Making Mistakes Saves the Single World of the Extended Wigner's Friend Experiment

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Article

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Making Mistakes Saves the Single World of the Extended Wigner's Friend Experiment

Szymon Łukaszyk

The Extended Wigner's Friend thought experiment comprising a quantum system containing an agent who draws conclusions, upon observing the outcome of a measurement of a qubit prepared in two non-orthogonal versions by another agent led its authors to conclude that quantum theory cannot consistently describe the use of itself. It has also been proposed that this thought experiment is equivalent to coherent entangled state (Bell type) experiments. It is argued in this paper that the assumption of the freedom of choice of the first Wigner's friend invalidates such equivalency. It is also argued that the assumption of locality (physical space) introduces superfluous identity of indiscernibles metric axiom, which is invalid in quantum domain and generally disproven by the Ugly duckling mathematical theorem.

Keywords: measurement problem, Wigner's friend, Bells' theorem, observer-independent facts, quantum contextuality, identity of indiscernibles, Łukaszyk-Karmowski metric, Ugly duckling theorem, locality, freedom of choice.

1. Introduction

The Extended Wigner's Friend thought experiment (EWF) contains a contradiction, which led its authors first to o give up the view that there is one single reality [9] and later to conclude that quantum theory cannot consistently describe the use of itself [10], as the contradiction appears within a single world. Indeed, many-worlds interpretation of quantum theory has paradoxical features of its own [8] and is not only counterfactually indefinite but factually indefinite [2]. However, one may wonder how a theory with such powerful predictive power that has led to so many awesome inventions can contain inconsistencies.

The EWF is allegedly not just a thought experiment. Its authors claim, for example, that if it was realised as a game between a gambler and a casino, both parties would likely end up in a dispute, putting forward contradicting arguments based on quantum-mechanical reasoning that should have been accepted as two alternative (observer-dependent) facts about what was the result of the first measurement in this thought experiment [10].

The paper aims to evaluate the prospects of the EWF implementation as a casino game. Feasibility of implementing Bell-Wigner type experiments [e.g. 4, 5, 17, 3] involving coherent entangled states as casino games is also discussed.

2. The EWF with a Super-Observer

In each round of the EWF in her sealed lab Alice prepares a first qubit

$$|\alpha\rangle = \sqrt{1/3}\,|h\rangle + \sqrt{2/3}\,|t\rangle \tag{1}$$

measures it at a certain time t_0 in a basis $\{|h\rangle, |t\rangle\}$, records the measurement and prepares a second state

$$|\beta\rangle = \begin{cases} |0\rangle & \text{iff } |\alpha\rangle = |h\rangle \\ \sqrt{1/2}\left(|0\rangle + |1\rangle\right) & \text{iff } |\alpha\rangle = |t\rangle \end{cases} \tag{2}$$

Alice hands the second state[1] to Bob (2) residing in another sealed lab[2]. At a later time $t_1 > t_0$ Bob measures the received second state in a basis $\{|0\rangle, |1\rangle\}$. At even later time $t_2 > t_1$ Charlie measures the first state (1) emitted from Alice's lab in a first Hadamard basis

$$|\square\rangle \doteq \sqrt{1/2}\left(|h\rangle + |t\rangle\right),\ |\bigcirc\rangle \doteq \sqrt{1/2}\left(|h\rangle - |t\rangle\right) \tag{3}$$

and the second state (2) emitted from Bob's lab in a second Hadamard basis

$$|+\rangle \doteq \sqrt{1/2}\left(|0\rangle + |1\rangle\right),\ |-\rangle \doteq \sqrt{1/2}\left(|0\rangle - |1\rangle\right) \tag{4}$$

and the round is completed. Since Charlie doesn't know the measurement of the first qubit (1) emitted from Alice's lab it is a mixed state with density matrix

$$\rho_\alpha = 1/3\,|h\rangle\langle h| + 2/3\,|t\rangle\langle t| \tag{5}$$

Charlie measures also mixed state (2) with density matrix

$$\rho_\beta = 1/3\,|0\rangle\langle 0| + 2/3\,|+\rangle\langle +| \tag{6}$$

There is nothing contradicting in this setup. Measurement probabilities for a large number of rounds are nonzero $p(|h\rangle) = p(|1\rangle) = 1/3$, $p(|t\rangle) = p(|0\rangle) = 2/3$; $p(|\square\rangle) = p(|\circ\rangle) = 1/2$; $p(|+\rangle) = 5/6$, $p(|-\rangle) = 1/6$.

Due to the assumption of locality, however, this kind of thought experiments containing a qubit and someone or something else that measures this qubit in a box or lab isolated from the environment (Schrödinger's cat, Wigner's friend [20], Deutsch's variant [7], and the EWF) are described as coherent *big quantum states* of those boxes or labs. In the case of Alice's lab, for example, this *big quantum state* is described as a tensor product of the basic quantum state (1), some device enabling for a measurement of this basic state and everything else connected with this device, and finally Alice herself in her lab including her sense organs, brain, etc. This indeed seems unrealistic (at least in the context of coherence), even if

[1] Describing gruesomely this preparation process: Alice puts a cat into a Schrödinger's box provided with all the necessary equipment (a Geiger counter, a flask of poison, etc.) if she measures $|t\rangle$ but she kills the cat before putting its corpse into the box if she measures $|h\rangle$. She delivers the box to Bob who will open it to find whether the cat is alive ($|1\rangle$) or dead ($|0\rangle$).

[2] This is the only action that violates otherwise perfect isolation of Alice's and Bob's labs.

not explicitly precluded by the laws of quantum theory as such [10]. Schrödinger's cat has nothing to do with quantum information science, even if the latter can be harnessed to kill the cat. Deutsch's variant, in which a friend informs Wigner that she has a definite measurement result, but does not reveal this result, so as not to accidentally destroy the superposition of the *big quantum state* from inside of the lab is particularly instructive.

Even if the assumptions (Q, C, S) of the EWF do not explicitly include locality [17], locality is used to model the enclosed spaces of the labs including Alice and Bob themselves and their actions in space and time as a coherent *big quantum state* that can be defined [6], from Charlie's perspective as

$$|\psi\rangle_{AB} = \sqrt{1/3}\,|h\rangle|0\rangle + \sqrt{1/3}\,|t\rangle|0\rangle + \sqrt{1/3}\,|t\rangle|1\rangle \quad (7)$$

after *factorizing out* Alice, Bob and their devices subsystems. Unlike Schrödinger's cat or Wigner's friend, only the EWF pretends to be a Bell type experiment.

In Charlie's bases $\{|\square\rangle, |\circ\rangle\}$ and $\{|+\rangle, |-\rangle\}$ $|\psi\rangle_{AB}$ is

$$\begin{aligned} |\psi\rangle_C &= \sqrt{9/12}\,|\square\rangle|+\rangle + \sqrt{1/12}\,|\square\rangle|-\rangle \\ &\quad -\sqrt{1/12}\,|\bigcirc\rangle|+\rangle + \sqrt{1/12}\,|\bigcirc\rangle|-\rangle \\ &= \sqrt{9/12}\,|\square\rangle|+\rangle + \sqrt{1/12}\,|\square\rangle|-\rangle - \sqrt{2/12}\,|\bigcirc\rangle|1\rangle \end{aligned} \quad (8)$$

in mixed bases $\{|h\rangle, |t\rangle\}$, $\{|0\rangle, |1\rangle\}$; $\{|\square\rangle, |\circ\rangle\}$, $\{|+\rangle, |-\rangle\}$ is

$$\begin{aligned} |\psi\rangle_{AC} &= \sqrt{1/6}\,|h\rangle|+\rangle + \sqrt{1/6}\,|h\rangle|-\rangle + \sqrt{4/6}\,|t\rangle|+\rangle = \\ &= \sqrt{1/3}\,|h\rangle|0\rangle + \sqrt{2/3}\,|t\rangle|+\rangle \end{aligned} \quad (9)$$

$$\begin{aligned} |\psi\rangle_{CB} &= \sqrt{4/6}\,|\square\rangle|0\rangle + \sqrt{1/6}\,|\square\rangle|1\rangle - \sqrt{1/6}\,|\bigcirc\rangle|1\rangle = \\ &= \sqrt{2/3}\,|\square\rangle|0\rangle + \sqrt{1/3}\,|t\rangle|1\rangle \end{aligned} \quad (10)$$

The following simple argumentation used to expose the contradiction of the EWF [6] is similar to the one used in Ref. [12] to illustrate the mystery of the quantum cakes (a simple "real-world" explanation of the proof of quantum mechanical nonlocality without the use of inequalities; I will further call it "quantum-cakes explanation"):

(#1) We know from $|\psi\rangle_C$ that measurements of $|\circ\rangle$ and $|-\rangle$ are possible with $p = 1/12$;
(#2) We know from $|\psi\rangle_{AC}$ that $|-\rangle \Rightarrow |h\rangle$[3];
(#2') We know from $|\psi\rangle_{CB}$ that $|\circ\rangle \Rightarrow |1\rangle$; and
(#3) We know from $|\psi\rangle_{AB}$ that $|h\rangle \Rightarrow |0\rangle$!
(In other words $|\circ-\rangle \Rightarrow |h1\rangle \cap |h\rangle \Rightarrow |0\rangle$).

It shall be appreciated by a skilled technician that state (7) in bases (3), (4) is more symmetric than the oven state and bases used in Ref. [12]. It shall be even more appreciated by a skilled technician that the concept of time is irrelevant in the quantum-cakes explanation.

The EWF contradiction is, however, derived in Table 3 of Ref. [10] on the grounds of the following predictions made by Alice, Bob and Charlie in different times t_0, t_1, t_2 during a happy round of this experiment. Each round goes like this:

(0) Alice and Bob prepare and/or measure their states (1) and (2) according to the procedure described in the outset and Charlie measures $|\psi\rangle_C$, that is to describe the *big quantum state* of both Alice's and Bob's labs (@ t_2);
☹ if Charlie's measurement result is $|\square+\rangle$, $|\square-\rangle$, or $|\circ+\rangle$ the round is completed with no contradiction and a new round begins;
☺ if Charlie's measurement result is $|\circ-\rangle$ then they have a happy round and
(A) Alice knows from (2) that $|t\rangle$ (@t_0) $\Rightarrow$ $|+\rangle$ (@t_2);
(B) Bob knows from (2) that $|1\rangle$ (@t_1) $\Rightarrow$ $|t\rangle$ (@t_0);
(C) Charlie knows from $|\psi\rangle_C$ that $|\circ\rangle$ (@t_2) $\Rightarrow$ $|1\rangle$ (@t_1);

All these four conditions in a happy round of the EWF also contradict each other: if ☺ then $|\circ\rangle \Rightarrow |1\rangle$ (C), $|1\rangle \Rightarrow |t\rangle$ (B) and $|t\rangle \Rightarrow |+\rangle$ (A) (In other words if ☺ then $C \Rightarrow B \cap B \Rightarrow A \cap A \Rightarrow \neg C$).

I will further call this argumentation "superposed-action explanation". It is grounded on the statements (A) and (B) that Alice's and Bob's states (1) and (2) have evolved unitarily to a composite entangled state (7) after Alice handed the second state to Bob.

3. Superposed Action

Let's have a closer look on how the state (7) could possibly be created using the procedure of the EWF under the standard assumptions of Wigner's friend thought experiments. Namely, it is assumed that from Charlie's super-observer perspective, after the state (1) is measured by Alice at time t_0, it becomes

$$|\alpha(t_0)\rangle = \sqrt{1/3}\,|h\rangle|\text{Alice knows } h\rangle + \sqrt{2/3}\,|t\rangle|\text{Alice knows } t\rangle \quad (11)$$

For the next stage of the EWF a usual definition of the

"Freedom of Choice". The choice of measurement settings is statistically independent from the rest of the experiment. (statement 3 in the no-go theorem of Ref. [5]),

seems, however, insufficient. Probability amplitudes are fixed in all the states and bases of the EWF (they don't need to be *chosen* in each round of this thought experiment). Proposed, amended definition is

"Freedom of Choice". Observer's opportunity and autonomy to perform an action selected from at least two available options is statistically independent from the rest of the experiment.

This definition, applies solely to Alice, but not necessarily to Alice, as a human being, but to any device that one would put into the lab in place of Alice to do her task (2), as long as Charlie does not have a control of this device from outside the lab. There is no guarantee that an agent or device inside the lab will act/function as expected. And that makes it impossible to derive arguments that implicitly assume that it operates properly.

Therefore assumption similar to (11) cannot be valid for Bob measuring the state (2) prepared by Alice at t_1

[3] Of course the notation "$|-\rangle \Rightarrow |h\rangle$" means that measurement of $|-\rangle$ implies measurement of $|h\rangle$, etc.

$$\left|\beta(t_1)\right\rangle \neq \sqrt{1/3}\,|0\rangle|\text{Bob knows 0 as Alice knew } h \text{ and did right}\rangle + \sqrt{1/3}\,|0\rangle|\text{Bob knows 0 as Alice knew } t \text{ and did right}\rangle + \sqrt{1/3}\,|1\rangle|\text{Bob knows 1 as Alice knew } t \text{ and did right}\rangle \quad (12)$$

as this would violate Alice's freedom of choice. The absence of observer-independent measurements [4, 5] allows one to discuss superpositions of observer-dependent measurements (11) of timeless quantum states but not to discuss superpositions of observer-dependent time specific actions (12).

An action of Alice is not only relative to Alice [4, 5]. It can be observed by Bob and Charlie.

Therefore, from Charlie's super-observer perspective quantum register containing the first qubit (1) and the second state $|\beta\rangle = |0\rangle$ or $|\beta\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ after it was prepared by Alice would initially contain either two separable pure states $\{|h0\rangle, |t0\rangle\}$ (if Alice measured $|h\rangle$)

$$|\psi\rangle_{h0} = \left(\sqrt{1/3}\,|h\rangle + \sqrt{2/3}\,|t\rangle\right) \otimes |0\rangle = \sqrt{1/3}\,|h0\rangle + \sqrt{2/3}\,|t0\rangle \quad (13)$$

or four separable pure states $\{|h0\rangle, |h1\rangle, |t0\rangle, |t1\rangle\}$ (if Alice measured $|t\rangle$)

$$|\psi\rangle_{t01} = \left(\sqrt{1/3}\,|h\rangle + \sqrt{2/3}\,|t\rangle\right) \otimes \left(\sqrt{1/2}\,|0\rangle + \sqrt{1/2}\,|1\rangle\right) = \sqrt{1/6}\,|h0\rangle + \sqrt{1/6}\,|h1\rangle + \sqrt{2/6}\,|t0\rangle + \sqrt{2/6}\,|t1\rangle \quad (14)$$

In order to affect the unitary evolution on such different quantum registers to bring them both to the entangled state $|\psi\rangle_{AB}$ that would be the same regardless of the initial state $|\psi\rangle_{h0}$ (13) or $|\psi\rangle_{t01}$ (14) Alice must use two different variants of some suitable 4×4 unitary matrices.

If she measures (1) as $|h\rangle$ she may use first the following unitary matrix A_{h0}

$$A_{h0}|\psi\rangle_{h0} = \begin{bmatrix} \sqrt{1/3} & 0 & \sqrt{2/3} & 0 \\ 0 & \sqrt{1/3} & 0 & -\sqrt{2/3} \\ \sqrt{2/3} & 0 & -\sqrt{1/3} & 0 \\ 0 & \sqrt{2/3} & 0 & \sqrt{1/3} \end{bmatrix} \begin{bmatrix} \sqrt{1/3} \\ 0 \\ \sqrt{2/3} \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = |\psi\rangle_{00} \quad (15)$$

while if she measures (1) as $|t\rangle$ she may use first the following unitary matrix A_{t01}

$$A_{t01}|\psi\rangle_{t01} = \begin{bmatrix} \sqrt{1/6} & \sqrt{1/6} & \sqrt{1/3} & \sqrt{1/3} \\ \sqrt{1/3} & \sqrt{1/3} & -\sqrt{1/6} & -\sqrt{1/6} \\ -\sqrt{1/6} & \sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/3} \\ \sqrt{1/3} & -\sqrt{1/3} & \sqrt{1/6} & -\sqrt{1/6} \end{bmatrix} \begin{bmatrix} \sqrt{1/6} \\ \sqrt{1/6} \\ \sqrt{1/3} \\ \sqrt{1/3} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = |\psi\rangle_{00} \quad (16)$$

Then she may use the following unitary matrix R

$$R|\psi\rangle_{00} = \begin{bmatrix} \sqrt{1/3} & 0 & \sqrt{1/2} & \sqrt{1/6} \\ 0 & 1 & 0 & 0 \\ \sqrt{1/3} & 0 & -\sqrt{1/2} & \sqrt{1/6} \\ \sqrt{1/3} & 0 & 0 & -\sqrt{2/3} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{1/3} \\ 0 \\ \sqrt{1/3} \\ \sqrt{1/3} \end{bmatrix} = |\psi\rangle_{AB} \quad (17)$$

to receive $|\psi\rangle_{AB} = R|\psi\rangle_{00}$ from $|\psi\rangle_{00} = A_{h0}|\psi\rangle_{h0}$ or $|\psi\rangle_{00} = A_{t01}|\psi\rangle_{t01}$.

There are obviously infinitely many possibilities of unitary transformations that would bring $|\psi\rangle_{h0}$ or $|\psi\rangle_{t01}$ to $|\psi\rangle_{AB}$, as groups of unitary 4×4 matrices act transitively on the unit vectors in Hilbert spaces over the complex numbers ($\mathbb{C}^4$). The above exemplary forms model the evolution of the EWF, provided they are correctly used.

Indeed, such an approach assumes that in order to arrive at $|\psi\rangle_{AB}$ Alice must after recording the outcome of her measurement of the first qubit to be either $|h\rangle$ or $|t\rangle$ act according to this outcome in a manner, predefined by (2) by applying an appropriate unitary matrix transformation.

But what if Alice makes a mistake and applies a "wrong" transformation RA_{h0} or RA_{t01} to a given input quantum register $|\psi\rangle_{t01}$ or $|\psi\rangle_{h0}$ respectively?

Assume, for example, that there is no agent (no Alice) performing the "actual" measurement of the first qubit (1) in Alice's isolated lab but just some mechanism applying randomly with a probability p matrix transformation RA_{h0} and with the probability $(1 - p)$ matrix transformation RA_{t01} to the initial separable state $|\psi\rangle_{h0}$ or $|\psi\rangle_{t01}$. Table 1 lists the results of such a mechanism operation.

initial state	initial state probability	applied transformation probability	applied transformation and the resultant state	resultant probability
$\lvert\psi\rangle_{h0}$	1/3	p	$RA_{h0}\lvert\psi\rangle_{h0} = \lvert\psi\rangle_{AB}$	$p/3$
		$1 - p$	$RA_{t01}\lvert\psi\rangle_{h0} = \lvert\psi\rangle_{ABth}$	$(1 - p)/3$
$\lvert\psi\rangle_{t01}$	2/3	p	$RA_{h0}\lvert\psi\rangle_{t01} = \lvert\psi\rangle_{ABht}$	$2p/3$
		$1 - p$	$RA_{t01}\lvert\psi\rangle_{t01} = \lvert\psi\rangle_{AB}$	$2(1 - p)/3$

Table 1. Random application of composite matrix transformations RA_{h0} and RA_{t01}

In this scenario state $|\psi\rangle_{AB}$ would be obtained with probability of $p/3 + 2(1 - p)/3 = 2/3 - p/3$ which has maximum of 2/3 for $p = 0$. But the following two[4] different states $|\psi\rangle_{ABth}$ and $|\psi\rangle_{ABht}$ would also be obtained

$$|\psi\rangle_{ABth} = 0.847|h0\rangle + 0.514|t0\rangle - 0.136|t1\rangle \quad (18)$$

$$|\psi\rangle_{ABht} = 0.680|h0\rangle - 0.236|h1\rangle + 0.680|t0\rangle - 0.136|t1\rangle \quad (19)$$

certainly with non-zero probability if one assumes that Alice has the freedom of choice in affecting the unitary evolution of the initial state $|\psi\rangle_{h0}$ (13) or $|\psi\rangle_{t01}$ (14). Alice's freedom of choice implies her fallibility. These states in mixed bases $\{|h\rangle, |t\rangle\}$, $\{|0\rangle, |1\rangle\}$; $\{|\square\rangle, |\circ\rangle\}$, $\{|+\rangle, |-\rangle\}$ are for $|\psi\rangle_{ABth}$

$$|\psi\rangle_{Cth} = 0.612|\square+\rangle + 0.749|\square-\rangle + 0.235|\bigcirc+\rangle + 0.099|\bigcirc-\rangle \quad (20)$$

$$|\psi\rangle_{ACth} = 0.599|h+\rangle + 0.599|h-\rangle + 0.267|t+\rangle + 0.460|t-\rangle \quad (21)$$

$$|\psi\rangle_{CBth} = 0.962|\square 0\rangle + 0.236|\bigcirc 0\rangle - 0.096|\square 1\rangle + 0.096|\bigcirc 1\rangle \quad (22)$$

and for $|\psi\rangle_{ABht}$

$$|\psi\rangle_{Cht} = 0.495|\square+\rangle + 0.866|\square-\rangle - 0.050|\bigcirc+\rangle + 0.050|\bigcirc-\rangle \quad (23)$$

$$|\psi\rangle_{ACht} = 0.315|h+\rangle + 0.648|h-\rangle + 0.385|t+\rangle + 0.577|t-\rangle \quad (24)$$

[4] At least $|\psi\rangle_{ABth}$ for $p = 0$.

$$|\psi\rangle_{CBht} = 0.962|\square 0\rangle - 0.263|\square 1\rangle - 0.070|\bigcirc 1\rangle \tag{25}$$

Quantum-cakes-explanation fails both for $|\psi\rangle_{ABth}$ and $|\psi\rangle_{ABht}$

(#1) We know from $|\psi\rangle_{C*}$ that measurements of $|\circ-\rangle$ are possible; but

(#2) We know from $|\psi\rangle_{AC*}$ that $|-\rangle \nRightarrow |h\rangle$. □

Superposed-action explanation also fails if Alice makes a mistake. Charlie can measure $|\psi\rangle_{C*}$ at time t_2 and observe $|\circ-\rangle$ but Alice's claim based on (2) that $|t\rangle$ measured at time t_0 implies $(|0\rangle + |1\rangle)/\sqrt{2} = |+\rangle$ measured at time t_2 is false merely due to her own mistake. Neither Bob's claim, also based on (2), that $|1\rangle$ measured at time t_1 implies $|t\rangle$ measured at time t_0 is true.

Elementary arithmetic shows that any normalised non-maximally entangled state

$$|\psi\rangle_{AB} = a|h0\rangle + c|t0\rangle + d|t1\rangle \tag{26}$$

measured in the EWF bases is

$$\begin{aligned} |\psi\rangle_{C} = &\frac{a+c+d}{2}|\square+\rangle + \frac{a+c-d}{2}|\square-\rangle \\ &+\frac{a-c-d}{2}|\bigcirc+\rangle + \frac{a-c+d}{2}|\bigcirc-\rangle \end{aligned} \tag{27}$$

$$|\psi\rangle_{AC} = \frac{a}{\sqrt{2}}\left(|h+\rangle + |h-\rangle\right) + \frac{c+d}{\sqrt{2}}|t+\rangle + \frac{c-d}{\sqrt{2}}|t-\rangle \tag{28}$$

$$|\psi\rangle_{CB} = \frac{a+c}{\sqrt{2}}|\square 0\rangle + \frac{a-c}{\sqrt{2}}|\bigcirc 0\rangle + \frac{d}{\sqrt{2}}\left(|\square 1\rangle - |\bigcirc 1\rangle\right) \tag{29}$$

where the normalisation constraint ($\langle\psi|\psi\rangle = 1$) and the surjective isometry constraints: $a - c + d \neq 0$ (27), $c - d = 0$ (28), and $a - c = 0$ (29), imposed to satisfy the conditions (#1)-(#3) of the quantum-cakes explanation, lead to the unique solution of $c = d$, $a = c = d$ with probability amplitude having modulus $|d| = 1/\sqrt{3}$. $|\psi\rangle_{ABth}$ and $|\psi\rangle_{ABht}$ do not belong to this solution.

Therefore the entangled quantum state that Alice delivers to Bob may not be $|\psi\rangle_{AB}$ but $|\psi\rangle_{ABth}$ and $|\psi\rangle_{ABht}$ as well, regardless of the outcome of her measurement of the first qubit (1). Thus the contradiction of the EWE cannot be discussed in isolation from Alice's freedom of choice understood as her fallibility.

4. Conclusion

Authors of the EWF argue to have arrived at the contradiction by *letting* (understood as affecting the unitary evolution of) the second state (2) that Bob receives from Alice to depend on a random value measured and known by Alice [10]. But since there is no unitary transformation that would bring a pure, single qubit state (1) and two versions of a mixed state (2) into an entangled pure two qubit state (7), as discussed above, this argumentation is false.

It is not the incompatibility of the unitary evolution and the measurement involved in the piecewise-defined function (2) that is not self-consistent [18, [1]] (although this incompatibility is the only source of contradictions in Bell type experiments) but this piecewise definition, as such. In this particular 1 out of 12 (on average) rounds of the experiment where $|\circ-\rangle$ is measured by Charlie the if-and-only-ifs in (2) cannot be guaranteed to hold and possible errors of Alice must be accounted for.

Presence of these errors should discourage a casino manager from offering a gambling game based on the principles of the EWF. In a dispute between a gambler and a casino, the judge should rule in favour of the gambler: shifting the responsibility for erroneous operation of this game to the gambler (consumer) appears as an unfair commercial practice.

Fortunately these considerations are academic and no judge will ever need to rule in such a case as the EWF is impossible to be implemented as a game in a casino. Coherent unitary transformations of the *big quantum state* (7) performed from within the lab by Alice herself [10] pursuant to the recipes of (1) and (2), or similar, are impossible.

Much of the essence of quantum theory already makes itself known in the case of just two non-orthogonal states [11]. But in the case of the EWF, the specific type of quantum states (1), (2), (7), measurements, outcomes and actions involved in the argument are relevant and should not be omitted [5]. This is important if one compares the EWF with Bell type experiments, and in particular with those belonging to their subset, which exclude coexistence of observer-independent measurements [4, 5, 17, 3] (Bell-Wigner type experiments [17]). Observer-independent measurements do not exist [4, 5] but this conclusion cannot be derived through the backdoor, at the price of inconsistency of quantum theory. It manifests in collected statistics of measurements of an entangled state but not in the flawed argument of superposed-action.

On the contrary to the EWF, a casino manager should not be discouraged from offering a gambling game based on the principles of the Bell-Wigner type experiments. They are by all means implementable in practice, while errors are relatively small[5]. 6-photon Bell-Wigner type experiment, violated the associated Bell type inequality by 5 standard deviations [17]. But here Alice, Bob and Charlie are photon detectors whose detections are processed by a classical computer to find 6-photon coincidence events. Thus Alice and Bob not only inform Charlie about obtaining a definite measurement results [7] (using heralding signals α' and β') but also reveal these results and yet do not destroy the superposition of the entangled state. As Bell-Wigner type experiments boil down to nonlocal correlations of observer-dependent measurements, which correlations are known at least from Bells' theorem, a judge would be presented with an easy task in any dispute between a gambler and a casino: lack of observer-independent measurements [4, 5] is a known, experimentally proven [17, 3], feature of quantum theory.

Different measurement times could be easily introduced in Bell-Wigner type setups. In the case of photonic implementation physical delays can be employed on particular light guides between a laser and detectors. On the other hand, in a relativistic frame of reference of a photon no time passes between an emission of the photon by a laser and its absorption by a detector, which is otherwise known as time dilation. Quantum state (in particular an entangled one) is, as such, time-independent.

[5] 95-99% polarizing efficiency is typical in quantum optics.

5. Loose Ends

Concepts of physical space, time, velocity, particles [16], position, momentum, etc., used to model perceived nature and express these models and observations in classical terms should be used, at least in explaining Bell type experiments, with extreme caution. Insofar as these concepts are experimentally verifiable (often only to a certain extent, like in the case of position and momentum, for example), they introduce axioms of their own and one runs into trouble trying to reconcile the results of these experiments with these axioms that are intuitively taken for granted. Intuitive axiomatization of time as a continued *progress of existence*, for example, does not hint at the solution of the arrow of time problem.

I think that one should rather take for granted universal validity of quantum theory (assumption Q of Ref. [10]) and try to find loopholes in the other axioms, than the other way round.

Assume, as another example, that physical space exists as three-dimensional extent in which objects and events (this includes time to this definition) have relative position and direction. This assumption is in fact an assumption of

"Locality". An object is directly influenced only by its immediate surroundings[6].

The notion of distance function (metric) d is required to define these immediate surroundings and time is included in the notion of influencing. It is required that a metric d satisfies three metric axioms. The 1st metric axiom

$$d(x,y)=0 \Leftrightarrow x=y \tag{30}$$

is known as the identity of indiscernibles principle, stating, in the context of spacetime, that there cannot be separate points x, y of spacetime that have all their spacetime coordinates in common.

The identity of indiscernibles, however, is not satisfied by the Łukaszyk-Karmowski metric D [14] defining a distance between two random variables or vectors X, Y given by their joint probability distributions for which even $D(X, X) > 0$ in general. This can be extended to square integrable quantum states in spacetime. $D(X_\delta, X_\delta) = 0$ if and only if X_δ is given by joint Dirac delta distributions over spacetime (there is a certain collision of definitions, as event in probability theory is not the same as event in relativity)

$$F_\delta(x,y,z,t)=\delta(x-\mu_x)\delta(y-\mu_y)\delta(z-\mu_z)\delta(t-t_0) \tag{31}$$

where μ_x, μ_y, μ_z are spatial coordinates and t_0 denotes measurement time.

These distributions are independent, since a dimension of a mathematical space is the minimum natural number of independent parameters (coordinates) needed to specify any point within it. Any dependence would introduce non-orthogonality (or a fractal dimension). Therefore X_δ corresponds to a measurement of a location of an object in a space performed by an observer at a given time t_0 (a fact about a location of an object in spacetime) and $D(X_\delta, X_\delta) = 0$ means that this measurement is relative only to this particular observer. Since facts are relative to an observer [4, 5], a different observer will measure different Y_δ, given by

$$G_\delta(x,y,z,t)=\delta(x-\nu_x)\delta(y-\nu_y)\delta(z-\nu_z)\delta(t-t_1) \tag{32}$$

This invalidates the identity of indiscernibles axiom, in the context of physical spacetime. Measurement times (t_0, t_1, etc.), in particular, are relative to an observer, which is known from special relativity.

Born rule is commonly described as saying that probability is equal to the *squared* probability amplitude. This is somehow incorrect in continuous spacetime settings, as in this case the probability of obtaining a measurement outcome of a continuous random variable X (for simplicity just one spatial dimension is considered) is calculated in an interval $[x_a, x_b]$ as Lebesgue-integral of a probability density function f_X of this random variable

$$P(x_a \le X \le x_b)=\int_{x_a}^{x_b} f_X(x)\,dx=\int_{x_a}^{x_b}|\psi(x)|^2\,dx \tag{33}$$

where $\psi(x)$ could be the spatial component of the quantum state of a particle in one-dimensional potential well [13].

Cardinality of a set of real numbers is $2^\wedge\aleph_0$, while the measurement results are eigenvalues λ of an observable A (Hermitian operator) corresponding to the position in this dimension, roots of a characteristic polynomial of this operator. They can be irrational numbers even if the characteristic polynomial has natural coefficients (e.g. x^2+7x+8) but the measurement result is always a definite (rational) number[7]. Cardinality of a set of rational numbers is $\aleph_0$ and by continuum hypothesis (or axiom) there is no set S having cardinality $|S|$ satisfying $\aleph_0 < |S| < 2^\wedge\aleph_0$. The concept of probability is not the same as the concept of probability density function

$$\left|\langle\lambda_i|\psi(x)\rangle\right|^2 \ne \int_{x_a}^{x_b}|\psi(x)|^2\,dx \tag{34}$$

even though Dirac delta distribution over space-time interval would *sift out* the eigenvalue(s) of an observable A in (33).

The identity of indiscernibles is also generally disproven [17] by the Ugly duckling mathematical theorem [19] stating that any two objects insofar as they are distinguishable (they do not have all their spacetime coordinates in common) are equally similar. No classification is possible without some sort of bias.

Observed anomalies in the cosmic microwave background radiation which are aligned with the plane of the Solar System, dubbed the "Axis of Evil" [sic], hint that our intuitive concept of physical space is incorrect. Also

[6] Alternatively and perhaps better suited for the EWF and Bell type experiments: The choice of the measurement settings of one observer has no influence on the outcomes of the other distant observer(s)" (statement 2 in f Ref. [5]).

[7] In Polish "liczba wymierna" ("measurable number").

quantum electrodynamics or sonoluminescence phenomenon, the mechanism of which remains unknown, preclude any naïve understanding of physical space.

There is no Heisenberg cut between quantum and classical description, as there is nothing that can be classically described[8] as *existing*, albeit everything remains to be modelled and "expressed in classical terms" (Niels Bohr, quoted from Ref. [6]). Attempts to resuscitate classicality on the grounds of alleged flaws of quantum description have proven unsuccessful so far. The argument of inconsistency of quantum theory [10] fails on a closer scrutiny. The argument of its incompleteness has ultimately (albeit only after more than 29 years between May 15, 1935 and November 4, 1964) also failed.

The absence of observer-independent measurements implicitly excludes any theory of everything, insofar as one demands that it will fully explain and link together observer-dependent measurements within a single, observer-independent framework [4, 5]. "It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature" (Niels Bohr, quoted from Ref. [4]). We can definitely say that there is no single observer-independently measurable classical world [4, 5]. That implies that the features of the observed nature (in a way a *single world*), such as dimensionality [15] in particular, do not correspond to any of the observer-dependent classical worlds, and that *Erare humanum est*.

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