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New Formulas for the Trigonometry of Isosceles Triangles

Noah King (king240@canton.edu)
SUNY Canton https://orcid.org/0000-0002-4613-5918

Research Article

Keywords: trigonometry, geometry, math, angle, triangle, applied, physics, computer, computational, efficient

Posted Date: April 8th, 2022

DOI: https://doi.org/10.21203/rs.3.rs-1445285/v4

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New Formulas for the Trigonometry of Isosceles Triangles

Noah G. King (Eski)

518-354-0396

king240@canton.edu

Abstract: This paper discusses a new formula to solve for the angle of isosceles triangles given proportion of differentiated unequivalent edge length versus the always-equivalent edges. The equation is equal to the arcsine of (X/2) multiplied by two. This formula is to be called 'isn^-1(x)', short for inverse isosceles sine equivalent. Also discussed is the non-inverse 'isn(x)' which is the cyclic function which is determined to correspond as the equivalent of the classic sine formula to isosceles triangles. The formula set is clearly superior and powerful at calculating the angle and measure of any given obtuse or acute triangle of an unclassified type, as well as unifying a simplest-fit formula across all types of triangle.

Equations:

The inverse isosceles sine, with the inequal over one of the equal edge lengths as x:

$$isn^{-1}(x) = \sum_{n=0}^{\infty}\left(\frac{(2n)!\,x^{2n+1}}{16^n(n!)^2(2n+1)}\right)$$
$$= \sum_{n=0}^{\infty}\left(\frac{(2n)! * x^{(2n+1)}}{\left(\left(2^{2n} * (n!)^2 * (2n+1)\right) * 2^{(2n)}\right)}\right)$$
$$\cong \left(\frac{1 * x^{(1)}}{\left((1*1*1)*1\right)}\right) + \left(\frac{2 * x^{(3)}}{\left((4*1*3)*4\right)}\right) + \left(\frac{24 * x^{(5)}}{\left((16*4*5)*16\right)}\right)$$
$$+ \left(\frac{720 * x^{(7)}}{\left((64*36*7)*64\right)}\right)\ldots$$

'Isosceles Inverse Cosine' ics^-1(x), to solve the angle of any triangle with three known side lengths:

$$ics^{-1}(x) = \left(\pi - \sum_{n=0}^{\infty}\left(\frac{(2n)!\,x^{2n+1}}{16^n(n!)^2(2n+1)}\right)\right) =$$

$$\pi - isn^{-1}(x) = (\pi - \sum_{n=0}^{\infty}\left(\frac{(2n)! * x^{(2n+1)}}{\left(\left(2^{2n} * (n!)^2 * (2n+1)\right) * 2^{(2n)}\right)}\right))$$
$$\cong \pi - \left(\left(\frac{1 * x^{(1)}}{((1*1*1)*1)}\right) + \left(\frac{2 * x^{(3)}}{((4*1*3)*4)}\right) + \left(\frac{24 * x^{(5)}}{((16*4*5)*16)}\right) + \left(\frac{720 * x^{(7)}}{((64*36*7)*64)}\right) \ldots\right)$$

$$\ldots where\ in\ any\ triangle\ besides\ isosceles:$$

$$x = \left(\frac{(adjacent\ 1)^2 + (adjacent\ 2)^2 - (opposite)^2}{(adjacent\ 1) * (adjacent\ 2)}\right)$$

For the cyclic function, i= sqrt(-1) and with isn^-1 angle theta as x:

$$isn(x) = \sum_{n=1}^{\infty}\left(\frac{x^{2n}}{(2n)!} * i^n\right) \cong \left(\frac{x^2}{2} - \frac{x^4}{4*3*2*1} + \frac{x^6}{6*5*4*3*2*1} - \frac{x^8}{8*7*6*5*4*3*2*1} \ldots\right) and: 1 - isn(x) = \sum_{n=0}^{\infty}\left(\left(\frac{x^{2n}}{(2n)!}\right) * i^{(n-1)}\right) \cong \left(1 - \frac{x^2}{2} + \frac{x^4}{4*3*2*1} - \frac{x^6}{6*5*4*3*2*1} + \frac{x^8}{8*7*6*5*4*3*2*1} \ldots\right)$$

The equations appear to exponentiate upon the exact values of the angular and proportionate measures for the given notation, though may take large numbers of iterations to become precise.

Another series of cyclic functions to be defined by modifications of isn(x) that will calculate and be equal to to proportion from the radius of the graph or angle to the respective x and y components on the cartesian plane is obviously managable from here. They are to be named isx(x) and isy(x).

$$isx(\gamma) = (1 - isn(\gamma))$$

$$isy(\gamma) = \left(1 - isn\left(\gamma - \left(\frac{\pi}{2}\right)\right)\right)$$

<u>Additional Applications</u>:

In physics, the isn^-1 equation is shown to be used to compute vector combinations and tensors more effectively. It can also be used in pure vector mathematics with ease.

Given right triangles as reference to x and y components plus magnitudes of forces, like so:

$$Given\ force\ or\ vector\ set\ F(n\ forces)\ and\ P's(n\ components)$$

$$of\ any\ vector\ for\ example:$$
$$Px = \{Px(n)\}$$
$$Py = \{Py(n)\}$$
$$Pf = \{Pf(n)\}$$

…the resultant force on a point or object will be given the components:

$$\gamma = \left(\left(\frac{Py}{\sqrt{Py^2}}\right) * ics^{-1}\left(\frac{2Px^2}{Px * Pf}\right)\right) or \ldots$$

$$\gamma = \left(isn^{-1}\left(\frac{Py\sqrt{(Pf - Px)^2 + (Py^2)}}{|Py| * Pf}\right)\right) and \ldots$$

$$Fx = \left(Pf * \left(isx(\gamma)\right)\right)$$
$$Fy = \left(Pf * \left(isy(\gamma)\right)\right)\ so\ that \ldots$$

$$The\ force's\ angle\ equals:$$

$$\theta = ics^{-1}\left(\frac{Fx^2 + \sqrt{Fx^2 + Fy^2}^2 - Fy^2}{Fx * \sqrt{Fx^2 + Fy^2}^2}\right)$$
$$\ldots and\ its\ magnitude\ equals:$$

$$\sqrt{Fx^2 + Fy^2}$$

This proves that the theorem is effective in calculating dimensional components of vectors in a vastly alternative and simplified method. Positive and negative x values are right at hand in this setup, while y values must be differentiated. This at least preserves the symmetry of the reference dimension and, using corresponding signs for if the angle to a radian is negative, one can obtain full 360 degree coordinates, or adapt the formula to more advanced purposes. Less formulas by far are needed for calculation of geometry and physics with this vector method, and the calculations can be put in one piece with less external operations or operators.

A calculation for the ideal circumference of the perfect circumscription of a known SSS triangle can be done as follows, where variable 'side' varies from the longest to shortest side of the triangle for the calculation of the correspondent circumscription, semiscription, or inscription:

$$x = \left(\frac{(adjacent\ 1)^2 + (adjacent\ 2)^2 - (opposite)^2}{(adjacent\ 1) * (adjacent\ 2)}\right)$$

$$C = \left((\pi + |isn^{-1}(x)|)\right) * side$$

Additionally, the ratio of a circle diameter to circumference PI = 3.14159… can be calculated using the infinite series simplified from isn^-1(x):

$$\pi = \sum_{n=0}^{\infty} \left(\frac{(2n)!\, 2^{-2n+1}}{(n!)^2 (2n+1)}\right)$$

One more theoretical set of expressions which works best using isn^-1(x) is the set which produces the average of all angles of x->y and x->z, as well as the one with the average of x->y and y->z, in a set of points within an object in equidistant and regular conditions. The average value of the sum pairs of these angles respectively condense unique irrational numbers that vary across a set of possible values based on the scale, rotation and perspective of an object or region in a graph. The single value produced by the set makes it possible to correspond a single number of many decimals to an expansion that reproduces the graph of the object, and in the latter case may be able to locate the object by its center of point sample density when it is part of a larger equation. This method relies

only on the average of the sum of the defining angles to a region R at its perspective, the equation and process can be expressed as:

$$Graph(x,y,z) = \sum_{x,y,z=(x_1,y_1,z_1)}^{(x_m,y_m,z_m)} \left(\frac{(isn\hat{}-1(x,y)+isn\hat{}-1(x,z))^{\sqrt{x^2+y^2+z^2}}}{m}\right)$$

Where m will represent the number of x,y,z coordinates to fill the region that are used as a 3-dimensional list.

Simply put, this method will return irrational numbers that correspond uniquely and identically to one particular graph with optional transformations.

Discussion:

The isn, isn^-1 and ics^-1 formulas provide a unifying method whereat it is optimally easy to solve any other type of triangles (in relation to the relative component of the positions or sides of the adjacent points to the angle). The sin and arcsin methods are specific to components and do not provide method to solve other triangles in a way where the modifications are significant to accessible properties of the triangles.

Since no other formula has the proper blend of consistency, reflexive clarity and ease with calculating all types of triangles, and this formula is computationally superior for obtaining transformed physical predictions and simulations as well as data on geometric systems, ics^-1 ought to be used as a unifying trigonometric equation in the process of relating expressions and triangles to each other or their other measurements. it may very well be the superior method of calculating any type of triangle from an educational and computational perspective, as well as from a standpoint in search of algebraic abstraction of geometric properties.

In ultimatum:

The expressional components of 'ics^-1 ready' triangular conversion coefficients for proper angle calculation are more universally supported by this formula than the cos^-1 theorem can rationally express to, making it computationally superior

for closed systems and computing large networks of geometrically or mathematically bound variable systems that would correspond to everyday applied mathematics. Using a simple script and algebraic rules every geometric and physical formulation may be given a uniform distribution and ease of reference between different theorems, potentially simplifying together series of expressions in systems that are closed or of high complexity, not easily possible with sin or sin^-1 on an input or any scale of algebraically easily transformable level. It is suggested that many computational systems and graphing calculator make an easy use of this function or its generated dataset and its conversion utilities for a geometric, spatial or coordinate network.

Data Availability Statement, and Conflicts of Interest:

The data and calculations generated during and/or analysed during this study are not included in this paper. The calculations are simple and are easily tested for proof that they converge and are equivalent to the arcsine of (X/2) multiplied by two. The example data is not included for the simple reason that it does not help demonstrate any further topics than is discussed so far in the paper. Due to the fact that any standard calculator or convergence test will yield the results which are claimed in this text, they are available from the corresponding author on reasonable request.

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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