2D Stream Depletion Problem in a Sloping Aquifer

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Abstract

Most of the analytical approaches for estimation of pumping induced drawdown and stream depletion are based on idealistic assumptions that the aquifer is resting on a perfectly horizontal base. In this research, new analytical expressions are derived and demonstrated to estimate stream depletion rate in response to uniform drawdown by a well. The mathematical set up consists of an unconfined aquifer resting on a sloping bed and hydrologically connected with a partially penetrating stream and an extraction well in the domain of the aquifer. The proposed analytical solution are obtained by using Laplace, Fourier and their inverse transforms. The solution developed herein is an advancement of the Hunt's analytical solution and can be used as screening tool for measuring stream depletion rate in sloping terrains. The results indicate that the analytical solution developed herein predicts significantly different rate of stream depletion from that of Hantush and Hunt model primarily due to the fact that the current study accounts for the bed slope of the aquifer.

1. Introduction

If pumping is done from well beside a stream, it lowers the ground water levels and reduces surface water flow within the stream. In the smaller streams, this decrease is highly remarkable and it can cause various quantative and qualitative hydrological effects on streams located in the aquifer's domain and also on the nearby areas. This study will facilitate in identifying the effects of aquifer's bed slope on the stream depletion rate and the local groundwater regime. In the past, several studies have been carried out for estimation of the stream depletion rate and successfully accomplished desired result. But none of the researchers have considered the study of stream depletion for sloping aquifer which is main focus of this research.

In, pumping induced groundwater flow, a large proportion of stream depletion is attributable to the bedrock pattern of the aquifer. Models based on varying geometry of the aquifer's bed can be used for the better estimation of the stream depletion produced by a well in the vicinity of a finite width stream of shallow penetration. Although, an analytical approach cannot represent stream-aquifer interaction to the same degree of detail as a numerical model, but it can provide a useful screening tool for measuring the influence of various factors, and to obtain the estimates which are comparable to available data.

The first unsteady solution for such problem was obtained by Theis (1941) [18]. He obtained the solution in the form of an integral which he evaluated with an infinite series. Later, same solution was restudied by Glover and Balmer (1954) [8]. Hydraulics of groundwater was studied in detail by Bear (1979) [4]. Jenkins(1968) [12] elaborated on some of the specific numerical details that are needed when applying equation of stream depletion to specific problems.

An analytical solutions for groundwater table drawdown and stream depletion which incorporates conductance and stream partial penetration was described by Hunt (1999) [11]. Spalding and Khaleel (1991) [16] and Sophocleous (1995) [17] studied the evaluation of simplified stream aquifer depletion models for water rights administration. The problems of sloping aquifer was discussed in detail by Chapman (1980) [7]. But Hunt's solution is based on approximation, and it assumes that the stream width is close to zero. The impact of groundwater pumping on nearby streams was described by Butler et. al. (2001, 2007)[5], [6]. Ramana, Rai and Singh (1995) [15] studied water table fluctuations due to transient recharge in a 2D aquifer system with inclined base.

Upadhyaya and Chauhan (2001) [19] studied interaction of stream and sloping aquifer receiving constant recharge. Analytical solutions have also been developed based on the assumptions of negligible drawdown in the sourcebed of a leaky aquifer and horizontal flow in an aquifer of infinite extent by Zlotnik (2001, 2008) [21], [22]. Rai, Mangalik and Singh (2006) [13] studied the water table fluctuation owing to time varying recharge whereas Ram and Chauhan (1987) [14] studied analytical and experimental solutions of drainage of sloping land with time varying recharge. Analytical solution of stream-aquifer interactions in partially penetrating streams was discussed by Huang (2010) [10]. In current paper, the mathematical formulation of Bansal [1], [2] is reconsidered in estimation of stream depletion rate in a patrially penetrating stream due to pumping of a well in an unconfined sloping aquifer. The proposed analytical solution is obtained using Laplace, Fourier transforms and their inverses. The analytical solution proposed is also compared with Hantush [9] and Hunt [11] existing solution to test the reliability of the solution. For the proposed solution, we have also considered the impact of streambed width, streambed thickness, streambed hydraulic conductivity, the distance between the well and streambed and the slope of the aquifer on stream depletion.

2. Mathematical Formulation

According to Bansal (2012), (2015) [1], [2] the mathematical formulation for this study consisting of a well beside a stream which reduces surface water flow within the stream in an unconfined sloping aquifer is given by following equation:

$$K_x\left\{\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right) - tan\theta\frac{\partial h}{\partial x}\right\}\cos^2\theta + K_y\frac{\partial}{\partial y}\left(h\frac{\partial h}{\partial y}\right) + W = S\frac{\partial h}{\partial t}$$

1

where $h = h(x, y, t)$ is the variable hydraulic head in the aquifer measured in the vertical direction from the sloping bed; K_x and K_y are components of hydraulic conductivity along x- axis and y- axis respectively and S is the specific yield of the aquifer. The term W simulates the combined effects of streambed seepage and withdrawal from the well. We write

$$W = \frac{k_s d_s}{b_s}(h_0 - h)\,\delta(x) - Q\delta(x - l)\,\delta(y)$$

2

where k_s, d_s and b_s are hydraulic conductivity, width and thickness of streambed layer. δ is the Dirac delta function. Assuming that the initial water table is parallel to the sloping impervious base, the initial condition can be described as

$$h(x, y, t = 0) = h_0 - \infty < x < \infty, \; - \infty < y < \infty$$

3

and the boundary conditions are

$$h(x \to \pm \infty, y, t) = h_0 - \infty < y < \infty$$

4

$$h(x, y \to \pm \infty, t) = h_0 - \infty < x < \infty$$

5

Following the analogy of Hunt, (1999) [11], the stream depletion rate q is defined as double integration of flux W over the streambed area. That is

$$q = \lim_{\epsilon \to \infty} \int_{-\infty}^{\infty} \int_{-\epsilon}^{\epsilon} W dx dy$$

6

According to Werner (1957) [20] linearization method, Eq. (1) can be linearlized in terms of h^2 and takes the form

$$K_r \left(\frac{\partial^2 h^2}{\partial x^2} - \frac{tan\theta}{D} \frac{\partial h^2}{\partial x} \right) + \frac{1}{cos^2\theta} \frac{\partial^2 h^2}{\partial y^2} + \frac{2W}{K_y cos^2\theta} = \frac{S}{K_y D cos^2\theta} \frac{\partial h^2}{\partial t}$$

7

The corresponding initial and boundary conditions are :

$$h^2(x, y, t = 0) = h_0^2 - \infty < x < \infty, \; - \infty < y < \infty$$

8

$$h^2(x \to \pm \infty, y, t) = h_0^2 - \infty < y < \infty$$

9

$$h^2(x, y \to \pm \infty, t) = h_0^2 - \infty < x < \infty$$

10

where$K_r = \frac{K_x}{K_y}$ is the anisotropic ratio(dimensionless) and $D = \frac{h_0 + h}{2}$ is the weighted mean of the water table height. We define the leakage factor as :

$$\lambda^2 = K_e D c = K_e \frac{h_0 + h}{2} c$$

11

where $K_e = \sqrt{K_x K_y}$ is the effective hydraulic conductivity in the horizontally anisotropic aquifer and $c = \frac{b_s}{k_s}$ is the hydraulic resistance. Introducing the new variables:

$$H(\xi, y, \tau) = h_0^2 - h^2(x, y, t)$$

12

$$\tau = \alpha t$$

13

$$\xi =x-v\tau$$

14

where $\alpha =\frac{{K}_{e}\sqrt{{K}_{r}}Dco{s}^{2}\theta }{S},v=\frac{tan\theta }{D}$.

As q is the contant well flow rate, H is the elevation of the free surface in the river, λ is the leakage factor between the seepage flow rate per unit distance (in the y direction) through the streambed and the difference between the river and the ground water level at x=0. Thus the integration of W defined in Eq. (2) over the streambed surface gives the total outflow seepage through the streambed. Hence (6) can be written as

$$q=-\lambda {\int }_{-\infty }^{\infty }\text{}[{h}_{0}^{2}-{h}_{(x,y,t)}^{2}]dy$$

which can be further written as

$$q=-\lambda {\int }_{-\infty }^{\infty }\text{}H(0,y,t)dy$$

15

Applying the chain rule, we obtain

$$\frac{\partial {h}^{2}}{\partial x}=-\frac{\partial H}{\partial \xi }$$

16

$$\frac{{\partial }^{2}{h}^{2}}{\partial {x}^{2}}=-\frac{{\partial }^{2}H}{\partial {\xi }^{2}}$$

17

$$\frac{{\partial }^{2}{h}^{2}}{\partial {y}^{2}}=-\frac{{\partial }^{2}H}{\partial {y}^{2}}$$

18

$$\frac{\partial {h}^{2}}{\partial t}=-\alpha \frac{\partial H}{\partial \tau }+\alpha v\frac{\partial H}{\partial \xi }$$

19

Now, substituting these derivatives in Eq. (7) and combining the resulting equation with Eq. (2) yields

$$\frac{{\partial }^{2}H}{\partial {\xi }^{2}}+G\frac{{\partial }^{2}H}{\partial {y}^{2}}-LH\delta \left(\xi \right)+\delta (\xi -l)\delta \left(y\right)=\frac{\partial H}{\partial \tau }$$

20

The contant λ has units of velocity, H(x,y,t) and *q* are the drawdown and the stream depletion rate respectively caused by the pumping of well. The problem formulation is completed by adding an initial condition and a boundary condition at infinity as given below.

$$H(\xi ,y,\tau =0)=0,-\infty <x<\infty ,-\infty <y<\infty$$

21

$$H(\xi \to \pm \infty ,y,\tau)=0,-\infty <y<\infty$$

22

$$H(\xi ,y\to \pm \infty ,\tau)=0,-\infty <\xi <\infty$$

23

where $G=\frac{1}{{K}_{r}co{s}^{2}\theta },L=\frac{{d}_{s}}{{\lambda }^{2}\sqrt{{K}_{r}}co{s}^{2}\theta },R=\frac{2Q}{{K}_{e}\sqrt{{K}_{r}}co{s}^{2}\theta }$

The solution domain for this problem consists of entire $x-y$ plane, which contrasts the semi definite domain used for the solutions obtained by Theis (1941) [18], Glover and Balmer (1954) [8] and Hantush (1965) [9]. Applying the Laplace transform to equations (20) and also to equations (21)–(23), we have

$$\frac{{\partial }^{2}\stackrel{-}{H}}{\partial {\xi }^{2}}+G\frac{{\partial }^{2}\stackrel{-}{H}}{\partial {y}^{2}}-p\stackrel{-}{H}=L\stackrel{-}{H}\delta \left(\xi \right)-\frac{R}{p}\delta (\xi -l)\delta \left(y\right)$$

24

$$\stackrel{-}{H}(\xi \to \pm \infty ,y,p)=0, -\infty <y<\infty$$

25

$$\stackrel{-}{H}(\xi ,y\to \pm \infty ,p)=0, -\infty <\xi <\infty$$

26

where $\stackrel{-}{H}(\xi ,y,p)$ denotes the Laplace transform of $H(\xi ,y,t)$ taken over the time domain. Furthermore, the exponential Fourier transform of $\stackrel{-}{H}(\xi ,y,p)$ is defined as

$$\mathcal{F}\left[\stackrel{-}{H}\right(\xi ,y,p\left)\right]= \stackrel{-}{\stackrel{-}{H}}(\xi ,\omega ,p)={\int }_{-\infty }^{\infty }\text{}{e}^{i\omega y}\stackrel{-}{H}(\xi ,y,p)d y \quad (27)$$

The exponential Fourier transform of Eq. (24) with respect to variable *y* yields following ordinary differential equation

$$\frac{{d}^{2}\stackrel{-}{\stackrel{-}{H}}}{d{\xi }^{2}}-{\beta }^{2}\stackrel{-}{\stackrel{-}{H}}=L\stackrel{-}{\stackrel{-}{H}}\delta \left(\xi \right)-\frac{R}{p}\delta (\xi -l)\delta \left(y\right)$$
28

$\text{w}\text{h}\text{e}\text{r}\text{e}{ \beta }^{2}={\omega }^{2}G+p$. The corresponding boundary condition (25) reduces to

$$\stackrel{-}{H}(\xi \to \pm \infty ,\omega ,p)=0$$
29

3. Development Of Analytical Solution

According to Hunt (1991) [11] methodology, solution for Eq. (28) is given by

$$\stackrel{-}{\stackrel{-}{H}}=\left\{\begin{array}{c}{c}_{1}{e}^{\beta \xi } -\infty <\xi <0\\ {c}_{2}cosh\left(\beta \xi \right)+{c}_{3}sinh\left(\beta \xi \right) 0<\xi <l\\ {c}_{4}{e}^{-\beta \xi } l<\xi <\infty \end{array}\right.$$
30

For simplicity, denote $\stackrel{-}{\stackrel{-}{H}}= {\stackrel{-}{\stackrel{-}{H}}}_{1}+ {\stackrel{-}{\stackrel{-}{H}}}_{2}+ {\stackrel{-}{\stackrel{-}{H}}}_{3}$

where ${\stackrel{-}{\stackrel{-}{H}}}_{1}={c}_{1}{e}^{\beta \xi } -\infty <\xi <0$, ${\stackrel{-}{\stackrel{-}{H}}}_{2}={c}_{2} \text{c}\text{o}\text{s}\text{h}\left(\beta \xi \right)+{c}_{3} \text{s}\text{i}\text{n}\text{h}\left(\beta \xi \right) 0<\xi <l \text{a}\text{n}\text{d}$

${\stackrel{-}{\stackrel{-}{H}}}_{3}={c}_{4}{e}^{-\beta \xi }l<\xi <\infty$. Integration of (28) with respect to *x* across two narrow regions containing $\xi =0$ and $\xi =l$ reveals that $\stackrel{-}{\stackrel{-}{H}}\left(\xi \right)$ is continuous at these two points but its first derivative has discontinuities of $L\stackrel{-}{\stackrel{-}{H}}(0,\omega ,p)$ and $-\frac{R}{p}$ and $\xi =0$ and $\xi =l$, respectively. Therefore, the constants in equations $\left(30\right)$ can be evaluated by using following additional conditions:

$$\left({\stackrel{-}{\stackrel{-}{H}}}_{1}\right){}_{\xi =0}=\left({\stackrel{-}{\stackrel{-}{H}}}_{2}\right){}_{\xi =0}$$
31

$$\left({\stackrel{-}{\stackrel{-}{H}}}_{2}\right){}_{\xi =l}=\left({\stackrel{-}{\stackrel{-}{H}}}_{3}\right){}_{\xi =l}$$
32

$$\left(\frac{d{\stackrel{-}{\stackrel{-}{H}}}_{2}}{d\xi }\right){}_{\xi =0}=\left(\frac{d{\stackrel{-}{\stackrel{-}{H}}}_{1}}{d\xi }\right){}_{\xi =0}+L\stackrel{-}{\stackrel{-}{H}}(0,\omega ,p)$$
33

$$\left(\frac{d{\stackrel{-}{\stackrel{-}{H}}}_{3}}{d\xi }\right){}_{\xi =l}=\left(\frac{d{\stackrel{-}{\stackrel{-}{H}}}_{2}}{d\xi }\right){}_{\xi =l}-\frac{R}{p}$$
34

Combining equations $\left(31\right)–\left(34\right)$ with Eq. (30), we have the following equations

$${c}_{1 }- {c}_{2 }=0$$
$${c}_{2 }{e}^{\beta l}+ {c}_{2 }{e}^{-\beta l}+ {c}_{3 }{e}^{\beta l} - {c}_{2 }{e}^{-\beta l} - {2c}_{4 }{e}^{-\beta l}=0$$
$${c}_{1 }\beta +L {c}_{1 }- {c}_{3 }\beta =0$$
$${c}_{2 }{e}^{\beta l} - {c}_{2 }{e}^{-\beta l}+ {c}_{3 }{e}^{\beta l} + {c}_{3 }{e}^{-\beta l} + {2c}_{4 }{e}^{-\beta l}=\frac{2R}{p\beta }$$

Solving above equations, we get

$${c}_{1}=\frac{R}{{e}^{\beta l}p(L+2\beta)}; {c}_{2}=\frac{R}{{e}^{\beta l}p(L+2\beta)}; {c}_{3}=\frac{R(L+\beta)}{\beta {e}^{\beta l}p(L+2\beta)}; {c}_{4}=\frac{- R{e}^{-\beta l}(L-2\beta {e}^{2\beta l}-L)}{2\beta p(L+2\beta)}$$
35

Inversion of (30) to calculate *H*(*x*, *y*, *t*) is relatively difficult problem. The stream depletion, however, can be obtained with far less difficulty. Taking the Laplace transform of Eq. (15), we get

$$\stackrel{-}{q}=-\lambda {\int }_{-\infty }^{\infty }\text{}\stackrel{-}{H}(0,y,p)dy$$
36

Now, setting $\xi =0 \text{a}\text{n}\text{d} \omega =0$ in Eq. (27), and comparing it with Eq. (36) we have

$$\stackrel{-}{q}=-\lambda \stackrel{-}{\stackrel{-}{H}}(\text{0,0},p)$$
37

Lastly, invoking Eq. (35) in (30) for $\xi =0$, and using the resulting equation in (37), it requires only inversion of Laplace transform of following equation:

$$\stackrel{-}{q}=\frac{\lambda R{e}^{-l\sqrt{p}}}{\text{L}p\left(\sqrt{p}+\frac{L}{2}\right)}$$
38

Using the solution from Bateman (1954) [3] and taking inverse Laplace transform, we have

$$q=\frac{\lambda R}{L}\left[erfc\frac{l}{2\sqrt{t}}-exp\left\{\frac{{L}^{2}t}{4}+\frac{Ll}{2}\right\}erfc\left(\frac{L\sqrt{t}}{2}+\frac{l}{2\sqrt{t}}\right)\right]$$
39

Substituting values for L and R in and using equation $\left(39\right)$ we have

$$\frac{q}{{Q}_{w}}=erfc\left(\frac{l}{2\sqrt{t}}\right)-\text{e}\text{x}\text{p}\left(\frac{{L}^{2}t}{4}+\frac{Ll}{2}\right)erfc\left(\frac{L\sqrt{t}}{2}+\frac{l}{2\sqrt{t}}\right)$$
40

where ${Q}_{w}=\frac{2{\lambda }^{3}Q}{{d}_{s}\sqrt{{K}_{x}{K}_{y}}}$

Stream depletion in Eq. (40) resembles Hantush solution from Hunt [11],

$$\frac{q}{{Q}_{w}}=erfc\left(\sqrt{\frac{S{l}^{2}}{4Tt}}\right)-\text{e}\text{x}\text{p}\left(\frac{Tt}{S{L}^{2}}+\frac{l}{L}\right)erfc\left(\sqrt{\frac{Tt}{S{L}^{2}}}+\sqrt{\frac{S{l}^{2}}{4t}}\right)$$
41

for S = Aquifer storage coefficient = 0.2, T = Transmissivity = 1

L = Stream leakance $=\frac{2{b}_{s}{K}_{x}co{s}^{2}\beta }{{k}_{s}{d}_{s}}$

The Glover and Balmer model [8] of stream depletion for fully penetrating stream can be written as:

$$\frac{q}{Q}=erfc\left(\frac{l}{2\sqrt{\gamma t}}\right)$$
42

where $\gamma ={k}_{a}{d}_{a}/{S}_{y}$ where ${S}_{y}$ =1,is the aquifer diffusivity.

The Hantush [9] model, which adds a vertical low permeability streambed to the fully penetrating stream model of Glover and Balmer [8], can be written as:

$$\frac{q}{Q}=erfc\left(\frac{l}{2\sqrt{\gamma t}}\right)-exp\left(\frac{l}{a}+\frac{\alpha t}{{l}^{2}}\right)erfc\left(\frac{l}{2\sqrt{\gamma t}}+\sqrt{\frac{\gamma t}{a}}\right)$$
43

where $a={k}_{s}{d}_{s}/{k}_{a}$ where is a retardation coefficient to incorporate the effects of the reduced permeability of the streambed.

4. Effects Of The Hydrogeological Parameters On Stream - Aquifer Interactions

The results developed in this study are illustrated with the help of a numerical example. A downward sloping aquifer is hydrologically connected with a partially penetrating stream, and depleted by an extraction well located in the domain of the aquifer. The values of aquifer parameters used in the illustrative example are listed in Table 1.

Table 1
Values assigned to the variables

Parameter symbol	Notation	Value	Dimension
K_x	Hydraulic conductivity of the aquifer along x-axis	2.5	m/d
K_y	Hydraulic conductivity of the aquifer along y-axis	2	m/d
S	Specific yield of the aquifer	0.2	dimensionless
k_s	Hydraulic conductivity of the streambed	0.1	m/d
d_s	Width of streambed	8	m
b_s	Thickness of the streambed	2	m
k_a	Hydraulic conductivity of the aquifer	0.1	m/d
l	Distance of pumping well from stream	10	m
K_r	K_x/K_y	1.25	dimensionless
h_0	Initial water level in the aquifer	10	m
θ	Bed sloping angle	5	deg
D	$(h_0 + h)/2$	10	m
Q	Quantity of water drawn from well	400	m^3/d
K_e	$(K_xK_y)^{1/2}$	2.236	
c	b_s/d_s	0.25	dimensionless
α	$(K_e\sqrt{K{}_{r}}D{\text{cos}}^{2}\theta)/S$	124.047	
λ	$(K_e D c)^{1/2}$	2.36	
G	$1/(K_r \cos^2\theta)$	0.806	
L	$d_s/(\lambda^2\sqrt{K{}_{r}}{\text{cos}}^{2}\theta)$	1.29	
R	$2Q/(K_e\sqrt{K{}_{r}}{\text{cos}}^{2}\theta)$	806.45	

It is easy to use these parameters to plot the graphs for equations (40), (41), and (42) as shown in Fig. 2.

Though, the model does show that Eqs. (40) and (41), (42) gives a physically reasonable behavior of the stream depletion for different days. The graphs obtained by (41) and (42) represents identical graph. It confirms the drawdown occuring in the water level however more useful results can be obtained for different values of k_s, b_s, d_s, l and the slope of the aquifer. We have studied and presented below the variation in stream depletion for different values.

We have also calculated the values of Proposed solution, Hantush solution and Glover & Balmer solution using the parameters given in Table 1 and the results are presented in Table 2 below.

Table 2
Comparison of Stream Depletion Rate for the present study (θ = 5 deg) with that of Hantush and Glover & Balmer Model

No. of days	Stream Depletion Rate		
	Current Model with θ = 5 deg	**Hantush Model for θ = 5 deg**	**Glover & Balmer Model for horizontal bedrock**
1	$5.5092*10^{-10}$	0.00019568	0.00024713
2	0.00025480	0.14995	0.16915
3	0.021586	1.4713	1.5982
4	0.20542	4.7588	5.0709
5	0.81427	9.7863	10.307
10	13.952	44.620	45.959
20	64.782	104.05	105.98
50	184.02	190.34	192.31
75	240.87	224.12	225.94
100	279.43	245.56	247.24
500	441.27	328.53	329.42
700	454.78	339.44	340.21
1000	485.93	349.23	349.89

4.1 Effect of the Stream Width (d_s)

Above graph shows that as the width of the streambed increases, the stream depletion decreases. It means that increase in the width of the streambed, resists the flow towards the extraction well.

4.2 Effect of the Stream Thickness (b_s)

Above graph shows that as the thickness of the streambed increases, the stream depletion increases. It means that thickness of the streambed provides a natural hydraulic gradient to the stream water to flow towards the extraction well.

4.3 Effect of the Stream hydraulic conductivity (k_s)

From the graph we can see that the stream depletion is not affected by the hydraulic conductivity of the streambed.

4.4 Effect of the Angle of aquifer (\theta)

From the graph, we can see that as the angle of the aquifer increases, stream depletion increases. It means that the bed slope increases the depletion of the stream by providing a natural hydraulic gradient to the stream water to flow towards the extraction well.

4.5 Effect of the distance between Well and Stream (l)

From the graph, we can see that, as the distance of the stream from the well increases, stream depletion decreases. It means the impact is more if the well is close to the stream.

5. Discussion Of Results

The mathematical model defined by the equations above was solved using Laplace, Fourier and its inverse transform. The stream depletion is computed using the solution proposed in this research. The values of the variables are chosen from Table 1 provided above. Further the results have been compared with existing numerical models given by Hantush [8] and Glover & Balmer [8] and are represented in the tabular form in Table 2. The dimensionless parameters are used to illustrates the result of these comparisons. Figures 2, 3, 4, 5, 6 and 7 represents the stream depletion for Eq. (40) plotted with respect to time in days which is similar to the Hantush model represented by Eq. (41).

The new analytical solution that is developed in this research is in agreement with Hantush solution and Glover & Balmer model [6].

6. Physical Interpretation Of The Result

From the tabular data and graphical representation, it is clear that pumping from well beside a stream lowers ground water levels and reduces surface water flow within the stream with the day. Though this analysis of interaction between groundwater and surface water is a very complicated problem in the evaluation and management of the water resources but has been worked in this research by approximation of some variables. The nature of stream depletion rate is analysed with respect to other aquifer parameters such as d_s, b_s, k_s, distance between the well & the stream and slope of the aquifer to provide a complete understanding of the groundwater regime. In this research it has been noted that stream depletion:

- decreases with the increase in width of the streambed,
- increases with the increase in the thickness of streambed,
- increases with the increase in angle of the aquifer and
- decreases with the increase in the distance between the stream and the well.

This research could prove to be a valuable new tool for water management design and other purposes by locating the areas affected by this stream depletion and identifying the radius of influence. This will not only help in preserving the aquatic ecosystem, but also the areas for water management can be planned and designed accordingly.

7. Conclusion

In this research, a new approximate analytical solution has been developed for estimation of the drawdown and stream depletion which is produced by pumping of an anisotropic unconfined sloping aquifer in the vicinity of the stream. The stream is considered to be partially penetrating the aquifer and dimensions of the streambed cross section are relatively small. A nonlinear advection-diffusion equation simulating the flow of subsurface seepage is linearized appropriately and solved by mixing Fourier and Laplace transform techniques. Solutions developed herein are indeed superior to the earlier known solutions particularely in sloping terrains. Application of new results is demonstrated using a set of hypothetical aquifer parameters, and it is observed that in the current setup, the bed slope accelerates the depletion of the stream by providing a natural hydraulic gradient to the stream water to flow towards the extraction well. Sensitivity of the stream depletion rate vis-à-vis other aquifer parameters such as d_s, b_s, k_s etc is analyzed so as to provide a holistic understanding of the groundwater regime. The results developed in this study can be used as test cases for numerical models and guidelines for experimental studies.

8. Statements And Declarations

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Competing Interest

We have no relevant financial or non-financial interests to disclose.

Author Contributions

Both authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by Manisha Kankarej and Rajeev Bansal. All authors read and approved the final manuscript.

Data Availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

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Figures

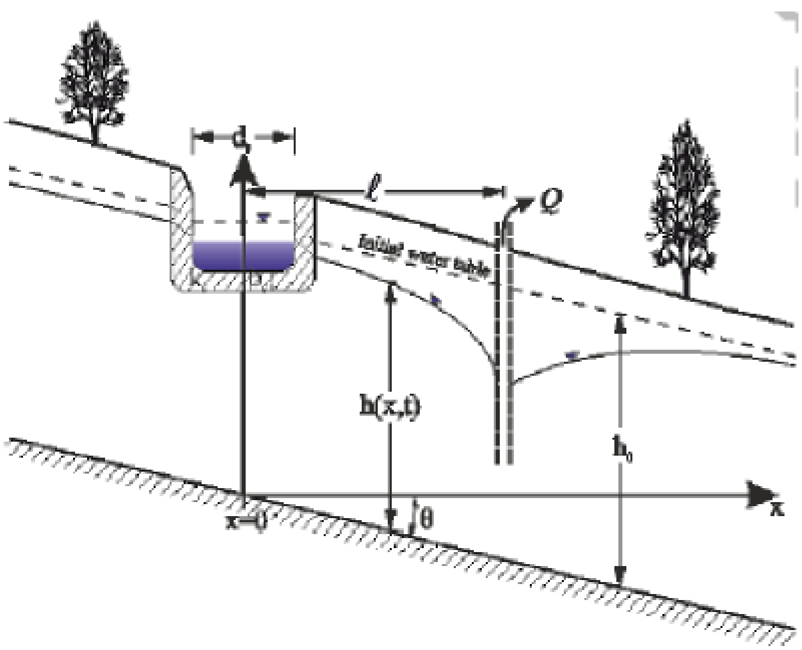

Figure 1

Definition sketch of imperfect hydraulic connection between partially penetrating stream and a downward sloping aquifer

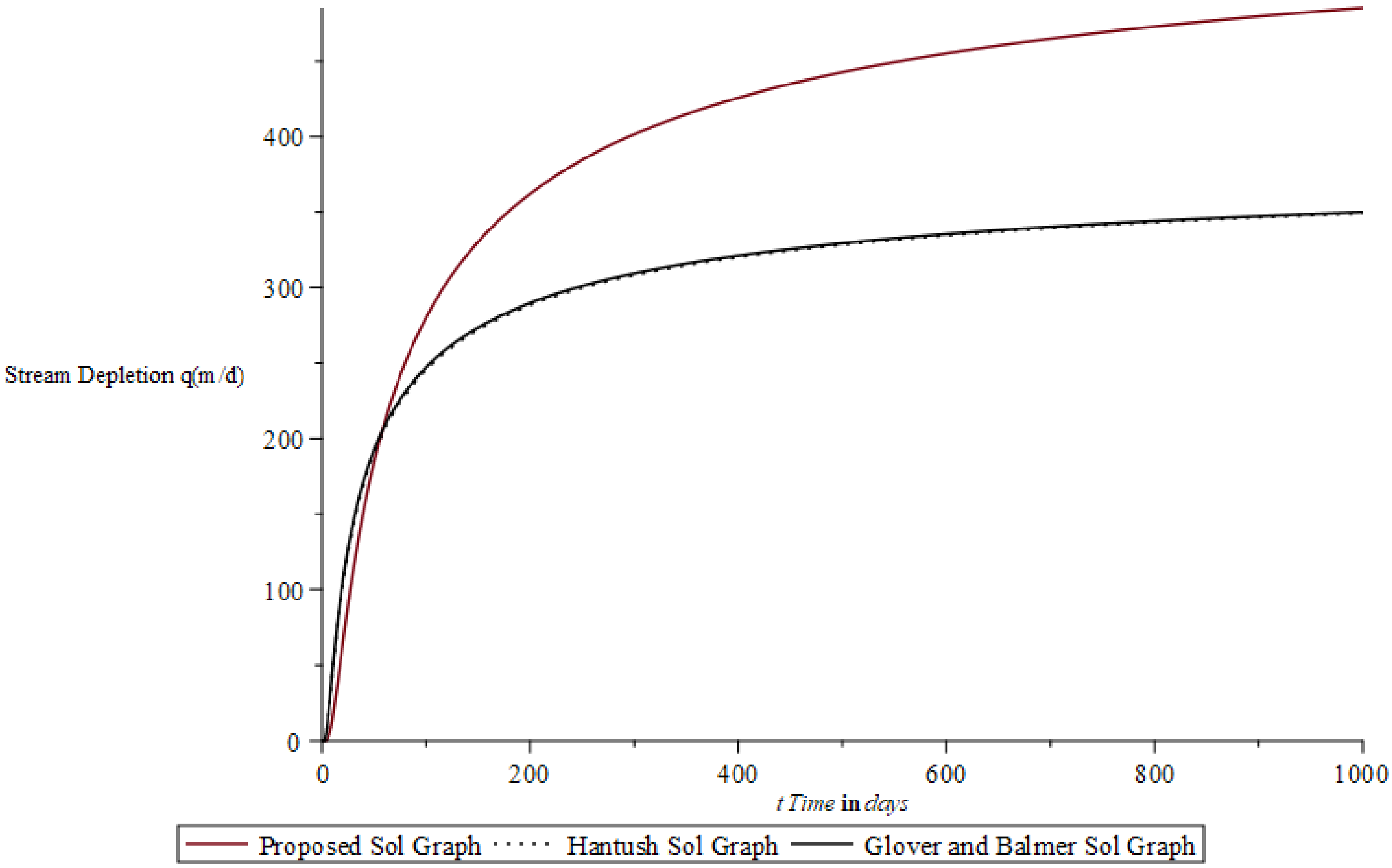

Figure 2

Comparison of stream depletion *q* predicted by the proposed model with that of Hantush model

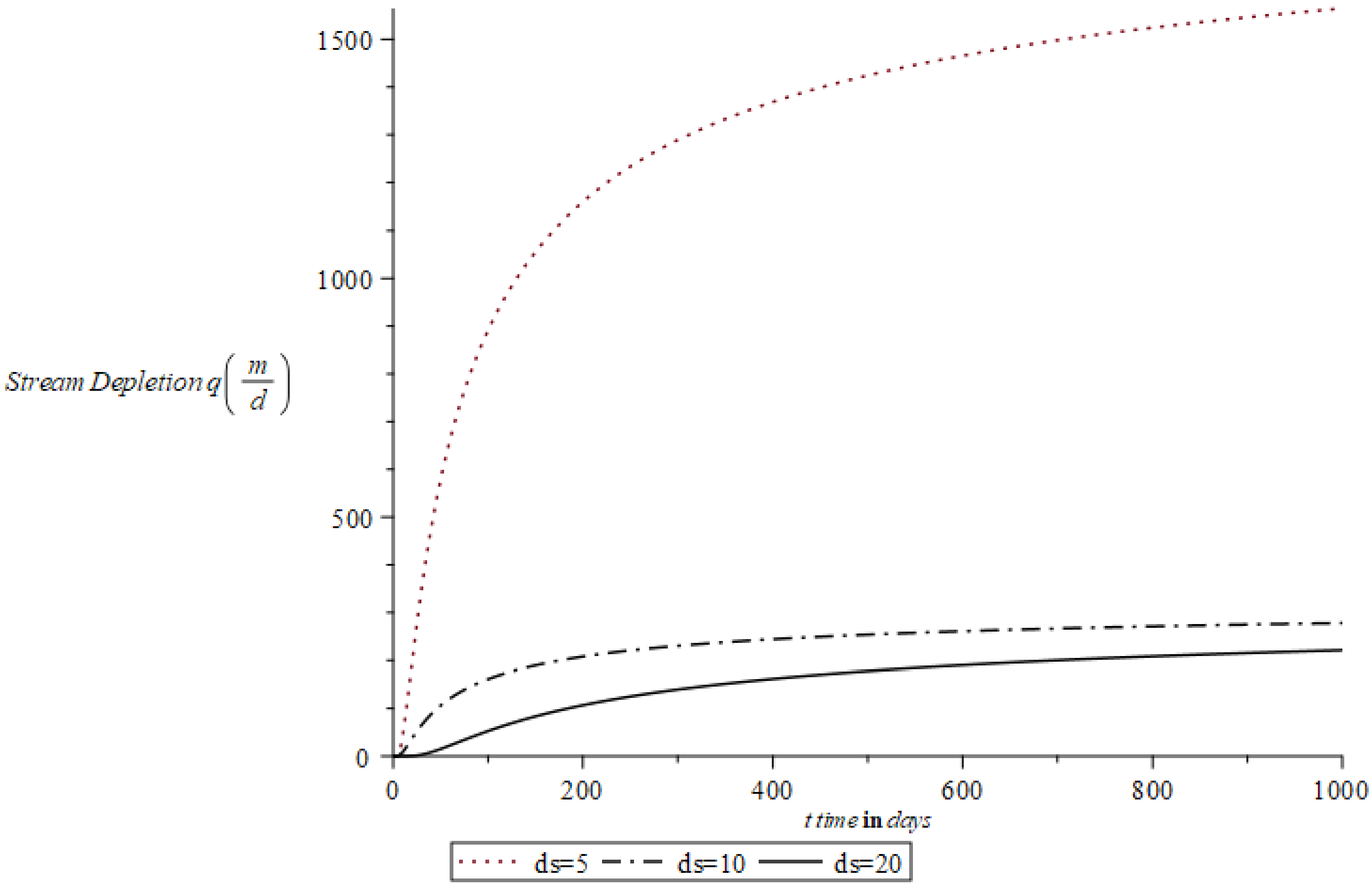

Figure 3

Effect of Stream Width on Stream Depletion

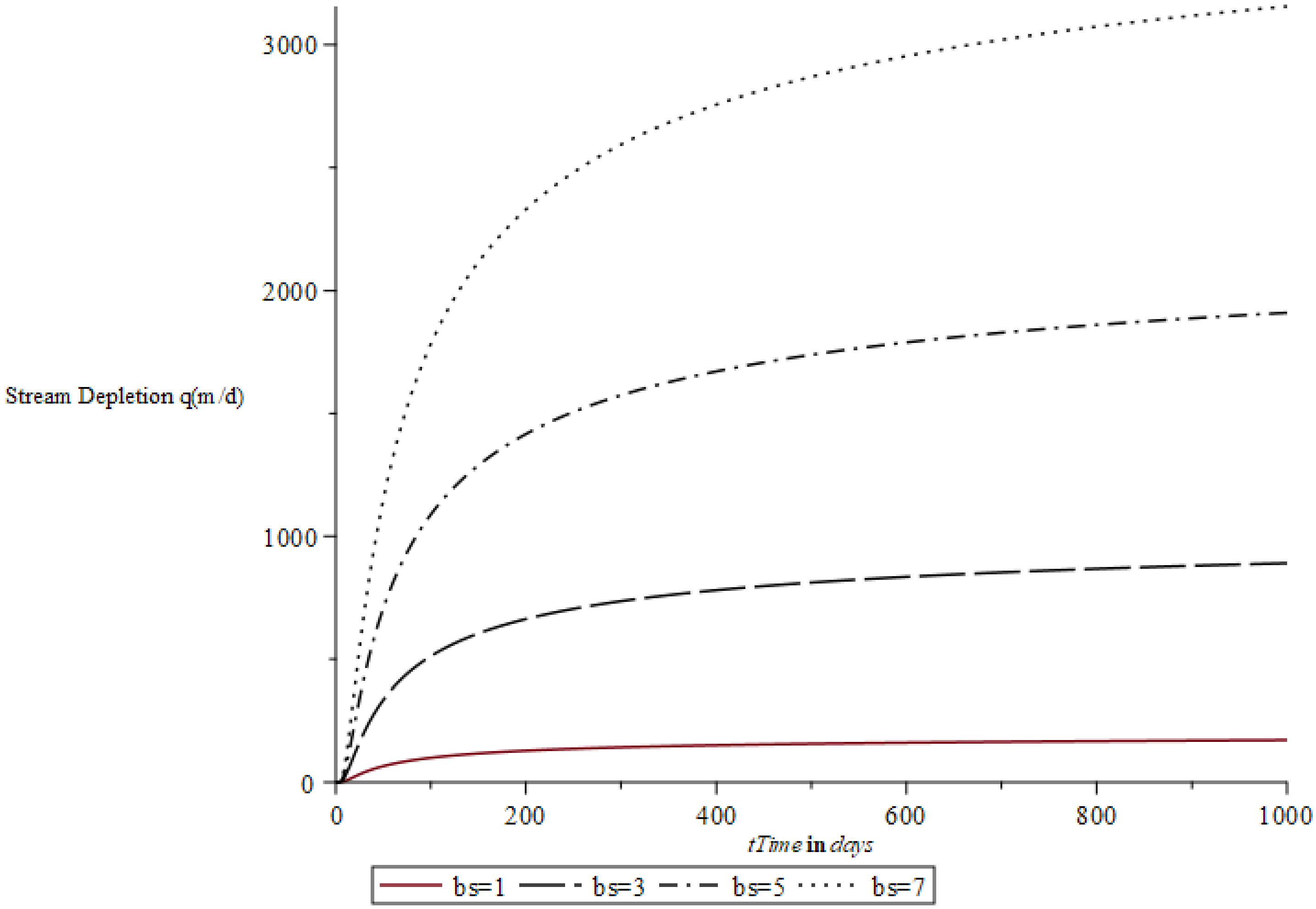

Figure 4

Effect of Stream Thickness on Stream Depletion

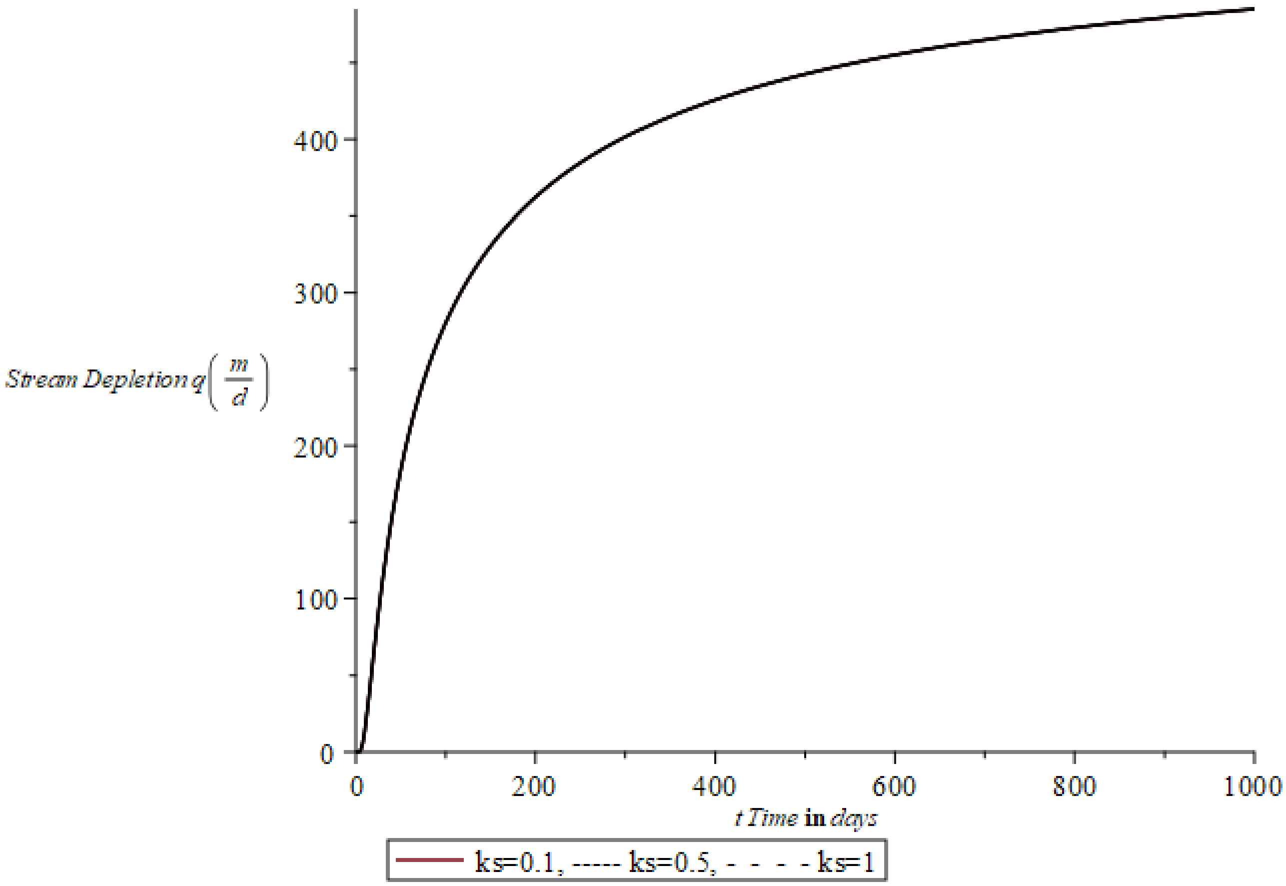

Figure 5

Effect of Stream Hydraulic Conductivity on Stream Depletion

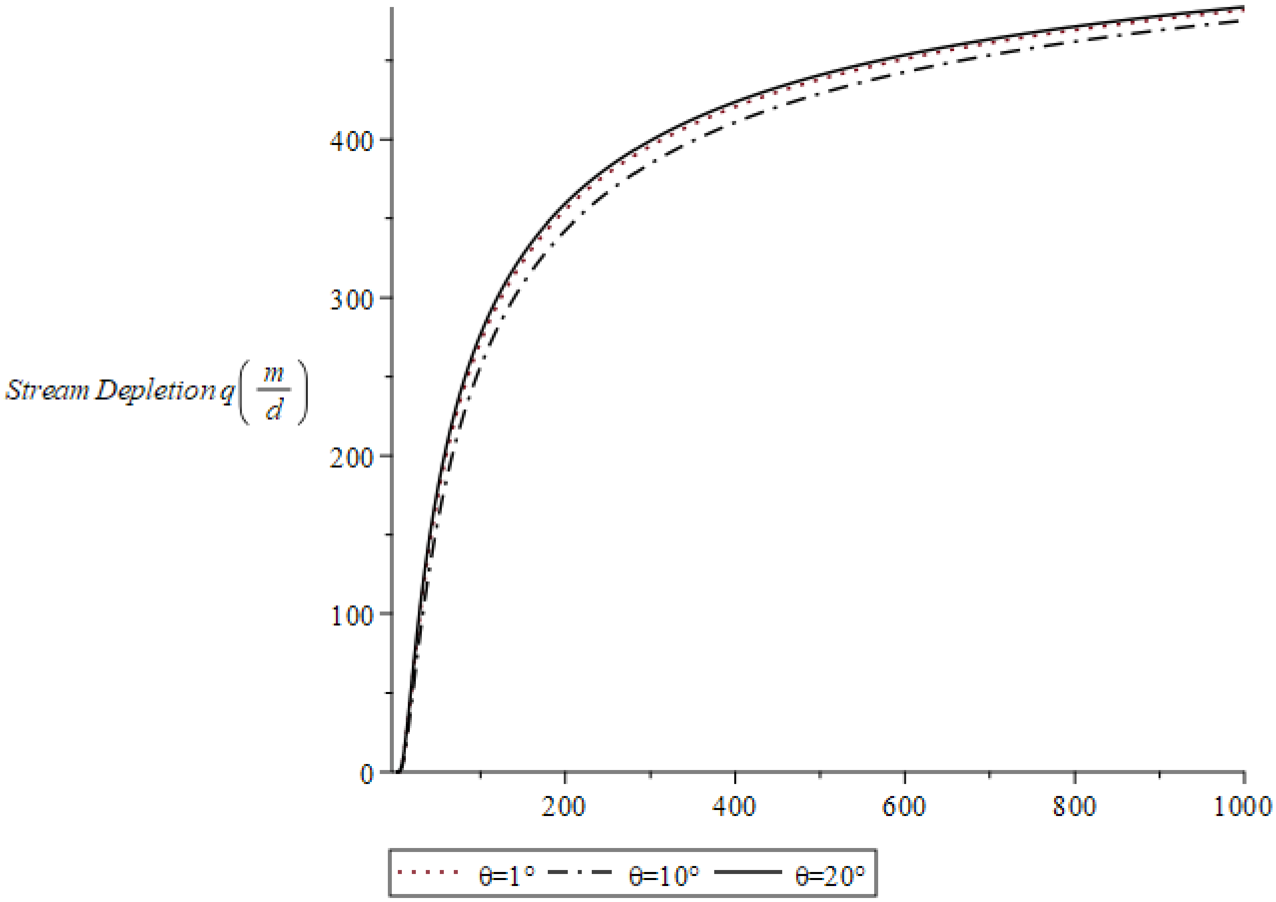

Figure 6

Effect of Angle of Aquifer on Stream Depletion

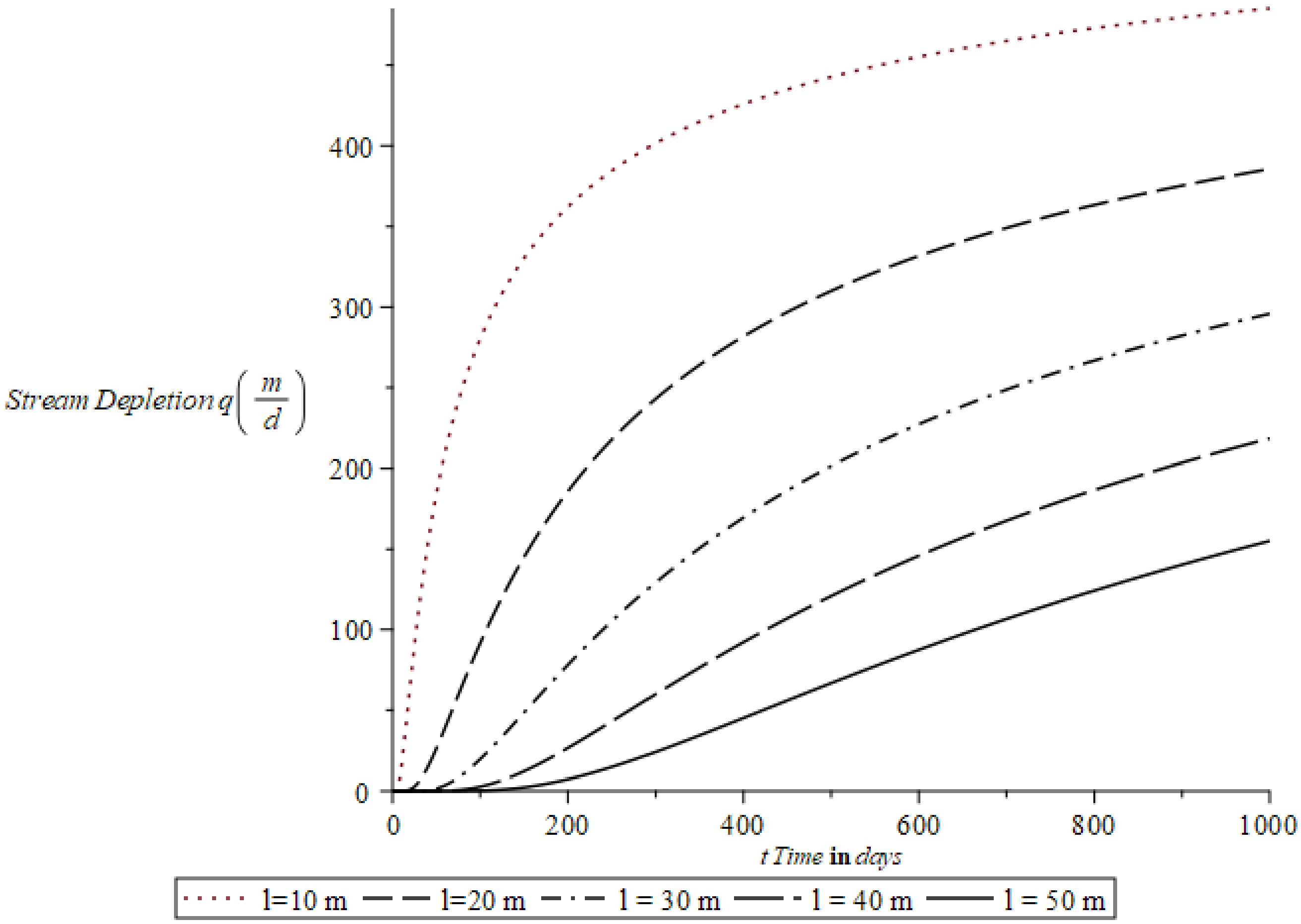

Figure 7

Effect of Distance of well from Aquifer on Stream Depletion