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Research Article

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Posted Date: March 18th, 2022

DOI: https://doi.org/10.21203/rs.3.rs-1456024/v1

Uncertainty analysis of water environmental capacity based on the copula model

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Abstract: Water environmental capacity is an important factor in total pollutant control and water environmental management. Due to the uncertainty of the water environment, the uncertainty analysis of water environmental capacity is crucial. Most uncertainty analyses of water environmental capacity do not consider the correlation between various parameters that were used to calculate water environmental capacity. In this study, the copula model of discharge and water quality was constructed to analyze the uncertainty of water environmental capacity. The risk threshold of each river section was calculated when the water environmental capacity was less than 0 in the wet, normal and dry seasons. The Yitong River Basin in Changchun city was taken as a case study. The results showed that the 68.27% confidence level was considered to reduce the uncertainty of water environmental capacity by more than 54.90%. The uncertainty of water environmental capacity in the wet season was the largest, and the risk threshold was the smallest. The risk thresholds for the water environmental capacity of each river reach were all greater than 90% in the dry season. The method of this study can be extended to other basins for uncertainty analysis of water environmental capacity.

Key words: Water environmental capacity; Uncertainty analysis; Copula model; Risk threshold

1. Introduction

Water environmental capacity (WEC) refers to the maximum limit value of pollutants that a water body can withstand under a certain target (Zhao et al. 2018). According to the calculated WEC and regional pollution status, the objectives of pollution reduction and environmental governance are determined (Xie et al. 2014). WEC is affected by water system characteristics, hydrological conditions, water quality targets, emissions of pollutants and other factors (Wang et al. 2021). In general, the calculation of the WEC is based on the determined design conditions, and the water quality model is established according to the actual situation (Zeng et al. 2021; Yan et al. 2019). Zhou et al. (2017) calculated the WEC through a one-dimensional water quality model, analyzed the spatial and temporal distribution of the WEC, and predicted the WEC in 2025 by using the system dynamics model and cellular automata. Huang et al. (2020) calculated the WEC of Qinhuangdao coastal waters by using the MIKE21 model, which provided a basis for the control of coastal water pollutants and water environmental management. Wang et al. (2019) calculated the daily WEC by using the Soil and Water Assessment Tool (SWAT) model, and the results showed that the WEC was positively correlated with rainfall intensity and that nonpoint source pollution had a great impact on the WEC. However, the random uncertainty of factors such as flow and background concentration that were used to calculate the WEC leads to the random uncertainty of the WEC (Chen et al. 2014).

The uncertainty analysis methods for WEC include the Monte Carlo method, Bayesian network method, blind number theory and so on (Zeng et al. 2021). Li et al.

(2009) established an extended-blind model based on blind number theory to calculate the WEC, and probable values of the WEC were expressed in the form of trapezoidal fuzzy numbers, and their corresponding reliabilities were computed. Zhu et al. (2021) used the Monte Carlo method to generate random rainfall and explored the impact of rainfall randomness on the uncertainty of the WEC. Zeng et al. (2021) constructed an uncertain accounting method for the WEC based on a Bayesian formula, and the probability distribution of the WEC was obtained by analyzing the random distribution characteristics of parameters such as flow and background concentration. Most current uncertainty analysis of WEC is a separate analysis of various factors, and few people have studied the correlation between various factors. Water quality and discharge are the basic and highly random factors for the calculation of WEC, and they are correlated. Rostami et al. (2020) showed that discharge was the main natural factor affecting river water quality. However, few people have considered the uncertainty analysis of WEC under the combined effects of water quality and discharge.

The copula model is an effective tool for exploring the correlation of random variables and constructing a joint probability distribution of multivariate variables (Su et al. 2017). The copula model is widely used in financial risk, resource shortage risk and flood risk analyses (Cai et al. 2019). Yu et al. (2020) analyzed the joint risk existing in the water-energy nexus system by using a copula-based fuzzy interval-random programming method, and the results provided optimal electricity supply schemes under water resource shortages. Ganguli et al. (2013) constructed a copula model of flood peak flow, duration and flow and obtained great flood risk analytical results.

There have been a few studies on copula functions for joint analysis of water quality and quantity (Park et al. 2019). Rehana et al. (2018) assessed the low water quality risk of rivers under an increase in river water temperature and altered river flows, and the results showed that a decrease in flow by 57% and an increase in temperature by 1.2° increased the risk of low water quality by 46%. WEC reflects the transformation, migration and accumulation of pollutants in water (Wang et al. 2021), which is an important factor limiting regional development. Therefore, the risk analysis of WEC is of great significance. The uncertainty analysis of WEC by using the copula model not only considers the relationship between water quality and discharge but can also calculate the risk threshold of the WEC. However, few researchers have analyzed the uncertainty of WEC by using copula modeling.

In this study, a copula model of discharge and water quality was constructed to analyze the uncertainty of WEC. The main objectives are as follows: 1) uncertainty analysis of WEC considering the relationship between discharge and water quality; and 2) risk analysis of WEC and calculation of the risk threshold.

2. Materials and Methods

2.1 Study area

The Changchun section in the Yitong River Basin is located at 124°32' ~ 125°46' E and 43°01' ~ 44°50' N (Fig. 1). The Yitong River is an important tributary of the Songhua River, which flows from south to north through Changchun city. The Changchun section of the Yitong River is 192.27 km long, and the area of the basin is

1712 km^2. The basin is located in the semihumid and semiarid monsoon climate zone of the northern temperate zone. The annual average temperature is 5.3 °C, and the annual average precipitation is 380 mm ~ 850 mm. The precipitation in the study area is mainly concentrated in July and August, which accounts for approximately 40%-53% of the annual precipitation. At present, Chemical Oxygen Demand (COD) and NH_3-N pollution is still serious. Therefore, they are selected as the research objects. The database used in this study is shown in **Table S1**.

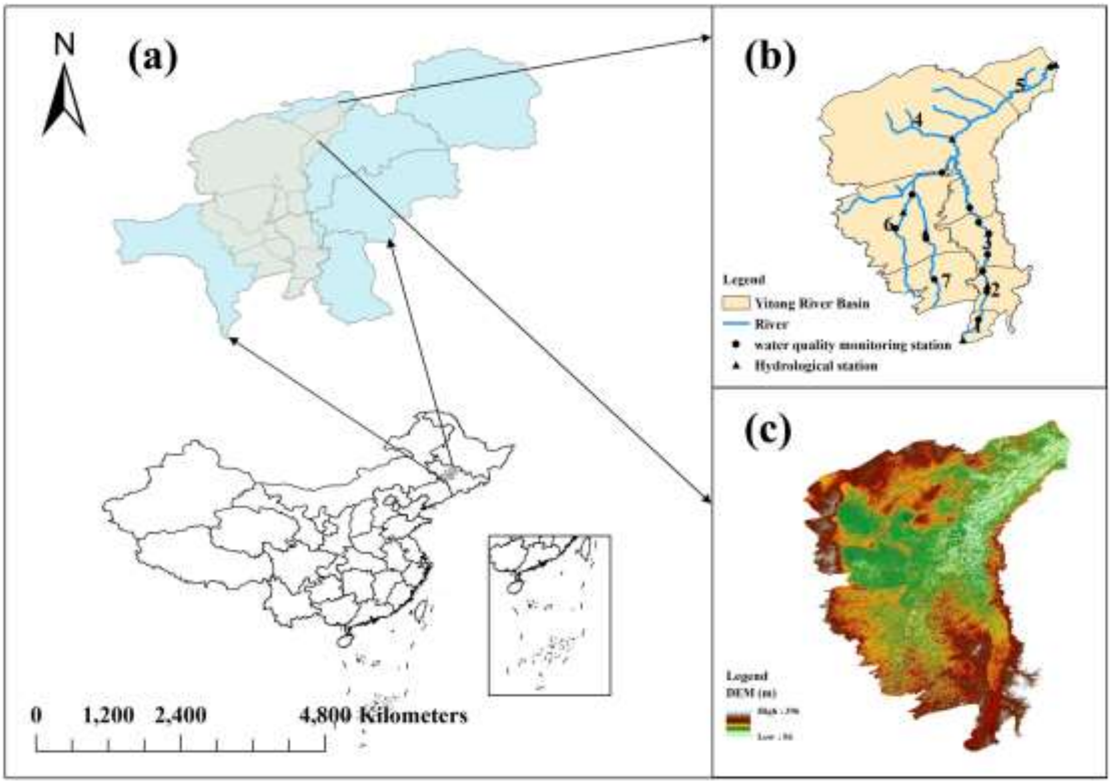

Fig. 1 The basic information of the study area. (a) Location of the study area; (b) division of control units in the study area; and (c) the DEM of the study area

2.2 The general framework

The framework of uncertain WEC mainly includes four steps: division of the control unit, calculation of the WEC, construction of the copula model, and uncertainty analysis of the WEC (**Fig. 2**).

Step I. Division of the control unit. The division of control units is a precondition for WEC calculation. Control units can be divided into three types based on administrative areas, hydrological units and aquatic ecosystems according to the different indicators (Deng et al. 2016). The division of control units based on hydrological units considers the natural catchment characteristics of the region. Effective improvement of water environmental management can be achieved by dividing the basin into multiple hydrological units and developing total pollutant control plans for the basin (Omernik et al. 2017). The division method based on water ecological zones is mainly based on the spatial heterogeneity of water ecological characteristics and ecological functions (Snelder et al. 2008). The division of control units based on administrative regions is mainly convenient for water quality management. The division method of the control unit is selected according to the different research purposes. The division of the control unit is the spatial basis for the uncertainty analysis of the WEC. The purpose is to calculate the WEC of each river reach.

Step II. Calculation of WEC. There are many methods to calculate WEC, which can be divided into simple mathematical models and complex mechanism models (Wang et al. 2021). A simple mathematical model has low requirements for data and high-calculation speed but low calculation accuracy (Yan et al. 2019). The complex mechanism model has high-calculation accuracy, but the data are difficult to obtain, and the calculation efficiency is low (Chen et al. 2014). Simple mathematical models include one-dimensional models and two-dimensional models (Zhou et al. 2017). Complex mechanism models include SWAT, MIKE, and the Environmental Fluid Dynamics Code (EFDC) (Huang et al. 2020; Wang et al. 2019). An appropriate WEC calculation model should be selected according to the data acquisition and calculation accuracy requirements of the study area.

Step III. Construction of the copula model. The copula model is mainly used to build the edge distribution function and joint distribution function. Common edge distribution functions are selected to fit data, and the fitting degree is tested by the K-S test or RMSE (Yu et al. 2014). The edge distribution function with a high-fitting degree is selected, and the parameters are solved by the maximum likelihood method or moment estimation method (Cai et al. 2019). The joint distribution function is constructed by the constructed edge distribution function. The squared Euclidean distance, Akaike information criterion (AIC) and other methods are used to test the fitting degree, and the maximum likelihood method or estimation of distribution method is used to solve the parameters (Reddy et al. 2012).

Step IV. Uncertainty analysis of WEC. Uncertainty analysis of WEC includes three parts: range of the WEC, range of the WEC at the confidence level and risk threshold of the WEC. The range of the WEC mainly determines the distribution interval of the WEC. The WEC is calculated according to the maximum and minimum values of water quality in the wet, normal and dry seasons, and the joint distribution probability of water quality and discharge is calculated. To narrow the range of the WEC, the range of the WEC at different confidence levels is considered. The water quality range is calculated according to the given confidence interval, and then the WEC and joint distribution probability are obtained. The risk threshold of the WEC is a probability of a WEC less than 0. When the WEC is 0, the corresponding water quality is obtained, and then the joint probability of water quality and discharge are calculated.

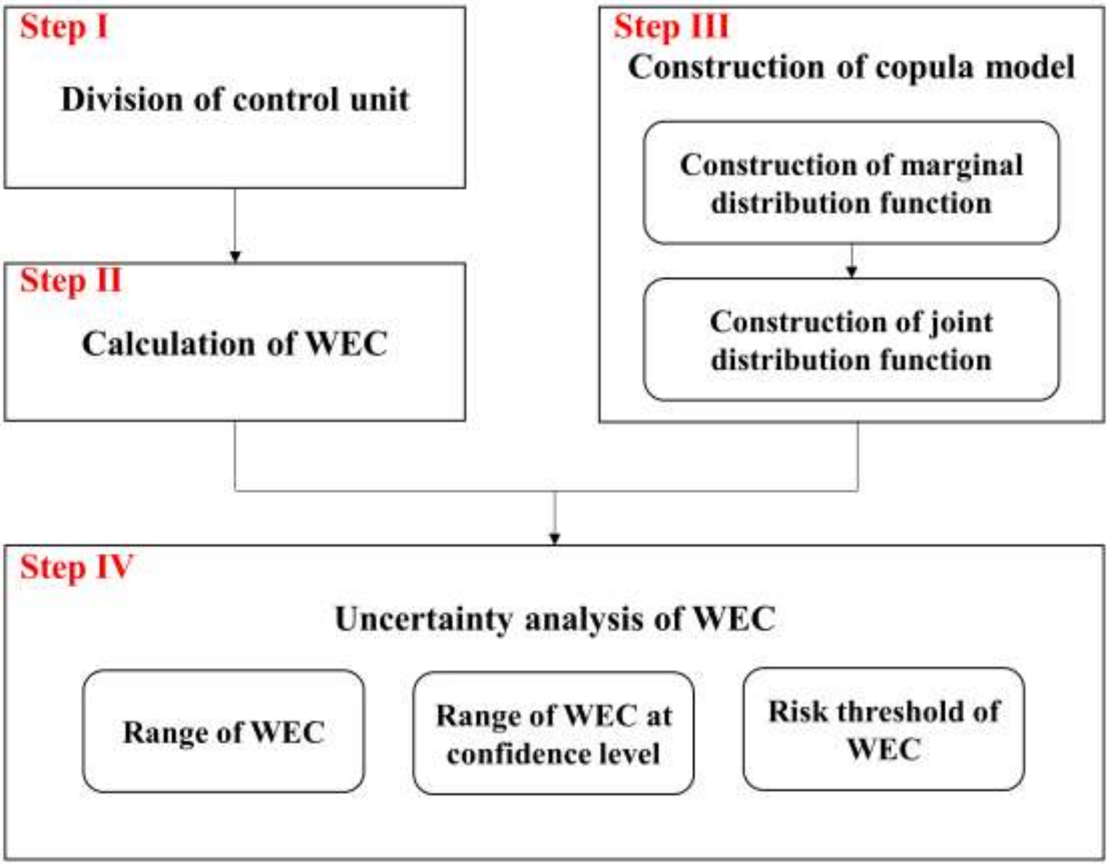

Fig. 2 Framework of uncertainty analysis for WEC

2.3 Division of the control unit

The control unit division method includes three steps: data collection, division principle and division of the control unit.

Step I. Data collection. The collected data include the digital elevation model (DEM), river network system, water environmental function zoning, location of hydrological stations and water quality monitoring stations (Li et al. 2020). The DEM and river network systems are used to divide catchment units. Water environmental function zoning ensures that each control unit has the same water quality objectives. The locations of hydrological stations and water quality monitoring stations are considered to ensure the rationality of the selection of the control section (Deng et al. 2016).

Step II. Division principle. The division of control units should follow the following principles (Wang et al. 2013): 1) Determining land by water. The control unit includes the water body and the land area of the water body, which reflects the water-land response relationship. 2) River basin integrity. Land entering the same water body is included in a control unit. 3) Considering existing sections. The control section should make use of the existing section to facilitate water environmental management and water pollution prevention.

Step III. Determination of the control unit. The catchment area is obtained by extracting the river system through the hydrology module of ArcGIS (Shen et al. 2013), and the catchment area is adjusted according to the actual river system in the plain river network area. The catchment area is superimposed with the distribution maps of hydrological stations and water quality monitoring stations to ensure that each control unit has a control section. Finally, it is superimposed with the water environmental function area to obtain the final control unit. In addition to the Yitong River and Xinkai River, the discharge of other rivers in the Yitong River Basin is small. Therefore, the Yitong River and Xinkai River are mainly considered in the division of control units, and other rivers are reflected in the confluence process of these two rivers. The division results are shown in **Fig. 1(b)** and **Table 1**.

Table 1 Control unit division results of the Yitong River Basin

ID	River	initial section	Termination section	length km	Water Quality Target	
					COD mg/L	NH3-N mg/L
1	Yitong River	Heng Yufeng	Zong Jiatun	24.84	20	1.0
2		Zong Jiatun	Highway Bridge Around City	14.50	20	1.0
3		Highway Bridge Around City	Sihua Bridge	23.18	20	1.0
4		Sihua Bridge	Wanjinta Highway Bridge	103.47	40	2.0
5		Wanjinta Highway Bridge	Kaoshan Town	49.58	30	1.5
6	Xinkai River	Jingtai Town	Yuan Jiatun	34.47	40	2.0
7		Yuan Jiatun	Lin Jiatun	66.27	40	2.0

2.4 Calculation of the WEC

The calculation of the WEC includes river division, model selection and parameter calculation.

Step I. River division. River division should meet the following requirements: 1) Pollution sources into the river can be evenly mixed. 2) The division of rivers should be consistent with hydrological stations and water quality monitoring stations. The division results are shown in Table 1.

Step II. Model selection. The river length is much larger than the river width, and the flow is smooth in the Changchun section of the Yitong River. Pollutants can be mixed uniformly over a short distance after entering the river, and the concentration of pollutants in the cross-section shows little change in the horizontal and vertical directions. The actual situation is considered, and the calculation conditions are simplified. A one-dimensional steady state model is selected to calculate the WEC. The WEC consists of two parts: a diluted WEC and a self-purified WEC (Dong et al.

2014).

Self-purified WEC:

$$E_{sp} = 86.4C_S(e^{\frac{kx}{u}} - 1) \cdot (Q + q) \quad (1)$$

Diluted WEC:

$$E_d = 86.4[(C_S(Q + q) - C_0Q)] \quad (2)$$

WEC:

$$E = E_{sp} + E_d \quad (3)$$

where E is the WEC (*kg/d*), C_s is the standard water quality concentration (*mg/L*), C_0 is the river background concentration (*mg/L*), Q is the river discharge (m^3/s), u is the river velocity (*m/s*), q is the inflow of water in the interval (m^3/s), k is the pollutant degradation coefficient (*1/d*), and x is the river distance (*km*).

Step III. Parameter calculation. The parameters include the water quality target, design discharge, flow rate and comprehensive degradation coefficient (Wang et al. 2011). Water quality targets are determined by water quality standards formulated by China's government (**Table 1**). The WEC is calculated in the wet season, normal season and dry season. The maximum monthly average discharge is selected as the design discharge in the wet season, the minimum monthly average discharge is selected as the design discharge in the dry season, and the annual average discharge is selected as the design discharge in the normal season. According to the section area of each river section, the corresponding design flow rate is calculated. The comprehensive degradation coefficient refers to the recommended value of the China Environmental Planning Institute.

2.5 Construction of the copula model

The copula model has strong flexibility and adaptability and can connect any form of marginal distribution function to obtain their joint distribution function (Yu et al. 2014; Reddy et al. 2012). The construction of the copula model includes three steps: correlation measurement of variables, construction of the marginal distribution function and construction of the joint distribution function (Sobkowiak et al. 2020).

Step I. Correlation measurement of variables. The Spearman rank correlation coefficient (ρ_n), Pearson correlation coefficient (r_n) and Kendall correlation coefficient (τ_n) were selected for correlation measurement. The calculation formula is as follows:

$$\rho_n = \frac{12}{n(n+1)(n-1)} \sum_{i=1}^{n} R_i S_i - \frac{3(n+1)}{n-1} \tag{4}$$

$$r_n = \sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y}) \,/ ((n-1)\sqrt{S_x{}^2 S_y{}^2}) \tag{5}$$

$$\tau_n = \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} sgn((X_i - X_j)(Y_i - Y_j)) \tag{6}$$

where n is the length of the sample, $\bar{X}$ and $\bar{Y}$ are the mean values of X_i and Y_i, respectively, $S_x{}^2$ and $S_y{}^2$ are the variances of X_i and Y_i, respectively, R_i is the rank of X_i in (X_1, X_2..., X_n), and S_i is the rank of Y_i in (Y_1, Y_2..., Y_n). sgn is the characteristic function: when $(X_i-X_j)(Y_i-Y_j)>0$, sgn =1; when $(X_i-X_j)(Y_i-Y_j) <0$, sgn =-1; and when $(X_i-X_j)(Y_i-Y_j) =0$, sgn =0.

Step II. Construction of the marginal distribution function. Common marginal probability functions (normal distribution, lognormal distribution, gamma distribution and Weibull distribution) were selected to fit the discharge and water quality. The maximum likelihood method is used to estimate the parameters of the marginal distribution function. The likelihood function can be expressed by L(θ):

$$L(\theta) = L(x_1, x_2, \dots, x_n; \theta) = \prod_{i=1}^{n} p(x_i; \theta) \tag{7}$$

$\hat{\theta}$ is the estimated value of θ. Let L(θ) reach the maximum, that is:

$$L(\theta) = L\big(x_1, x_2, \dots, x_n; \hat{\theta}\big) = max \;\; L(x_1, x_2, \dots, x_n; \theta) \tag{8}$$

When $dlnL(\theta)/d(\theta) = 0$, the maximum value $\hat{\theta}$ is obtained, where X is a discrete random variable, $p(x,\theta)$ is a probability function, and θ is a parameter to be solved.

The Kolmogorov–Smirnov (K-S) test was used to test the fitting degree of the marginal distribution function. If $F_n(X)$ represents the cumulative frequency distribution of the sample and $F_t(X)$ represents the assumed theoretical distribution, then the test statistics are constructed:

$$D = max\{|F_n(X_i) - F_t(X_i))|\} \tag{9}$$

If *D>D(n,a)*, then reject H_0; otherwise, accept H_0, where *D(n,a)* is the rejection threshold, *a* is the explicit level, and *n* is the sample size.

Step III. Construction of the joint distribution function. Common copula functions (Normal copula, Gumbel copula, Clayton copula, and Frank copula) were selected to construct a multivariate joint distribution function. The parameters of the copula function were estimated by the maximum likelihood method. Suppose the marginal distribution functions of X and Y are $F(x;\theta_1)$ and $G(y;\theta_2)$, and the marginal probability functions are $f(x;\theta_1)$ and $g(y;\theta_2)$, respectively. θ_1 and θ_2 are the unknown parameters of the marginal distribution, respectively.

Assuming that the copula distribution function is $C(u,v;\alpha)$ and the copula density function is $c(u,v;\alpha)$, then the joint distribution and joint density functions of (X,Y) are:

$$H(x,y;\theta_1,\theta_2,\alpha) = C(F(x;\theta_1),G(y;\theta_2);\alpha) \tag{10}$$

$$h(x,y;\theta_1,\theta_2,\alpha) = c(F(x;\theta_1),G(y;\theta_2);\alpha)f(x;\theta_1)g(y;\theta_2) \tag{11}$$

The likelihood function of sample (X_i,Y_i), i=1,2,…,n is

$$L(\theta_1,\theta_2,\alpha) = \prod_{i=1}^{n} c(F(x;\theta_1),G(y;\theta_2);\alpha)f(x;\theta_1)g(y;\theta_2) \tag{12}$$

Solving functions $\hat{\theta}_1,\hat{\theta}_2,\alpha = argmaxlnL(\theta_1,\theta_2,\alpha)$

The empirical copula function method is used to test the fitting degree.

Assuming (x_i, y_i) as the sample, the empirical distribution functions of X and Y are $F_n(X)$ and $G_n(Y)$, respectively, and the empirical copula of the sample is defined as follows:

$$C_0(U,V) = \frac{1}{n}\sum_{i=1}^{n} I[F_n(X_i) \leq U] I[G_n(Y_i) \leq V] \quad U,V \in [0,1] \tag{13}$$

$$D^2 = \sum_{i=1}^{n} |C(U,V) - C_0(U,V)|^2 \tag{14}$$

where D^2 is the square Euclidean distance. The smaller D^2 is, the better the fitting effect. $C(U,V)$ is the selected copula function, and $C_0(U,V)$ is the empirical copula function.

3. Results

3.1 Deterministic WEC accounting results

Table 2 shows that the WEC_{COD} of the Yuan Jiatu-Lin Jiatun reach was the smallest, which was 9421.01 kg/d. This was mainly due to a large amount of industrial point source pollution from the Huaide Industrial Park in this section. The WEC_{COD} of the Jingtai town-Yuanjiatu reach was the largest, which was 17663.02 kg/d. The water quality requirement and the background concentration were low, which made the WEC_{COD} the largest. The $WEC_{NH3\text{-}N}$ of other reaches was less than 0, except Zongjiatun-Highway Bridge around the city. This was because Xinlicheng Reservoir, one of the drinking water sources in Changchun city, is located in the reach, resulting in better water quality. The absolute value of the WEC in the wet season was greater than that in the normal and dry seasons.

The flow in the wet season was much larger than that in the normal and dry seasons so that it can accommodate more pollutants when the water quality meets the standards. However, if the water quality exceeds the standard, it also leads to more pollution. The water quality requirement of the Yitong River in the Hengyufeng-Sihua Bridge section was high, resulting in a low water environmental capacity. The Sihua Bridge-Wanjinta Highway Bridge section had a large WEC_{COD} due to its low water quality requirements and long river distance, while the NH_3-N pollution in this section was serious, resulting in $WEC_{NH3\text{-}N}$ still being negative. The Xinkai River, as a tributary of the Yitong River, had a lower COD background concentration and larger WEC when entering Changchun city. With the discharge of industrial point source and agricultural nonpoint source pollution, the water quality of the downstream deteriorated, and WEC_{COD} and $WEC_{NH3\text{-}N}$ were both negative.

Table 2 WEC_{COD} and $WEC_{NH3\text{-}N}$ in the wet, normal and dry seasons

ID	WEC_{COD} kg/d			$WEC_{NH3\text{-}N}$ kg/d		
	wet season	normal season	dry season	wet season	normal season	dry season
1	-9058.38	-2555.80	-117.22	-75.55	-20.81	-0.32
2	1984.69	611.68	83.67	365.53	105.71	7.79
3	-6390.10	-1093.50	28.42	-421.50	-88.94	0.05
4	14016.41	9870.59	6531.94	-12412.30	-4330.45	-934.88
5	-24904.25	-13270.81	-4012.99	-14914.58	-8877.96	-3974.36
6	17663.02	4531.03	1291.91	-4338.99	-1052.87	-245.00
7	-9421.01	-3358.03	-1885.68	-5734.00	-2219.27	-1363.78

3.2 Evaluation of the copula model

Pearson, Spearman and Kendall tests were used to indicate the correlation between discharge and water quality. **Table S2** shows that nearly half of the river reaches were negatively correlated with discharge and water quality, and they are far from the urban area and mainly affected by nature. The discharge-water quality of highway bridges around the city–Kaoshan town and Jingtai town–Yuanjiatun reach were positively correlated because these reaches are located in urban areas and are greatly affected by human factors.

In this study, four common distribution functions (normal distribution, lognormal distribution, gamma distribution and Weibull distribution) were selected to fit the observed data. The maximum likelihood method was used to estimate the parameters of the distribution function, and the fitting results were tested by the K–S test. From **Table S3**, h was greater than 0, indicating that the four distribution functions could be used to characterize the distribution of observation data. The greater p is, the better the fitting effect. The lognormal distribution had the best fitting effect on most discharge and water quality. **Fig. 3** shows the copula model construction of the Hengyufeng-Zongjiatun (ID=1) reach, and the other sections are shown in **Fig. S1-Fig. S6**. **Fig. 3 (a) shows** that COD conformed to a lognormal distribution, and the maximum probability density was 36.6 mg/L. When COD was 100 mg/L, the cumulative probability was close to 1 (**Fig. 3 (b)**). These results showed that the section pollution was serious.

Four common copula functions (Normal copula, Clayton copula, Frank copula and Gumbel copula) were selected to establish the joint distribution function of discharge and water quality. The maximum likelihood method was used to estimate the correlation coefficient, and the empirical copula was introduced to calculate the Euclidean distance (D^2) of the four copula models. According to **Table S4**, the Clayton copula and Frank copula had the best fitting effect on the joint distribution of discharge and water quality, and the expression is shown in **Table 3**. V represents discharge, Q represents water quality, and the joint distribution is shown in **Fig. 3(e)** and **Fig. 3(f)**. With the increase in the probability of water quality and discharge, the joint probability increased.

Table 3 Joint distribution expression of discharge-water quality in the Yitong River Basin

ID	Discharge-water quality	Function expressions
1	Discharge-COD	$C^{Fr}(Q,V;6.377) = -\frac{1}{6.377}\ln\left(1+\frac{(\exp(-6.377Q)-1)(\exp(-6.377V)-1)}{(\exp(-6.377)-1)}\right)$
	Discharge-NH_3-N	$C^{Fr}(Q,V;22.656) = -\frac{1}{22.656}\ln\left(1+\frac{(\exp(-22.656Q)-1)(\exp(-22.656V)-1)}{(\exp(-22.656)-1)}\right)$
2	Discharge-COD	$C^{Cl}(Q,V;0.014) = (Q^{-0.014}+V^{-0.014}-1)^{\frac{-1}{0.014}}$
	Discharge-NH_3-N	$C^{Fr}(Q,V;21.791) = -\frac{1}{21.791}\ln\left(1+\frac{(\exp(-21.791Q)-1)(\exp(-21.791V)-1)}{(\exp(-21.791)-1)}\right)$
3	Discharge-COD	$C^{Cl}(Q,V;0.094) = (Q^{-0.094}+V^{-0.094}-1)^{\frac{-1}{0.094}}$
	Discharge-NH_3-N	$C^{Cl}(Q,V;0.691) = (Q^{-0.691}+V^{-0.691}-1)^{\frac{-1}{0.691}}$
4	Discharge-COD	$C^{Cl}(Q,V;1.095) = (Q^{-1.095}+V^{-1.095}-1)^{\frac{-1}{1.095}}$
	Discharge-NH_3-N	$C^{Cl}(Q,V;0.229) = (Q^{-0.229}+V^{-0.229}-1)^{\frac{-1}{0.229}}$
5	Discharge-COD	$C^{Cl}(Q,V;5.222) = (Q^{-5.222}+V^{-5.222}-1)^{\frac{-1}{5.222}}$
	Discharge-NH_3-N	$C^{Fr}(Q,V;16.847) = -\frac{1}{16.847}\ln\left(1+\frac{(\exp(-16.847Q)-1)(\exp(-16.847V)-1)}{(\exp(-16.847)-1)}\right)$

6	Discharge-COD	$C^{Cl}(Q,V;0.071)=(Q^{-0.071}+V^{-0.071}-1)^{\frac{-1}{0.071}}$
	Discharge-NH_3-N	$C^{Cl}(Q,V;3.218)=(Q^{-3.218}+V^{-3.218}-1)^{\frac{-1}{3.218}}$
7	Discharge-COD	$C^{Fr}(Q,V;30.096)=-\frac{1}{30.096}\ln\left(1+\frac{(\exp(-30.096Q)-1)(\exp(-30.096V)-1)}{(\exp(-30.096)-1)}\right)$
	Discharge-NH_3-N	$C^{Fr}(Q,V;4.960)=-\frac{1}{4.96}\ln\left(1+\frac{(\exp(-4.96Q)-1)(\exp(-4.96V)-1)}{(\exp(-4.96)-1)}\right)$

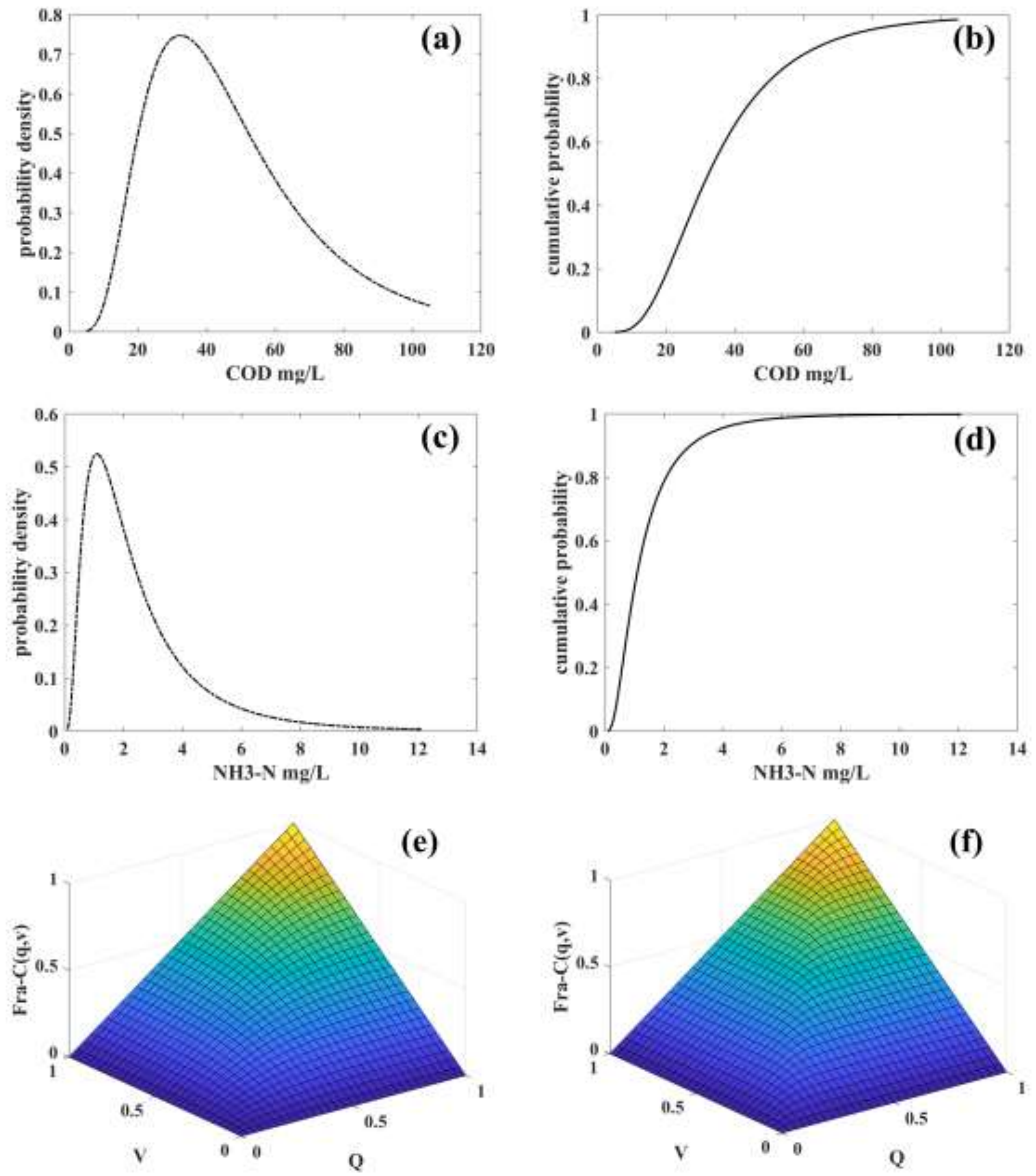

Fig. 3 Construction of the copula model for the Hengyufeng-Zongjiatun reach ((a)-(b): the probability density and cumulative probability distribution of COD; (c)-(d): the probability density and cumulative probability distribution of COD; and (e)-(f): the joint distribution of discharge-COD and discharge-NH_3-N)

4. Discussion

4.1 Uncertainty analysis of the WEC

The calculation results of the deterministic WEC are unique and do not conform to the stochastic characteristics of water systems (Zeng et al. 2021). The W_{COD} of the Hengyufeng-Zongjiatun reach in the wet season was taken as an example. The deterministic WEC was -9058.38 kg/d, the range of the WEC obtained by the copula model was -48712~5947 kg/d, and the corresponding probability was 0.33%~97.53% (**Table 4**). The range of the WEC was the largest in the wet season and the smallest in the dry season. The reason for this was that when the water quality was certain, the larger the discharge was, the larger the range. This indicated that the uncertainty of the WEC was maximum in the wet season and minimum in the dry season. The effectiveness of water quality management was effectively improved by establishing the relationship between uncertainty and estimation probability (Shi et al. 2015). However, uncertainty analysis of WEC mostly analyzes the influence of a single factor or the probability combination of various factors (Liu et al. 2012). There was a close relationship between water quality and discharge, and the correlation between them was not considered in the current analysis of the WEC. The copula model could be used to obtain the joint probability of water quality and quantity, and then the uncertainty analysis of the WEC was carried out, which made the uncertainty analysis of the WEC more accurate and closer to reality.

The calculation of the WEC at the confidence level can effectively reduce the range of the WEC (Liu et al. 2012). The range of the WEC of the Hengyufeng-Zongjiatun reach (ID=1) was -16893~490 kg/d (**Table 5**) at a confidence level of 68.27% in the wet season, which reduced the uncertainty by 68.20%. **Table 5 shows** that the range of the WEC of the Zongjiatun-Highway Bridge around the city reach (ID=2) was greater than 0. The reason for this was that the self-purification capacity of the river was large, which could offset the lack of a diluted WEC. The lower limit of the WEC of other reaches in the Yitong River Basin was less than 0, indicating that there was a possibility that the reach had no pollutant carrying capacity. Although the deterministic WEC of some reaches in the basin was greater than 0, it could not indicate that the reaches had complete pollutant carrying capacity. Only when the lower limit of the uncertain WEC was greater than 0 could the reach have the ability to absorb pollution.

Table 4 Range of the WEC and joint probability of discharge-water quality

ID		1	2	3	4	5	6	7
	wet	-48712	-2560	-179620	-101920	-117410	-21969	-94669
	season	5947	5420	21453	85915	28685	33068	24321
WEC_{COD}	normal	-13814	-678	-65238	-58167	-68774	-5079	-36586
kg/d	season	1704	1587	8733	54535	18883	8405	9794
	dry	-720	15	-16181	-23074	-29414	-944	-22449
	season	111	136	3163	28505	10702	2319	6253
	wet	-3105	-55	-128270	-84131	-30213	-28324	-25935
	season	407	466	2936	3568	-5000	1509	567
$WEC_{NH3\text{-}N}$	normal	-881	-14	-47155	-50329	-18057	-6927	-10093
kg/d	season	116	134	1114	2290	-2929	382	237
	dry	-46	1	-12295	-22856	-8175	-1664	-6237
	season	7	9	328	1226	-1252	105	156
	wet	0.33	0.33	0.25	0.22	2.57	1.01	0.93
	season	97.53	97.61	97.29	97.46	96.81	99.20	98.99
	normal	0.32	0.22	0.19	0.22	2.57	0.72	0.93
P_{COD} %	season	61.22	60.97	60.09	53.70	53.78	62.72	56.94
	dry	0.06	0.01	0.02	0.19	1.10	0.07	0.84
	season	3.35	3.33	1.70	1.10	1.10	2.63	8.07
$P_{NH3\text{-}N}$ %	wet	0.46	0.33	0.76	0.41	1.29	0.09	0.68

season	97.74	97.83	96.80	97.33	97.32	98.97	98.52
normal	0.46	0.33	0.74	0.34	1.29	0.09	0.64
season	61.28	61.28	59.97	53.57	53.81	62.82	56.77
dry	0.24	0.17	0.41	0.07	0.20	0.09	0.22
season	3.35	3.35	1.71	1.10	1.10	2.64	8.06

Table 5 The range of the WEC at a confidence level of 68.27%

ID		1	2	3	4	5	6	7
WEC$_{COD}$ kg/d	wet season	-16893	407	-12697	-28338	-57860	6687	-35367
		490	3438	-878	49165	8030	25513	9481
	normal season	-4765	164	-2884	-14017	-33044	1941	-13471
		155	1024	471	32485	6490	6554	4010
	dry season	-235	60	-67	-2868	-13062	755	-8144
		28	106	112	18413	5030	1871	2674
WEC$_{NH3-N}$ kg/d	wet season	-635	307	-1712	-18850	-22420	-9357	-11378
		235	435	468	363	-11626	-987	-1796
	normal season	-180	89	-455	-11161	-13381	-2280	-4419
		67	125	164	356	-6905	-229	-684
	dry season	-9	7	-20	-4930	-6035	-540	-2725
		4	9	14	343	-3071	-43	-414

4.2 Risk analysis of WEC

Table 6 shows that the larger the discharge and water quality are, the larger the joint probability. This was consistent with some studies (Park et al. 2019; Liu et al. 2018; Wang et al. 2017). When the joint probability was constant, the greater the discharge was, the smaller the water quality, which indicated that the increase in discharge could reduce the risk of water quality exceeding the standard (Hou et al. 2021; Rehana et al. 2018). When the discharge was constant, the lower the water quality target was, the greater the joint probability and the smaller the risk of water quality exceeding the standard. The risk of water quality exceeding the standard in the Zongjiatun-Highway Bridge around the city (ID = 2) reaches was the lowest, which was mainly due to the low agricultural nonpoint source and industrial point source pollution. However, the risk of excessive water quality was not completely consistent with the WEC. WEC is affected by water quality, discharge, water quality targets and other factors (Yuan et al. 2019).

When the WEC was 0, the joint probability of discharge and water quality was calculated (Table 7). The W_{COD} of the Hengyufeng-Zongjiatun reach (ID = 1) was taken as an example. The risks of WEC_{COD} less than 0 in the wet, normal and dry seasons were 81.65%, 82% and 97.36%, respectively. The probability of the dry season was much greater than that of the wet season. The reason for this was that the small discharge could not effectively dilute pollutants in the river (Wang et al. 2016). The risk of WEC_{COD} less than 0 in the Jingtai town-Yuanjiatun section (ID = 6) was the smallest, which was 7.17%. The low water quality targets allowed rivers to contain more pollutants. The risk of a WEC less than 0 in each reach of the Yitong River Basin was above 90% in the dry season. Therefore, the control of pollutant discharge should be strengthened in the dry season.

Table 6 Joint probability of discharge-water quality in the Yitong River Basin under different water quality targets

WEC/P	hydrographic period	water quality target mg/L	ID 1	2	3	4	5	6	7
P_{COD} %	wet season	40	65.07	98.11	64.29	54.61	44.79	92.45	32.82
		30	44.15	98.09	32.94	33.68	19.50	68.74	12.86
		20	18.24	86.71	5.53	14.97	3.73	19.12	1.77

	normal season	40	53.17	61.28	40.43	37.98	42.19	58.58	32.82
		30	40.21	61.27	21.30	26.74	19.48	43.97	12.86
		20	17.63	54.20	3.82	13.51	3.73	12.71	1.77
	dry season	40	3.30	3.35	1.28	1.09	1.10	2.49	8.07
		30	3.13	3.35	0.80	1.08	1.10	1.98	7.42
		20	2.23	2.98	0.22	1.05	1.10	0.72	1.57
$P_{NH3\text{-}N}$ %	wet season	2.0	78.75	98.10	68.89	15.31	0.14	5.92	2.29
		1.5	66.29	98.04	63.26	8.72	0.07	2.13	0.68
		1.0	45.53	97.66	54.76	3.36	0.04	0.37	0.09
	normal season	2.0	61.20	61.28	46.60	10.26	0.14	5.92	2.16
		1.5	60.05	61.28	43.64	6.12	0.07	2.13	0.64
		1.0	45.41	61.28	38.98	2.52	0.04	0.37	0.08
	dry season	2.0	3.35	3.35	1.67	0.52	0.02	2.58	0.73
		1.5	3.35	3.35	1.66	0.40	0.01	1.88	0.22
		1.0	3.35	3.35	1.64	0.24	0.01	0.37	0.03

Table 7 Risk threshold for the WEC in the Yitong River Basin

ID	P_{COD} %			$P_{NH3\text{-}N}$ %		
	wet season	normal season	dry season	wet season	normal season	dry season
1	81.65	82.00	97.36	54.37	54.26	96.65
2	12.56	44.32	96.65	2.32	38.72	96.65
3	93.69	94.70	99.54	44.84	60.42	98.35
4	37.85	56.86	98.90	82.02	87.14	99.37

5	75.60	72.50	98.90	99.92	99.91	99.98
6	7.17	40.56	97.42	93.95	93.53	97.38
7	66.14	64.54	91.93	97.60	97.57	99.13

5. Conclusions

In this study, the uncertainty of the WEC was analyzed by using a copula model. Other uncertainty analysis methods rarely consider the relationship between the various factors used to calculate the WEC and to analyze the risk of WEC. The copula model considers the relationship between water quality and discharge and calculates the risk threshold of the WEC. The range of the WEC and corresponding probability of each section of the Yitong River Basin were obtained. A 68.27% confidence level was considered to reduce the uncertainty of the WEC by more than 54.90%. The uncertainty of the WEC in the wet season was the largest, but the risk of a WEC less than 0 was the smallest. The risk of a WEC less than 0 in each section of the Yitong River Basin in the dry season was greater than 90%. The WEC of the Zongjiatun-Highway Bridge around the city reach (ID=2) and Jingtai town-Yuanjiatun section (ID=6) was larger, and the risk threshold of the WEC was smaller. Although the deterministic WEC of some reaches was greater than 0, this did not indicate that the reach was fully capable of receiving pollution. Only when the range of the WEC was greater than 0 and the risk threshold was less than 0 could the reach completely absorb pollutants.

The parameters of the WEC calculation also include the pollutant degradation coefficient and background concentration. However, this study only considered the uncertainty of the WEC caused by water quality and discharge. In the future, a multivariate copula model will be considered to analyze the uncertainty of the WEC.

Acknowledgements

This research was funded by the Fund for Innovative Research Group of the National Natural Science Foundation of China (No. 51721093) and the Inter-discipline Research Funds of Beijing Normal University.

Conflict of interest

The authors declare that they have no competing interests.

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