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Yafei Hu (✉ 540924782@qq.com)

PetroChina Research Institute of Petroleum Exploration & Development, China

Junshi Li

Petrochina Consulting Center, China

Zhiying WU

Reservoir Reconstruction Department of SRIPE, China

Jianfeng Hou

PetroChina Research Institute of Petroleum Exploration & Development, China

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Application of Extended Finite Element Method in Fracture Propagation Simulation of Plastic Formation

Yafei Hu¹, Junshi Li², Zhiying Wu³, Jianfeng Hou¹

¹PetroChina Research Institute of Petroleum Exploration & Development, Beijing 100083, China

²Petrochina Consulting Center, Beijing 100120, China

³Reservoir Reconstruction Department of SRIPE, Beijing 100101, China

*Correspondence to 540924782@qq.com

Abstract: Current hydraulic fracture models are mainly based on elastic theories, which fail to give accurate prediction of fracture parameters in plasticity formation. This paper proposed a fluid–solid coupling model for fracture propagation in elastoplastic formations. The rock plastic deformation in the model satisfied the Mohr-Coulomb yield criterion and plastic strain increment theory. The coupled model is solved by using extended finite-element method (XFEM) and the cohesive zone method (CZM). The accuracy of the model is verified by comparing the calculated results with existing models. The influences of stress difference, friction angle and dilation angle on fracture shape (length, width), injection pressure, plastic deformation, induced stress and pore pressure are investigated. The results indicate that compared with elastic formation, fracture shape in elastoplastic formation is wider and shorter and fracture propagation is more difficult with greater breakdown pressure and extending pressure. Plastic deformation also cause blunt fracture tip. High stress difference or low friction angle formations tend to occur large plastic deformation area and form wide and short fracture. Compared with friction angle, dilation angle is less sensitive to plastic deformation and fracture parameters and geometry. For the formation with high stress difference and friction angle, plasticity effects on fracture propagation should not be ignored.

Keywords: Elastic-plastic model; Cohesive element method; Fracture parameter; Injection pressure; Plastic deformation; Friction angle; Dilation angle

1. Introduction

With the increasing energy consumption, efficient development of oil and gas resources has become the focus of attention. Hydraulic fracturing (HF), as a widely used technology, has become increasingly important in oil and gas stimulation (Hu et al, 2019; Zhao et al, 2019). Studying fracture propagation mechanism has a great significance for fracture parameter (with, length, height) optimization. The classical fracture propagation models treated formation as an elastic medium without considering the plasticity effects and fluid-solid coupling. (Perkins and Kern, 1961; Geertsma and Klerk, 1969; Nordgren, 1972; Saouma et al, 1987; Dean and Schmidt, 2009; Dahi-Taleghani and Olson JE, 2011; Leonhart and Meschke, 2011). However, plastic failure usually occurs in soft rocks of high temperature and pressure formation and unconsolidated formation during fracturing (Brünig, 1999; Abu Al-Rub and Voyiadjis, 2003; Bohloli and de Pater, 2006). Various experiment have been conducted on fracture propagation in soft rocks. The results indicate that with the increase of confining pressure, the rock property gradually appear plastic character, which has great influence on fracture pressure and fracture shape (Sone and Zoback, 2011; Busetti et al, 2012a, 2012b; Wang, 2015; Shojaei et al, 2016). Complex fracture and fluid lag near the fracture was also observed in soft rock during fracturing. Initiation pressure and fracture width was also affected by the confining stresses in unconsolidated formation (Van Dam et al, 2002; De Pater et al, 2007; Golovin et al, 2010; Germanovich et al, 2012; Omori et al, 2013). Owing to rock plasticity and strong fluid-solid coupling, traditional fracture propagation models fail to accurately

predict the fracture parameters in soft formation.

Previous studies on the effect of plasticity in HF were just based on the static fracture and constant injection pressure in fracture without considering the coupling between the pressure diffusion and rock deformation. Papanastasiou (Papanastasiou, 1997, 1999, 2012) proposed that plastic yielding was affected by rock dilation, stress difference and the cohesive zone, and utilized the Mohr-Coulomb criterion to study the effect of pore pressure and permeability on fracture parameters without considering the fluid–solid coupling in porous formation. Zaki (Zaki et al, 2004) used the cam-clay model to assess the influence of rock plasticity on fracture volumes based on the simple models that would rely in using lower modulus or higher fracture toughness to account for plasticity. Wang (Wang et al, 2016) proposed the poroelastic and poroplastic HF model in brittle and ductile formations. Liu (Liu et al., 2019) established fracture propagation model based on the Drucker-Prager yield criterion and the associated flow law. Lin (Lin et al, 2018) investigated the effects of young's model, Poisson's ratio and injection rate on fracture propagation based on the modified cam-clay model. However, the influence of rock plasticity on the pore pressure and induced stress distribution based on fluid–solid coupling and the effects of rock friction angle, dilation angle and principal stress difference on fracture propagation were not considered in the above studies. Therefore, it is necessary to establish a fluid-solid coupling model for fracture propagation in elastoplastic formations.

In this study, a fluid–solid coupling elastoplasticity HF model is proposed. The rock plastic deformation conforms to the Mohr-Coulomb criterion and plastic strain increment theory. Fluid flows within the fracture follows the mass conservation law. The fracture propagation criterion is based on the traction/separation law of cohesive element. By combining the extended finite-element method (XFEM) and the cohesive zone method (CZM), Abaqus 6.14 software has been utilized to simulate fracture initiation and growth in elastoplastic formation. The simulation results are also compared with existing fracture expansion models. Meanwhile, the influences of principal stress difference, friction angle and dilation angle on fracture length, width, injection pressure, plastic deformation, pore pressure and induced stress are further investigated.

2 Mathematical model

Fracture propagation involves rock deformation and failure, fluid flowing in fracture and formation, the dynamic variation of pore pressure and induced stress, which is a very complex process, involving multiple coexisting and interdependent sub-processes (Wu, 2014). In addition, formation heterogeneity and plasticity make modeling more complex. In order to simplify the process of crack propagation, the following assumptions are made:

- 1) Formation is a homogeneous, porous elastic-plastic rock medium;
- 2) Fracture extending without proppant transport;
- 3) Fluid flows within the fracture existing fluid leakoff;
- 4) Fluid flows through the formation in accordance with the Darcy law;
- 5) Formation temperature is constant during fluid injection;

2.1 Fluid-solid coupling equation of rock deformation

Based on the above assumptions, rock deformation conforms to pore-elastoplastic theory. According to the stress balance equation, the stress equilibrium, strain displacement, and strain–stress equations can be written as follows (Ma et al, 2016):

$$\begin{cases} d\sigma_{ij,j} + df_i = 0 \\ d\varepsilon_{ij} = \frac{1}{2}(du_{i,j} + du_{j,i}) \\ d\sigma_{ij} = D_{ijkl}^{ep} d\varepsilon_{kl} \end{cases} \quad (1)$$

Where $d\sigma_{ij}$ is the stress increment, $d\varepsilon_{ij}$ is the strain increment, $du_{i,j}$ is the medium displacement increment, df_i is the body force increment, D_{ijkl}^{ep} is elastic-plastic coefficient matrix.

The displacement and stress at the outer boundary and fluid pressure at the fracture surface are

$$\begin{cases} du = du^*, & \text{on } \Gamma_u \\ d\sigma \bullet n = dt^*, & \text{on } \Gamma_t \\ dn^T (\sigma \bullet n) = -dp, & \text{on } \Gamma_{cr} \end{cases} \quad (2)$$

where Γ_u , Γ_t , and Γ_{cr} are the displacement boundary condition, stress boundary condition, and fracture pressure boundary condition, respectively; u^* and t^* are the corresponding displacement and stress increment on the boundary; and dp is fluid pressure increment on the fracture.

2.2 The elastic-plastic increment constitutive model

When the rock begins to occur plastic failure, the strain increment of the rock is composed of elastic strain increment and plastic strain:

$$d\varepsilon_{ij} = d\varepsilon_{ij}^e + d\varepsilon_{ij}^p \quad (3)$$

The elastic strain increment $d\varepsilon_{ij}^e$ and the stress increment $d\sigma_{ij}$ satisfy the Hooke's law:

$$d\sigma_{ij} = D_{ijkl}^e d\varepsilon_{kl}^e \quad (4)$$

The plastic strain increment follows the flow rule, which is defined as (Vermeer and Borst, 1984):

$$d\varepsilon_{ij}^p = \lambda \frac{\partial F}{\partial \sigma_{ij}} \quad (5)$$

In the process of plastic deformation, the yield surface of the rock is a function of strengthening parameters, plastic strain and stress:

$$F = F(\sigma_{ij}, \varepsilon_{ij}^p, \kappa) \quad (6)$$

$$dF = \frac{\partial F}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial F}{\partial \kappa} d\kappa + \frac{\partial F}{\partial \varepsilon_{ij}^p} d\varepsilon_{ij}^p = 0 \quad (7)$$

Where ε_{ij} is the total strain; ε_{ij}^e is the elastic strain; ε_{ij}^p is the plastic strain; σ_{ij} is the stress tensor; D_{ijkl}^e is the elastic constitutive tensor; F is the load surface equation; κ is the strengthening parameter of rock; λ is a quantity related to the slope of the uniaxial curve of stress σ -plastic strain ε^p , which can be determined by yield criteria and strengthening theory.

The stress increment is obtained by substituting equation (4) into (3).

$$d\sigma_{ij} = D_{ijkl}^e (d\varepsilon_{kl} - d\varepsilon_{kl}^p) \quad (8)$$

Multiplying both sides of equation (8) by $\partial F / \partial \sigma_{ij}$ and substituting it into equation (7):

$$\frac{\partial F}{\partial \sigma_{ij}} D_{ijkl}^e (d\epsilon_{kl} - d\epsilon_{kl}^p) = -\frac{\partial F}{\partial \kappa} d\kappa - \frac{\partial F}{\partial \epsilon_{ij}^p} d\epsilon_{ij}^p \quad (9)$$

The hardening parameter is a function of the equivalent plastic strain and temperature, which is determined as:

$$\begin{aligned} \frac{\partial F}{\partial \kappa} d\kappa &= \frac{\partial F}{\partial \kappa} \left(\frac{\partial \kappa}{\partial \bar{\epsilon}^p} d\bar{\epsilon}^p + \frac{\partial \kappa}{\partial T} dT \right) = \frac{\partial F}{\partial \kappa} \left(\frac{\partial \kappa}{\partial \bar{\epsilon}^p} + \frac{\partial \kappa}{\partial T} \frac{\partial T}{\partial \bar{\epsilon}^p} \right) d\bar{\epsilon}^p \\ &= \frac{\partial F}{\partial \kappa} \left(\frac{\partial \kappa}{\partial \bar{\epsilon}^p} + \frac{\partial \kappa}{\partial T} \frac{\partial T}{\partial \bar{\epsilon}^p} \right) \frac{d\bar{\epsilon}^p}{d\epsilon_{ij}^p} d\epsilon_{ij}^p \end{aligned} \quad (10)$$

The equivalent plastic strain increment is defined as:

$$\begin{aligned} d\bar{\epsilon}^p &= \left[\frac{2}{3} d\epsilon_{ij}^p d\epsilon_{ij}^p \right]^{1/2} = \frac{\sqrt{2}}{3} \{ (d\epsilon_x^p - d\epsilon_y^p)^2 + (d\epsilon_y^p - d\epsilon_z^p)^2 + (d\epsilon_z^p - d\epsilon_x^p)^2 \\ &\quad + \frac{3}{2} [(d\gamma_{xy}^p)^2 + (d\gamma_{yz}^p)^2 + (d\gamma_{xz}^p)^2] \}^{1/2} \end{aligned} \quad (11)$$

Where $\bar{\epsilon}^p$ is the equivalent plastic strain; T is temperature; ϵ_x^p 、 ϵ_y^p 、 ϵ_z^p are the plastic strain on the x, y and z axis respectively; γ_{xy}^p 、 γ_{yz}^p 、 γ_{xz}^p is the shear stress component.

Substituting equation (10) and (5) into equation (9)

$$\lambda = \frac{\frac{\partial F}{\partial \sigma_{ij}} D_{ijkl}^e d\epsilon_{kl}}{A} \quad (12)$$

Where

$$A = \left[\frac{\partial F}{\partial \sigma_{ij}} D_{ijkl}^e - \frac{\partial F}{\partial \kappa} \left(\frac{\partial \kappa}{\partial \bar{\epsilon}^p} + \frac{\partial \kappa}{\partial T} \frac{\partial T}{\partial \bar{\epsilon}^p} \right) \frac{d\bar{\epsilon}^p}{d\epsilon_{ij}^p} - \frac{\partial F}{\partial \epsilon_{kl}^p} \right] \frac{\partial F}{\partial \sigma_{kl}} \quad (13)$$

Substituting equation (12) and (13) into equation (3) and then substituting equation (5) into equation (8), the elastic-plastic matrix can be obtained.

$$d\sigma_{ij} = D_{ijkl}^{ep} d\epsilon_{kl} \quad (14)$$

$$D_{ijkl}^{ep} = D_{ijkl}^e - \frac{D_{ijkl}^e \left[\frac{\partial F}{\partial \sigma_{pq}} \frac{\partial F}{\partial \sigma_{rs}} D_{pqrs}^e \right]}{A} \quad (15)$$

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132 2.3 Plastic yield criteria for rocks

133 Rock deformation obeys the Mohr-Coulomb yield criterion, and we stipulates that the tensile
134 stress is positive and the compressive stress is negative:

$$135 \tau = c - \sigma \tan \varphi \quad (16)$$

136 According to the geometry of the mole circle, shear stress τ and normal stress σ can be
137 expressed as following:

$$\begin{cases} \tau = \frac{\sigma_1 - \sigma_3}{2} \cos \varphi \\ \sigma = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \sin \varphi \end{cases} \quad (17)$$

Where τ is the shear stress; σ is the normal stress; φ is the friction degree; σ_1 and σ_3 are the maximum and minimum principal stresses, respectively; σ_2 are the Intermediate principal stress; Substitute equation (17) into equation (16), we have:

$$\frac{\sigma_1 - \sigma_3}{2} \cos \varphi = c - \left[\frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \sin \varphi \right] \tan \varphi \quad (18)$$

Principal stress and principal deviatoric stress satisfy the following equation:

$$\begin{cases} \sigma_1 = \frac{2}{\sqrt{3}} \sqrt{J_2} \cos \Theta + \frac{1}{3} I_1 \\ \sigma_2 = \frac{2}{\sqrt{3}} \sqrt{J_2} \cos \left(\frac{2\pi}{3} - \Theta \right) + \frac{1}{3} I_1 \\ \sigma_3 = \frac{2}{\sqrt{3}} \sqrt{J_2} \cos \left(\frac{2\pi}{3} + \Theta \right) + \frac{1}{3} I_1 \end{cases} \quad (19)$$

Where σ_1 and σ_3 are the maximum and minimum principal stresses, respectively; σ_2 are the Intermediate principal stress; I_1 is the first invariant of stress tensor; J_2 is the second invariant of deviatoric stress; Θ is the deviatoric polar angle.

Substitute equation (19) into equation (18):

$$F = \frac{1}{3} I_1 \sin \varphi + \sqrt{J_2} \sin \left(\Theta + \frac{\pi}{3} \right) + \frac{\sqrt{J_2}}{\sqrt{3}} \cos \left(\Theta + \frac{\pi}{3} \right) \sin \varphi - c \cos \varphi = 0 \quad (20)$$

Equivalent compressive stress p , Mises equivalent stress q , the first invariant of stress tensor I_1 and the second invariant of deviatoric stress J_2 satisfy the following equation:

$$\begin{cases} p = -\frac{1}{3} \text{trace}(\sigma) = -\frac{I_1}{3} \\ q = \sqrt{\frac{3}{2} s_{ij} s_{ij}} = \sqrt{3 J_2} \\ \cos(3\Theta) = \frac{r^3}{q^3} \end{cases} \quad (21)$$

Where p is the equivalent pressure stress; q is the Mises equivalent stress; r is the third invariant of deviatoric stress; s is the deviatoric stress.

Substitute equation (21) into equation (20):

$$F = f(p, q, \Theta) = R_{mc} q - p \tan \varphi - c = 0 \quad (22)$$

Where

$$R_{mc} = \frac{1}{\sqrt{3} \cos \varphi} \sin \left(\Theta + \frac{\pi}{3} \right) + \frac{1}{3} \cos \left(\Theta + \frac{\pi}{3} \right) \tan \varphi \quad (23)$$

In the case of equation (23), there will be sharp angles on the yield surface, leading to not unique plastic flow direction, which leads to tedious calculation and slow convergence. In order to solve this problem, Menetrey and Willam (Menetrey and Willam, 1995) proposed a continuous smooth elliptic function as a plastic potential surface.

$$G = \sqrt{(\varepsilon c_0 \tan \psi)^2 + (R_{mw} q)^2} - p \tan \psi \quad (24)$$

Where

$$R_{mw} = \frac{4(1-e^2)\cos^2 \Theta + (2e-1)^2}{2(1-e^2)\cos \Theta + (2e-1)\sqrt{4(1-e^2)\cos^2 \Theta + 5e^2 - 4e}} \frac{3-\sin \varphi}{6\cos \varphi} \quad (25)$$

$$e = \frac{3-\sin \varphi}{3+\sin \varphi} \quad (26)$$

Where ψ is the dilation angle measured in the p - $R_{mw}q$ plane at high confining pressure and can depend on temperature and predefined field variables; c_0 is the initial cohesion yield stress; ε is the eccentricity ratio, here is assumed to be 0.1; e is the “out of roundedness” of the deviatoric section in terms of the ratio between the shear stress along the extension meridian and the shear stress along the compression meridional.

2.4 Fluid flow in fracture and formation

Once the fracture is formed, fluid will flow into the fracture at once, fluid flow in the fracture follows the mass conservation law (Yao, 2012):

$$\frac{\partial w}{\partial t} + \frac{\partial q_t}{\partial s} + v_t + v_b = 0 \quad (27)$$

Where q_t is the injection flow rate, w is the fracture width, s is the fracture length, t is injection time, v_t and v_b are the fracturing fluid leakoff rate through the top and bottom fracture surfaces, respectively.

Both tangential flow along and normal flow across the fracture surfaces can be calculated using the Poiseuille equation (Boone and Ingraffea, 1990; Chen Z, Bunger et al, 2009):

$$q_t = \frac{w^3}{12\mu} \frac{\partial p}{\partial s} \quad (28)$$

Where μ is the fluid viscosity, p is the fluid pressure.

Fluid leakoff rate is mainly affected by pressure and leakoff coefficient, which satisfy the following equation (Elguedj, 2006; Zielonka et al, 2014):

$$\begin{cases} v_t = c_t (p - p_{top}) \\ v_b = c_t (p - p_{bot}) \end{cases} \quad (29)$$

Where c_t is the fluid leakoff coefficients, p_{top} and p_{bot} are pressure in the top and bottom fracture surface, respectively.

When the fluid leaks off from the fracture, it will flows through the formation in accordance with the Darcy law. The mass conservation equation is expressed as:

$$\nabla \left(\frac{\rho_w k_w}{b_w \mu_w} \nabla p_w \right) + q_w = \frac{\partial}{\partial t} \left(\frac{\phi \rho_w s_w}{b_w} \right) \quad (30)$$

Where ρ_w is the water permeability; k_w is the water permeability; p_w is water pressure in formation; b_w is the water compression coefficient; μ_w is the water viscosity; q_w are the leakoff fluid; S_w is the water saturation; ϕ is the effective porosity.

2.5 Cohesive crack model

The singularity of the fracture tip stress field will affect the convergence of the calculation of the model. In order to solve this problem, CZM is adopted to simulate the initiation and propagation of fractures. This method is implemented by embedding an artificially predefined path of cohesive element into the formation. The cohesive element glues two conjugated cohesive surfaces of the reservoir pressure/displacement element. The failure of these element obeys the traction/separation law in the vertical or shear direction (Tvergaard and Hutchinson, 1992; Camacho and Ortiz, 1996). When the cohesive element is completely destroyed, the two bound cohesive surfaces are separated and hydraulic fracture is generated. When the maximum nominal stress criterion is adopted for rock failure, initiation propagation occurs as the maximum nominal stress ratio reaches a critical value (Nguyen et al, 2017; Sobhaniragh et al, 2018a, 2018b):

$$f_c = \max \left\{ \frac{\langle T_n \rangle}{T_n^0}, \frac{T_s}{T_s^0} \right\} \quad (31)$$

$$1 \leq f_c \leq 1 + f_t \quad (32)$$

Where δ_n , δ_s are, respectively the normal and tangential displacement jump vector; δ_{eq} are effective displacement; T_n^0, T_s^0 are, respectively the normal and tangential stress component of the stress element under a linear elastic condition; $\langle \rangle$ denotes the Macaulay bracket; f_c is a parameter evaluating the fracture failure, f_t is assured to be 0.05;

The fracture damage evolution is determined by damage variables, and the expressions of damage effects on the normal and tangential stress components of the element can be obtained as (Zhu et al, 2015; Sobhaniragh et al, 2018a, 2018b)

$$T_n = \begin{cases} (1-d)T_n^0, & T_n^0 \geq 0 \\ T_n^0, & T_n^0 < 0 \end{cases} \quad (33)$$

$$T_s = (1-d)T_s^0 \quad (34)$$

Where d is the scalar damage variable between fractures, $0 < d < 1$; T_n, T_s are, respectively, the values of the normal stress component and the tangential stress component of the traction/separation model.

When the critical fracture energy of rock material along the first and second shear directions are similar, the damage evolution during fracture propagation can also be determined by the Benzeggagh–Kenane fracture criterion (Benzeggagh and Kenane, 1996). The combined energy dissipated by failure G_c is defined as

$$G^c = G_I^c + (G_{II}^c - G_I^c) \left(\frac{G_{II}}{G_I + G_{II}} \right)^\eta \quad (35)$$

where G_c is the critical fracture energy release rate, G_I^c and G_{II}^c are the normal and first shear direction fracture toughness values, G_I and G_{II} are the normal and shear direction fracture energy

release rate, and η is a material properties parameter, here is assumed to be 2.3.

3、Model verification

In order to simulate HF in elastic-plastic formation, the size of geological model is set as 50 m x 50 m with a cohesive element embedded in the central of the model. The injection point is in the center of the cohesive zone. The minimum principal stress is in y direction. All the external boundary displacement are fixed in normal directions. The geological model is shown in figure 1 and the input parameters of simulation model is presented in table 1. The simulation calculation process consists of two steps: geostatic step and HF step. The purpose of geostatic step is to balance the applying initial formation stress and fluid pressure. HF step is to simulate the change of fluid injection, rock deformation and fracture initiation and expansion during fracturing.

Figure.1. Geological model for hydraulic fracture simulation

Table. 1. Input parameters for hydraulic fracture model

properties	Value
Young modulus (MPa)	15000
Poisson ratio	0.25
Material cohesion (MPa)	1.5
Material friction angle ($^{\circ}$)	28
Material dilation angle ($^{\circ}$)	28
Formation permeability coefficients (m/s)	1×10^{-7}
Maximum horizontal stress (MPa)	20
Minimum horizontal stress (MPa)	15
Porosity	0.1
Fluid viscosity (cp)	1
Specific weight of fluid (KN/m ³)	9.8
Initial pore pressure (MPa)	0.1
Injection rate (m ³ /s)	5×10^{-3}
Pressure dependent leak-off coefficient (m ³ /MPa.s)	1×10^{-14}
Critical fracture energy (KN/m ³)	28
Damage initiation stress (MPa)	0.32

To verify the accuracy of the model, we compared the calculated results with the model of Papanastasiou. Papanastasiou's model is a simple fluid-solid coupling model without considering fluid leak off and is solved numerically by a combined finite difference and finite element method. The change of injection pressure, fracture width and length and the equivalent plastic strain zone with time in elastic and elastic-plastic formation is also analyzed by the model. The calculated results are demonstrated in figure 1-4.

Fig.2a and Fig.3a exhibit that the injection pressure and fracture width at the wellbore in elastic-plastic formation is greater than that in elastic formation, resulting greater breakdown pressure and propagation pressure in elastic-plastic formation, which is similar to results of Papanastasiou model shown in Fig.2b and Fig.3b. Moreover, in the early stage of fracture initiation, the change trend of injection pressure and fracture width at injection point are also in accordance with Papanastasiou model. Injection pressure in both models is first increased and then decreased to a constant value. Wellbore fracture width are also both gradually increasing with injection time.

Fig.4 presents that fracture in elastic-plastic formation is incline to be wider and shorter than elastic formation. Because plastic damage requires more energy and plastic deformation of rock also lead the fracture tip to be blunt, resulting fracture more difficult to extend. Fig.5 displays that the equivalent plastic strain area increases with injection time, and largest plastic strain appears near the fracture tip. The plastic deformation also cause rock compaction, resulting in the porosity and permeability loss around the fracture with less fluid leak off and higher injection pressure.

(a) Our model (b) Papanastasiou's model (Papanastasiou, 1997)
Fig.2. Comparison of injection pressure calculated by different models

(a) Our model

(b) Papanastasiou's model (Papanastasiou, 1997)

Fig.3. Comparison of wellbore fracture width with different time calculated by poroelastic and poroelastoplastic models

Fig.4. Comparison of fracture parameters with different time calculated by poroelastic and poroelastoplastic models

(a) Injection time at 20s

(b) Injection time at 40s

(c) Injection time at 60s

(d) Injection time at 80s

Fig.5. Equivalent plastic strain zone of fracture varies with injection time

4、 Numerical results and discussion

4.1 Effect of stress difference on fracture propagation

The effects of horizontal principal stress differences on fracture width, length, injection pressure, fracture width at the wellbore, equivalent plastic strain of fracture, pore pressure and induced stress are simulated. It is assumed that the minimum horizontal principal stress is constant at 30MPa, the horizontal principal stress difference is 5MPa, 8MPa and 10MPa respectively, the pore pressure of the formation is 20MPa, the injection time is 80s, and the friction angle and dilation

angle of the rock are both 15° . The calculation results are presented in figure 6-10.

Fig. 6 demonstrates that crack width increases and crack length decreases with the increasing stress difference. Fig. 7 indicates that the crack width at the wellbore and injection pressure increases with the increase of horizontal stress difference. Figure 8 displays that the area of equivalent plastic strain around the crack also increases with the increase of horizontal stress difference. Figure 9-10 present that pore pressure and induced stress in x direction surround the fracture also grows with the increasing stress difference. Because high stress difference is easy to generate large plastic zones, and rock compaction caused by the plastic deformation around the fracture leads to a decrease in porosity and permeability, which in turn makes it difficult for the fracturing fluid to flow into the formation, resulting greater pore pressure near the fracture surface. Meanwhile, the induced stress on the fracture surface also makes crack difficult to extent, owing to plastic effect. Therefore, plastic formations with high horizontal stress difference is benefit to form wide and short crack.

Fig.6. Effect of stress difference on fracture width and length

Fig.7. Effect of stress difference on wellbore fracture width and injection pressure

(a) Stress difference 5MPa

(b) Stress difference 8MPa

(c) Stress difference 10MPa

Fig.8. Effect of stress difference on equivalent plastic strain zone of fracture

(a) Stress difference 5MPa

(b) Stress difference 8 MPa

(c) Stress difference 10 MPa

Fig.9. Effect of stress difference on pore pressure

Fig.10. Effect of stress difference on stress on x direction

4.2 Effect of friction angle on fracture propagation

In order to study the influence of friction angle on hydraulic fracture propagation, the effects of friction angle on fracture width and length, injection pressure, fracture width at the wellbore, equivalent plastic strain, pore pressure and induced stress are simulated. It is assumed that the minimum horizontal principal stress is constant at 10MPa, the horizontal principal stress difference is 5MPa, and the injection time is 80s. The friction angle of the rock is 10°, 20° and 28°, respectively, and dilation angle is equal to friction angle. The calculation results are presented in figure 11-15.

Fig. 11 demonstrates that crack width decreases with the increasing friction angle, while crack length increases with the increasing friction angle. Lower friction angle cause the fracture tip to be blunter. When friction angle is 10°, plastic deformation at the injection point has obviously appeared in the early stage of crack expansion, leading to a significant increase of crack width near the wellbore. Figure 12 indicates that the crack width at the wellbore and injection pressure increases with friction angle decrease. When the friction angle decreases to 10°, fracture breakdown pressure and propagation pressure grow significantly, which leads to rapid increasing fracture width near the injection point. Figure 13 displays that the equivalent plastic strain area around the crack increases with the decreasing friction angle. Plastic strain occur obviously at the fracture entrance with friction angle of 10°, which is difficult for crack to extent. Figure 14-15 present that pore pressure and stress in x direction surround the fracture also increases with the decreasing friction angle.

Rock with low friction angle are easy to occur plastic deformation and form wide and short crack.

Fig.11. Effect of stress difference on fracture width and length

Fig.12. Effect of stress difference on wellbore fracture width and injection pressure

(a) Friction angle 10°

(b) Friction angle 20°

(c) Friction angle 28°

Fig.13. Effect of friction angle on equivalent plastic strain zone of fracture

(a) Friction angle 10°

(b) Friction angle 20°

(c) Friction angle 28°

Fig.14. Effect of friction angle on pore pressure

(a) Friction angle 10°

(b) Friction angle 20°

(c) Friction angle 28°

Fig.15. Effect of friction angle on stress on x direction

4.3 Effect of dilation angle on fracture propagation

The dilation angle reflects the change rate of rock volume, normal displacement and tangential displacement in the shearing process. To study the influence of dilation angle on hydraulic fracture propagation, the effects of dilation angle on fracture width and length, injection pressure, fracture width at the wellbore, equivalent plastic strain, pore pressure and induced stress are simulated. It is assumed that the dilation angle of the rock is 10°, 20° and 28°, respectively, and friction angle is 28°. The other input parameters are the same as section 4.2. The calculation results are presented in figure 16-20.

Fig. 16 demonstrates that crack width decreases with the increasing dilation angle and this trend is not obvious. Figure 17 indicates that with injection time more than 20s, dilation angle has little influence on injection pressure and fracture width at the wellbore. Figure 18 displays that the equivalent plastic strain around the crack increases with the decrease of dilation angle and the value of plastic strain is small. Figure 19-20 present that dilation angle has also little effect on pore pressure and stress on x direction. Compared with friction angle, dilation angle is less sensitive to fracture width and length, as well as injection pressure.

Fig.16. Effect of dilation angle on fracture width and length

Fig.17. Effect of dilation angle on wellbore fracture width and injection pressure

(a) Dilation Angle 10°

(b) Dilation Angle 20°

(c) Dilation Angle 28°

Fig.18. Effect of dilation angle on equivalent plastic strain zone of fracture

(a) Dilation angle 10°

(b) Dilation angle 20°

(c) Dilation angle 28°

Fig.19. Effect of dilation angle on pore pressure

(a) Dilation angle 10°

(b) Dilation angle 20°

(c) Dilation angle 28°

Fig.20. Effect of dilation angle on stress on x direction

5、Conclusions

A fluid–solid coupling elastoplasticity HF model is proposed. Calculation results of the model is compared with Papanastaiou's model and they have good agreements.

In comparison to elastic formation, elastic-plastic formation exhibit greater breakdown and propagation pressure, resulting wider and shorter fracture. Plastic deformation also lead to the fracture tip to be blunt. Plastic strain area grows with increasing injection time, and the largest plastic strain appears near the fracture tip.

Fracture width increases and fracture length decreases with the increasing stress difference. High stress difference or low friction angle are easy to occur large plastic deformation area, resulting great breakdown pressure, propagation pressure, induced stress in x direction and pore pressure. Soft formations with great stress difference or low friction angle is inclined to form wide and short crack and blunt fracture tip.

Compared with friction angle, dilation angle is less sensitive to plastic deformation and fracture parameters. For the formation with high stress difference and friction angle, the influence of plasticity effects on fracture propagation should not be ignored.

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Figures

Figure 1

Geological model for hydraulic fracture simulation

Figure 2

Comparison of injection pressure calculated by different models

(a) Our model

(b) Papanastasiou's model (Papanastasiou, 1997)

Figure 3

Comparison of wellbore fracture width with different time calculated by poroelastic and poroelastoplastic models

Figure 4

Comparison of fracture parameters with different time calculated by poroelastic and poroelastoplastic models

Figure 5

Equivalent plastic strain zone of fracture varies with injection time

Figure 6

Effect of stress difference on fracture width and length

Figure 7

Effect of stress difference on wellbore fracture width and injection pressure

Figure 8

Effect of stress difference on equivalent plastic strain zone of fracture

Figure 9

Effect of stress difference on pore pressure

Figure 10

Effect of stress difference on stress on x direction

Figure 11

Effect of stress difference on fracture width and length

Figure 12

Effect of stress difference on wellbore fracture width and injection pressure

(a) Friction angle 10°

(b) Friction angle 20°

(c) Friction angle 28°

Figure 13

Effect of friction angle on equivalent plastic strain zone of fracture

Figure 14

Effect of friction angle on pore pressure

Figure 15

Effect of friction angle on stress on x direction

Figure 16

Effect of dilation angle on fracture width and length

Figure 17

Effect of dilation angle on wellbore fracture width and injection pressure

(a) Dilation Angle 10°

(b) Dilation Angle 20°

(c) Dilation Angle 28°

Figure 18

Effect of dilation angle on equivalent plastic strain zone of fracture

Figure 19

Effect of dilation angle on pore pressure

Figure 20

Effect of dilation angle on stress on x direction