Low-Resolution ADCs for Two-Hop Massive MIMO Relay System under Rician Channel

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RESEARCH

Low-resolution ADCs for Two-hop Massive MIMO Relay System under Rician Channel

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Abstract

Advantages of the system in this Paper: This paper investigates the uplink of a two-hop massive Multi-input Multi-output (MIMO) relaying system with low-resolution Analog-to-Digital Converters (ADCs) at both relays. Significantly different from previous work, we design a two-hop MIMO system to shorten the distance between each hop, so that there will be Line-of-Sight (LOS) path between relays. This allows signals to be transmitted under Rician channels instead of Rayleigh channels and thus increases the sum achievable rate. Second, we apply low-resolution ADCs at both relays of the system to reduce the energy burden.

The main work of this paper: The main work of this paper is as follows. First, assuming the Channel State Information (CSI) is perfect, we use the higher-order statistics to derive the closed-form expression of the uplink sum achievable rate of the two-hop low-resolution ADCs massive MIMO relay system under one-hop Rayleigh channels and two-hop Rician channels. Next, supposing that the number of antennas tends to infinity, we derive the law of power scaling and further achieve the asymptotic closed-form expressions and the asymptotic values under different power scales. Next, we verify the correctness of theoretical analysis with numerical simulation results, and compare the results under one-hop Rayleigh channels and two-hop Rician channels. Finally, we conclude that transmitting signal under two-hop Rician channels can achieve a lower sum achievable rate compared with transmitting under one-hop Rayleigh channels, which means it's more effective to apply two-hop Rician MIMO system. Besides, we conclude that deploying low-cost and energy-efficient low-resolution ADCs at a large-scale relay system can improve the energy efficiency and achieve a fairly considerable sum achievable rate at the same time.

Keywords: two-hop massive MIMO relaying system; Rician channel; low-resolution ADCs; sum achievable rate; energy efficiency

Introduction

In the field of wireless communication, MIMO systems have been widely used for their superior performance like increasing channel capacity and improving user anti-interference performance [1, 2]. However, MIMO systems also have the following disadvantages. First, when the distances between users and targets are long, the signal cannot reach the targets directly due to the heavy shadow and path loss [3, 4, 5, 6]. Therefore, there is no LOS between users and targets and we can only apply Rayleigh channels to the system. However, its sum achievable rate is lower than the Rician system. Second, when a large number of transmitting and receiving antennas are equipped with high-resolution ADCs, the system will consume

tremendous energy. To be specific, if a high-resolution ADC is with b -bit precision and the sampling frequency is f_s , $f_s * 2^b$ conversions will be required per second, which means the energy consumption of the system increases exponentially with the quantization accuracy [3].

With the development of multi-hop communication, we can decrease the distances between relays so that LOS can appear. Then we can apply Rician channels instead of Rayleigh channels to reduce the sum achievable rate. Besides, using low-resolution ADC instead of high-resolution ADC can alleviate the energy consumption burden of MIMO system and achieve a fairly considerable sum achievable rate at the same time.

In order to reduce the power consumption of ADCs, scholars have made plenty of attempts with the idea of using low-resolution ADCs. [7] applies hybrid ADCs to the relay system and proposes the power scaling law. This law reveals that the transmission power can be reduced inversely proportional to the number of relay antennas, and an effective power allocation scheme is further proposed based on this law. [8] studies the one-bit low-resolution ADCs relay system for MIMO system, which is a special case of low-resolution ADCs, and proves one-bit low-resolution ADCs is effective for reducing energy consumption. However, no paper has applied low-resolution ADCs to two-hop Rician channel and study its superiority of sum achievable rate over one-hop Rayleigh channel, which is more widely applied to MIMO systems.

This paper considers the uplink of two-hop low-resolution ADCs massive MIMO relaying system over the Rician channel. Firstly, we derive the closed-form expression of the sum achievable rate of the uplink of two-hop low-resolution ADCs massive MIMO relay system over two-hop Rician channel and one-hop Rayleigh channel based on the higher-order statistics of perfect CSI. Secondly, we derive the asymptotic closed-form expressions when the number of antennas tends to infinity. Next, we further achieve the law of power scaling and asymptotic values under different power scales, and conclude that the transmission power scaled down inversely proportional to the number of antennas at the relay. Finally, we compare the sum achievable rates of the two-hop Rician system and one-hop Rayleigh system, and verify the correctness of theoretical analysis with numerical simulation results.

Notation: The superscripts $(\cdot)^T$, $(\cdot)^H$, $tr(\cdot)$ and $diag(\cdot)$ represent the transpose, Hermitian transpose, trace of the matrix, and diagonal matrix, respectively. $\|\cdot\|$ represents the Euclidean norm. $CN(*, \sigma^2)$ represents the complex Gaussian distribution with the mean of $*$ and the variance of σ^2 . $E[\cdot]$ represents the expectation. I_N denotes an $N \times N$ identity matrix. $\mathbf{X}_{ij}$ or $[\mathbf{X}]_{ij}$ represents the $(i, j)_{th}$ entry of $\mathbf{X}$.

Methods/Experimental

Aim of Our Study

The aim of our study is to design a massive MIMO system with a higher sum achievable rate and higher energy efficiency. In order to achieve a higher sum achievable rate, we design a two-hop low-resolution ADCs massive MIMO relaying system over the Rician channel. Usually, signals are transmitted over Rayleigh channels in massive MIMO systems due to the heavy shadow and path loss generated by long

transmitting distances. However, converting the system from one-hop to two-hop can decrease signal transmission distances and create LOS between relays. As a result, we can apply Rician channels to the massive MIMO system and increase the sum achievable rate.

In order to increase the energy efficiency, we substitute the high-resolution ADCs with low-resolution ADCs. This method enables us to improve the energy efficiency and achieve a fairly considerable sum achievable rate at the same time.

Processes of Our Study

The specific process of our study is as follows

Firstly, we derive the closed-form expression of the sum achievable rate of the uplink of two-hop low-resolution ADCs massive MIMO relay system over two-hop Rician channel and one-hop Rayleigh channel based on the higher-order statistics of perfect CSI.

Secondly, we derive the asymptotic closed-form expressions of the two systems when the number of antennas tends to infinity.

Thirdly, we further achieve the law of power scaling and asymptotic values under different power scales, and conclude that the transmission power scaled down inversely proportional to the number of antennas at the relay.

Finally, we compare the sum achievable rates of the two-hop Rician system and one-hop Rayleigh system, and verify the correctness of theoretical analysis with numerical simulation results.

System Model and Signal Processing

One-hop Rayleigh Channel

System Model

In Figure 1, we use $\mathbf{G}_{R1} \in \mathbb{C}^{N_{R1} \times K}$ to denote the MIMO channel matrix. $\mathbf{G}_{R1}$ can be represented as

$$\mathbf{G}_{R1} = \mathbf{H}_{R1} \mathbf{D}_{R1}^{1/2} \quad (1)$$

where $\mathbf{D}_{R1}$ is a K -order diagonal matrix representing the large-scale fading between K users and the K randomly selected antennas in relay R_1 , and $[\mathbf{D}_{R1}]_{KK} = \alpha_k$. $\mathbf{H}_{R1} \in \mathbb{C}^{N_{R1} \times K}$ denotes the fading matrix of the Rayleigh channels.

Signal processing

Assume that the signal transmitted by K single antenna users is $\mathbf{x}_S \in [x_1, x_2, \dots, x_K]^T$, where $E[\mathbf{x}_S \mathbf{x}_S^T] = \mathbf{I}_K$. After one time slot, the signal received by relay R_1 can be expressed as

$$\mathbf{y}_{R1} = \sqrt{P_u} \mathbf{G}_{R1} \mathbf{x}_S + \mathbf{n}_{R1} \quad (2)$$

where P_u is the transmission power of each user, $\mathbf{n}_{R1}$ is the i.i.d complex Gaussian white noise at relay R_1 , $\mathbf{n}_{R1} \sim CN(0, \sigma_{R1}^2)$.

$\mathbf{y}_{R1}$ is then quantized by low-resolution ADCs at R_1 . Based on Additive Quantization Noise Model (AQNM), the quantized signal can be represented as

$$\tilde{\mathbf{y}}_{R1} = Q[\mathbf{y}_{R1}] = k\mathbf{y}_{R1} + \tilde{\mathbf{n}}_{R1} \quad (3)$$

where $\tilde{\mathbf{n}}_{R1}$ denotes the additive quantization noise vector and is independent from the received signal $\mathbf{y}_{R1}$, k denotes the linear quantization gain. According to [7, 9, 10], k satisfies the following equation $k = 1 - \rho$, where ρ represents the quantization distortion factor and equals the ratio of the quantizer error variance over received signal variance. For relay R_1 , $\rho = \frac{E[|\mathbf{y}_{R1} - \tilde{\mathbf{y}}_{R1}|^2]}{E[|\mathbf{y}_{R1}|^2]}$. When the number of quantization bits $q \leq 5$, the values of ρ is shown in Table 1. When $q > 5$, $\rho \approx \frac{\pi\sqrt{3}}{2}$.

According to [11], the covariance matrix of the quantization noise can be expressed as

$$\mathbf{R}_{\tilde{\mathbf{n}}_{R1}} = k\rho \text{diag}(P_u \mathbf{G}_{R1} \mathbf{G}_{R1}^H + \sigma_{R1}^2 \mathbf{I}_{N_{R1}}) \quad (4)$$

Because Maximum Ratio Combining (MRC) has low-complexity and is able to achieve the optimal reception performance, we use MRC to linear process the quantized signal $\tilde{\mathbf{y}}_{R1}$, where the MRC matrix $\mathbf{W}_{R1}^H = \mathbf{G}_{R1}^H$. Therefore, the processed signal $\mathbf{x}_{R1}$ can be written as

$$\mathbf{x}_{R1} = \mathbf{W}_{R1}^H \tilde{\mathbf{y}}_{R1} = k\sqrt{P_u} \mathbf{G}_{R1}^H \mathbf{G}_{R1} \mathbf{x}_S + k\mathbf{G}_{R1}^H \mathbf{n}_{R1} + \mathbf{G}_{R1}^H \tilde{\mathbf{n}}_{R1} \quad (5)$$

Noticing that the signal of the k_{th} user and the other users in (5) are uncorrelated, the received signal of the the k_{th} user at relay R_1 can be written as

$$\mathbf{x}_{R1,k} = \underbrace{k\sqrt{P_u} \mathbf{g}_{R1,k}^H \mathbf{G}_{R1} \mathbf{x}_{S,k}}_{\text{desired signal}} + \underbrace{k\sqrt{P_u} \sum_{j \neq k}^K \mathbf{g}_{R1,j}^H \mathbf{G}_{R1} \mathbf{x}_{S,j}}_{\text{interference}} + \underbrace{k\mathbf{g}_{R1,k}^H \mathbf{n}_{R1} + \mathbf{g}_{R1,k}^H \tilde{\mathbf{n}}_{R1}}_{\text{noise}} \quad (6)$$

System Achievable Rate Analysis

In this section, supposing that we have perfect CSI, we will derive a closed-form expression for the achievable rate of the one-hop low-precision ADCs MIMO relay system over Rayleigh channels. When the number of antennas tends to infinity, we will achieve power scaling laws and asymptotic system achievable rates under different power scales.

Closed-form Expression for the Achievable Rate Supposing that the CSI is perfect, based on Shannon Entropy and according to (6), we can get the rate of the k_{th} user in one-hop low-precision ADCs MIMO relay system over Rayleigh channels as

$$R_k^{Rayleigh} = \frac{1}{2} E[\log_2(1 + \frac{P_k^{Rayleigh}}{N_k^{Rayleigh}})] \quad (7)$$

where $P_k^{Rayleigh}$ represents the power of desired signal of the k_{th} user, $P_k^{Rayleigh} = k^2 P_u |\mathbf{g}_{R1,k}^H \mathbf{G}_{R1}|^2$. $N_k^{Rayleigh}$ represents the power of interference signal and the power of noise of the k_{th} user, $N_k^{Rayleigh} = k^2 P_u \sum_{j \neq k}^K |\mathbf{g}_{R1,j}^H \mathbf{G}_{R1}|^2 + k^2 \sigma_{R1}^2 |\mathbf{g}_{R1,k}^H|^2 + |\mathbf{g}_{R1,k}^H \mathbf{R}_{\tilde{\mathbf{n}}_{R1}} \mathbf{g}_{R1,k}|$.

According to [12], the rate of k_{th} user can also be denoted as

$$R_k^{Rayleigh} = \frac{1}{2} \log_2(1 + SNR_k) \quad (8)$$

where SNR_k represents the Signal-to-noise Ratio (SNR) of the k_{th} user at the receiving end R_1 .

$$SNR_k = \frac{P_k^{Rayleigh'}}{N_k^{Rayleigh'}} \quad (9)$$

Based of (8) and (9), we can derive the closed-form expression for the achievable rate of the k_{th} user in the one-hop low-precision ADCs MIMO relay system over Rayleigh channels is

$$R_k^{Rayleigh} = \frac{1}{2} \log_2\left(1 + \frac{P_k^{Rayleigh'}}{N_k^{Rayleigh'}}\right) \quad (10)$$

In formula (10), $P_k^{Rayleigh'}$ and $N_k^{Rayleigh'}$ can be represented as follows, the proof is attached in *Appendix 1*.

$$\begin{aligned} P_k^{Rayleigh'} &= k^2 P_u E[|\mathbf{g}_{R1,k}^H \mathbf{G}_{R1}|^2] \\ &= k^2 P_u \alpha_k^2 N_{R1} (N_{R1} + 1) \end{aligned} \quad (11)$$

$$\begin{aligned} N_k^{Rayleigh'} &= k^2 P_u \sum_{j \neq k}^K E[|\mathbf{g}_{R1,j}^H \mathbf{G}_{R1}|^2] \\ &\quad + k^2 \sigma_{R1}^2 E[|\mathbf{g}_{R1,k}^H|^2] + E[|\mathbf{g}_{R1,k}^H \mathbf{R}_{\tilde{\mathbf{n}}_{R1}} \mathbf{g}_{R1,k}|] \\ &= k^2 P_u N_{R1} \sum_{j \neq k}^K \alpha_k \alpha_j + k^2 \sigma_{R1}^2 N_{R1} \alpha_k \\ &\quad + k \rho \alpha_n N_{R1} (P_u \sum_{i=1}^K \alpha_i + P_u \alpha_n + \sigma_{R1}^2) \end{aligned} \quad (12)$$

Power Scaling Laws and Asymptotic Analysis Based on the closed-form expression over Rayleigh channel given by (10), we further analyze their performances and derive the law of energy scaling in different conditions.

Suppose that the transmit power at the user end is $P_u = \frac{E_u}{N_{R1}^a}$, a is the power scaling constant. When the number of antennas tends to infinity, the limit of SNR_k in (9) can be represented as

$$\lim_{N_{R1} \rightarrow \infty} SNR_k = \lim_{N_{R1} \rightarrow \infty} \frac{\alpha_k k^2 E_u N_{R1}^{2-a}}{k^2 \sigma_{R1}^2 N_{R1} + k \rho \sigma_{R1}^2 N_{R1}} = \lim_{N_{R1} \rightarrow \infty} \frac{\alpha_k k E_u N_{R1}^{1-a}}{\sigma_{R1}^2} \quad (13)$$

When a takes different values, we can obtain the following power scaling law

$$\lim_{N_{R1} \rightarrow \infty} SNR_k = \begin{cases} \infty & a < 1 \\ \frac{\alpha_k k E_u}{\sigma_{R1}^2} & a = 1 \\ 0 & a \geq 1 \end{cases} \quad (14)$$

As can be seen from (14), when N_{R1} tends to infinity, the transmit power at the user end can be scaled down by $\frac{1}{N_{R1}^a}$. When the scale index a satisfies $a = 1$, the achievable rate remains stable. Based on (9), (10) and (14), when the perfect CSI exists from users to R_1 , we assume that $N_{R1} \gg K \gg 1$, the large-scale fading between users and R_1 satisfies $\alpha_1 = \alpha_2 = \dots = \alpha_k = \alpha$, the user transmit power $P_u \gg \sigma_{R1}^2$, SNR_k can be approximated as

$$SNR_k \approx \frac{k_1 N_{R1}}{K} \quad (15)$$

The proof is attached in *Appendix 2*.

The uplink sum achievable rate can be approximated as

$$R_{sum}^{Rayleigh} = \frac{K}{2} \log_2 \left(1 + \frac{k_1 N_{R1}}{K} \right) \quad (16)$$

Two-hop Rician Channel

In order to increase the sum achievable rate, we convert the one-hop MIMO system to two-hop MIMO system, so that the distance between users and relays can be reduced. As a result, LOS will appear between users and relays and Rician channels can be applied to the MIMO system to increase the system achievable rate.

System Model

In Figure 2, we use $\mathbf{G}_{R1} \in \mathbb{C}^{N_{R1} \times K}$ to denote the MIMO channel matrix between users, $\mathbf{G}_{R2} \in \mathbb{C}^{N_{R2} \times K}$ to denote the MIMO channel matrix between R_1 and R_2 . $\mathbf{G}_{R1}$ can be represented as

$$\mathbf{G}_{R1} = \mathbf{H}_{R1} \mathbf{D}_{R1}^{1/2} \quad (1)$$

$\mathbf{G}_{R2}$ can be represented as

$$\mathbf{G}_{R2} = \mathbf{H}_{R2} \mathbf{D}_{R2}^{1/2} \quad (17)$$

where $\mathbf{D}_{R1}$ is K -order diagonal matrices representing the large-scale fading between K users and the K randomly selected antennas in relay R_1 , and $[\mathbf{D}_{R1}]_{KK} = \alpha_k$. $\mathbf{D}_{R2}$ is K -order diagonal matrices representing the large-scale fading between K users and the K randomly selected antennas in relay R_2 , and $[\mathbf{D}_{R2}]_{KK} = \beta_k$. $\mathbf{H}_{R1} \in \mathbb{C}^{N_{R1} \times K}$ and $\mathbf{H}_{R2} \in \mathbb{C}^{N_{R2} \times K}$ denote the fading matrix of Rician channels.

Signal processing

Same as the previous chapter, we can get the quantized signal $\tilde{\mathbf{y}}_{R1}$ at R_1 as

$$\tilde{\mathbf{y}}_{R1} = Q[\mathbf{y}_{R1}] = k\mathbf{y}_{R1} + \tilde{\mathbf{n}}_{R1} \quad (3)$$

From (4), we can get the covariance matrix of the quantization noise $\mathbf{R}_{\tilde{\mathbf{n}}_{R1}}$ at R_1 as

$$\mathbf{R}_{\tilde{\mathbf{n}}_{R1}} = k_1 \rho_1 \text{diag}(P_u \mathbf{G}_{R1} \mathbf{G}_{R1}^H + \sigma_{R1}^2 \mathbf{I}_{N_{R1}}) \quad (18)$$

where k_1 denotes the linear quantization gain at R_1 , ρ_1 denotes the quantization distortion factor at R_1 .

The processed signal $\mathbf{x}_{R1}$ can be expressed as

$$\mathbf{x}_{R1} = \mathbf{W}_{R1}^H \tilde{\mathbf{y}}_{R1} = k\sqrt{P_u} \mathbf{G}_{R1}^H \mathbf{G}_{R1} \mathbf{x}_S + k\mathbf{G}_{R1}^H \mathbf{n}_{R1} + \mathbf{G}_{R1}^H \tilde{\mathbf{n}}_{R1} \quad (5)$$

Then, we apply the technique of Amplify-and-Forward (AF) to signal $\mathbf{x}_{R1}$ and transmit the processed signals to R_2 with K randomly selected antennas. The signal $\mathbf{y}_{R2}$ received at relay R_2 can be denoted as

$$\mathbf{y}_{R2} = \gamma \mathbf{G}_{R2} \mathbf{x}_{R1} + \mathbf{n}_{R2} \quad (19)$$

where $\mathbf{G}_{R2}$ is the Rayleigh fading channel between relay R_1 and R_2 , $\mathbf{n}_{R2}$ is the i.i.d Gaussian white noise at relay R_2 , $\mathbf{n}_{R2} \sim CN(0, \sigma_{R2}^2)$. γ is an amplification factor at relay R_1 , which satisfies the power constraint $E[\|\gamma \mathbf{x}_{R1}\|^2] = P_R$. Therefore, γ can be expressed as

$$\gamma = \sqrt{\frac{P_R}{E[\|\mathbf{x}_{R1}\|^2]}} \quad (20)$$

where P_R represents the transmit power at relay R_1 ,

$$E[\|\mathbf{x}_{R1}\|^2] = k_1^2 [p_u \text{tr}(E[\mathbf{G}_{R1}^H \mathbf{G}_{R1} \mathbf{G}_{R1}^H \mathbf{G}_{R1}]) + \sigma_{R1}^2 \text{tr}(E[\mathbf{G}_{R1}^H \mathbf{G}_{R1}])] + \text{tr}(E[\mathbf{G}_{R1}^H \mathbf{R}_{\tilde{\mathbf{n}}_{R1}} \mathbf{G}_{R1}]).$$

To simplify the expression, we make the following definition.

$$\Delta_{R1,k} = \frac{2\mu_k + 1}{(\mu_k + 1)^2} \Phi_{R1,ki} = \frac{\sin(N_{R1}\pi(\sin\theta_{R1} - \sin\theta_{R1,i})/2)}{\sin(\pi(\sin\theta_{R1,k} - \sin\theta_{R1,i})/2)} \mathbf{Q}_{ki} = \frac{\frac{\mu_k \mu_i \Phi_{ki}^2}{N_{R1}} + \mu_k + \mu_i + 1}{(\mu_k + 1)(\mu_i + 1)}$$

$$\Delta_{R2,k} = \frac{2\varepsilon_k + 1}{(\varepsilon_k + 1)^2} \Phi_{R2,ki} = \frac{\sin(N_{R2}\pi(\sin\theta_{R2} - \sin\theta_{R2,i})/2)}{\sin(\pi(\sin\theta_{R2,k} - \sin\theta_{R2,i})/2)} \mathbf{R}_{ki} = \frac{\frac{\varepsilon_k \varepsilon_i \Phi_{ki}^2}{N_{R2}} + \varepsilon_k + \varepsilon_i + 1}{(\varepsilon_k + 1)(\varepsilon_i + 1)}$$

Therefore, γ can be expressed as follows, the proof is attached in *Appendix 3*.

$$\begin{aligned}\gamma &= \sqrt{\frac{P_R}{k_1^2(P_u S_1 + N_{R1} \sigma_{R1}^2 \sum_{i=1}^K \alpha_i) + k_1 \rho_1 S_2}} \\ S_1 &= N_{R1} \sum_{i=1}^K \alpha_i^2 (N_{R1} + \Delta_{R1,i}) + N_{R1} \sum_{i=1}^K \alpha_i \sum_{l=1}^K \alpha_l \mathbf{Q}_{il} \\ S_2 &= p_u N_{R1} \sum_{n=1}^K \alpha_n (\alpha_n + \sum_{i=1}^K \alpha_i) + N_{R1} \sigma_{R1}^2 \sum_{n=1}^K \alpha_n\end{aligned}\quad (21)$$

Similar to the quantization at relay R_1 , the quantized signal $\tilde{\mathbf{y}}_{R2}$ at relay R_2 can be modeled as

$$\tilde{\mathbf{y}}_{R2} = Q[\mathbf{y}_{R2}] = k_1 \mathbf{y}_{R2} + \tilde{\mathbf{n}}_{R2} \quad (22)$$

The covariance matrix of the quantization noise $\tilde{\mathbf{n}}_{R1}$ can be written as

$$\mathbf{R}_{\tilde{\mathbf{n}}_{R2}} = k_2 \rho_2 \text{diag}(\gamma^2 \mathbf{R}_{y_{R2}} + \sigma_{R2}^2 \mathbf{I}_{N_{R2}}) \quad (23)$$

where k_2 denotes the linear quantization gain at R_2 , ρ_2 denotes the quantization distortion factor at R_2 , $\mathbf{R}_{y_{R2}} = \mathbf{G}_{R2} \mathbf{G}_{R1}^H \mathbf{R}_{y_{R1}} \mathbf{G}_{R1} \mathbf{G}_{R2}^H$, $\mathbf{R}_{\tilde{\mathbf{y}}_{R2}} = k_1^2 (P_u \mathbf{G}_{R1} \mathbf{G}_{R1}^H + \sigma_{R1}^2 \mathbf{I}_{N_{R1}}) + k_1 \rho_1 \text{diag}(P_u \mathbf{G}_{R1} \mathbf{G}_{R1}^H + \sigma_{R1}^2 \mathbf{I}_{N_{R1}})$.

Same as MRC processing at relay R_1 , we also use MRC to process signals at R_2 , where the MRC matrix $\mathbf{W}_{R2}^H = \mathbf{G}_{R2}^H$. Therefore, the processed signal $\mathbf{x}_{R2}$ can be written as

$$\begin{aligned}\mathbf{x}_{R2} &= \mathbf{W}_{R2}^H \tilde{\mathbf{y}}_{R2} = \gamma k_1 k_2 \sqrt{P_u} \mathbf{G}_{R2}^H \mathbf{G}_{R2} \mathbf{G}_{R1}^H \mathbf{G}_{R1} \mathbf{x}_S + \gamma k_1 k_2 \mathbf{G}_{R2}^H \mathbf{G}_{R2} \mathbf{G}_{R1}^H \mathbf{n}_{R1} \\ &\quad + \gamma k_2 \mathbf{G}_{R2}^H \mathbf{G}_{R2} \mathbf{G}_{R1}^H \tilde{\mathbf{n}}_{R1} + k_2 \mathbf{G}_{R2}^H \mathbf{n}_{R2} + \mathbf{G}_{R2}^H \tilde{\mathbf{n}}_{R1}\end{aligned}\quad (24)$$

Noticing that the signal of the k_{th} user and the other users in (24) are uncorrelated, the received signal of the the k_{th} user at relay R_2 can be written as

$$\begin{aligned}\mathbf{x}_{R2,k} &= \underbrace{\gamma k_1 k_2 \sqrt{P_u} \mathbf{g}_{R2,k}^H \mathbf{G}_{R2} \mathbf{G}_{R1}^H \mathbf{g}_{R1,k} \mathbf{x}_{S,k}}_{\text{desired signal}} + \underbrace{\gamma k_1 k_2 \sqrt{P_u} \sum_{j \neq k}^K \mathbf{g}_{R2,k}^H \mathbf{G}_{R2} \mathbf{G}_{R1}^H \mathbf{g}_{R1,j} \mathbf{x}_{S,j}}_{\text{interference}} \\ &\quad + \underbrace{\gamma k_1 k_2 \mathbf{g}_{R2,k}^H \mathbf{G}_{R2} \mathbf{G}_{R1}^H \mathbf{n}_{R1} + \gamma k_2 \mathbf{g}_{R2,k}^H \mathbf{G}_{R2} \mathbf{G}_{R1}^H \tilde{\mathbf{n}}_{R1} + k_2 \mathbf{g}_{R2,k}^H \mathbf{n}_{R2} + \mathbf{g}_{R2,k}^H \tilde{\mathbf{n}}_{R1}}_{\text{noise}}\end{aligned}\quad (25)$$

System Achievable Rate Analysis

In this section, supposing that we have perfect CSI, we will derive a closed-form expression for the achievable rate of the two-hop low-precision ADCs MIMO relay

system over Rician channels. When the number of antennas tends to infinity, we will achieve power scaling laws and asymptotic system achievable rates under different power scales.

Closed-form Expression for the Achievable Rate Similar to (7), the achievable rate of the k_{th} user in two-hop low-precision ADCs MIMO relay system over Rician channels can be represented as

$$R_k^{Rician} = \frac{1}{2} E[\log_2(1 + \frac{P_k^{Rician}}{N_k^{Rician}})] \quad (26)$$

where P_k^{Rician} and represents the power of desired signal of the k_{th} user, and N_k^{Rician} represents the power of interference signal and the power of noise of the k_{th} user.

The detailed formula of P_k^{Rician} and N_k^{Rician} can also be represented as

$$P_k^{Rician} = \gamma^2 k_1^2 k_2^2 P_u |\mathbf{g}_{R2,k}^H \mathbf{G}_{R2} \mathbf{G}_{R1}^H \mathbf{g}_{R1,k}|^2 \quad (27)$$

$$\begin{aligned} N_k^{Rician} = & \gamma^2 k_1^2 k_2^2 P_u \sum_{j \neq k}^K |\mathbf{g}_{R2,k}^H \mathbf{G}_{R2} \mathbf{G}_{R1}^H \mathbf{g}_{R1,j}|^2 + \gamma^2 k_1^2 k_2^2 \sigma_{R1}^2 |\mathbf{g}_{R2,k}^H \mathbf{G}_{R2} \mathbf{G}_{R1}^H|^2 \\ & + \gamma^2 k_2^2 |\mathbf{g}_{R2,k}^H \mathbf{G}_{R2} \mathbf{G}_{R1}^H \mathbf{R}_{\tilde{\mathbf{n}}_{R1}} \mathbf{G}_{R1} \mathbf{G}_{R2}^T \mathbf{g}_{R2,k}|^2 + k_2^2 \sigma_{R2}^2 |\mathbf{g}_{R2,k}|^2 \\ & + |\mathbf{g}_{R2,k}^H \mathbf{R}_{\tilde{\mathbf{n}}_{R2}} \mathbf{g}_{R2,k}|^2 \end{aligned} \quad (28)$$

Similar to (8), the rate of k_{th} user can be denoted as

$$R_k^{Rician} = \frac{1}{2} \log_2(1 + SNR_k) \quad (29)$$

where SNR_k represents the SNR of the k_{th} user at the receiving end R_2 .

$$SNR_k = \frac{P_k^{Rician'}}{N_k^{Rician'}} \quad (30)$$

Based of (29) and (30), we can derive the closed-form expression for the achievable rate of the two-hop low-precision ADCs MIMO relay system over Rician channels is

$$R_k^{Rician} = \frac{1}{2} \log_2(1 + \frac{P_k^{Rician'}}{N_k^{Rician'}}) \quad (31)$$

In formula (31), $P_k^{Rayleigh'}$ and $N_k^{Rayleigh'}$ can be represented as follows, the proof is attached in *Appendix 4*.

$$\begin{aligned}
P_k^{Rician'} &= \gamma^2 k_1^2 k_2^2 P_u E[|\mathbf{g}_{R2,k}^H \mathbf{G}_{R2} \mathbf{G}_{R1}^H \mathbf{g}_{R1,k}|^2] \\
&= \gamma^2 k_1^2 k_2^2 P_u \alpha_k \beta_k N_{R1} N_{R2} [\alpha_k \beta_k (N_{R1} + \Delta_{R1,k})(N_{R2} + \Delta_{R2,k}) + \sum_{i \neq k}^K \alpha_i \beta_i \mathbf{Q}_{ki} \mathbf{R}_{ki}]
\end{aligned} \tag{32}$$

$$\begin{aligned}
N_k^{Rician'} &= A_k^{Rician} + B_k^{Rician} + C_k^{Rician} + D_k^{Rician} + E_k^{Rician} \\
A_k^{Rician} &= \gamma^2 k_1^2 k_2^2 P_u \sum_{j \neq k}^K E[|\mathbf{g}_{R2,k}^H \mathbf{G}_{R2} \mathbf{G}_{R1}^H \mathbf{g}_{R1,j}|^2] \\
&= \gamma^2 k_1^2 k_2^2 P_u \beta_k N_{R1} N_{R2} \sum_{j \neq k}^K \alpha_j [\alpha_k \beta_k \mathbf{Q}_{kj} (N_{R2} + \Delta_{R2,k}) \\
&\quad + \alpha_j \beta_j \mathbf{R}_{kj} (N_{R1} + \Delta_{R1,j}) + \sum_{i \neq k}^K \alpha_i \beta_i \mathbf{R}_{ki} \mathbf{Q}_{ij}] \\
B_k^{Rician} &= \gamma^2 k_1^2 k_2^2 \sigma_{R1}^2 E[|\mathbf{g}_{R2,k}^H \mathbf{G}_{R2} \mathbf{G}_{R1}^H|^2] \\
&= \gamma^2 k_1^2 k_2^2 \sigma_{R1}^2 \beta_k N_{R1} N_{R2} [\alpha_k \beta_k (N_{R2} + \Delta_{R2,k}) + \sum_{i \neq k}^K \alpha_i \beta_i \mathbf{R}_{ki}] \\
C_k^{Rician} &= \gamma^2 k_2^2 E[\mathbf{g}_{R2,k}^H \mathbf{G}_{R2} \mathbf{G}_{R1}^H \mathbf{R}_{\tilde{n}R1} \mathbf{G}_{R1} \mathbf{G}_{R2}^H \mathbf{g}_{R2,k}] \\
&= \gamma^2 k_1 \rho_1 k_2^2 \beta_k N_{R1} N_{R2} [P_u ((N_{R2} + \Delta_{R2,k}) \alpha_k \beta_k (\alpha_k + \sum_{l \neq i}^K \alpha_l) \\
&\quad + \sum_{i \neq k}^K \alpha_i \beta_i \mathbf{R}_{ki} (\alpha_i + \sum_{l \neq i}^K \alpha_l)) + \sigma_{R1}^2 (\alpha_k \beta_k (N_{R2} + \Delta_{R2,k}) + \sum_{i \neq k}^K \alpha_i \beta_i \mathbf{R}_{ki})] \\
D_k^{Rician} &= k_2^2 \sigma_{R2}^2 E[|\mathbf{g}_{R2,k}|^2] = k_2^2 \sigma_{R2}^2 N_{R2} \beta_k \\
E_k^{Rician} &= E[\mathbf{g}_{R2,k}^H \mathbf{R}_{\tilde{n}R1} \mathbf{g}_{R2,k}] \\
&= \gamma^2 k_1 k_2 \rho_2 N_{R1} N_{R2} \beta_k (\alpha_k \beta_k \Delta_{R2,k} (P_u \sum_{i=1}^K \alpha_i + k_1 N_{R1} P_u \alpha_k + \rho_1 P_u \alpha_k + \sigma_{R1}^2) \\
&\quad + \sum_{n=1}^K \alpha_n \beta_n (p_u \sum_{i=1}^K \alpha_i + k_1 N_{R1} P_u \alpha_n + \rho_1 p_u \alpha_n + \sigma_{R1}^2)) + k_2 \rho_2 \sigma_{R2}^2 N_{R2} \beta_k
\end{aligned} \tag{33}$$

Power Scaling Laws and Asymptotic Analysis Based on the closed-form expressions over Rician channel given by (30) and (31), we further analyze their performances and derive the law of energy scaling in different conditions.

Suppose that the transmit power at the user end is $P_u = \frac{E_u}{N_{R1}^a}$, the transmit power at relay R_1 is $P_R = \frac{E_u}{N_{R2}^b}$, a and b are power scaling constants, $\lambda = \frac{N_{R2}}{N_{R1}} < \infty$. When the N_{R1} and N_{R2} tend to infinity, the limit of SNR_k in (30) can be represented as

$$\begin{aligned}
\lim_{N_{R1} \rightarrow \infty} SNR_k &= \lim_{N_{R1} \rightarrow \infty} \frac{k_1^2 k_2 \gamma^2 P_u N_{R1}^2 N_{R2} \alpha_k^2 \beta_k}{k_1 k_2 \gamma^2 N_{R1} N_{R2} \alpha_k \beta_k \sigma_{R1}^2 + \sigma_{R2}^2} \\
&= \lim_{N_{R1} \rightarrow \infty} \frac{k_1^2 k_2 \lambda E_u \alpha_k N_{R1}^{1-a}}{k_1 k_2 \lambda \sigma_{R1}^2 + \frac{\sigma_{R2}^2}{\gamma^2 \alpha_k \beta_k N_{R1}^2}}
\end{aligned} \tag{34}$$

When a and b take different values, we can obtain the following power scaling law, the proof is attached in *Appendix 5*.

$$\lim_{N_{R1} \rightarrow \infty} SNR_k = \begin{cases} \infty & a, b < 1 \\ \frac{k_2 E_R \alpha_k^2 \beta_k}{\sigma_{R2}^2 \sum_{i=1}^K \alpha_i^2} & a < b = 1 \\ \frac{k_1 E_u \alpha_k}{\sigma_{R1}^2} & b < a = 1 \\ \frac{k_1 k_2 E_u E_R \alpha_k^2 \beta_k}{\tau} & a = b = 1 \\ 0 & a > 1 \text{ or } b > 1 \end{cases} \quad (35)$$

where $\tau = k_2 E_R \alpha_k \beta_k \sigma_{R1}^2 + \sigma_{R2}^2 (k_1 E_u \sum_{i=1}^K \alpha_i^2 + \sigma_{R1}^2 \sum_{i=1}^K \alpha_i)$. As can be seen from (35), when N_{R1} tends to infinity, the transmit power at the user end can be scaled down by $\frac{1}{N_{R1}^a}$ and $\frac{1}{N_{R2}^b}$. When the scale index a and b satisfy $a < b = 1$, $b < a = 1$ or $a = b = 1$, the achievable rate remains stable.

Based on (35) and according to [13], when the perfect CSI exists from users to R_1 and from R_1 to R_2 , we assume that $N_{R2} > N_{R1} \gg K \gg 1$, the large-scale fading between users and $R1$ satisfies $\alpha_1 = \alpha_2 = \dots = \alpha_k = \alpha$, the fading between R_1 and R_2 satisfies $\beta_1 = \beta_2 = \dots = \beta_k = \beta$, the user transmit power $p_u \gg \sigma_{R1}^2$, R_1 transmit power $p_R \gg \sigma_{R2}^2$, $\lambda = \frac{N_{R2}}{N_{R1}} < \infty$, SNR_k can be approximated as

$$SNR_k \approx \frac{k_1 N_{R1}}{K} \quad (36)$$

Therefore, no matter the system is under Rayleigh channel or Rician channel, we can derive the approximate uplink sum achievable rate as

$$R_{sum}^{Rician} = \frac{K}{2} \log 2 \left(1 + \frac{k_1 N_{R1}}{K} \right) \quad (37)$$

The proof is attached in *Appendix 6*.

Results and Discussion

In this section, we use system 1 to represent the one-hop Rayleigh system, system 2 to represent the two-hop Rician system. We set different experiments and visualize the Monte Carlo simulation results and the sum achievable rate calculated from the closed-form expression. Then we compare the results of system 1 and system 2 to verify the correctness of theoretical analysis. In our experiments, we set the number of users $K = 10$, the transmission power of users $P_u = 20dB$, the transmission energy $P_R = 25dB$, the noise energy at $\sigma_{R1}^2 = 1dB$, $\sigma_{R2}^2 = 1dB$. We assume $N_{R2} = 4N_{R1}$, the large-scale fading coefficient $\alpha_k = (\frac{d_{ref}}{d_{u-R1}})^v$, $\beta_k = (\frac{d_{ref}}{d_{R1-R2}})^v$, where d_{ref} represents the reference distance, d_{i-j} represents the distance from node i to node j , v is power exponent coefficient. During the simulation, we set $d_{ref} = 100m$, $d_{R1-R2} = 150m$, $v = 2.4$. In system 1, we set $d_{u-R1} = [700, 1136, 1096, 694, 285, 872, 531, 489, 440, 356]m$. In system 2, we set $d_{R1-R2} = [550, 986, 946, 544, 135, 722, 381, 339, 290, 206]m$. We use $\theta_{R2,j}$ to represent the arrival angle from R_1 to R_2 , $\theta_{R2,j}$ obeys a uniform distribution on $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

Experiment 1: Sum Achievable Rates with Different N_{R1}

As is shown in Figure 3, the curve of the Monte Carlo simulations perfectly matches the curve derived from the closed-form expressions, which proves the correctness of the derived closed-form expressions (10) and (31). Obviously, when the simulation parameters are the same, the sum achievable rate of the two-hop Rician system is higher than the sum achievable rate of the one-hop Rayleigh system. This result proves that converting one-hop Rayleigh system to two-hop Rician system can improve the sum achievable rate. It is in consistent with the actual communication process where there is LOS signal, and the communication quality under Rician channel is better.

Experiment 2: Sum Achievable Rates with Different q

Figure 4 shows the variation of sum achievable rate when q increases. Apart from previous findings, we can also discover that the low-resolution quantization brings performance loss. It is because when the quantization occurs, it reduces the SNR and causes performance degradation. We can also discover that when the number of quantization bits $q > 3$, the sum achievable rate can maintain a stable rate.

Experiment 3: ADC Energy Efficiency with Different q

In Figure 5, the ADC energy efficiency can be obtained by $EE = \frac{R}{P}$, where R represents the sum achievable rate, P represents the energy loss. According to [12] and [14], $P = c_0 N_{R1} * 2^q + c_1$, $c_0 = 0.0001W$, $c_1 = 0.02W$. The result shows that in both systems, the energy efficiency of the ADC shows a logarithmic downtrend when q increases, which denotes that the low-resolution ADC can improve the energy efficiency and reduce the energy consumption during signal transmission. Besides, we can clearly find that when q is small, system 2 has a higher ADC energy efficiency compared with system 1, which proves the superiority of system 2.

Experiment 4: Asymptotic Sum Achievable Rates in Two Systems

Figure 6 shows the simulation result of sum achievable rates with different scaling indexes. When N_{R1} is relatively small, the results of the 1000 Monte Carlo simulation do not match with the analytical results precisely. However, as the number of antennas continues to increase, the 1000 Monte Carlo simulation results can perfectly match the analytical results. It is because the law of power scaling is derived when N_{R1} is large enough.

Besides, Figure 6 shows that in system 1, P_u can be scaled down in inversely proportional to N_{R1} when scaling index $a = 1$ while maintain a desirable sum achievable rate when N_{R1} grows large. In system 2, P_u and P_R can be scaled down in inversely proportional to N_{R1} and N_{R2} when $a = 0, b = 1$, or $a = 1, b = 0$, or $a = 1, b = 1$ and maintain desirable sum achievable rates when N_{R1} grows large. The results shown in Figure 6 are consistent with the theoretical analysis given by equation (14) and (35).

Conclusions

In this paper, we investigate the uplink of two-hop low-resolution ADCs massive MIMO relaying system over Rician channel and compare its superiority of the sum achievable rate with the one-hop Rayleigh channel system. Firstly, we use the higher-order statistics to derive the closed-form expression of sum achievable rate. From the simulation results, we discover that converting one-hop Rayleigh channel system to two-hop Rician channel system can increase the sum achievable rate. Besides, the use of low-resolution ADCs only causes limited loss of sum achievable rate, but greatly improves the energy efficiency. Secondly, we discover that as the number of relay antennas continues to increase, the sum achievable rate eventually reaches a stable state. Finally, the power scaling law shows that when the number of antennas at the relay grows large, both P_u and P_R can be scaled down inversely proportional to N_{R1} and N_{R2} , while maintaining a desirable sum achievable rate.

Appendix

Appendix 1

According to [10, 12, 15], we can get the higher-order statistics

$$E[||\mathbf{g}_{R1,k}||^2] = \alpha_k N_{R1}$$

$$E[||\mathbf{g}_{R1,k}, \mathbf{g}_{R1,i}||^2] = \begin{cases} \alpha_k^2 N_{R1} (N_{R1} + 1) & i = k \\ \alpha_k \alpha_i N_{R1} & i \neq k \end{cases}$$

Substitute high-order statistics into the origin formula, we can get

$$P_k^{Rayleigh'} = k^2 P_u \alpha_k^2 N_{R1} (N_{R1} + 1)$$

$$N_k^{Rayleigh'} = k^2 P_u N_{R1} \sum_{j \neq k}^K \alpha_k \alpha_j + k^2 \sigma_{R1}^2 N_{R1} \alpha_k$$

$$+ k_1 \rho_1 \sum_{n=1}^K \alpha_n N_{R1} (P_u \sum_{i=1}^K \alpha_i + P_u \alpha_n + \sigma_{R1}^2)$$

Appendix 2

$$SNR_k = \frac{k^2 P_u \alpha_k^2 N_{R1} (N_{R1} + 1)}{k^2 P_u N_{R1} \sum_{j \neq k}^K \alpha_k \alpha_j + k^2 \sigma_{R1}^2 N_{R1} \alpha_k + k_1 \rho_1 \sum_{n=1}^K \alpha_n N_{R1} (P_u \sum_{i=1}^K \alpha_i + P_u \alpha_n + \sigma_{R1}^2)}$$

$$= \frac{k^2 P_u \alpha^2 N_{R1}^2}{k^2 P_u \alpha^2 K N_{R1} + k(1-k) P_u \alpha^2 K N_{R1}}$$

$$= \frac{k^2 P_u \alpha^2 N_{R1}^2}{k P_u \alpha^2 K N_{R1}}$$

$$= \frac{k N_{R1}}{K}$$

Appendix 3

According to [12], we can get the higher-order statistics

$$E[||\mathbf{g}_{R1,k}||^2] = \alpha_k N_{R1}$$

$$E[||\mathbf{g}_{R2,k}||^2] = \beta_k N_{R2}$$

$$E[||\mathbf{g}_{R1,k}, \mathbf{g}_{R1,i}||^2] = \begin{cases} \alpha_k^2 N_{R1} (N_{R1} + \Delta_{R1,k}) & i = k \\ \alpha_k \alpha_i N_{R1} \mathbf{Q}_{ki} & i \neq k \end{cases}$$

$$E[||\mathbf{g}_{R2,k}, \mathbf{g}_{R2,i}||^2] = \begin{cases} \beta_k^2 N_{R2} (N_{R2} + 1) & i = k \\ \beta_k \beta_i N_{R2} \mathbf{R}_{ki} & i \neq k \end{cases}$$

Substitute higher-order statistics into (20), the first term in the denominator $tr(E[\mathbf{G}_{R1}^H \mathbf{G}_{R1} \mathbf{G}_{R1}^H \mathbf{G}_{R1}])$ can be represented as

$$\begin{aligned} tr(E[\mathbf{G}_{R1}^H \mathbf{G}_{R1} \mathbf{G}_{R1}^H \mathbf{G}_{R1}]) &= tr(E[|\mathbf{g}_{R1,i}^H|^2] + E[\sum_{i \neq l}^K |\mathbf{g}_{R1,i}^H \mathbf{g}_{R1,l}^H|^2]) \\ &= \sum_{l=1}^K \alpha_i^2 N_{R1} (N_{R1} + \Delta_{R1,i}) + \sum_{i=1}^K \sum_{l \neq i}^K \alpha_i \alpha_l N_{R1} \mathbf{Q}_{il} \triangleq S_1 \end{aligned}$$

The second term in the denominator $\sigma_{R1}^2 tr(E[\mathbf{G}_{R1}^H \mathbf{G}_{R1}])$ can be represented as

$$\begin{aligned} \sigma_{R1}^2 tr(E[\mathbf{G}_{R1}^H \mathbf{G}_{R1}]) &= \sum_{l=1}^K E[|\mathbf{g}_{R1,l}^H|^2] \\ &= N_{R1} \sum_{l=1}^K \alpha_l \end{aligned}$$

The simplification process of the third term in the denominator $tr(E[\mathbf{G}_{R1}^H \mathbf{R}_{\bar{n}R1} \mathbf{G}_{R1}])$ is as follows

$$\begin{aligned} tr(E[\mathbf{G}_{R1}^H \mathbf{R}_{\bar{n}R1} \mathbf{G}_{R1}]) &= k \rho tr(E[\mathbf{G}_{R1}^H diag(P_u \mathbf{G}_{R1} \mathbf{G}_{R1}^H + \sigma_{R1}^2 \mathbf{I}_{N_{R1}}) \mathbf{G}_{R1}]) \\ &= k \rho tr(E[\sum_{m=1}^{N_{R1}} |\mathbf{g}_{m,i}|^2 (\sigma_{R1}^2 + P_u \sum_{i \neq n}^K |\mathbf{g}_{m,i}|^2 + P_u |\mathbf{g}_{m,n}|^2)]) \\ \text{when } \sigma_{R1}^2 &= 1 \\ tr(E[\mathbf{G}_{R1}^H \mathbf{R}_{\bar{n}R1} \mathbf{G}_{R1}]) &= \sum_{m=1}^{N_{R1}} E[|\mathbf{g}_{m,n}|^2] + \sum_{m=1}^{N_{R1}} \sum_{i \neq n}^K E[|\mathbf{g}_{m,n}|^2] E[|\mathbf{g}_{m,i}|^2] + P_u \sum_{m=1}^{N_{R1}} E[|\mathbf{g}_{m,n}|^4] \end{aligned}$$

According to [13], we can get the higher-order statistics

$$\begin{aligned} E[|\mathbf{g}_{R1,mn}|^2] &= \alpha_n \\ E[|\mathbf{g}_{R1,mn}|^4] &= 2\alpha_n^2 \end{aligned}$$

Substitute the higher-order statistics back to the previous calculation, we can get

$$\begin{aligned} tr(E[\mathbf{G}_{R1}^H \mathbf{R}_{\bar{n}R1} \mathbf{G}_{R1}]) &= tr(N_{R1} (\sigma_{R1}^2 \alpha_n + P_u \alpha_n \sum_{i=1}^K \alpha_i + P_u \alpha_n^2)) \\ &= P_u N_{R1} \sum_{n=1}^K \alpha_n (\alpha_n + \sum_{i=1}^K \alpha_i) + N_{R1} \sigma_{R1}^2 \sum_{n=1}^K \alpha_n \triangleq S_2 \end{aligned}$$

Therefore, we can get the amplification factor γ as

$$\gamma = \sqrt{\frac{PR}{k_1^2 (p_u S_1 + N_{R1} \sigma_{R1}^2 \sum_{i=1}^K \alpha_i) + k_1 \rho_1 S_2}}$$

Appendix 4

Formula (32) represents the power of the desired signal of the $k_t h$ user, the calculation process is as follows

$$\begin{aligned} P_k^{Rician'} &= \gamma^2 k_1^2 k_2^2 P_u E[|\mathbf{g}_{R2,k}^H \mathbf{G}_{R2} \mathbf{G}_{R1}^H \mathbf{g}_{R1,k}|^2] \\ &= E[|\sum_{i=1}^K \mathbf{g}_{R2,k}^H \mathbf{g}_{R2,i} \mathbf{g}_{R1,i}^H \mathbf{g}_{R1,k}|^2] \\ &= \sum_{i=1}^K E[|\mathbf{g}_{R2,k}^H \mathbf{g}_{R2,i} \mathbf{g}_{R1,i}^H \mathbf{g}_{R1,k}|^2] + \sum_{i=1}^K \sum_{l \neq i}^K \underbrace{E[\mathbf{g}_{R2,k}^H \mathbf{g}_{R2,i} \mathbf{g}_{R2,l}^H \mathbf{g}_{R2,k}] E[\mathbf{g}_{R2,i}^H \mathbf{g}_{R2,k} \mathbf{g}_{R2,l}^H \mathbf{g}_{R2,i}]}_0 \\ &= E[|\mathbf{g}_{R2,k}^H \mathbf{g}_{R2,k}|^2] E[\mathbf{g}_{R1,k}^H \mathbf{g}_{R1,k}] + \sum_{i \neq k}^K E[|\mathbf{g}_{R2,k}^H \mathbf{g}_{R2,i}|^2] E[|\mathbf{g}_{R1,i}^H \mathbf{g}_{R1,k}|^2] \end{aligned}$$

Substitute the higher-order statistics back to the previous calculation, we can get

$$P_k^{Rayleigh'} = \gamma^2 k_1^2 k_2^2 P_u \alpha_k \beta_k N_{R1} N_{R2} [\alpha_k \beta_k (N_{R1} + \Delta_{R1,k}) (N_{R2} + \Delta_{R2,k}) + \sum_{i \neq k}^K \alpha_i \beta_i \mathbf{Q}_{ki} \mathbf{R}_{ki}]$$

Formula $N_k^{Rician'}$ in (33) represents the power of interference signal and the power of noise of the $k_t h$ user and it contains four terms. The calculation process is as follows

1) A_k^{Rician}

Substitute higher-order statistics into A_k^{Rician} , we can get the power of interference

$$A_k^{Rician} = \gamma^2 k_1^2 k_2^2 p_u \beta_k N_{R1} N_{R2} \sum_{j \neq k}^K \alpha_j [\beta_k \alpha_k \mathbf{Q}_{kj} (N_{R2} + \Delta_{R2,k})]$$

2) B_k^{Rician}

Substitute higher-order statistics into B_k^{Rician} , we can get the power of gaussian white noise at $R1$

$$B_k^{Rician} = \gamma^2 k_1^2 k_2^2 \sigma_{R1}^2 \beta_k N_{R1} N_{R2} [\alpha_k \beta_k (N_{R2} + \Delta_{R2,k}) + \sum_{i \neq k}^K \alpha_i \beta_i \mathbf{R}_{ki}]$$

3) C_k^{Rician}

$$\begin{aligned} C_k^{Rician} &= k_1 \rho_1 E[\mathbf{g}_{R2,k}^H \mathbf{G}_{R2} \mathbf{G}_{R1}^H \text{diag}(P_u \mathbf{G}_{R1} \mathbf{G}_{R1}^H + \sigma_{R1}^2 I_{N_{R1}}) \mathbf{G}_{R1} \mathbf{G}_{R2}^H \mathbf{g}_{R2,k}] \\ &= k_1 \rho_1 P_u E[\mathbf{g}_{R2,k}^H \mathbf{G}_{R2} \mathbf{G}_{R1}^H \text{diag}(\mathbf{G}_{R1} \mathbf{G}_{R1}^H) \mathbf{G}_{R1} \mathbf{G}_{R2}^H \mathbf{g}_{R2,k}] + k_1 \rho_1 \sigma_{R1}^2 E[\mathbf{g}_{R2,k}^H \mathbf{G}_{R2} \mathbf{G}_{R1}^H \mathbf{G}_{R1} \mathbf{G}_{R2}^H \mathbf{g}_{R2,k}] \end{aligned}$$

We use $E[S_1]$ to represent the first term in C_k^{Rician} , $E[S_2]$ to represent the second term in C_k^{Rician} . Substitute higher-order statistics into C_k^{Rician} , we can get the power of quantization noise at $R1$. The calculation process is as follows

$$\begin{aligned} E[S_1] &= E[\mathbf{g}_{R2,k}^H \mathbf{G}_{R2} \mathbf{G}_{R1}^H \text{diag}(\mathbf{G}_{R1} \mathbf{G}_{R1}^H) \mathbf{G}_{R1} \mathbf{G}_{R2}^H \mathbf{g}_{R2,k}] \\ &= \sum_{m=1}^{N_{R1}} (\sum_{i \neq k}^K E[|\mathbf{g}_{R2,k}^H \mathbf{g}_{R2,i}|^2] E[|\mathbf{g}_{R1,mi}|^4] + \sum_{i=1}^K \sum_{l \neq i}^K E[|\mathbf{g}_{R2,k}^H \mathbf{g}_{R2,i}|] E[|\mathbf{g}_{R1,mi}|^2 |\mathbf{g}_{R1,ml}|^2]) \\ &= \sum_{m=1}^{N_{R1}} (\sum_{i \neq k}^K E[|\mathbf{g}_{R2,k}^H \mathbf{g}_{R2,i}|^2] E[|\mathbf{g}_{R1,mi}|^4] + E[|\mathbf{g}_{R2,k}|^4] E[|\mathbf{g}_{R1,k}|^4] \\ &\quad + \sum_{i \neq k}^K \sum_{l \neq i}^K E[|\mathbf{g}_{R2,k}^H \mathbf{g}_{R2,i}|^2] E[|\mathbf{g}_{R2,mi}|^2 |\mathbf{g}_{R2,ml}|^2] \\ &\quad + \sum_{l \neq i}^K E[|\mathbf{g}_{R2,k}|^4] E[|\mathbf{g}_{R1,mi}|^2 |\mathbf{g}_{R1,ml}|^2]) \\ &= \beta_k N_{R1} N_{R2} ((N_{R2} + \Delta_{R2,k}) \alpha_k \beta_k (\alpha_k + \sum_{l \neq i}^K \alpha_l) + \sum_{i \neq k}^K \alpha_i \beta_i \mathbf{R}_{ki} (\alpha_i + \sum_{l \neq i}^K \alpha_l)) \\ E[S_2] &= E[\mathbf{g}_{R2,k}^H \mathbf{G}_{R2} \mathbf{G}_{R1}^H \mathbf{G}_{R1} \mathbf{G}_{R2}^H \mathbf{g}_{R2,k}] = E[|\mathbf{g}_{R2,k} \mathbf{G}_{R2} \mathbf{G}_{R1}^H|^2] \\ &= \beta_k N_{R1} N_{R2} \sigma_{R1}^2 (\alpha_k \beta_k (N_{R2} + \Delta_{R2,k}) + \sum_{i \neq k}^K \alpha_i \beta_i \mathbf{R}_{ki}) \\ \therefore C_k^{Rician} &= \gamma^2 k_1 \rho_1 k_2^2 \beta_k N_{R1} N_{R2} [P_u ((N_{R2} + \Delta_{R2,k}) \alpha_k \beta_k (\alpha_k + \sum_{l \neq i}^K \alpha_l) + \sum_{i \neq k}^K \alpha_i \beta_i \mathbf{R}_{ki} (\alpha_i + \sum_{l \neq i}^K \alpha_l)) \\ &\quad + \sigma_{R1}^2 (\alpha_k \beta_k (N_{R2} + \Delta_{R2,k}) + \sum_{i \neq k}^K \alpha_i \beta_i \mathbf{R}_{ki})] \end{aligned}$$

4) D_k^{Rician}

Substitute higher-order statistic into D_k^{Rician} , we can get

$$D_k^{Rician} = k_2^2 \sigma_{R2}^2 E[|\mathbf{g}_{R2,k}|^2] = k_2^2 \sigma_{R2}^2 N_{R2} \beta_k$$

5) E_k^{Rician}

Combine (23) and higher-order statistics, we can get

$$\begin{aligned}
 k_1^2 E[\mathbf{g}_{R1,n}^H (P_u \mathbf{G}_{R1} \mathbf{G}_{R1}^H + \sigma_{R1}^2 \mathbf{I}_{N_{R1}}) \mathbf{g}_{R1,n}] &= k_1^2 \alpha_n N_{R1} (P_u \sum_{i=1}^K \alpha_i + N_{R1} P_u \alpha_n + \sigma_{R1}^2) \\
 k_1 \rho_1 E[\mathbf{g}_{R1,n}^H \text{diag}(P_u \mathbf{G}_{R1} \mathbf{G}_{R1}^H + \sigma_{R1}^2 \mathbf{I}_{N_{R1}}) \mathbf{g}_{R1,n}] \\
 &= P_u (\sum_{m=1}^{N_{R1}} E[|\mathbf{g}_{R1,mn}|^4] + \sum_{m=1}^{N_{R1}} \sum_{i \neq k}^K E[|\mathbf{g}_{R1,mi}|^2]) + \sigma_{R1}^2 \sum_{n=1}^K E[|\mathbf{g}_{R1,mn}|^2] \\
 &= k_1 \rho_1 \alpha_n N_{R1} (P_u \sum_{i=1}^K \alpha_i + P_u \alpha_n + \sigma_{R1}^2)
 \end{aligned}$$

Combine the two items together, we can get

$$\begin{aligned}
 E[\mathbf{g}_{R1,n}^H (k_1^2 (P_u \mathbf{G}_{R1} \mathbf{G}_{R1}^H + \sigma_{R1}^2 \mathbf{I}_{N_{R1}}) + k_1 \rho_1 \text{diag}(P_u \mathbf{G}_{R1} \mathbf{G}_{R1}^H + \sigma_{R1}^2 \mathbf{I}_{N_{R1}})) \mathbf{g}_{R1,n}] \\
 = k_1 \alpha_n N_{R1} (P_u \sum_{i=1}^K \alpha_i + k_1 N_{R1} P_u \alpha_n + \rho_1 P_u \alpha_n + \sigma_{R1}^2)
 \end{aligned}$$

According to [14], we can get the higher-order statistics

$$\begin{aligned}
 \sum_{m=1}^{N_{R2}} E[|\mathbf{g}_{R2,mk}|^4] &= \beta_k^2 N_{R2} (1 + \Delta_{R2,k}), \quad \sum_{m=1}^{N_{R2}} E[|\mathbf{g}_{R2,mk}|^4] = N_{R2} \beta_k. \text{ Assume that} \\
 \nabla_n &= k_1 \alpha_n N_{R1} (P_u \sum_{i=1}^K \alpha_i + k_1 N_{R1} P_u \alpha_n + \rho_1 P_u \alpha_n + \sigma_{R1}^2), \text{ we can get}
 \end{aligned}$$

$$\begin{aligned}
 E_k^{Rician} &= E[\mathbf{g}_{R2,k}^H \mathbf{R}_{R1} \mathbf{g}_{R2,k}] \\
 &= k_2 \rho_2 \sigma_{R2}^2 E[|\mathbf{g}_{R2,k}|^2] + k_2 \rho_2 \sum_{m=1}^{N_{R2}} E[|\mathbf{g}_{R2,mk}|^4] \nabla_k + k_2 \rho_2 \sum_{m=1}^{N_{R2}} \sum_{n \neq k}^K E[|\mathbf{g}_{R2,mk}|^2] E[|\mathbf{g}_{R2,mn}|^2] \nabla_n
 \end{aligned}$$

Therefore, the quantization noise power D_k^{Rician} at R_2 can be represented as

$$\begin{aligned}
 E_k^{Rician} &= E[\mathbf{g}_{R2,k}^H \mathbf{R}_{R1} \mathbf{g}_{R2,k}] \\
 &= \gamma^2 k_1 k_2 \rho_2 N_{R1} N_{R2} \beta_k (\alpha_k \beta_k \Delta_{R2,k} (p_u \sum_{i=1}^K \alpha_i + k_1 N_{R1} p_u \alpha_k + \rho_1 p_u \alpha_k + \sigma_{R1}^2) \\
 &\quad + \sum_{n=1}^K \alpha_n \beta_n (p_u \sum_{i=1}^K \alpha_i + k_1 N_{R1} p_u \alpha_n + \rho_1 p_u \alpha_n + \sigma_{R1}^2)) + k_2 \rho_2 \sigma_{R2}^2 N_{R2} \beta_k
 \end{aligned}$$

Appendix 5

Substitute γ into (34), the second term in the denominator can be expressed as

$$\frac{\sigma_{R2}^2}{\gamma^2 \alpha_k \beta_k N_{R1}^2} = k_1 \lambda^b \sigma_{R2}^2 N_{R1}^{b-2} \times \frac{k_1 (\Delta_1 + \Delta_2 + \Delta_3) + (1 - k_1) (\Delta_3 + \Delta_4)}{E_R \alpha_k \beta_k}$$

where $\Delta_1 = E_u N_{R1}^{2-a} \sum_{i=1}^K \alpha_i^2$, $\Delta_2 = E_u N_{R1}^{-a} (\sum_{i=1}^K \alpha_i)^2$, $\Delta_3 = N_{R1} \sigma_{R1}^2 \sum_{i=1}^K \alpha_i$, $\Delta_4 = E_u N_{R1}^{-a} \sum_{i=1}^K \alpha_i (\alpha_i + \sum_{l=1}^K \alpha_l)$. When the value of a and b are different, the limit values are different.

1) $a < 1, b < 1$

$$\lim_{N_{R1} \rightarrow \infty} \frac{k_1^2 k_2 \lambda E_u \alpha_k N_{R1}^{1-a}}{k_1 k_2 \lambda \sigma_{R1}^2 + \frac{\sigma_{R2}^2}{\gamma^2 \alpha_k \beta_k N_{R1}^2}} = \lim_{N_{R1} \rightarrow \infty} \frac{k_1^2 k_2 \lambda E_u \alpha_k N_{R1}^{1-a}}{k_1 k_2 \lambda \sigma_{R1}^2 + \frac{k_1^2 \lambda^b \sigma_{R2}^2}{E_R \alpha_k \beta_k} N_{R1}^{b-1} \Delta_1} = \infty$$

2) $b = 1, a < b$

$$\lim_{N_{R1} \rightarrow \infty} \frac{k_1^2 k_2 \lambda E_u \alpha_k N_{R1}^{1-a}}{k_1 k_2 \lambda \sigma_{R1}^2 + \frac{\sigma_{R2}^2}{\gamma^2 \alpha_k \beta_k N_{R1}^2}} = \lim_{N_{R1} \rightarrow \infty} \frac{k_1^2 k_2 \lambda E_u \alpha_k N_{R1}^{1-a}}{\frac{k_1^2 \lambda \sigma_{R2}^2}{E_R \alpha_k \beta_k} \Delta_1} = \frac{k_2 E_R \alpha_k^2 \beta_k}{\sigma_{R2}^2 \sum_{i=1}^K \alpha_i^2}$$

3) $a = 1, a > b$

$$\lim_{N_{R1} \rightarrow \infty} \frac{k_1^2 k_2 \lambda E_u \alpha_k N_{R1}^{1-a}}{k_1 k_2 \lambda \sigma_{R1}^2 + \frac{\sigma_{R2}^2}{\gamma^2 \alpha_k \beta_k N_{R1}^2}} = \lim_{N_{R1} \rightarrow \infty} \frac{k_1^2 k_2 \lambda E_u \alpha_k N_{R1}^{1-a}}{k_1 k_2 \lambda \sigma_{R1}^2} = \frac{k_1 E_u \alpha_k}{\sigma_{R1}^2}$$

4) $a = b = 1$

$$\lim_{N_{R1} \rightarrow \infty} \frac{k_1^2 k_2 \lambda E_u \alpha_k N_{R1}^{1-a}}{k_1 k_2 \lambda \sigma_{R1}^2 + \frac{\sigma_{R2}^2}{\gamma^2 \alpha_k \beta_k N_{R1}^2}} = \lim_{N_{R1} \rightarrow \infty} \frac{k_1^2 k_2 \lambda E_u \alpha_k N_{R1}^{1-a}}{k_1 k_2 \lambda \sigma_{R1}^2 + \frac{k_1^2 \lambda \sigma_{R2}^2 (k_1 \Delta_1 + \Delta_3)}{E_R \alpha_k \beta_k}} = \frac{k_1 k_2 E_u E_R \alpha_k^2 \beta_k}{\tau}$$

where $\tau = k_2 E_R \alpha_k \beta_k \sigma_{R1}^2 + \sigma_{R2}^2 (k_1 E_u \Sigma_{i=1}^K \alpha_i^2 + \sigma_{R1}^2 \Sigma_{i=1}^K \alpha_i)$

5) $a > 1$

$$\lim_{N_{R1} \rightarrow \infty} \frac{k_1^2 k_2 \lambda E_u \alpha_k N_{R1}^{1-a}}{k_1 k_2 \lambda \sigma_{R1}^2 + \frac{\sigma_{R2}^2}{\gamma^2 \alpha_k \beta_k N_{R1}^2}} = \lim_{N_{R1} \rightarrow \infty} \frac{k_1^2 k_2 \lambda E_u \alpha_k N_{R1}^{1-a}}{k_1 k_2 \lambda \sigma_{R1}^2 + \lambda^b \sigma_{R2}^2 N_{R1}^{b-1} \mathcal{O}(1)} = 0$$

6) $a \leq 1 < b$

$$\lim_{N_{R1} \rightarrow \infty} \frac{k_1^2 k_2 \lambda E_u \alpha_k N_{R1}^{1-a}}{k_1 k_2 \lambda \sigma_{R1}^2 + \frac{\sigma_{R2}^2}{\gamma^2 \alpha_k \beta_k N_{R1}^2}} = \lim_{N_{R1} \rightarrow \infty} \frac{k_1^2 k_2 \lambda E_u \alpha_k N_{R1}^{1-a}}{k_1 \lambda^b \sigma_{R2}^2 N_{R1}^{b-1} \mathcal{O}(N_{R1}^{1-a})} = 0$$

Appendix 6

Supposing that we have perfect channel state information (CSI), $N_{R1} > N_{R2} \gg K \gg 1$, $\alpha_1 = \alpha_2 = \dots = \alpha_k = \alpha$, $\beta_1 = \beta_2 = \dots = \beta_k = \beta$, $P_u \gg \sigma_{R1}^2$, $P_R \gg \sigma_{R2}^2$, SNR_k can be approximated as follows

$$SNR_k \approx \frac{k_1^2 k_2 P_u \alpha^2 \beta N_{R1} N_{R2}}{k_1^2 k_2 N_{R1} N_{R2} \alpha \beta \sigma_{R2}^2 + k_1 (1 - k_1) K^2 N_{R1} N_{R2} \sigma_{R1}^2 \alpha \beta} \approx \frac{k_1 N_{R1}}{K}$$

Therefore, we derive the asymptotic expression as

$$R_{sum} \approx \frac{K}{2} \log_2 \left(1 + \frac{k_1 N_{R1}}{K} \right)$$

Abbreviations

MIMO: Multi-input Multi-output

LOS: Light of Sight

ADC: Analog-to-Digital Converters

CSI: Channel State Information

AQNM: Additive Quantization Noise Model

SNR: Signal-to-noise Ratio

MRC: Maximum Ratio Combining

AF: Amplify-and-Forward

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Figures

Figure 1 System model of a one-hop MIMO system under Rayleigh channels

This figure shows the system model of the uplink of a one-hop massive MIMO relay with low-resolution ADCs under Rayleigh channels. This system contains relay R_1 with N_{R1} antennas, and K users with single antenna. The system works under Rayleigh channels because the distances between users and the relay is very large. As a result, there is no LOS between users and the relay and the signals reach the targets only by reflections and refractions from obstacles.

Figure 2 System model of a two-hop MIMO system under Rician channels

This figure shows the system model of the uplink of a two-hop massive MIMO relay with low-resolution ADCs under Rician channels. This system contains relay R_1 with N_{R1} antennas, relay R_2 with N_{R2} antennas and K users with single antenna. The system works under Rician channels because LOS exists between users and R_1 and between R_1 and R_2 .

Figure 3 Sum Achievable Rate vs N_{R1}

This figure shows the variation curve of the sum achievable rate in System 1 and System 2 with the variation of N_{R1} . The asterisks indicate the experimental result obtained through 1000 times Monte Carlo simulations, and the circles indicate the simulation result of the sum achievable rate calculated by the closed-form expression 10 and 31.

Figure 4 Sum Achievable Rate vs Quantization Bits q

This figure shows the variation curve of the sum achievable rate in System 1 and System 2 with the variation of q when $N_{R1} = 200, 400, 800$.

Figure 5 ADC Energy Efficiency vs Quantization Bits q

This figure shows the variation curve of the ADC Energy Efficiency in System 1 and System 2 with the variation of q when $N_{R1} = 200, 400, 800$.

Figure 6 Asymptotic Sum Achievable Rates with Different Scaling Indexes

This figure shows the variation curve of sum achievable rates when N_{R1} increases with different scaling indexes and the corresponding asymptotic values.

Table 1 Quantization distortion factor ρ under Different ADC quantization bits q . According to [7, 9, 10], the values of ρ (ρ_1, ρ_2) when the number of quantization bits $q \leq 5$ is as follows

q	1	2	3	4	5
ρ, ρ_1, ρ_2	0.3634	0.1175	0.03454	0.009497	0.002499

Tables**Declarations****Ethics approval and consent to participate**

Not applicable.

Consent for publication

Not applicable.

Availability of data and materials

1. The data in Table 1 is from paper [7, 9, 10].
2. The data of energy loss formula is from [12] and [14].
3. The data in simulation experiments is generated randomly by Matlab.

Competing interests

The authors declare that they have no competing interests.

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Authors' contributions

Shujuan Yu and Yun Zhang are the instructors of this project. They guided and determined the direction of this project, and put forward opinions and amendments to the simulation experiments. They also help polish the manuscript. Xinyi Liu participates in formula derivation and is in charge of drawing simulation diagrams and organize the manuscript. Jian Cao participates in formula derivation and experiment designing.

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- * Simulation Result
- Analytical Result ($N_{R1} = 200$)
- Analytical Result ($N_{R1} = 400$)
- Analytical Result ($N_{R1} = 800$)

Figures

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