New formulas for the trigonometry of isosceles triangles

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Abstract: *This paper discusses a new formula to solve for the angle of isosceles triangles given proportion of differentiated unequivalent edge length versus the always-equivalent edges. The equation is equal to the arcsine of (X/2) multiplied by two. This formula is to be called 'isn^-1(x)', short for inverse isosceles sine equivalent. Also discussed is the non-inverse 'isn(x)' which is the cyclic function which is determined to correspond as the equivalent of the classic sine formula to isosceles triangles. The formula set is clearly superior and powerful at calculating the angle and measure of any given obtuse or acute triangle of an unclassified type, as well as unifying a simplest-fit formula across all types of triangle.*

Equation:

The inverse isosceles sine, with the inequal over one of the equal edge lengths as x:

$$isn^{-1}(x) = \sum_{n=0}^{\infty}\left(\frac{(2n)!\,x^{2n+1}}{16^n(n!)^2(2n+1)}\right)$$

$$= \sum_{n=0}^{\infty}\left(\frac{(2n)! * x^{(2n+1)}}{\left(\left(2^{2n} * (n!)^2 * (2n+1)\right) * 2^{(2n)}\right)}\right)$$

$$\cong \left(\frac{1 * x^{(1)}}{((1 * 1 * 1) * 1)}\right) + \left(\frac{2 * x^{(3)}}{((4 * 1 * 3) * 4)}\right)$$

$$+ \left(\frac{24 * x^{(5)}}{((16 * 4 * 5) * 16)}\right) + \left(\frac{720 * x^{(7)}}{((64 * 36 * 7) * 64)}\right) \ldots$$

'Isosceles Inverse Cosine' ics^-1(x), to solve the angle of any triangle with three known side lengths:

$$ics^{-1}(x) = \left(\pi - \sum_{n=0}^{\infty}\left(\frac{(2n)!\,x^{2n+1}}{16^n(n!)^2(2n+1)}\right)\right) =$$

$$\pi - isn^{-1}(x) = (\pi - \sum_{n=0}^{\infty}\left(\frac{(2n)! * x^{(2n+1)}}{\left(\left(2^{2n} * (n!)^2 * (2n+1)\right) * 2^{(2n)}\right)}\right))$$

$$\cong \pi - \left(\left(\frac{1 * x^{(1)}}{((1 * 1 * 1) * 1)}\right) + \left(\frac{2 * x^{(3)}}{((4 * 1 * 3) * 4)}\right) + \left(\frac{24 * x^{(5)}}{((16 * 4 * 5) * 16)}\right) + \left(\frac{720 * x^{(7)}}{((64 * 36 * 7) * 64)}\right) \ldots\right)$$

$\ldots$*where in any triangle besides isosceles*:

$$x = \left(\frac{(adjacent\ 1)^2 + (adjacent\ 2)^2 - (opposite)^2}{(adjacent\ 1) * (adjacent\ 2)}\right)$$

For the cyclic function, i= sqrt(-1) and with isn^-1 angle theta as x:

$$isn(x) = \sum_{n=1}^{\infty}\left(\frac{x^{2n}}{(2n)!} * i^{n}\right) \cong \left(\frac{x^2}{2} - \frac{x^4}{4*3*2*1} + \frac{x^6}{6*5*4*3*2*1} - \frac{x^8}{8*7*6*5*4*3*2*1} \dots\right) \; and$$

$$: \; 1 - isn(x) = \sum_{n=0}^{\infty}\left(\left(\frac{x^{2n}}{(2n)!}\right) * i^{(n-1)}\right) \cong \left(1 - \frac{x^2}{2} + \frac{x^4}{4*3*2*1} - \frac{x^6}{6*5*4*3*2*1} + \frac{x^8}{8*7*6*5*4*3*2*1} \dots\right)$$

The equations appear to exponentiate upon the exact values of the angular and proportionate measures for the given notation, though may take large numbers of iterations to become precise.

Another series of cyclic functions to be defined by modifications of isn(x) that will calculate and be equal to to proportion from the radius of the graph or angle to the respective x and y components on the cartesian plane is obviously managable from here. They are to be named isx(x) and isy(x).

$$isx(\gamma) = (1 - isn(\gamma))$$

$$isy(\gamma) = \left(\left(1 - isn\left(\gamma - \left(\frac{\pi}{2}\right)\right)\right)\right)$$

Additional Applications:

In physics, the isn^-1 equation is shown to be used to compute vector combinations and tensors more effectively. It can also be used in pure vector mathematics with ease.

Given right triangles as reference to x and y components plus magnitudes of forces, like so:

$$Given\ force\ or\ vector\ set\ F(n\ forces)\ and\ P's(n\ components)$$

$$of\ any\ vector\ for\ example:$$
$$Px = \{Px(n)\}$$
$$Py = \{Py(n)\}$$
$$Pf = \{Pf(n)\}$$

…the resultant force on a point or object will be given the components:

$$\gamma = \left(\left(\frac{Py}{\sqrt{Py^2}}\right) * ics^{-1}\left(\frac{2Px^2}{Px * Pf}\right)\right) or \ldots$$

$$\gamma = \left(isn^{-1}\left(\frac{Py\sqrt{(Pf - Px)^2 + (Py^2)}}{|Py| * Pf}\right)\right) and \ldots$$

$$Fx = \left(Pf * \left(isx(\gamma)\right)\right)$$
$$Fy = \left(Pf * \left(isy(\gamma)\right)\right)\ so\ that \ldots$$

$$The\ force's\ angle\ equals:$$

$$\theta = ics^{-1}\left(\frac{Fx^2 + \sqrt{Fx^2 + Fy^2}^2 - Fy^2}{Fx * \sqrt{Fx^2 + Fy^2}^2}\right)$$

$$\ldots \textit{and its magnitude equals}:$$

$$\sqrt{Fx^2 + Fy^2}$$

This proves that the theorem is effective in calculating dimensional components of vectors in a vastly alternative and simplified method. Positive and negative x values are right at hand in this setup, while y values must be differentiated. This at least preserves the symmetry of the reference dimension and, using corresponding signs for if the angle to a radian is negative, one can obtain full 360 degree coordinates, or adapt the formula to more advanced purposes. Less formulas by far are needed for calculation of geometry and physics with this vector method, and the calculations can be put in one piece with less external operations or operators.

A calculation for the ideal circumference of the perfect circumscription of a known SSS triangle can be done as follows, where variable 'side' varies from the longest to shortest side of the triangle for the calculation of the correspondent circumscription, semiscription, or inscription:

$$x = \left(\frac{(adjacent\ 1)^2 + (adjacent\ 2)^2 - (opposite)^2}{(adjacent\ 1) * (adjacent\ 2)}\right)$$

$$C = \left((\pi + |isn^{-1}(x)|)\right) * side$$

Additionally, the ratio of a circle diameter to circumference PI = 3.14159… can be calculated using the infinite series simplified from isn^-1(x):

$$\pi = \sum_{n=0}^{\infty}\left(\frac{(2n)!\,2^{-2n+1}}{(n!)^2(2n+1)}\right)$$

<u>Discussion</u>: The isn, isn^-1 and ics^-1 formulas provide a unifying method whereat it is optimally easy to solve any other type of triangles (in relation to the relative component of the positions or sides of the adjacent points to the angle). The sin and arcsin methods are specific to components and do not provide method to solve other triangles in a way where the modifications are significant to accessible properties of the triangles. Since no other formula has the proper blend of consistency, reflexive clarity and ease with calculating all types of triangles, and this formula is computationally superior for obtaining transformed physical predictions and simulations as well as data on geometric systems, ics^-1 ought to be used as a unifying trigonometric equation in the process of relating expressions and triangles to each other or their other measurements. it may very well be the superior method of calculating any type of triangle from an educational and computational perspective, as well as from a standpoint in search of algebraic abstraction of geometric properties.

In ultimatum; the expressional components of 'ics^-1 ready' triangular conversion coefficients for proper angle calculation are more universally supported by this formula than the cos^-1 theorem can rationally express to, making it computationally superior for closed systems and computing large networks of geometrically or mathematically bound variable systems that would correspond to everyday applied mathematics. Using a simple script and algebraic rules every geometric and physical formulation may be given a uniform distribution and ease of reference between different theorems, potentially simplifying together series of expressions in systems that are closed or of high complexity, not easily possible with sin or sin^-1 on an input or any scale of algebraically easily transformable level. It is suggested that many computational systems and graphing calculator make an easy use of this function or its generated dataset and its conversion utilities for a geometric, spatial or coordinate network.

<u>References</u>: *<u>No references or sources on this formula or the new information on this paper were used in the process of finding this formula, though the sin and sin^-1 formulas are a similar work in right triangles that has been used in mathematics in previous years and are given by various sources.</u>*