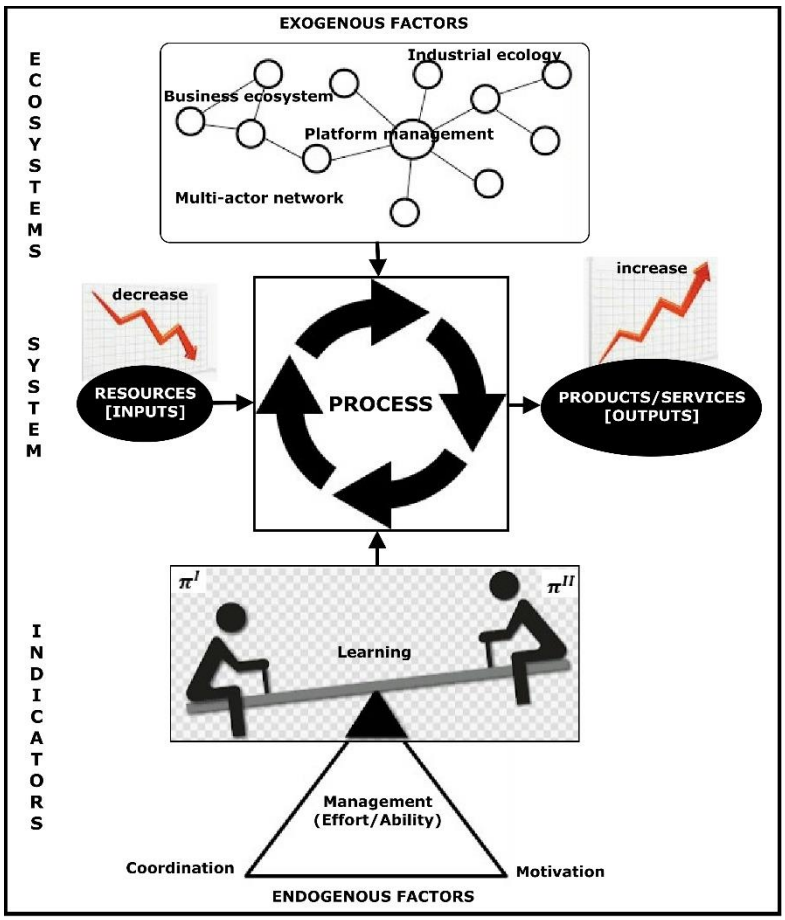


Fig. 14 - Dual performance in sustainable ecosystems

As in the DEA and SFA approaches, the MDA has the possibility of expanding to other contexts that use the input-output relationship in economic, social and environmental processes, with the limitation of working with time series. The advantage lies in the flexibility of the measurement and the decision-making duality in each ecosystem. The results seem to demonstrate that although the MDA has a high correlation with efficiency (DEA, SFA) the comparison must consider that the MDA is a dual benchmarking system.

6. Conclusion

We conclude that the MDA might be as stable as the DEA and SFA approaches, but captures the dual performance, measuring the Leontief partial equilibrium of the variables of interest, enabling the practitioners and decision-makers to make necessary adjustments. The MDA has the capacity to identify if a given ecosystem is in equilibrium. The instability can be caused by the excessive use of resources or abnormal productivity. When two entities are compared, it is evident if one or the other phenomenon is occurring. This is what we called dual benchmarking. Some aspects can be highlighted in relation to the results: (i) Only relative changes in input and output can change the indicators, therefore multiplying the

input/output by any scalar does not change the indicator; (ii) The distribution of the results of the MDA shows greater asymmetries when compared to the DEA/SFA, which on the one hand provides greater discrimination, but on the other there is a need to examine the cases of outliers through Effect I and Effect II; (iii) There is correlation between the MDA and the efficiency models (DEA, SFA), but the sign depends on a certain threshold in the use of resources. For this study, the threshold occurs when the aggregate activity effect becomes negative; (iv) The MDA does not start from optimization assumptions like DEA and SFA, being a Leontief partial equilibrium model that produces dual indicators, offering a ranking system capable of detecting relative changes in the use of resources. The MDA enables the conflict resolution in terms of gains and losses over time.

The main limitation of this article is related to the MDA model concerning DEA-SFA. MDA works only with time series data and is not operationalized for cross-sections. Another limitation is that MDA works with the decomposition principle and DEA and SFA with the optimization principle, which brings challenges to the comparison. Although the directions are different, the models are based on the theory of production (MDA Leontief's general equilibrium production model – DEA-SFA production function of the theory of the firm.

We believe that future studies can be developed using other techniques of comparison such as simulation. Given the stability of the MDA evidenced in this work, a promising area for future research would probably be in applied studies to measure the dynamic equilibrium of the sustainable ecosystem variables. In practice, the objective of this work was to verify if the MDA has as much stability as the DEA and the SFA; instead of using the simulation, we used the "marginal exponentiation" that is not common in the literature. For future studies, a comparison of marginal exponentiation with a Monte Carlo simulation model is challenging. On the other hand, the development of MDA to work with cross-sectional data could bring good results. We also suggest the development of axioms of mathematical theory that support the empirical study carried out.

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Availability of data and materials The datasets used and/or analyzed during the curent study are available in the supplementay material.

Author Contributions

Conceptualization, M. G. P and F. M. R.; methodology, M. G. P., and C. P. V; software, M. G. P., and W. V. S.; validation, M. G. P., S. Z., and C. P. V.; resources, C.P.V; data curation, M. G. P and W.V. S.; writing, original draft preparation, M.G. P., S. Z., and W. V. S.; writing, review and editing, M. G. P., F. M. R., S.Z., C. P. V., and W. V. S.; visualization, S. Z. and W. V. S. C. P. V.; supervision, C. P. V; project administration, M. G. P and C. P. V.

DECLARATIONS

Ethical Approval Not applicable

Consent to Participate Not applicable

Consent to Publish Not applicable

Competing Interests The authors declare no competing interests.

566 Table 1 – toy example for π^I and π^{II} (MDA)

	OUTPUT		INPUT1	INPUT2	INPUT3
			1,4,9,16,25,36,49,64	1,8,27,64,125,216,343,512	1,16,81,256,625,1296,2401,4096
A	1,2,3,4,5,6,7,8	π^I	0,70	0,79	0,85
		π^{II}	0,30	0,21	0,15
			1,2,3,4,5,6,7,8	2,4,6,8,10,12,14,16,	3,6,9,12,15,18,21,24
B	1,2,3,4,5,6,7,8	π^I	0,50	0,50	0,50
		π^{II}	0,50	0,50	0,50
			8,7,6,5,4,3,2,1	16,14,12,10,8,6,4,2	24,21,18,15,12,9,6,3
C	8,7,6,5,4,3,2,1	π^I	5,00	5,00	5,00
		π^{II}	-4,00	-4,00	-4,00
			64,49,36,25,16,9,4,1	512,343,216,125,64,27,8,1	4096,2401,1296,625,81,16,1
D	8,7,6,5,4,3,2,1	π^I	7,00	8,80	10,54
		π^{II}	-6,00	-7,80	-9,54
			1,1,1,1,1,1,1,1	2,2,2,2,2,2,2,2	3,3,3,3,3,3,3,3
E	1,2,3,4,5,6,7,8	π^I	0,00	0,00	0,00
		π^{II}	1,00	1,00	1,00
			1,2,3,4,5,6,7,8	1,4,9,16,25,36,49,64	1,8,27,64,125,216,343,512
F	10,10,10,10,10,10,10,10	π^I	1,00	1,00	1,00
		π^{II}	0,00	0,00	0,00

567 Note: A - input=output[^2, ^3, ^4]; B - input=output[x1, x2, x3]; C - input=output[x1, x2, x3]; D - input=output[^2, ^3, ^4]; E - input=[1,2,3]; F -
568 input=output(E)[^2, ^3, ^4]. The example can be found in the supplementary materials.
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572 Table 2 – LDA Confusion Matrix

Indicators	SFA-TRANSLOG	SFA-LOG	DEA-OUT	DEA-IN	π^I	π^{II}
SFA-TRANSLOG	**33,50**	4,23	0,50	1,53	0,00	0,23
SFA-LOG	4,03	**33,47**	0,27	2,10	0,00	0,13
DEA-OUT	0,73	0,87	**36,67**	1,50	0,00	0,23
DEA-IN	0,43	0,17	0,03	**39,20**	0,00	0,17
π^I	0,00	0,00	0,00	0,00	**40,00**	0,00
π^{II}	1,33	0,63	0,07	2,77	0,00	**35,20**

573 *Note: (a) Accuracy =0.9084; AccuracyPValue ≅ 0; (b) Average of 30 models, (c) implemented in the R caret package in chunk 16*
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