

Effect of Wall Shear Stress on Two Phase Fluctuating Flow of Dusty Fluids by Solving Light Hill Technique

Dolat khan

King Mongkut's University of Technology Thonburi

Ata ur Rahman

City University of Science and Information Technology

Gohar ali

City University of Science and Information Technology

Poom kummam (✉ poom.kum@kmutt.ac.th)

King Mongkut's University of Technology Thonburi

Research Article

Keywords: Wall shear stress, Oscillating two-phase fluctuation flow, heat transfer, Magnetohydrodynamic (MHD), dust particles.

Posted Date: January 22nd, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-145804/v1>

License:  This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

EFFECT OF WALL SHEAR STRESS ON TWO PHASE FLUCTUATING FLOW OF DUSTY FLUIDS BY SOLVING LIGHT HILL TECHNIQUE

Dolat khan¹, Ata ur Rahman², Gohar Ali², Poom Kumam^{1,3,4,*}

¹Fixed Point Research Laboratory, Fixed Point Theory and Applications Research Group, Center of Excellence in Theoretical and Computational Science (TaCS-CoE), Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), 126 Pracha Uthit Rd., Bang Mod, Thung Khru, Bangkok 10140, Thailand; dolat.ddk@gmail.com

² Department of Mathematics, City University of Science & Information Technology, Peshawar, KPK, Pakistan.

³ Center of Excellence in Theoretical and Computational Science (TaCS-CoE), Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), 126 Pracha Uthit Rd., Bang Mod, Thung Khru, Bangkok 10140, Thailand

⁴ Department of Medical Research, China Medical University Hospital, China Medical University, Taichung 40402, Taiwan

* Correspondence: Poom Kumam; poom.kum@kmutt.ac.th

Abstract

On the importance of wall shear stress effect and dust fluid in the fluid problems. The aim of this paper to discover the influence of wall shear stress on dust fluids of fluctuating flow. The flow is consider between two parallel plates which are non-conducting. Due to the transformation of heat the fluid flow is generated. We consider every dust particle having spherical uniformly disperse in the base fluid. The perturb solution is obtained by applying Poincare-Lighthill perturbation technique (PLPT). The fluid velocity along with shear stress is discussed for the different parameters like Grashof number, magnetic parameter, radiation parameter and dusty fluid parameter. Graphical results for fluid and dust particles are plotted through Mathcad-15. The behavior of base fluid and dusty fluid is matching for different embedded parameters.

Keywords: Wall shear stress, Oscillating two-phase fluctuation flow, heat transfer, Magnetohydrodynamic (MHD), dust particles.

Introduction

Fluid flow inserted with similar non-miscible inert solid particles is admitted as the two-phase structure of fluid. The fields of technology and engineering accommodate various uses of the flow of the gas-particles mixture. Nuclear reactors with gas-solid feeds, cooling of nuclear reactors, solid rocket exhaust nozzles, electrostatic drizzle, ablation cooling, polymer technology, the moving blast waves over the Earth's surface, the distillation of crude oil, environmental poison, the industry of petroleum, fluidized beds, transmit of powdered materials, physiological flows and various other fields of technologies are the practical examples where the dusty fluid are mostly applicable [1-2]. Different types of multiphase flows can be seen in the literature, although the most familiar type of such flows which are multi-phase are two-phase flows, in these multi-phase flows liquid and liquid flow, gas and liquid flow, liquid and solid flow, solid and gas flow are frequently discussed by the researchers [3-4]. In turbulent flows the liquid droplets vaporization is an immensely essential multiphase flow area due to an extensive measure of energy which

is used for heating, electrical and impulse power generation is borrowed against liquid fuels which have been transformed into atomized sprays. The cooperation of the discharged vapors of droplets with the turbulent flow formed immensely complicated problems in such situations. A comprehensive explanation of the fundamental flow processes and droplets involved in these situations can be initiated in [5-7].

The researchers are very keen to study the multi-phase dusty flows over the last few decades because of their considerable applications. In this regard, various researchers investigate the evaluation of preliminary and theoretical modeling of particle phase viscosity in a multi-phase dusty fluid [8-11], but keep in mind Soo [12] is the first who presented the fundamental theory of multi-phase flows. Other useful applications consist of dust particles in nuclear processing, in boundary layers contains soil emancipation which is occurred by natural winds, in aerodynamic refusal of plastic sheets, landing vehicle in a cloud designed during a nuclear detonation, and lunar surface extinction by the dust entrainment are investigated many researchers in the literature [13-16]. Zhou et al. [17] discuss the fluid particles in translational motion and also study the converging and diverging behavior in a microchannel. In another paper, Zhau et al. [18] established a model in which he examines the deformable interaction of particles and he studies thoroughly the contact of dielectrophoresis (DEP) in the presence of an electric field generated by the alternating current. Recently, Ali et al. [19] present an article on fluctuating flow of absorbing heat viscoelastic dusty fluid with free convection and MHD effect past in a horizontal channel. In this reported study the researcher discussed the consequence of different parameters on fluid velocity and particles and they investigate the solution for the fluid velocity and also for the velocity of the dust particle by applying the technique of Poincare-Light Hill. Similarly, in another paper, Ali et al. [20] established the study about the dusty viscoelastic fluid of two-phase fluctuating flow with heat transfer between rigid plates which are non-conducting and examine the connected effect of the heat transfer and magnetic field on the viscoelastic dusty fluid which is conducting electrically with the help of Light-Hill technique. Furthermore, Attia et al. [21] work out the fallout of different substantial parameters on the steady flow of MHD incompressible non-Newtonian Oldroyd dusty fluid in a circular pipe and also study the consequence of Hall current. In this particular article, the author investigates the characteristics of the particle phase viscosity and non-Newtonian fluid, flow rates in terms of volume, and the coefficient of skin fraction for both the particle phase and fluid. Keeping in mind Ali et al. [22] present an article about second-grade fluid in which they present closed-form solutions of free convection unsteady flow. In this article, the author investigates the influence of different parameters on the velocity profile of second-grade fluid like Grashof number, Prandtl number, and viscoelastic parameter.

Multi-phase flow regimes have considerable consideration because of their useful applications [23]. There is two specific access that has been used commonly. According to this specific access, if we study the first way, we can observe that the motion of the continuous phase does not extremely affect by the dispersed phase because the motion of this phase is so minute. This approach is commonly admitted as the 'alter phased approach' and this approach also familiar with the Lagrangian approach. This approach is applied broadly in such situations where the particles are dealing with dispersed phase like in sprays, atomization, droplets [23]. On the other hand, the second approach is related to the two phases which are combined in such a way that each phase precisely manipulates the altitude and the motion of the other phase [23]. The second approach is called the approach of dense phase. It is also known as the Eulerian approach. This approach is very useful in solid-gas flows [24], aerial transmitting [25], in fluidization and is defined for a variety of uses and applications in suspensions [26-27].

The Light-Hill method was introduced by M. J. Light-Hill in 1949 [28]. He obtained uniformly valid approximate solutions for various classes of partial and ordinary differential equations. After that, this method was used by many researchers successfully in the research field and solve ordinary and partial differential equations [29]. As far as wall-shear stress is concerned many researchers present different problems in which they discuss wall-shear stress and interpret different behaviors of the different parameters on the fluid velocity. In this regard, Grobe et al. [30] study the boundary layer wall shear-stress of the turbulent wind tunnel with a high Reynolds number. In a similar way, Amili et al. [31] represent wall shear-stress distribution in a turbulent flow in the channel and discuss the different influence of parameters on the velocity of the fluid. Keeping in mind Orlu et al. [32] described the study on the fluctuating wall shear stress in zero pressure-gradient turbulent boundary layers flow. Recently, Mob et al. [33] present a paper in which they describe the study of the impact of wall-shear stress on the evolution and execution of electrochemically alive biofilm.

In the above-mentioned literature, the authors have considered different types of dusty fluids, some of the dusty fluids are electrically conducting and some of them are investigated with heat transfer and energy equations. According to our utmost knowledge, no investigation has been disclosed about the effect of wall shear stress on two phases of the fluctuating flow of dusty fluid by solving the Light-Hill technique. Therefore, we have approved to examine and study the different behavior of velocity of the problem theoretically and graphically in the present work. The purpose of this study is, hence, to interrogate the flow of fluid ingrained with dust particles along with transfer heat over the bounded plates.

Formulation of the problem

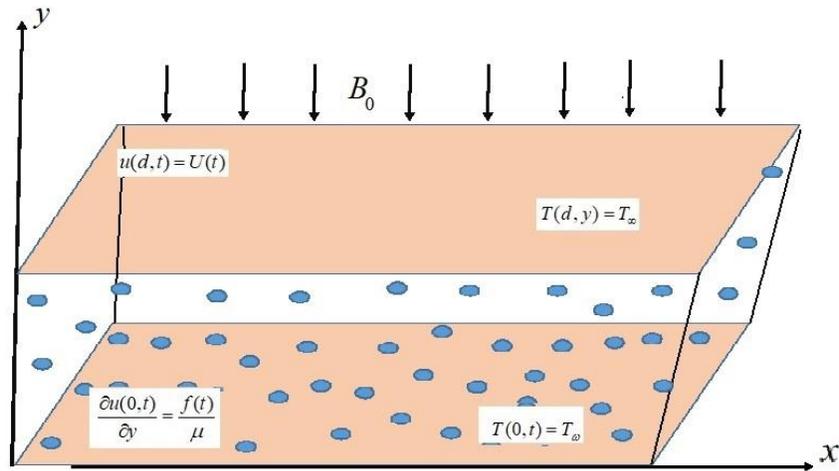


Figure (1): Geometry of the problem

The incompressible, unidirectional, and one-dimensional electrically conducting the unsteady flow of dusty fluid along the x-axis has been considered between two plates that are parallel. The magnetic field B_0 is transversely applied to the fluid and due to the small size of the induced magnetic field, the emission is so small therefore, the electric field inside is ignored. The motion of the upper plate with free stream

velocity ($U(t)$), is free of space variable the flow is generated. Also, flow origination is induced by heat transfer, the upper plate temperature is T_∞ and the lower plate temperature is T_ω . The wall shear stress is applied to the lower plate, where the upper plate is fluctuating with free stream velocity $U(t) = u_0 \left(1 + \frac{\varepsilon}{2} (e^{i\omega t} + e^{-i\omega t}) \right)$. $u(y,t)$, $v(y,t)$ denoted the velocities of fluid and dust particles respectively. As shown in figure 1. The effect of the equation of energy radiation is also getting hold of into description. By apply the assumptions of Bossiness resemblance and in sequence to escape similarities, the equation of momentum and energy is.

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + \frac{K_0 N_0}{\rho} (v - u) - \frac{\sigma B_0^2 u}{\rho} + g \beta_T (T - T_\infty), \quad (1)$$

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y}, \quad (2)$$

Where

$$-\frac{\partial q_r}{\partial y} = 4\alpha_0 (T - T_\infty).$$

In equation (1) and (2), which are constantly spreader in the viscoelastic fluid, q_r , ν , c_p , k , g , ρ , α_0 , σ , B_0 , N_0 , β_T and K_0 are show the radiation heat flux, kinematic viscosity, specific heat capacity, thermal conductivity, gravitational acceleration, fluid density, mean radiation absorption coefficient, electrical conductivity, magnetic field, number of density of the dust particle which is supposed to be constant, coefficient of thermal expansion, and stock's resistance coefficient respectively.

The following equation can show Newton's law of motion.

$$m \frac{\partial v}{\partial t} = K_0 (u - v), \quad (3)$$

Where equation (3) m represent the average mass of dust particles.

The physical boundary conditions are;

$$\left. \begin{aligned} \frac{\partial u(0,t)}{\partial y} &= \frac{f(t)}{\mu} & t > 0, & u(d,t) = U(t) \\ T(0,t) &= T_\omega, & T(d,t) &= T_\infty \end{aligned} \right\}, \quad (4)$$

Where $U(t) = u_0 \left(1 + \frac{\varepsilon}{2} (e^{i\omega t} + e^{-i\omega t}) \right)$.

The answer of the dust particles velocity we suppose the velocity as [34]:

$$v(y,t) = v_0(y)e^{i\omega t}, \quad (5)$$

From equation (4), we have

$$v(y,t) = \left(\frac{K_0}{mi\omega + K_0} \right) u(y,t), \quad (6)$$

Put equation (6) In equation (1). So equation (1) becomes,

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + \frac{K_0 N_0}{\rho} \left\{ \left(\frac{K_0}{mi\omega + K_0} \right) u - u \right\} - \frac{\sigma B_0^2 u}{\rho} + g\beta_T (T - T_\infty). \quad (7)$$

Using dimensionless variables.

$$u^* = \frac{u}{u_0}, y^* = \frac{y}{d}, t^* = \frac{u_0 t}{d}, \theta^* = \frac{T - T_\infty}{T_\omega - T_\infty}. \quad (8)$$

For simplicity (*) sign has been ignored. So equation (2), and equation (7) becomes,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - (M + K_1 - K_2)u + Gr\theta, \quad (9)$$

$$Pe \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + N^2 \theta, \quad (10)$$

with dimensionless physical conditions are:

$$\left. \begin{aligned} \frac{\partial u(0,t)}{\partial y} = f(t), u(1,t) = U(t) = 1 + \frac{\varepsilon}{2} (e^{i\omega t} + e^{-i\omega t}) \\ \theta(0,t) = 1, \quad \theta(1,t) = 0 \end{aligned} \right\}. \quad (11)$$

Where,

$$M = \frac{\sigma B_0^2 d^2}{\rho \nu}, \quad Gr = \frac{g \beta_T d^2 (T_\omega - T_\infty)}{u_0 \nu}, \quad K_2 = \frac{K_0^2 N_0 d^2}{\rho \nu (m_i \omega + K_0)},$$

$$K_1 = \frac{K_0 N_0 d^2}{\rho \nu}, \quad Pe = \frac{\rho c_p u_0 d}{k}, \quad N^2 = \frac{4 \alpha_0^2 d^2}{k}.$$

Where h_s is a heat transfer coefficient, M is a magnetic variable, Gr is Grashof number, K_2 dusty fluid variable, K_1 second-grade fluid variable, Pe Peclet number, N radiation variable.

The solution of equation (10) we suppose [34] the solution will be:

$$\theta(y, t) = \theta_0(y) + \theta_1(y) e^{i\omega t}, \quad (12)$$

From equation (12) we get,

$$\theta(y, t) = \frac{\sin(N - Ny)}{\sin(N)}. \quad (13)$$

By integrating equation (13) in equation (12), we get;

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - (M + K_1 - K_2)u + Gr \left\{ \frac{\sin(N - Ny)}{\sin(N)} \right\}. \quad (14)$$

Solution of the Problem

The solution of equation (14) can be find by applying the Light Hill Technique [35].

$$u(y, t) = F_0(y) + \frac{\varepsilon}{2} (F_1(y) e^{i\omega t} + F_2(y) e^{-i\omega t}), \quad (15)$$

To find out the results for $F_0(y)$, $F_1(y)$, and $F_2(y)$, we integrate equations (14) and (15) together and get:

$$F_0(y) = \left(1 - \frac{(f(t) + H) \sinh(\sqrt{m_1})}{\sqrt{m_1} \cdot \cosh(\sqrt{m_1})} \right) \cosh(y \sqrt{m_1}) + \left(\frac{f(t) + H}{\sqrt{m_1}} \right) \sinh(y \sqrt{m_1}) + A \left\{ \frac{\sin(N - Ny)}{\sin(N)} \right\}, \quad (16)$$

Where $H = \frac{A \cdot N \cdot \cos(N)}{\sin(N)}$, and $m_1 = M + K_1 - K_2$, $A = \frac{Gr}{m_1}$.

$$F_1(y) = \frac{\cosh\left(y\sqrt{m_2}\right)}{\cosh\left(\sqrt{m_2}\right)}. \quad (17)$$

Where $m_2 = m_1 + i\omega$

$$F_2(y) = \frac{\cosh\left(y\sqrt{m_3}\right)}{\cosh\left(\sqrt{m_3}\right)}. \quad (18)$$

Where $m_3 = m_1 - i\omega$,

In last we putting the results in equation (15) from equations (16), (17) and (18), we obtained the following result:

$$u(y,t) = \left\{ \left(1 - \frac{(f(t)+H)\sinh\left(\sqrt{m_1}\right)}{\sqrt{m_1} \cdot \cosh\left(\sqrt{m_1}\right)} \right) \cosh\left(y\sqrt{m_1}\right) + \left(\frac{f(t)+H}{\sqrt{m_1}} \right) \sinh\left(y\sqrt{m_1}\right) + \right. \\ \left. A \left\{ \frac{\sin(N - Ny)}{\sin(N)} \right\} + \frac{\varepsilon}{2} \left(\frac{\cosh\left(y\sqrt{m_2}\right)}{\cosh\left(\sqrt{m_2}\right)} \right) e^{i\omega t} + \frac{\varepsilon}{2} \left(\frac{\cosh\left(y\sqrt{m_3}\right)}{\cosh\left(\sqrt{m_3}\right)} \right) e^{-i\omega t} \right\}. \quad (19)$$

Graphical Results and Discussion

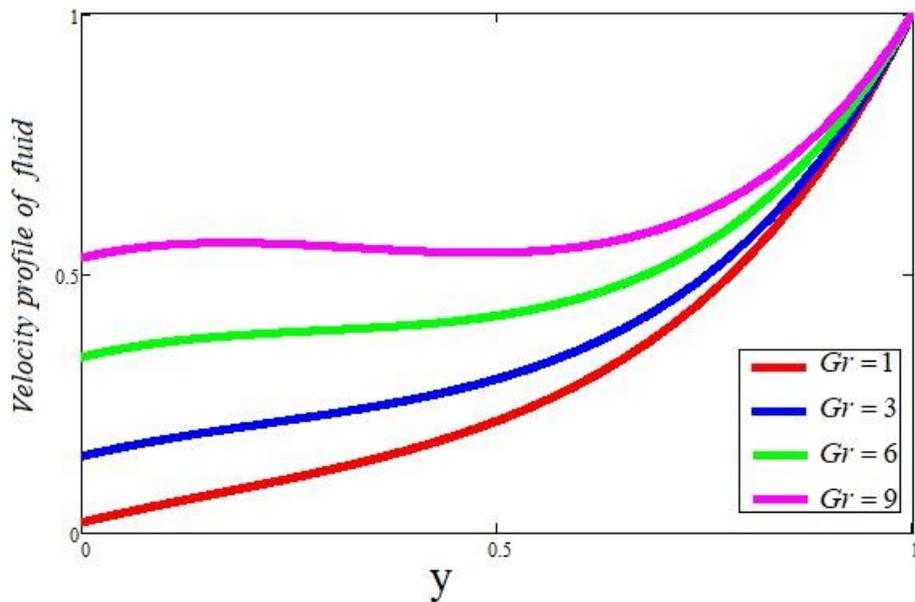


Figure (2): Impact of Gr on the profile of fluid velocity.

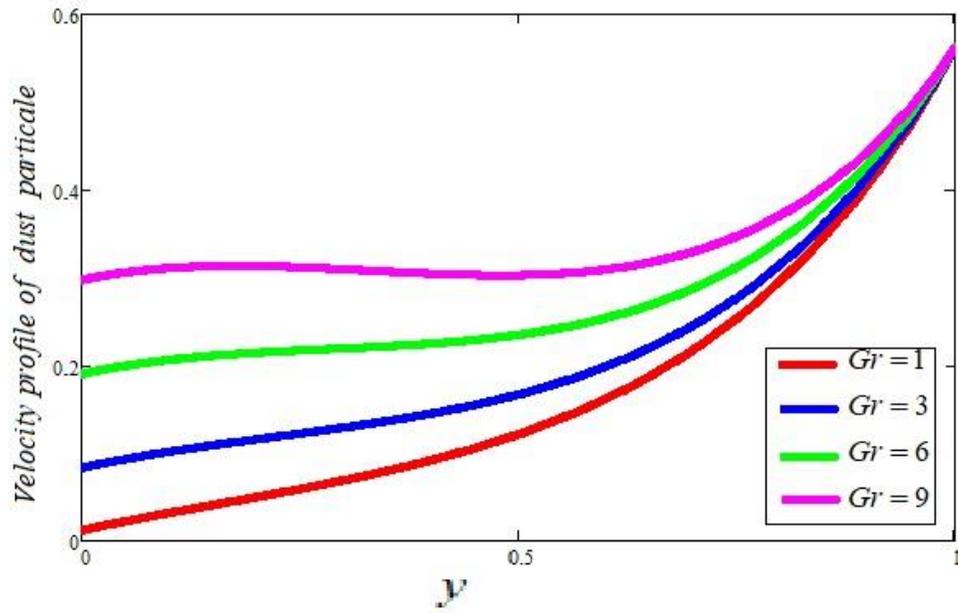


Figure (3): Impact of Gr on the profile of dust particles velocity.

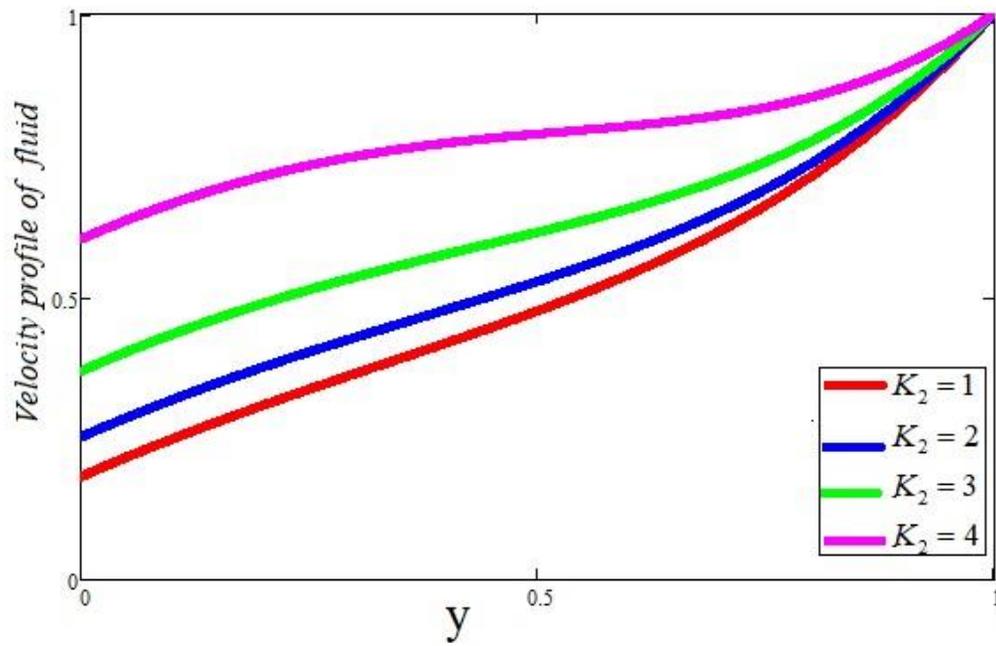


Figure (4): Impact of K_2 on the profile of fluid velocity.

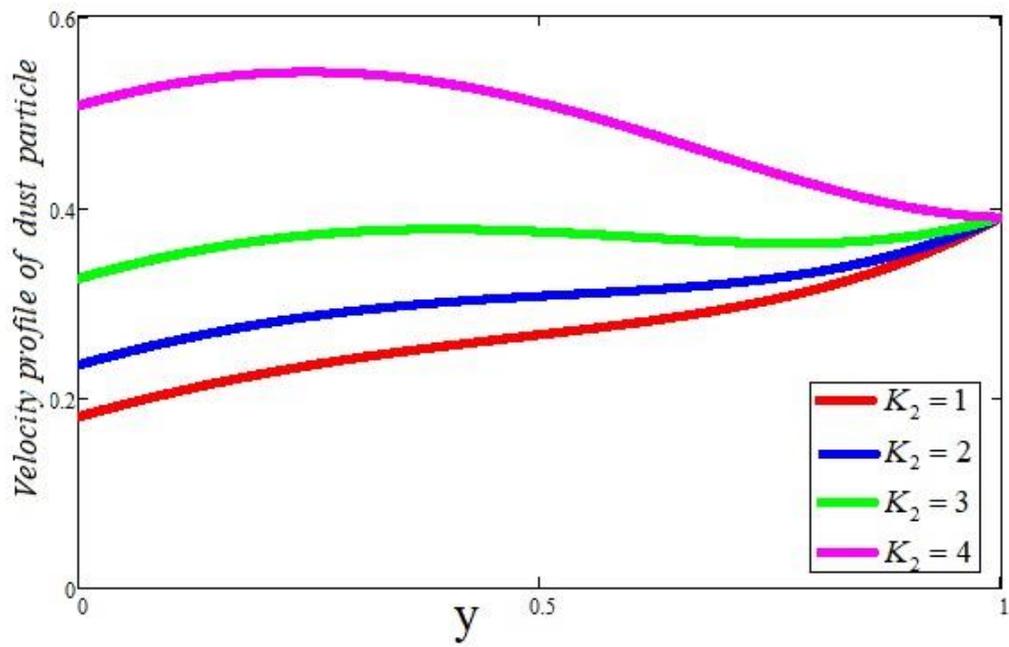


Figure (5): Impact of K_2 on the profile of dust particles velocity.

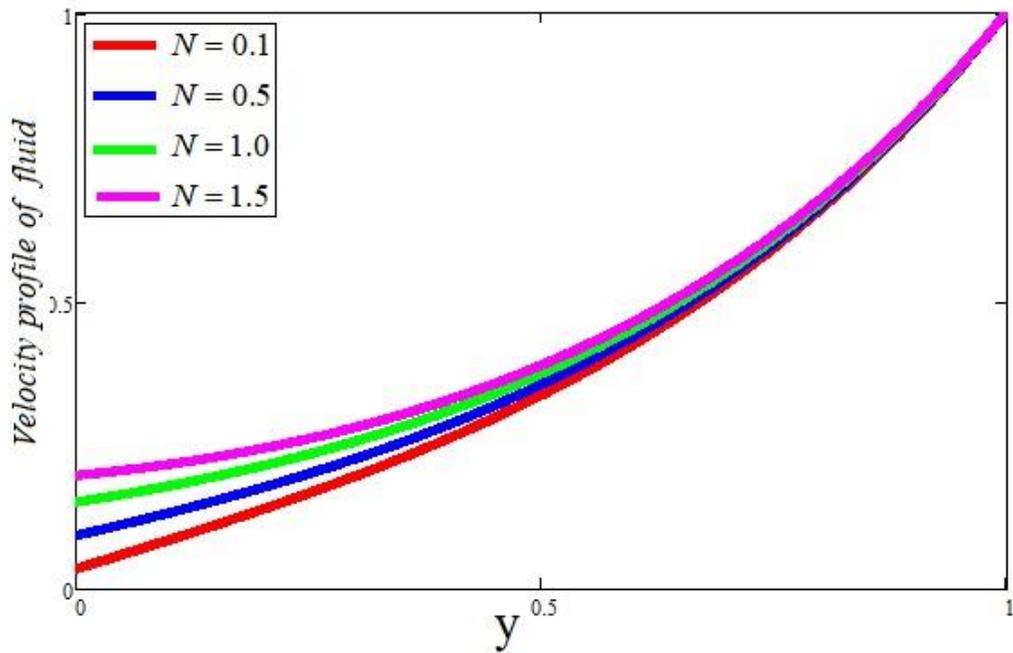


Figure (6): The behavior of N on velocity profile (fluid).

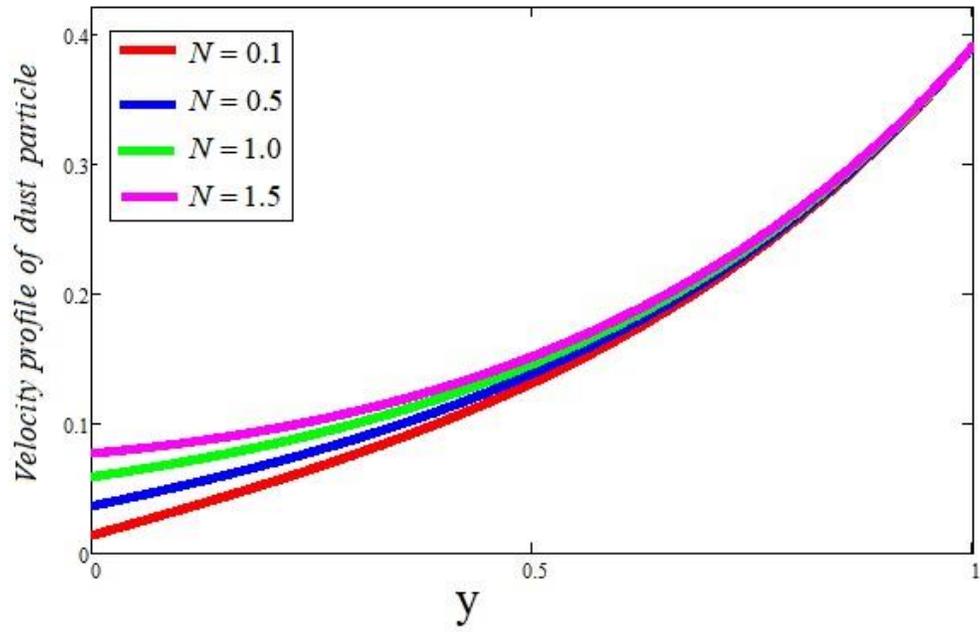


Figure (7): Impact of N on the profile of dust particle velocity.

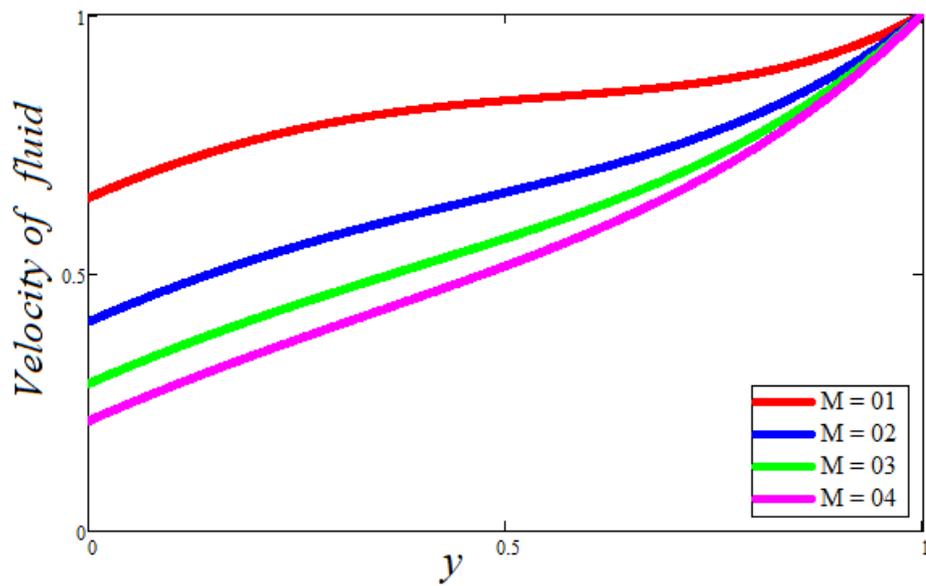


Figure (8): Impact of M on the profile of fluid velocity.

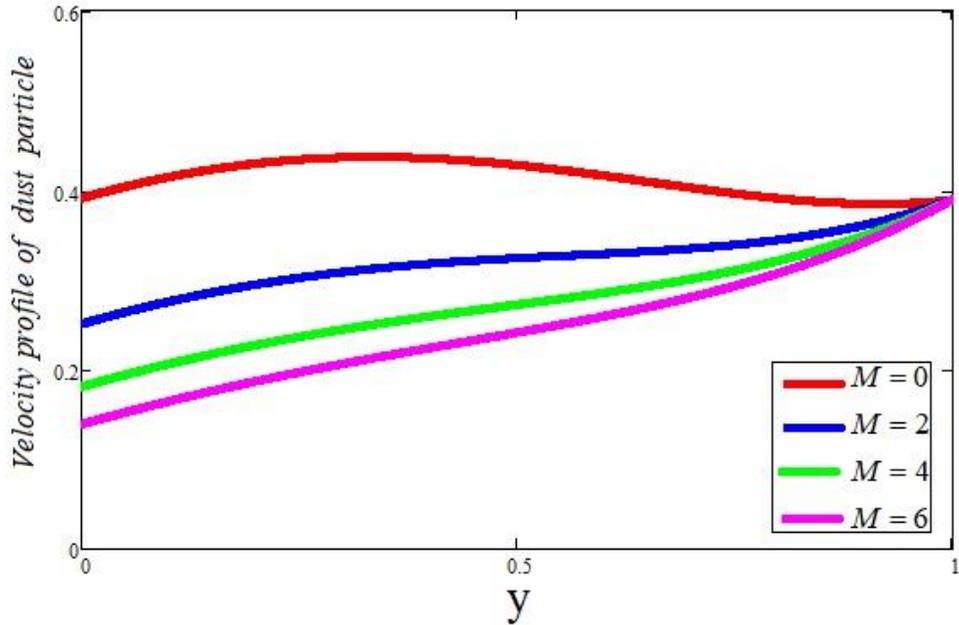


Figure (9): The impact of M on the profile velocity of dust particles.

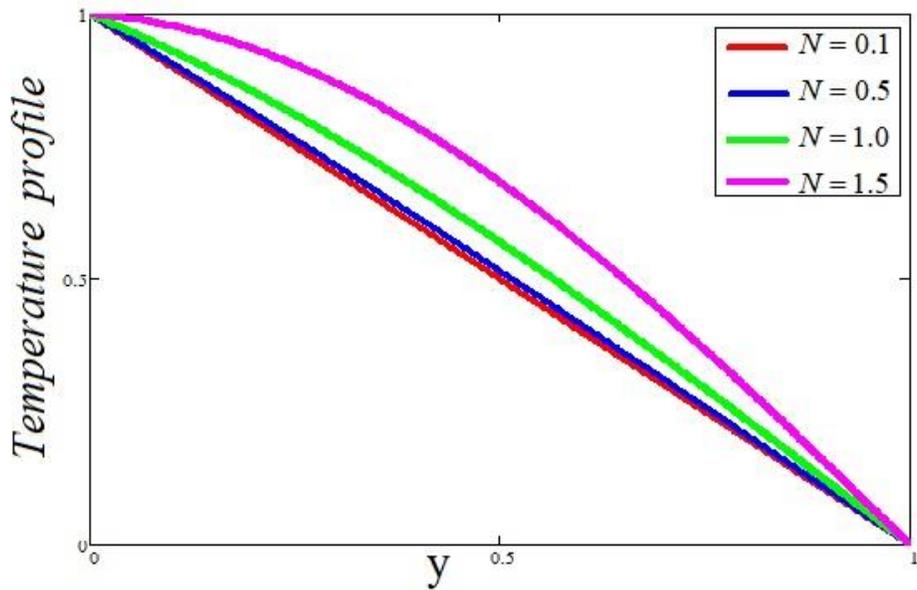


Figure (10): Impact of N on temperature.

In this work, the unsteady motion because of an infinite plate that addresses wall-shear stress to a two-phase fluctuating flow of dusty fluids is considered by means of the light hill technique. The solutions that have been achieved satisfy all the introduced initial and boundary conditions. To find out some specific and important information about the repercussion of different parameters of flow on the velocities of magnetic dusty particles and fluid, certain numerical simultaneous results have been made with the help of Mathcad-15 software. These influences of different physical parameters like magnetic parameter M , dusty

fluid parameter K_2 , Grashof number Gr and radiation variable N are graphically shown in this section. According to these graphs, we get different results for the profile of fluid velocities and also for the velocities of dusty particles which is briefly discussed in the following paragraph.

The obtained results are shown in figures 2 and 3 respectively, which reflect the behavior of Grashof number on the velocity profile of fluid and dusty particles. In these graphs, we observed the direct variation between both velocity of the base fluid and as well as the velocity of dust particles that is by increasing the Grashof number the velocity of fluid and dust particles also increases. According to the physics of Grashof number, we know that it is the ratio of buoyancy forces and drag forces therefore, by increasing this number the buoyancy forces increase and the viscosity decreases, and this why the velocities of fluid and dust particles increase. Figure 4 and 5 displayed the fluid and dust particles velocities against the dusty parameter K_2 , these graphs show that by increasing the dusty parameter the velocity of the fluid and dusty particles are also increase respectively. To illustrate the effect of radiation on both the velocities of dusty particles and base fluid, for this behavior one can observe figures 6 and 7 respectively. These figures show that when the increase occurs in the radiation, the velocity of fluid and dust particles also increases. It is obvious from the physics of radiation that by increasing the radiation the temperature of the fluid increases, because of this increase in temperature the kinetic energy of the fluid and dust particles are also increase. The corresponding graphs 8 and 9 are plotted respectively for the investigation of the influence of magnetic parameter M on the profile of fluid velocity and also on the velocity of dusty particles. It is noted from these figures that the velocity of the fluid shortens monotonically due to the rise in magnetic parameter M . This reduction in the velocity of the fluid is actually the implementation of magnetic force against the direction of fluid flow. It is also clear from these figures that velocity profiles for the velocity of fluid are much larger than those for the velocity of dusty particles. The affiliation of the radiation and the temperature also check out in this article. Figure 10 tells about the relation of radiation variable parameter N and temperature, it is clear from this specific graph that an increase in radiation variable parameter N occurs an increase in the fluid temperature.

Conclusion

The graphical and theoretical analysis of the consequence of the various physical parameters on the MHD flow of two phases fluctuating flow of dusty fluid is examined along x-direction between two plates which are parallel with the help of Light hill technique. It is pretended that the fluid flow is unidirectional, one dimensional, incompressible, free stream fluctuating, conducting electrically and heat convection with heat transfer is also appropriated in this problem. The ingrained dust particles are also pretended to be conducting and homogeneously dispersed in the second-grade dusty fluid. The parametric consequence of the physical parameters on the profile of fluid velocity and dusty particles, temperature, and radiation are examined comprehensively. It is observed that increase in Grashof number Gr , dusty parameter K_2 , radiation variable parameter N occurs an increase in both the velocities of fluid and dusty particles. The increase in magnetic parameter M occurs a decrease in the velocities of fluid and dusty particles. The relation of radiation and temperature are also discussed graphically in this article and according to this relation the increase in radiation cause the increase in temperature.

References

- [1] Crowe, C. T., Troutt, T. R., & Chung, J. N. (1996). Numerical models for two-phase turbulent flows. *Annual Review of Fluid Mechanics*, 28(1), 11-43.
- [2] Amkadni, M., Azzouzi, A., & Hammouch, Z. (2008). On the exact solutions of laminar MHD flow over a stretching flat plate. *Communications in Nonlinear Science and Numerical Simulation*, 13(2), 359-368.
- [3] Michael, D. H., and D. A. Miller. "Plane parallel flow of a dusty gas." *Mathematika* 13.1 (1966): 97-109.
- [4] Healy, J. Val. "Perturbed Two-Phase Cylindrical Type Flows." *The physics of fluids* 13.3 (1970): 551-557.
- [4] Chigier N. 1981. *Energy, Combustion and Environment*. New York: McGraw-Hill. 496 pp.
- [5] Sirignano W.A. 1993. *Fluid dynamics of sprays-1992 Freeman scholar lecture*. *J. Fluid Mech.* 115:345-78
- [6] Ghosh, S., and Julian Charles Roland Hunt. "Induced air velocity within droplet driven sprays." *Proceedings of the Royal Society of London. Series A: Mathematical and Physical Sciences* 444.1920 (1994): 105-127.
- [7] Ahmed, N., H. K. Sarmah, and D. Kalita. "Thermal diffusion effect on a three-dimensional MHD free convection with mass transfer flow from a porous vertical plate." *Latin American applied research* 41.2 (2011): 165-176.
- [8] Takhar, H. S., Roy, S. & Nath, G. Unsteady free convection flow over an infinite vertical porous plate due to the combined effects of thermal and mass diffusion, magnetic field and Hall currents. *Heat and mass transfer* 39(10), 825–834 (2003)
- [9] Wilks, Graham. "Magnetohydrodynamic free convection about a semi-infinite vertical plate in a strong cross field." *Zeitschrift für angewandte Mathematik und Physik ZAMP* 27.5 (1976): 621-631.
- [10] Gupta, A. S. "Steady and transient free convection of an electrically conducting fluid from a vertical plate in the presence of a magnetic field." *Applied Scientific Research* 9.1 (1960): 319.
- [11] Gupta, Anadi Shankar. "Laminar free convection flow of an electrically conducting fluid from a vertical plate with uniform surface heat flux and variable wall temperature in the presence of a magnetic field." *Zeitschrift für angewandte Mathematik und Physik ZAMP* 13.4 (1962): 324-333.
- [12] Soo, Shao Lee. "Fluid dynamics of multiphase systems." WALTHAM, MASS, BLAISDELL PUBLISHING CO, 1967. 524 P, 206 FIG, 8 TAB, 886 REF. (1967).
- [13] Michael, D. H., and D. A. Miller. "Plane parallel flow of a dusty gas." *Mathematika* 13.1 (1966): 97-109.
- [14] Saffman, P. G. "On the stability of laminar flow of a dusty gas." *Journal of fluid mechanics* 13.1 (1962): 120-128.
- [15] Vimala, C. S. "Flow of a dusty gas between two oscillating plates." *Defence Sci. J* 22 (1972): 231-236.
- [16] Venkateshappa, V., et al. "Viscous dusty fluid flow with constant velocity magnitude." *Electronic Journal of Theoretical Physics* 5.17 (2008): 237-252.

- [17] Zhou, Teng, et al. "Dielectrophoretic choking phenomenon of a deformable particle in a converging-diverging microchannel." *Electrophoresis* 39.4 (2018): 590-596.
- [18] Zhou, Teng, et al. "AC dielectrophoretic deformable particle-particle interactions and their relative motions." *Electrophoresis* 41.10-11 (2020): 952-958.
- [19] Ali, Farhad, et al. "A Report on fluctuating free convection flow of Heat Absorbing Viscoelastic Dusty fluid past in A Horizontal channel With MHD Effect." *Scientific Reports* 10.1 (2020): 1-15.
- [20] Ali, Farhad, et al. "Two-Phase Fluctuating Flow of Dusty Viscoelastic Fluid between Non-conducting Rigid Plates with Heat Transfer." *IEEE Access* 7 (2019): 123299-123306.
- [21] Attia, H. A., A. M. A. Al-Kaisy, and K. M. Ewis. "MHD Couette flow and heat transfer of a dusty fluid with exponential decaying pressure gradient." *Tamkang Journal of Science and Engineering* 14.2 (2011): 91-96.
- [22] Ali, Farhad, Ilyas Khan, and Sharidan Shafie. "Closed form solutions for unsteady free convection flow of a second grade fluid over an oscillating vertical plate." *Plos One* 9.2 (2014): e85099.
- [23] Massoudi, M.: Constitutive relations for the interactions force in multicomponent particulate flows. *Int. J. Non-Linear Mech.* 38, 313–336 (2006)
- [24] Fan, L. S., and C. Zhu. "Chapter 9 Dense-phase fluidized beds." *Principles of gas-solid flows*, Cambridge University Press, Cambridge; New York (1998): 371-415.
- [25] Anthony, E. J. "Fluidized bed combustion of alternative solid fuels; status, successes and problems of the technology." *Progress in Energy and Combustion Science* 21.3 (1995): 239-268.
- [26] Gidaspow, D.: *Multiphase Flow and Fluidization*. Academic Press, San Diego (1994)
- [27] Ungarish, Marius. "Hydrodynamics of Suspensions." *Journal of Fluid Mechanics* 290 (1995): 406-406.
- [28] Lighthill, Michael James. "CIX. A Technique for rendering approximate solutions to physical problems uniformly valid." *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science* 40.311 (1949): 1179-1201.
- [29] Comstock, Craig. "The Poincaré–Lighthill perturbation technique and its generalizations." *SIAM Review* 14.3 (1972): 433-446.
- [30] Große, Sebastian, and Wolfgang Schröder. "High Reynolds number turbulent wind tunnel boundary layer wall-shear stress sensor." *Journal of Turbulence* 10 (2009): N14.
- [31] Amili, Omid, and Julio Soria. "Wall shear stress distribution in a turbulent channel flow." *15th International Symposium on Applications of Laser Techniques to Fluid Mechanics*, Lisbon, Portugal. 2010.
- [32] Örlü, Ramis, and Philipp Schlatter. "On the fluctuating wall-shear stress in zero pressure-gradient turbulent boundary layer flows." *Physics of fluids* 23.2 (2011): 021704.
- [33] Moß, Christopher, et al. "Studying the impact of wall shear stress on the development and performance of electrochemically active biofilms." *ChemPlusChem* 85.10 (2020): 2298-2307.
- [34] Hussanan, A., Anwar, M. I., Ali, F., Khan, I., & Shafie, S. (2014). Natural convection flows past an oscillating plate with Newtonian heating. *Heat Transfer Research*, 45(2).
- [35] Comstock, C. (1972). The Poincaré–Lighthill perturbation technique and its generalizations. *SIAM Review*, 14(3), 433-446.

Figures

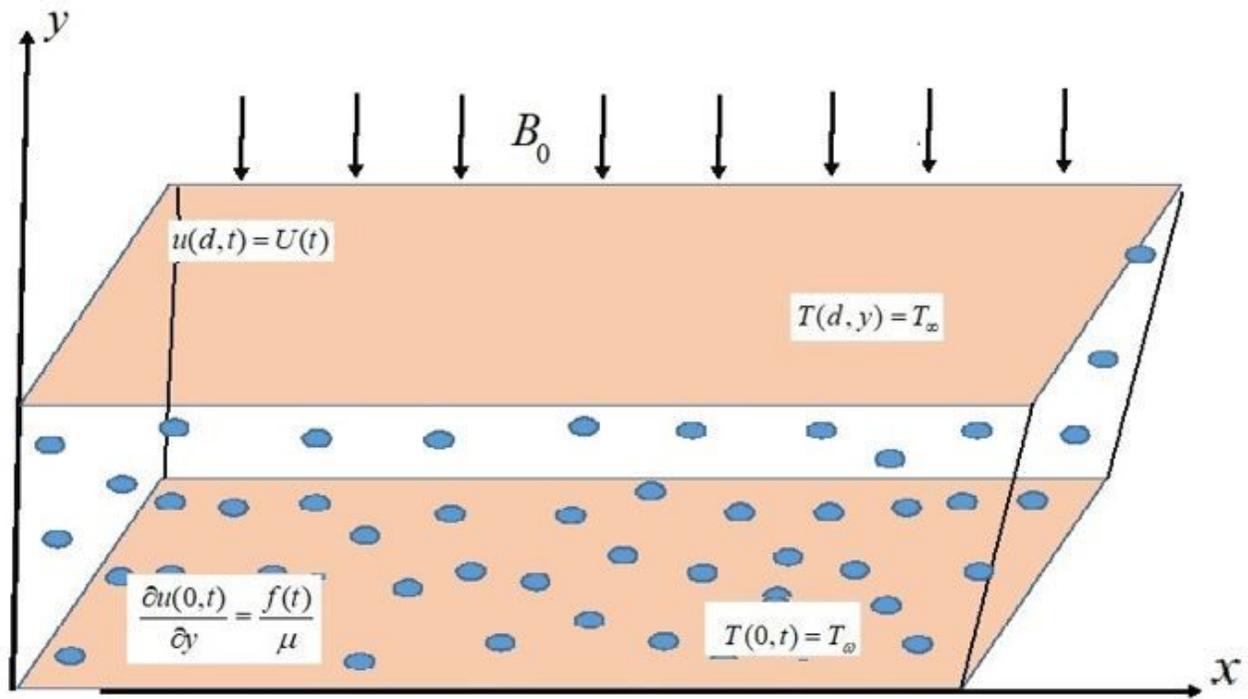


Figure 1

Geometry of the problem

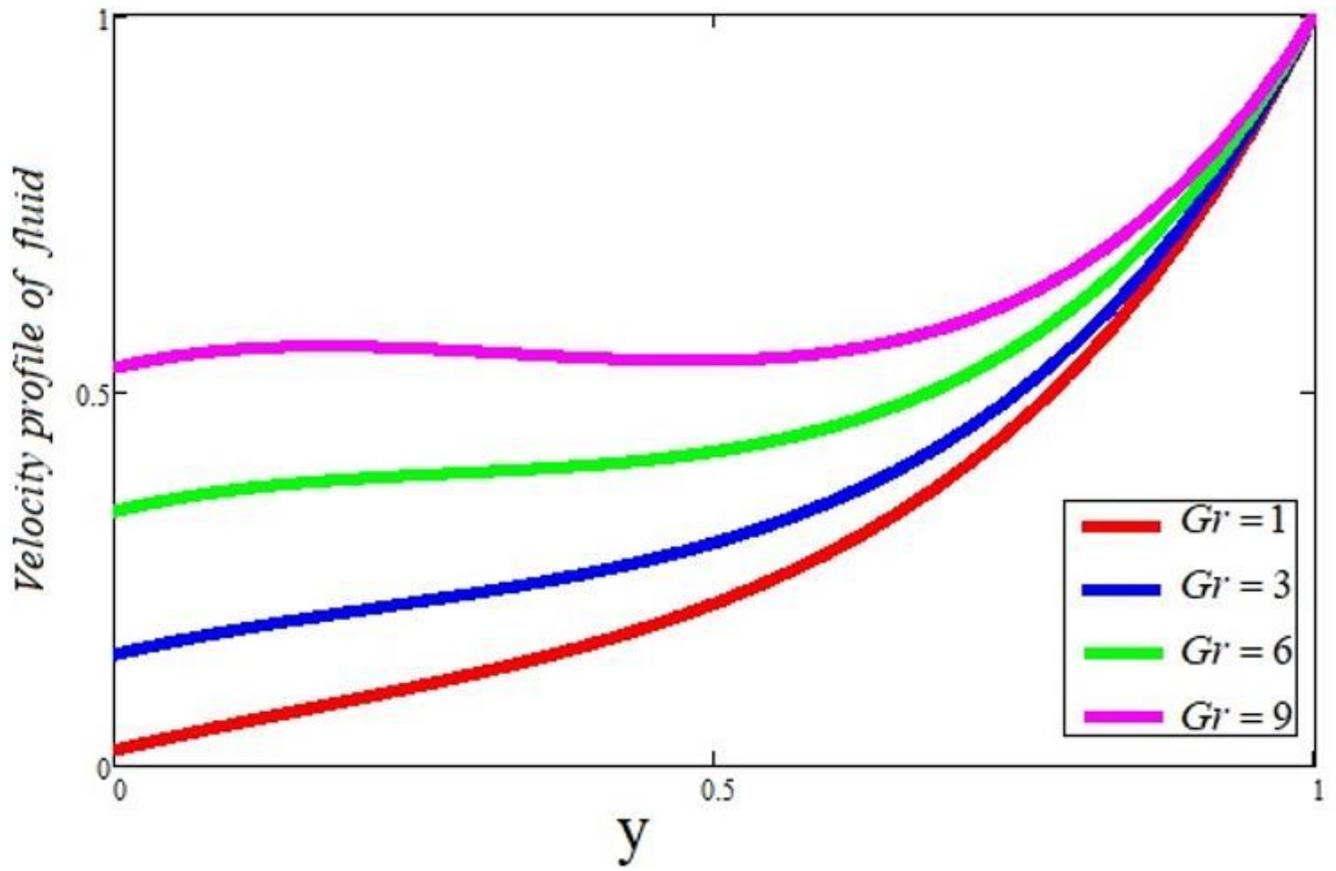


Figure 2

Impact of Gr on the profile of fluid velocity.

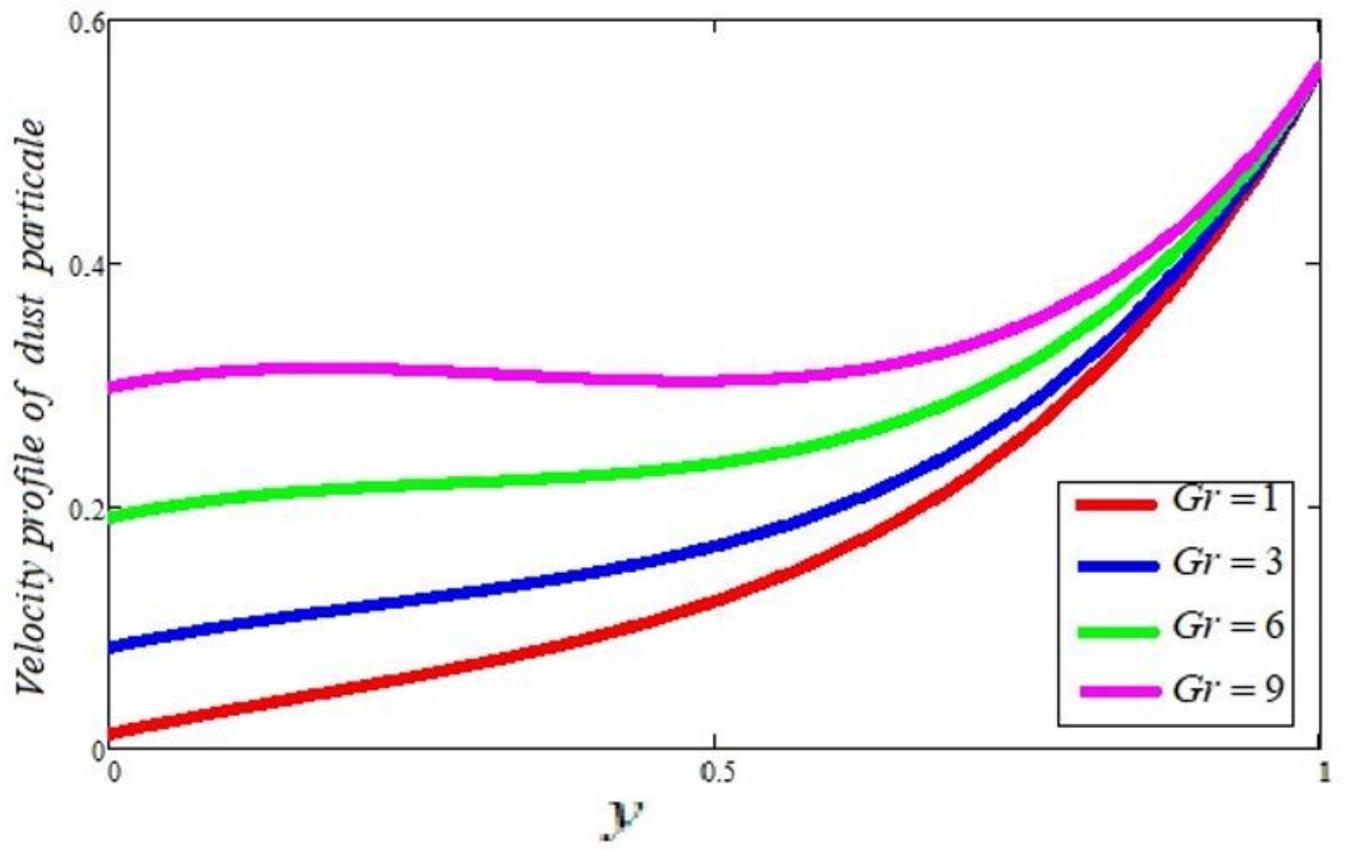


Figure 3

Impact of Gr on the profile of dust particles velocity.

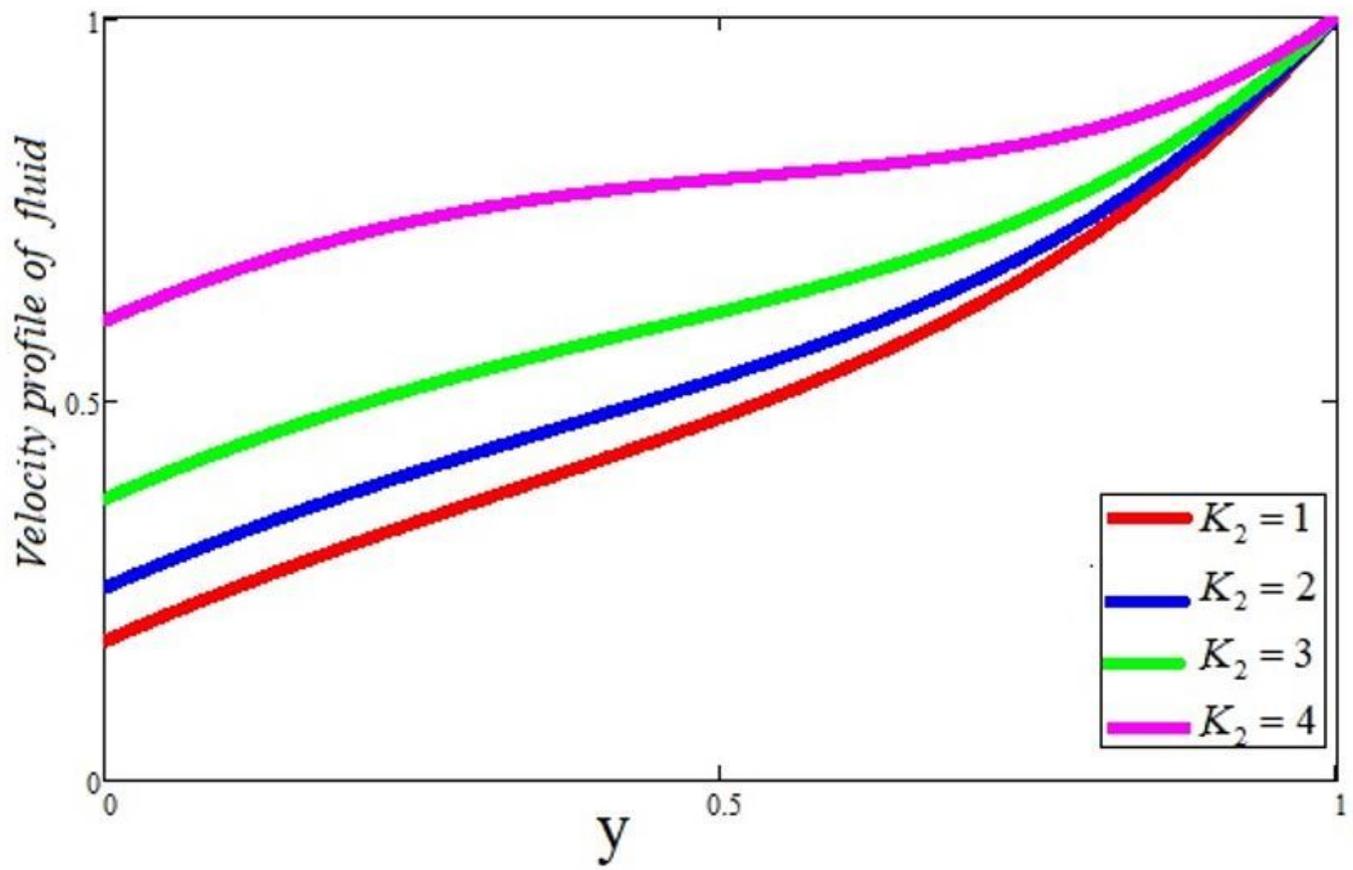


Figure 4

Impact of K_2 on the profile of fluid velocity.

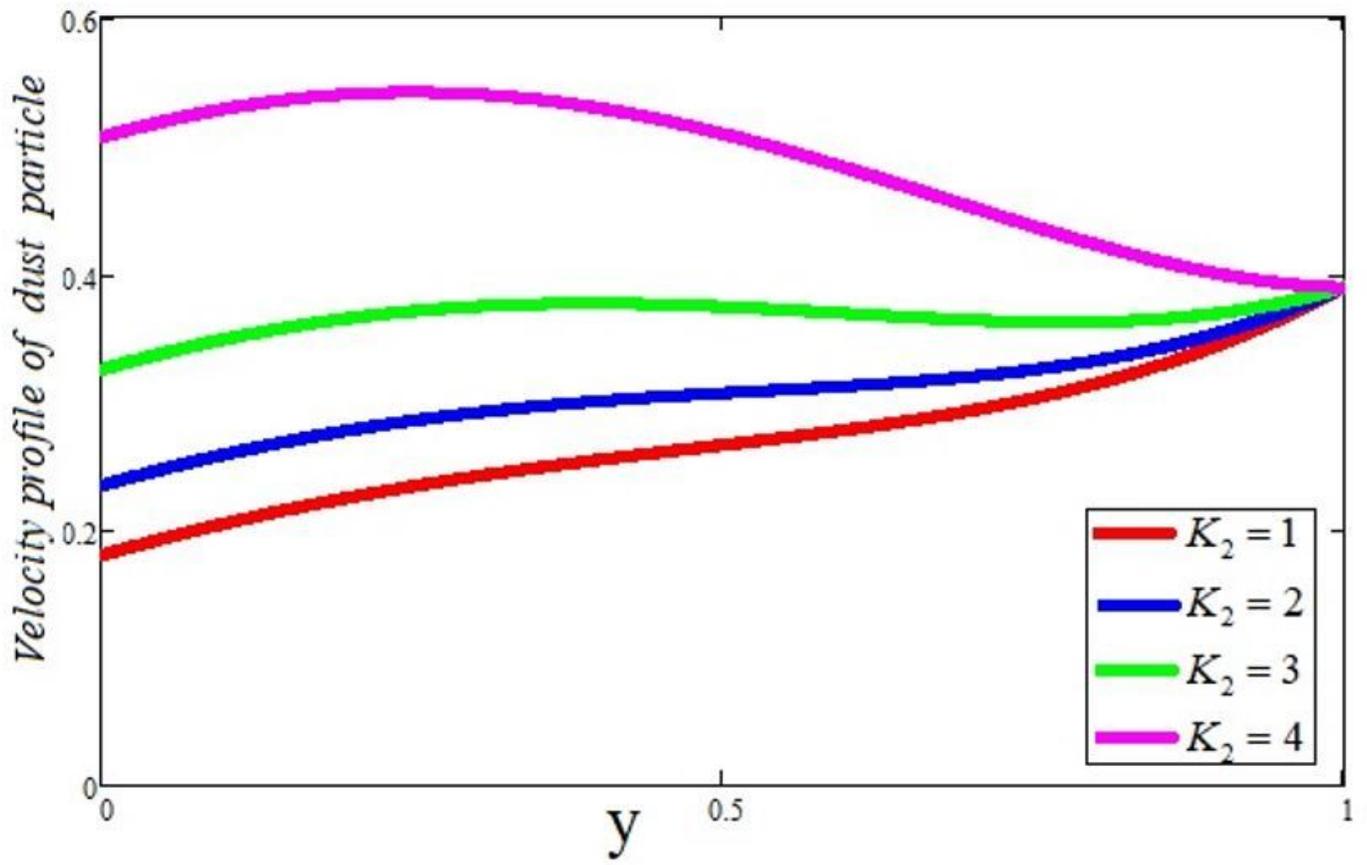


Figure 5

Impact of K_2 on the profile of dust particles velocity.

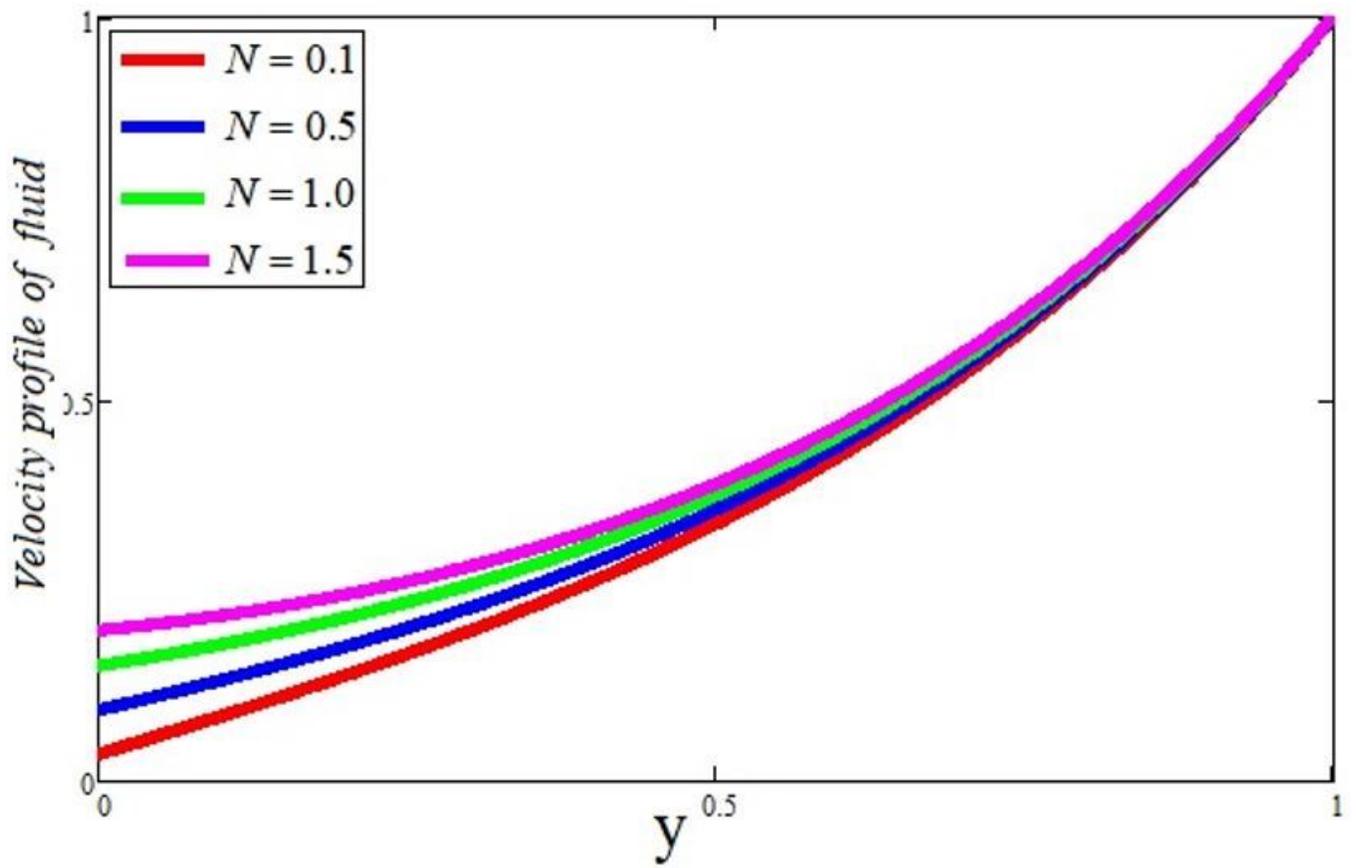


Figure 6

The behavior of N on velocity profile (fluid).

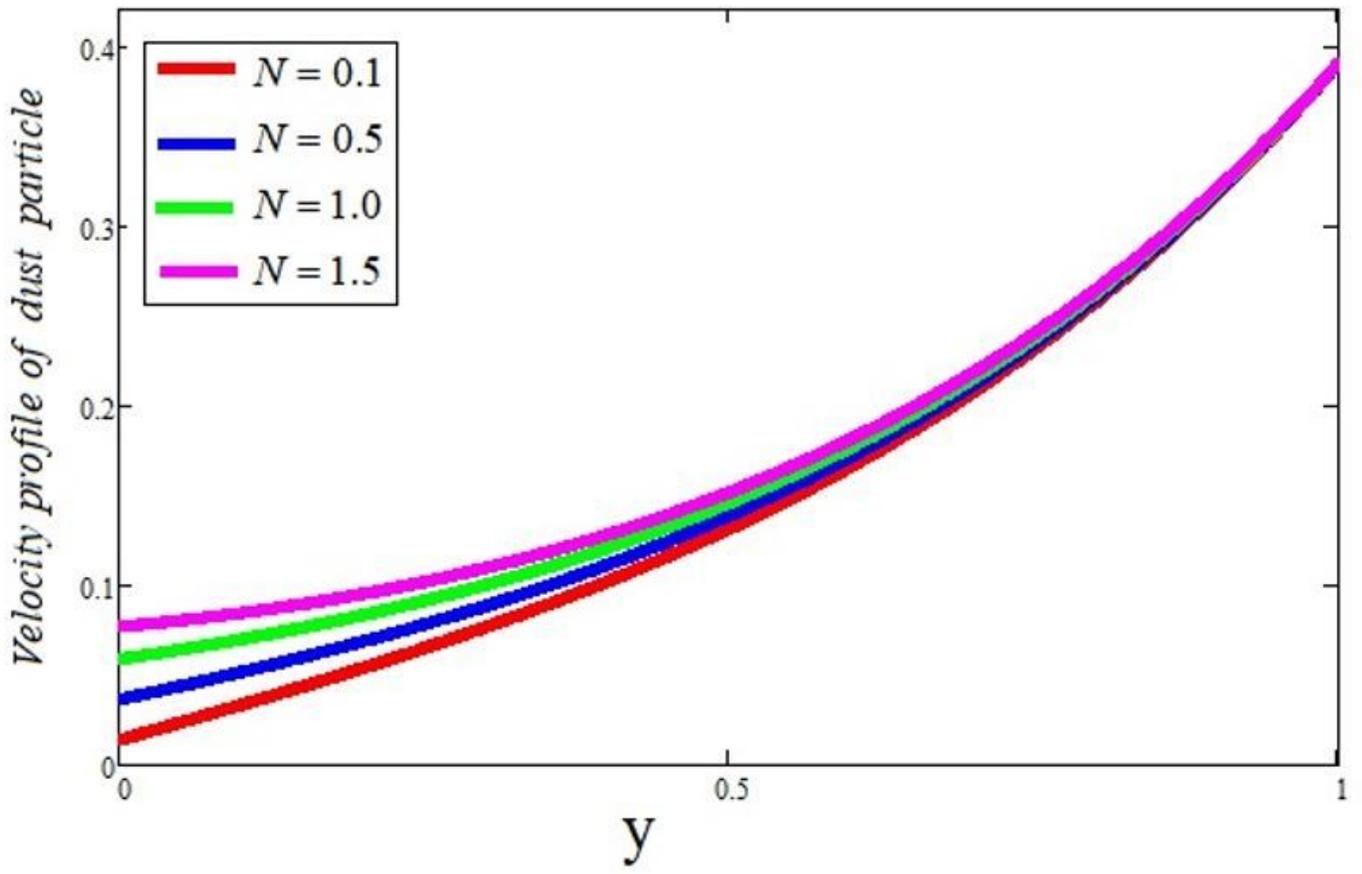


Figure 7

Impact of N on the profile of dust particle velocity.

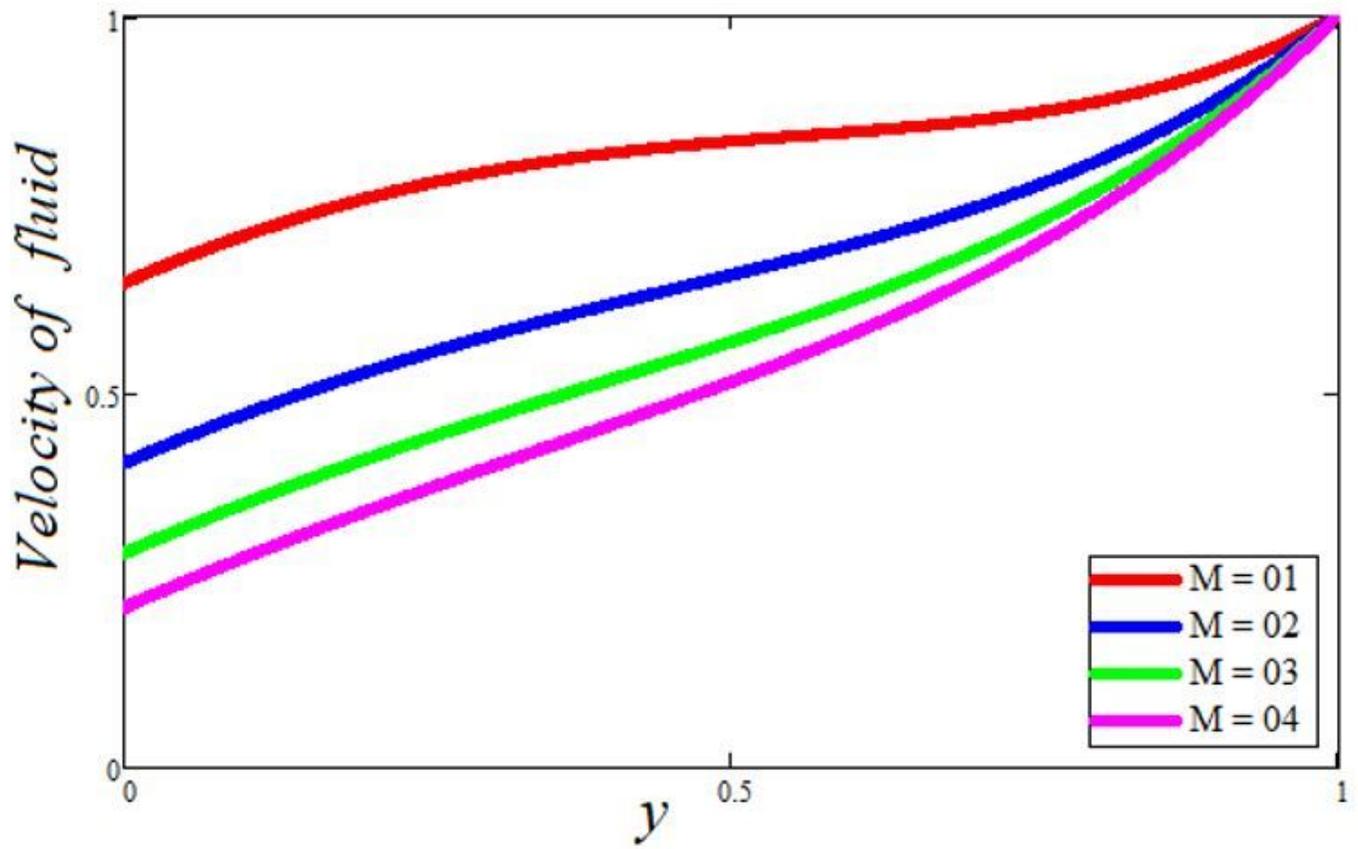


Figure 8

Impact of M on the profile of fluid velocity.

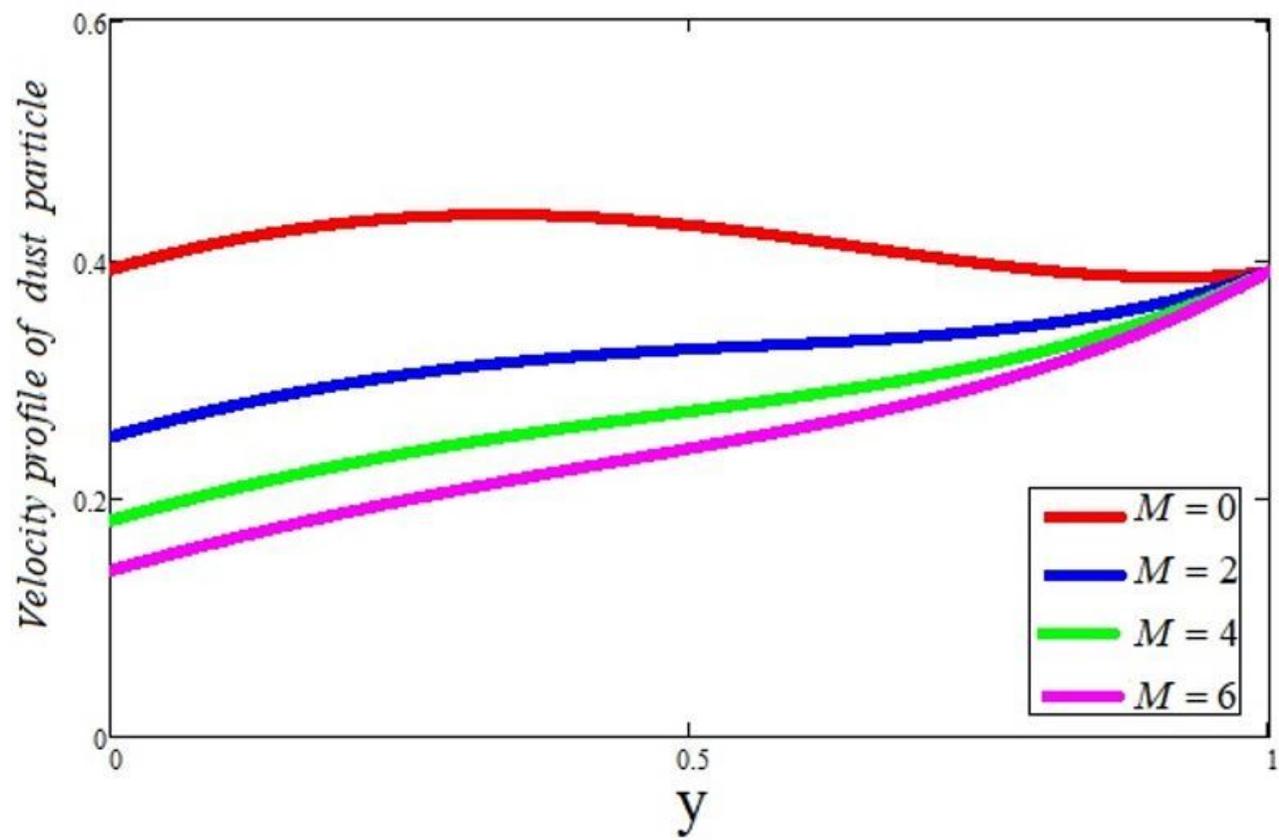


Figure 9

The impact of M on the profile velocity of dust particles.

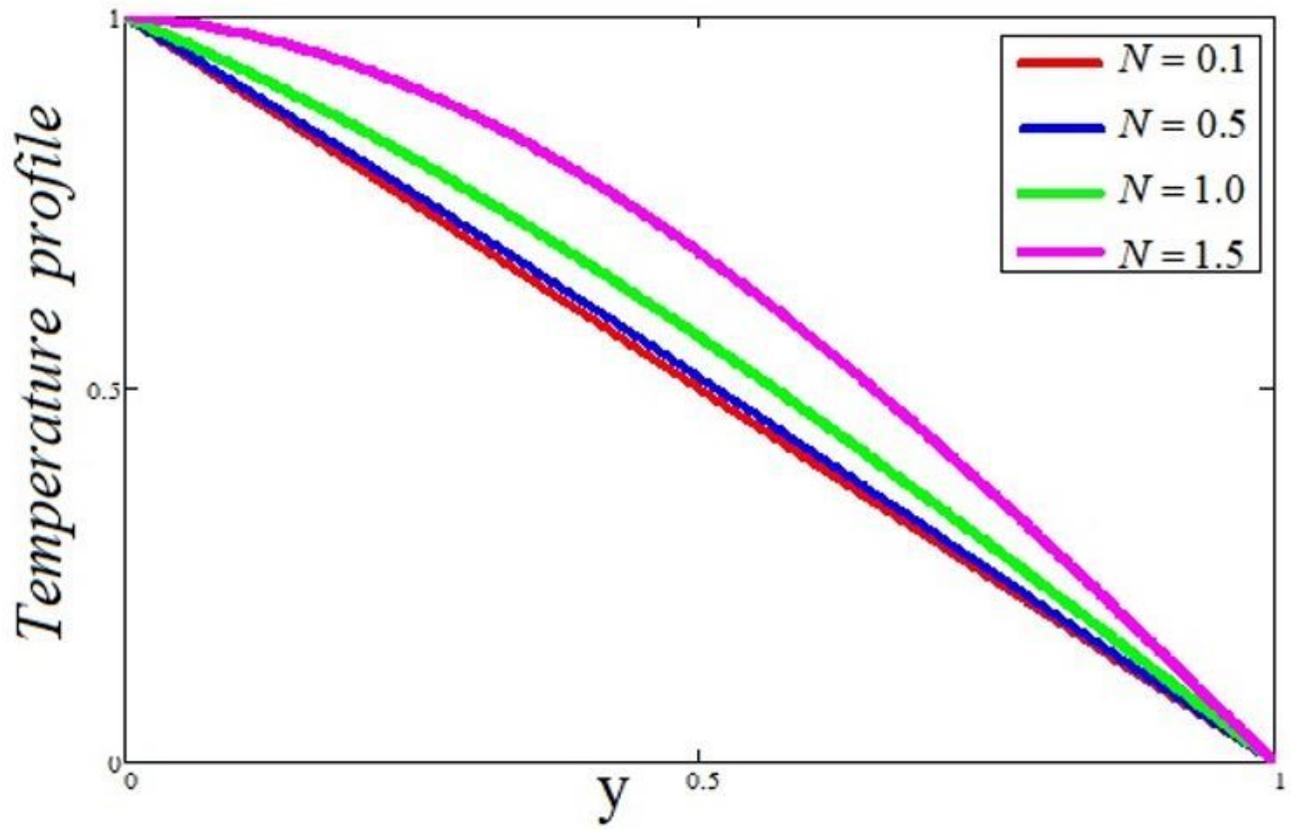


Figure 10

Impact of N on temperature.