

Research on Improved DS Evidence Theory

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Research on improved DS evidence theory

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Abstract: In view of the lack of effective information fusion model for heterogeneous multi-sensor, an improved DS evidence theory algorithm is designed to fuse heterogeneous multi-sensor information. The algorithm first introduces the compatibility coefficient to characterize the compatibility between the evidence, obtains the weight matrix of each proposition, and then redistributes the BPA of each focal element to obtain a new evidence source. Then, the concept of credibility is introduced, and the synthetic rules are improved by using the credibility of evidence and the average support of evidence focal elements, so as to obtain the fusion results. The results show that compared with other algorithms, this algorithm can solve the problems of DS evidence theory in dealing with highly conflicting evidence to a certain extent, and the fused results are more reasonable and converge faster.

Key Words: Information fusion, DS evidence theory, Evidence credibility

1. INTRODUCTION

At present, there is no effective fusion model and algorithm for information fusion of heterogeneous multi-sensor information [1-15], but with the help of modern statistical theory, this problem can be effectively solved to a certain extent [16,22]. As an important branch of statistical theory, DS evidence theory is widely used in the field of multi-sensor information fusion due to its ability to represent uncertain and unknown information clearly and its strong operability. Although Dempster's rule of combination in this theory can reasonably combine the evidence and fuse the expected experimental results, it has a high requirement on the degree of evidence conflict. When the degree of conflict is high, the fusion results are often inconsistent with the actual situation, and some fusion results even seriously violate common sense.

DS evidence theory was proposed by Dempster firstly, then further developed by his student Shafer as an uncertainty reasoning method with confidence function as the measurement standard. Due to the existence of general conflict problems, one vote rejection problems, robustness problems and invalidation of synthesis rules in DS evidence theory, the results of evidence synthesis will deviate from the facts or cannot be synthesized, and then lead to wrong reasoning results, which are affecting the reliability of fusion results. Therefore, it is necessary to improve the DS evidence theory to form an improved DS evidence theory that can overcome the above four problems, so a fusion result is get which conforms to common sense and can provide decision-makers with higher reliability.

2. IMPROVE DS EVIDENCE THEORY

In this study, an improved DS evidence theory algorithm is designed, which starts from both the improvement of combination rules and the modification of evidence source.

First, the compatibility coefficient is introduced to modify the evidence source, and then the reliability and the average support degree of evidence focal elements are introduced to improve the combination rules.

2.1 Calculation of compatibility coefficient between evidences

Based on the basic idea of evidence support and evidence distance, this study believes that although there is a conflict between conflicting evidence, such a conflict is not absolute, and there must be consistent information that can be used effectively. Based on these ideas, this paper introduces the concept of inter-evidence compatibility coefficient. The definition is as follows.

Identify frame $\theta = \{A_1, A_2, \dots, A_n\}$, take any $\forall A_k$, and BPAF of the two evidences are Respectively $m_i(A_k)$ and $m_j(A_k)$, then the compatibility coefficient of the two evidences with respect to A_k is:

$$R_{i,j}(A_k) = \frac{m_i(A_k) * m_j(A_k)}{\frac{1}{2}(m_i(A_k)^2 + m_j(A_k)^2)} \quad (1)$$

According to the above equation, when the BPA of any focal elements in $m_i(A_k)$ and $m_j(A_k)$ are zero, the value of $R_{i,j}(A_k)$ is zero, indicating that in the evidence set, when one piece of evidence supports the focal element A_k , that is, when there is a high conflict between the two pieces of evidence, the compatibility coefficient is zero. When the BPA of $m_i(A_k)$ and $m_j(A_k)$ is equal, the $R_{i,j}(A_k)$ value is 1, indicating that the compatibility coefficient is 1 in the evidence set when the support degree of the focal element A_k of the two evidences is consistent, that is, when there is no conflict between the two evidences. Therefore, the compatibility coefficient can represent the degree of mutual support between two pieces of evidence.

The degree of consistency between the evidence in the evidence set can usually be expressed by the expected

value, which can be expressed by the average evidence. Therefore, the higher the mutual support between the expected value and the evidence, the higher the credibility of the evidence and the corresponding weight should be.

Using the calculation idea of compatibility coefficient, when there are n pieces of evidence in the evidence set, the mutual support degree between each piece of evidence and the average evidence can be expressed as:

$$w_{ik}(A_k) = \bar{R}_{i,k}(A_k) = \frac{2m_i(A_k)\bar{m}(A_k)}{m_i(A_k)^2 + \bar{m}(A_k)}, k = 1, 2, \dots, n \quad (2)$$

Where, $\bar{m}(A_k) = \frac{1}{n} \sum_{i=1}^n m_i(A_k)$ represents the average evidence, i.e. the overall expectation of the evidence.

It can be seen that the weight matrix of each evidence regarding each proposition is:

$$w = \begin{bmatrix} \bar{R}_{1,1}(A_1) & \bar{R}_{1,2}(A_2) & \dots & \bar{R}_{1,n}(A_n) \\ \bar{R}_{2,1}(A_1) & \bar{R}_{2,2}(A_2) & \dots & \bar{R}_{2,n}(A_n) \\ \vdots & \vdots & \ddots & \vdots \\ \bar{R}_{n,1}(A_1) & \bar{R}_{n,2}(A_2) & \dots & \bar{R}_{n,n}(A_n) \end{bmatrix} \quad (3)$$

Then, each focal element of BPA in each piece of evidence is redistributed using the above weights, namely:

$$m_i'(A_k) = \bar{R}_{i,k}(A_k) m_i(A_k) \quad (4)$$

BPA distribution for focal elements of evidence is as follows:

$$m_i'(\theta) = 1 - \sum_{k=1}^n (1 - \bar{R}_{i,k}(A_k)) m_i(A_k), i = 1, 2, \dots, n \quad (5)$$

2.2 Calculation of compatibility coefficient between evidences

Set the evidence set, m_1, m_2, \dots, m_n , and set the focal element conflict size between winning game I and J as k_{ij} , then:

$$k_{ij} = \sum_{A_i \cap A_j = \emptyset} m_i(A_i) m_j(A_j) \quad (6)$$

k_{ij} is the size of conflict between every two evidences in the evidence set, which reflects the sum of the magnitude of conflict between a certain focal element in evidence i and other focal elements in evidence j, and can effectively reflect the degree of conflict between evidences.

c is defined as the average conflict degree of the evidence set, then:

$$c = \frac{1}{n(n-1)/2} \sum_{i < j} k_{ij}, i, j \leq n \quad (7)$$

Where, n is the number of evidence sources

ε is defined as the credibility of evidence, then:

$$\varepsilon = \frac{1}{e^c} \quad (8)$$

Where: c is defined in equation 7.

Analysis of equations 6 and 7 shows that ε is the decreasing function of c. When the conflict among evidences increases, when C increases, the credibility of evidences will decrease. Therefore, ε can represent the credibility of evidences.

2.3 Improvement of fusion rules

This paper introduces inter-evidence credibility and improves Dempster's rule of combination as follows:

$$p(A) = \sum_{\substack{A_i \in M_i \\ \cap_{i=1}^n A_i = A}} m_1(A_1) m_2(A_2) \dots m_n(A_n), q(A) \quad (9)$$

$$\begin{cases} m(A) = P(A) + k * \varepsilon * (q(A) + G(A)), A \neq \theta \\ m(\theta) = P(\theta) + k * \varepsilon * q(\theta) / (k + \varepsilon) \end{cases} \quad (10)$$

Where, $P(A)$, $Q(A)$ are equation 9, K is the discrepancy factor, is equation 8, $G(A)$ is the amount of evidence that can be trusted to A in an unknown focal element.

$$G(A) = \sum_{i \neq j} m_i(A) m_j(\theta) m_j(\theta) \dots m_n(\theta) / n \quad (11)$$

According to Equation 10, when k is small, $P(A)$ plays major role in the equation, and the resultant result is similar to that of DS evidence theory. When k=0, the new combination equation is the same as Dempster's combination rule. When $K \rightarrow 1$, the combination result will be mainly determined by $\varepsilon * (q(A) + G(A))$. Therefore, when the evidence is highly conflicting, the fusion result of the improved Dempster combination rule will be mainly determined by $\varepsilon * (q(A) + G(A))$.

Against sun quan combination rules, the k increases or ε decreases, and $k(1 - \varepsilon)$ will increase, which in turn leads to increasing synthetic results of unknown problems. In this paper, the BPAF of unknown focal element is improved. According to $m(\theta)$ in Equation 10, when the inconsistency factor K increases or the evidence credibility ε decreases, $k * \varepsilon / (k + \varepsilon)$ can more reasonably balance the conflict between K and ε .

2.4 Steps of conflict evidence processing and fusion

This study improves the DS evidence theory from two aspects. On the one hand, it introduces the compatibility coefficient to assign different weights to the focal elements that cause conflicts in the consistency information among evidences, so as to minimize the influence of evidence with low reliability on the fusion results. For evidence, on the other hand, the conflict between information, introducing the credibility of evidence, the evidence for integration, support for highly give fully affirmation of evidence, the evidence than major, of low trust evidence for the separation, to make it small in proportion of evidence, in highly conflict evidence, greatly reduce the uncertainty of the fusion results, reduce the correlation between low interference in the process of integration of evidence, makes the result more clear and reasonable.

According to above analysis, the processing of conflict evidence in this study should be started with revised data first. Then it will use compatible coefficient to modify source of conflict evidence. After the new source of evidence is get, using introduction credibility of evidence and evidence focal element average support synthesis rules in the new source of conflict evidence combination of

evidence, concrete evidence conflict process is shown in fig 1.

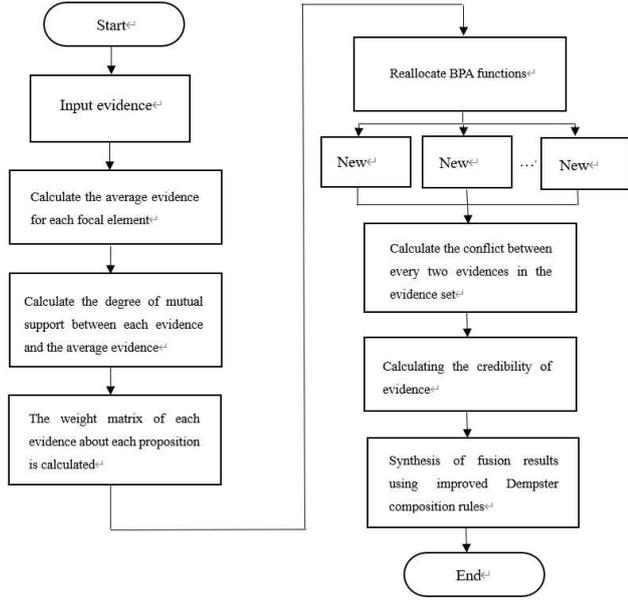


Fig 1. Conflict evidence processing flow chart.

It can be seen from the above processing process that the steps of processing conflict evidence are summarized as follows:

- Step 1: Calculate the weight coefficient corresponding to each focal element in each piece of evidence by Formula 2;
- Step 2: According to the weight coefficient of each focal element in each evidence, BPAF is redistributed according to Equations 4 and 5 to obtain a new BPA;
- Step 3: Calculate the conflicts between every two pieces of evidence in the evidence set k_{ij} and ε ;
- Step 4: Obtain the fusion result through Equation 9.

3. Experimental comparison and example verification

3.1. Experimental comparison of the synthesis of conflicting evidence

In this paper, Dempster's rule, Murphy's method, Yager method, Sun Quan method and this research method were respectively used to fuse conflicting evidence of the four problems existing in D-S evidence theory, and the fusion results were compared and analyzed to verify the effectiveness of this algorithm. Taking the following evidence as an example, the evidence includes general conflict evidence, focal element rejection evidence, focal element fine-tuning evidence and complete conflict evidence.

Set the identification framework $\theta = \{A, B, C\}$, and the evidence is as follows:

$$m_1: m_1(A) = 0.99, m_1(B) = 0.01, m_1(C) = 0, m_1(\theta) = 0$$

$$m_2: m_2(A) = 0, m_2(B) = 0.01, m_2(C) = 0.99, m_2(\theta) = 0$$

$$m_3: m_3(A) = 0.98, m_3(B) = 0.01, m_3(C) = 0.01, m_3(\theta) = 0$$

$$m_4: m_4(A) = 0, m_4(B) = 1, m_4(C) = 0, m_4(\theta) = 0$$

$$m_5: m_5(A) = 1, m_5(B) = 0, m_5(C) = 0, m_5(\theta) = 0$$

In the evidence set, $m(A), m(B), m(C)$ respectively represent BPAF as the BPA allocated to identify each focal element in the framework, and $m(\theta)$ represents the probability of assigning evidence to unknown focal elements.

3.1.1 For general conflict problems

Table1. The processing results of different algorithms for general conflict problems

m_1, m_2	$m(A)$	$m(B)$	$m(C)$	$m(\theta)$	k
Dempster	0	1	0	0	0.9999
Murphy	0.4998	0.0003	0.4999	0	0.5099
Yager	0	0.0002	0	0.9999	0.9999
Sun quan	0.1822	0.0033	0.1824	0.6328	0.9999
this paper	0.4586	0.0102	0.4585	0.0732	0.6436

The evidence collection of m_1, m_2 shows that when general conflict problem, that is when focal elements A and C of BPA in evidence m_1 were 0.99 and 0, focal elements A and C of BPA in evidence m_2 were 0 and 0.99 respectively. Obviously, the two evidences are conflicting, and coefficient of evidence conflict approaches to be 1, and the fusion results of focal element A and C should be equal, but not 0. From the table1, the fusion results by Dempster rule and Yager method assign focal element A and C to 0, so these two methods cannot effectively fusion data conflict, But Murphy, Sun Quan method and the method in this paper can solve the problem of general conflict to some extent, but Murphy method only make weighted average to conflict simply, and the convergence speed is slow. The value of $m(A)$ and $m(C)$ in the fusion results of the Sun Quan method are not zero, but the weight which the conflict evidence divided into the unknown field is higher in this method, the $m(\theta)$ value is higher than the values of $m(A)$ and $m(C)$, it could not judge the fusion results, so the fusion results failed to meet expectations. In this paper, $m(A)$ and $m(C)$ values are not 0, and the weight of evidence divided into unknown focal elements is small, which conforms to the expected results. Therefore, the method of this study can solve the general conflict problem to a certain extent.

3.1.2 The question of one veto

Table2. Different algorithms deal with the problem of one vote rejection

m_1, m_2	$m(A)$	$m(B)$	$m(C)$	$m(\theta)$	k
Dempster	0	1	0	0	1

Murphy	0.8845	0	0.1159	0	0.6797
Yager	0	0	0	1	1
Sun quan	0.3340	0.0055	0.1702	0.4906	1
this paper	0.7723	0.0075	0.1579	0.0636	0.6120

From the evidence sets m_1 , m_2 , m_3 , the BPA of the focal element A in m_1 and m_3 is 0.99 and 0.98 respectively, and the BPA of the focal element A in m_2 is 0. Obviously, although the evidence sets m_1 and m_3 both support the focal element A, the evidence sets m_2 opposes the focal element A. As can be seen from Table 2, the fusion results of Dempster's rule and Yager method both have 0 assignment of focal element A, and the fusion results are not as expected. So these two methods cannot effectively fuse conflicting evidence. The values of $m(A)$ in Murphy, Sun Quan method and the method in this paper are not zero, so can solve the Veto problems to some extent, but Murphy method only make weighted average to conflict simply, and the convergence speed is slow, the value of $m(\theta)$ in fusion result of Sun Quan is still the highest values, unable to judge the fusion results, so the fusion results failed to meet expectations. The $m(A)$ value obtained in the method in this paper is the highest, and the fusion target points to the focal element A, which meets the expected result. Therefore, the method of this study can solve the problem of one vote veto to some extent.

3.1.3 Robustness

Table3. The processing results of different algorithms for robustness problems

m_1, m_2	$m(A)$	$m(B)$	$m(C)$	$m(\theta)$	k
Dempster	0	0.01	0.99	0	0.99
Murphy	0.4896	0.0002	0.5102	0	0.5098
Yager	0	0.0001	0.0096	0.99	0.99
Sun quan	0.1801	0.0031	0.1932	0.6225	0.99
this paper,	0.4494	0.0105	0.4678	0.0726	0.6404

The evidence collection of m_2 , m_3 shows that the BPA of the focal element A in m_3 is 0.98, which is reduced 0.01 than BPA of the focal element A in m_1 focal element A. But focal elements BPA from Dempster rule that is compared between the table 1 and table 3, shows that the fusion results change from pointing to focal element B to the focal element C. This method takes great changes in the recognition of the target object, so it can not the conflict

evidence for effective synthesis. The results of Murphy method, Yager method, Sun Quan method and the method in this paper are not much different from the results of dealing with general conflict problems. Therefore, the method in this paper can solve the robustness problem to some extent.

3.1.4 Rule failure Problem

Table4. The processing results of different algorithms to rule failure problem

m_1, m_2	$m(A)$	$m(B)$	$m(C)$	$m(\theta)$	k
Dempster	-	-	-	-	-
Murphy	0.5	0.5	0	0	0.5
Yager	0	0	0	1	1
Sun quan	0.1838	0.1837	0	0.6328	1
this paper	0.4971	0.4974	0	0.002	0.6406

From the evidence sets m_4 , m_5 , it can be known that the values of inconsistent factor k in m_4 and m_5 are all 1, that is, evidence m_4 , m_5 completely conflict, but the BPA of focal element B in evidence m_4 and the BPA of focal element A in evidence m_5 are 1, so the fusion results should point to focal element A and focal element B on average. It can be seen from Table 4 that Dempster's rule cannot be used in this case. The Yager method completely allocates the conflict result to the unknown focal element, and the obtained result is inconsistent with the actual situation. Murphy method, Sun Quan method and the method in this paper can solve the rule failure problem to some extent, and the obtained result points to the BPA of focal element A and focal element B. However, Murphy's method only made a simple weighted average of the conflicting evidence. The weight of the Sun Quan method divided the conflicting evidence into unknown fields was higher than other focal elements, which failed to effectively identify the target object. The value of $m(\theta)$ obtained by the method in this paper was small, and the $m(A)$ and $m(B)$ values met the expected results. Therefore, the method in this paper can solve the problem of rule failure to some extent.

4. Conclusion

In this paper, based on the analysis of the improved algorithm of classical evidence combination rules, an improved DS evidence theory combination algorithm is designed to solve the problems of general conflict, one vote rejection, poor robustness and failure of synthesis rules existing in DS evidence theory algorithm. This algorithm introduces the concepts of compatibility coefficient and credibility. On the one hand, it uses compatibility coefficient to modify the original evidence, and on the other hand, it uses credibility to improve Dempster's combination

rule. Finally, through experimental comparison and example verification, the effectiveness of this algorithm is verified. The experimental results show that compared with other algorithms, the proposed algorithm can solve the problems existing in the classical DS evidence theory to some extent, and the inference results are more reasonable and the convergence speed is faster.

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Figures

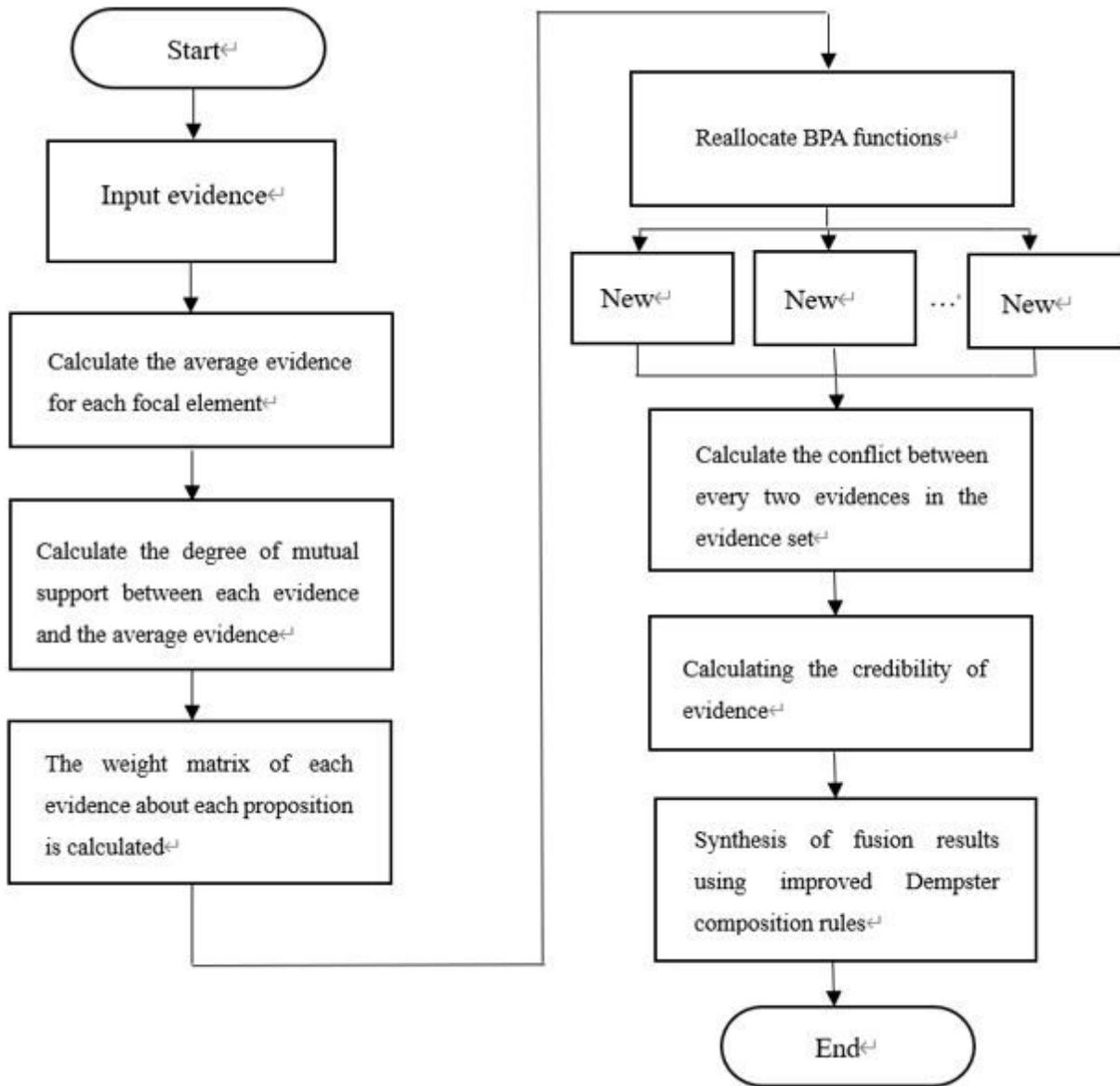


Figure 1

Conflict evidence processing flow chart