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2D and 3D Potential Flow Simulations around NACA 0012 with Ground Effect

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Abstract. The purpose of this paper is to develop a Boundary Element Based Method (BEM) for determining the steady potential about two and three dimensional airfoil. The numerical investigation of NACA 0012 airfoil with using Boundary Element Method is utilized. Two different physical problems of the NACA 0012 airfoil are examined: potential flow around airfoil in an unbounded fluid and potential flow prediction with ground effect. Computation of potential flow around the airfoil is investigated by the mixed constant strength source and constant strength dipole based panel method. Boundary Element Code is written in FORTRAN. To check the accuracy of the 2D boundary element based code, the validation studies are carried out by comparing the present results obtained for the NACA 0012 airfoil from the XFOIL and other published simulation results. 3D results are also evaluating with the available experimental and other numerical simulation results. The numerical outcomes are examined in terms of pressure distribution and lifting force on the foil.

Keywords: Boundary element method; Potential flow; XFOIL; Ground effect; NACA 0012

1. Introduction

It is known that for an airfoil flying close to ground clearance can produce a considerable lift growth as compared to airfoil flying out of the ground effect. In order to investigate the ground effect, it is essential to first calculate the aerodynamic forces on a wing. Dionne and Lee [1] experimentally investigated ground effect on aerodynamics of airfoil. In this study lift increment was found to increase with reducing ground distance.

Numerical simulations are developing a widespread technique to assess the aerodynamic and hydrodynamic performance in preliminary design stage. The model experiments are still very valuable, hence time and cost encourage the use of Computational Fluid Dynamics (CFD). With

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the improvements in computer technology, scientists have started to study flow around the airfoil with solving Navier-Stokes equations. By the help of computers, it has been able to model the flow around airfoil bodies by using suitable turbulence model. RANS based numerical approaches are widely used in airfoil design. Winslow *et al.* [2] investigated airfoil characteristics at low Reynolds number. In this study, the computational fluid dynamics tool used is a Reynolds-averaged Navier–Stokes solver with a Spalart–Allmaras turbulence model. Rubel *et al.*, [3], Sohaib Obeid *et al.* [4] have carried out some numerical analyses in order to decide airfoil performance. Alsarheed and Sedaghat [5] computationally investigated ground effects using both the panel and finite volume methods. Their study show that the results of both panel and finite volume numerical methods are consistent are in agreement with the experimental results. The flow is incompressible, irrotational and inviscid. Some CFD methods based on potential flow theory because the Reynolds Averaged Navier-Stokes Equations (RANSE) is both computer power and time consuming.

The prediction of pressure distribution of an airfoil is important for the airfoil's performance. Some methods can be found in the literature, but the most reliable ones are developed by Hess and Smith [6]. They used the panel method in solving the problem of lifting and non-lifting bodies by distribution of constant sources and vortices on them. Several references that have been printed such as Morino and Kuo [7], Kerwin [8] and the books by (Anderson [9], Moran [10], Newmann [11], Katz and Plotkin [12], Katsikadelis [13]), so these will not be examined any further here.

Over the last few decades, Potential based numerical approaches are widely used in aerodynamic design. Tarafder *et al.* [14] and Tarafder *et al.* [15] studied on the computation of potential flow around 2D and 3D NACA 0012 airfoil by the source based panel methods respectively. They evaluated their results with the analytical and experimental data. Chaves [16] analysed 2D airfoil by using the panel method. His work showed that trends of lift coefficient and pressure distribution results follow the experimental and CFD results. Farinha *et al.* [17] examined the application of panel method, which is able to solve steady, two-dimensional, potential flow around airfoils including the ground effect. Their study emphasized that at small angles of attack, the findings obtained from this simple approach are qualitatively similar to experimental results.

Potential theory not only used for aerodynamic design but also used for hydrodynamic calculations. Theoretical analyses on estimation of wave pattern generated by hydrofoils and ships have been carried out for along time. As a pioneering study, Dawson [18] suggested an efficient linearized method, which gives convincing results. Nowadays, Dawson's method successfully using for simulation of free surface flow. Tarafder and Suzuki [19] modeled a potential-based boundary element method for solving a nonlinear free-surface flow problem for a Wigley catamaran moving with a uniform speed in deep water. Their study successfully presents a potential-based boundary element method for predicting the hydrodynamic characteristics of a catamaran hull. Xie and Vassalos [20] develop a potential-based panel method for determining the steady potential flow about three dimensional hydrofoil under free surface. An IBEM [iterative boundary element (panel) method] for the solution of hydrofoil transferring under a free surface was explained in detail in Bal and Kinnas [21]. Their iterative method is extended to apply to the surface piercing bodies and some numerical results of the method are given in Bal [22], Bal [23].

The main purpose of this study is to generate the boundary element based solution code. 2D and 3D simulations for various angle of attack in order to be able to evaluate results from the Xfoil and the other published results from literature. The model was solved with a range of different attack of angles from 0° to 10° . 3D simulations were carried out different span length. In the following sections, the computational results of aerodynamic coefficient and surface pressure

distribution for NACA 0012 are presented. The boundary element code written in FORTRAN, the program is able to find the lift force and pressure distribution. The ability of the panel methods in solving the potential flow problem around 2-D and 3-D lifting airfoils is experienced in this study.

2. Geometry and conditions

In this study, the NACA 0012, the well-documented airfoil was used. For the four digit numbers, the first number describes the maximum camber as a percentage of chord length. The second number is the position is the maximum tenths of the chord and third and fourth number corresponds to the maximum thickness of the foil as a percent of the chord length. This airfoil is symmetrical as both first and second number is zero, and maximum thickness of 12% of the chord length. The thickness of the NACA 0012 airfoil can be calculated from a well-known equation (Abbott and Von Doenhoff [24]) and geometry of the airfoil is given in Fig. 1. In order to validate the present simulation process, the operating conditions are mirrored to equalled the operating conditions of NASA Langley Research Center validation cases (Abbott and Von Doenhoff [24]).

Fig. 1 NACA 0012 symmetric airfoil geometry

3. Mathematical formulations

It is accepted that the fluid is incompressible, inviscid and that the flow is irrotational. A Cartesian coordinate system is placed on foil surface the components of the free stream velocity U in the x - z frame of reference. The angle of attack α is defined as the angle between the free stream velocity and the x -axis. The total velocity potential is separated into sum of a free stream potential and a perturbation potential (Bal [25], Lee *et al.* [26])

$$\Phi = Ux + \varphi, \quad (1)$$

Where φ is the perturbation potential. The total velocity potential Φ should satisfy the Laplace's equation in the fluid domain Ω .

$$\nabla^2 \Phi = 0,$$

$$\nabla^2 (Ux + \varphi) = 0, \quad (2)$$

The domain Ω is bounded by the airfoil surface S_A , wake surface S_W and an outer control surface S_∞ enclosing the body and the wake surfaces. The definition of notations, airfoil and the fluid domain is depicted in Fig. 2. The problem can be constructed by specifying the boundary conditions as follows:

Kinematic body boundary condition:

$$\frac{\partial \phi}{\partial n} = \bar{U} \cdot \bar{n}, \text{ on } S_A \quad (3)$$

Where $\bar{n}$ is the unit normal vector, which points out of the fluid domain.

Radiation condition at infinity:

$$\nabla \phi \rightarrow 0, \quad (4)$$

Kutta condition at the trailing edge of a foil:

$$\nabla \phi = \text{finite}, \quad (5)$$

Assumption of the wake surface:

$$P_U - P_L = P \quad (6)$$

streamwise velocity must be continuous across the surface:

$$\bar{t} \cdot (\bar{V}_U - \bar{V}_L) = 0, \quad (7)$$

subscript U and L are the upper and lower surfaces of the wake, and V and P are the velocity and pressure respectively. $\bar{t}$ is a unit vector in the direction of the mean velocity. The wake surface has zero thickness and the pressure jump across S_w is zero, while there is a jump in the potential:

$$\Delta \phi = \phi_U - \phi_L \quad (8)$$

Fig. 2 Definition of notations for airfoil in unbounded fluid

4. Numerical implementation

The velocity potential ϕ can be written an integral equation on the airfoil surface by applying Green's theorem:

$$\frac{1}{2} \phi = \int_{S_A} \phi \frac{\partial G}{\partial n} + G \frac{\partial \phi}{\partial n} dS + \int_{S_w} \Delta \phi_w \frac{\partial G}{\partial n} dS, \quad (9)$$

where S_A and S_W are the boundaries of the airfoil surface and wake surface respectively. G is the Green function given as $G = \frac{1}{4\pi r}$ in 3D and $G = \frac{1}{2\pi} \ln r$ in 2D, r is the location vector of the singularities and the field point.

To construct a numerical solution, the surface S_A divided into N panels and the discretized form of Eq. (9) is expressed on the airfoil surface:

$$\frac{1}{2} \phi^i = \sum_{j=1}^N \int_{S_A + S_W} \phi \frac{\partial G}{\partial n} dS - \sum_{j=1}^N \int_{S_A} G \frac{\partial \phi}{\partial n} dS, \quad (10)$$

The values of ϕ and $\frac{\partial \phi}{\partial n}$ are constant on each element, so they can be moved outside the integral. Eq. (10) may be written as:

$$\frac{1}{2} \phi^i - \sum_{j=1}^N \int_{S_A + S_W} \frac{\partial G}{\partial n} dS \phi^j = - \sum_{j=1}^N \int_{S_A} G dS \frac{\partial \phi^j}{\partial n} \quad (11)$$

$$\int_{S_A + S_W} \frac{\partial G}{\partial n} dS \equiv \hat{H}_{ij}, \quad (12)$$

$$\int_{S_A} G dS \equiv G_{ij}, \quad (13)$$

Fig. 3 Discretization of 2D NACA 0012 airfoil by 34 constant lines

In order to employ the 2-D boundary element method the airfoil is discretized by 34 line elements in which the mesh is clustered near the leading edge of foil. Constant line elements are shown in Fig. 3. The numbering system starts at lower surface trailing edge. $N+1$ points define N panels. 3D airfoil NACA 0012 is chosen in order to compare present numerical results with the other printed results from literature. The airfoil is of unit chord length (c) and its span (S) lengths are chosen 5, 10 and 15 respectively. The body is discretized into 10 strips in span wise and 34 strips in chord wise direction. Fig. 4 shows the surface panels of the NACA 0012 airfoil and span length ($S=10$).

These elements estimate the actual geometry by a constant panel while the unknown quantity supposed to be constant. Both G_{ij} and H_{ij} is calculated analytically. A more detailed discussion of this point is provided in Katz and Plotkin (2001). Eq. (13) may be written as:

$$\sum_{j=1}^N H_{ij} \varphi_j = - \sum_{j=1}^N G_{ij} \frac{\partial \varphi_j}{\partial n} \quad (14)$$

$$H_{ij} = \frac{1}{2} \delta_{ij} - \hat{H}_{ij} \quad (15)$$

Where δ_{ij} is the Kronecker delta, which is defined as $\delta_{ij} = 0$ for $i \neq j$ and $\delta_{ij} = 1$ for $i = j$. Determination of the influence coefficient the perturbation potentials H_{ij} at the each of the collocation point will result in an $N \times N$ influence matrix, with $N+1$ unknown (where the wake potential $\Delta\phi_W$ is the $(N + 1)$ -th unknown). The additional equation is provided by using the Kutta condition:

$$(\varphi_1 - \varphi_N) - \Delta\phi_W = 0, \quad (16)$$

The above set of algebraic equations is solved by using Gaussian elimination method.

Fig. 4 Discretization of 3D NACA 0012 airfoil by 34x10 panels (S=5)

5. Numerical results and discussion

After perturbation potential on hydrofoil is calculated, the local external tangential velocity on each collocation point can be computed by differentiating the velocity potential along the tangential direction as follows Katz and Plotkin (2001):

$$V_{tj} = \frac{\varphi_{j+1} - \varphi_j}{\Delta t_j} + U \vec{t}, \quad (17)$$

Pressure coefficient can be obtained by using Bernoulli Equation:

$$C_{pj} = \frac{P_j - P_\infty}{\frac{1}{2} \rho U^2} = 1 - \frac{V_{tj}^2}{U^2}, \quad (18)$$

Airfoils have different geometry and dimensions. So, non-dimensional coefficients (lift coeffic

ients) were considerate to evaluate the advantages and disadvantages of airfoils. The non-dimensional lift and drag coefficients were given as below:

$$C_L = \frac{L}{0.5\rho c U^2}, \quad (19)$$

$$C_D = \frac{D}{0.5\rho c U^2},$$

Where L is the lift force and D is the drag force. The contribution of element to the aerodynamic loads ΔF_j :

$$\Delta F_j = C_{pj} \frac{1}{2} \rho U^2 \Delta S_j \vec{n}_j \quad (20)$$

Where ΔS_j is the panel area. The total aerodynamic loads are then obtained by adding the contributions of the individual panels.

5.1 Unbounded fluid

We examine the dependency of the solution to the number of panels by comparing force results with increasing numbers of panels.

Fig. 5 Change of drag coefficient with number of panels

Fig. 6 Change of lift coefficient with number of panels

Fig. 5 and Fig.6 show the change of drag and lift respectively. Potential flow around 2D NACA 0012 is computed at using a 30 uniform spacing, 34 non-uniform spacing, 60, 90, 120 uniform spacing panels. Drag coefficients found by integrating the pressures over the airfoil is an indication of error in the numerical method. The figures show the drag reaching to zero and the lift extending

constant as the number of panels increase. It can be seen also from this figure non-uniform spacing with 34 panels arrangement gives accurate results

Simulation for several angle of attack were performed in order to able to compare the results from the present simulations and numerical results from the Xfoil potential based code and the validate them with existing experimental data from reliable sources. Table 1 shows the computed lift coefficient with using both present study and Xfoil results for 2D NACA 0012. The comparison present simulation with Xfoil is very good. It is accepted that incoming velocity is unity, because the potential solution in coefficient form is independent of scale. The lift coefficients increased linearly with the increasing angles of attack. Present BEM model had a good agreement with Xfoil results angles of attacks from 0° to 10° . The lift coefficients are shown for various angle of attack, computed with present study, and compared with the includes numerical results from the Xfoil are given Fig. 7.

Table 1 Computed lift coefficient both Xfoil and BEM simulations for NACA 0012

Attack of Angle	XFOIL C_L	BEM C_L			
		2D	S=5	S=10	S=15
0	0.000	0.000	0.000	0.000	0.000
1	0.120	0.117	0.072	0.088	0.096
2	0.242	0.235	0.143	0.176	0.191
3	0.362	0.353	0.214	0.265	0.286
4	0.483	0.470	0.285	0.353	0.382
5	0.603	0.588	0.356	0.440	0.476
6	0.724	0.705	0.427	0.527	0.570
7	0.843	0.822	0.496	0.613	0.664
8	0.963	0.939	0.566	0.699	0.756
9	1.083	1.055	0.634	0.783	0.848
10	1.202	1.171	0.702	0.867	0.938

Fig. 7 Comparison between experimental data from Abbott & von Doenhoff [24] and Xfoil and present simulation of lift coefficient for 2D airfoil

2D simulation results are illustrated in Fig. 7. From the given figure, it is observed that some distinctions are observed with comparison of experimental results of Abbott & von Doenhoff [14]. For the attack of angle lower than 10° agreement is quite impressive with the present numerical simulation and other numerical computations and experimental results. Flow is attached to the airfoil through this regime. For the attack of angle that are greater than 10° some peculiarities can be seen and viscous effects begin to show up as a reduction in lift with increasing angle of attack. Figure 7 shows that potential theory cannot estimate stall angle. At angle of attack roughly $14-17^\circ$, stall began to develop.

Fig. 8 Comparison between potential flow simulation data from Tarafder et al. [15] and present study of lift coefficient for 3D airfoil

Fig. 9 Variation of surface pressure coefficient for flow past 2D NACA 0012 airfoil, $\alpha=4^\circ$, $\alpha=8^\circ$

Variations of computed lift coefficient C_L with angle of attack for 3D NACA 0012 for different span wise length are depicted in Fig.8. The triangle in the figure is the numerical results of Tarafder *et al.* [15]. Some distinctions are seen from this figure. Our explanation for this discrepancy is that the mesh structure is insufficient in the study of Tarafder *et al.* They used 18 chord wise panels and 5 span wise panels for discretization of geometry.

As the angle of attack is increased, greater values of C_P are viewed on the lower surfaces and the region of the high pressure extends towards to the trailing edge. It can be seen from Fig. 9 and Fig. 10. The areas with greater pressure coefficient values have a significant contribution to lift coefficient.

Fig. 10 Variation of surface pressure coefficient for flow past 3D NACA 0012 airfoil, $\alpha=4^\circ$, $\alpha=8^\circ$ and $S=10$

5.1 Ground effect

In this part our numerical results are compared with experimental simulation for symmetrical 3D NACA 0012 geometries. To provide certainty in the current results, a validation has been completed against printed results for similar studies. Experimental measurements were carried out by N. Moore et al. [27] in Southampton University. The main dimension of test case is given Table 2. In Boundary element method, the ground effect simulated with the method of images, which consist on placing a mirrored image of the airfoils with the wall as a symmetry axis, as shown in Figure 11.

Figure 12 shows the measured and computed lift coefficient values for given speeds for the NACA 0012 airfoil model. For ground effect, the comparison with the experimental measures of N. Moore *et al.* [27] is very good for attack of angles between 1 and 6. Both experimental and numerical results reveal that for attack of angle between 3° and 6° , lift coefficient decrease with the growth in the altitude. This is due to the greater pressure between airfoil and ground. The BEM computations generally predicted higher lift coefficient values.

Fig. 11 Image method

Fig. 12 Variation of C_L with altitude for NACA 0012 airfoil

The lift curve slope gradients ($\partial C_L / \partial \alpha$) are summarized in Table 3, showing comparison with the experimental data Moore *et al.* [27] in Figure 13. Comparisons between experimental and numerical results reveal that for altitude (h/c) between 0.1 and 0.3, curve slope gradient differences ($\partial C_L / \partial \alpha$) are increased. The obtained differences are believed due to the potential theories deficiency. Figure 14 illustrates the comparisons of lift performance of the NACA 0012 section between h/c 0.3 to 0.1. An remarkable observation is reported by Moore *et al.* When the NACA 0012 is at $h/c=0.1$ large downward forces produced when AoA reduces flow the three degrees. Potential theory cannot simulate this forces but good agreement the other altitudes. Differences between our numerical results and experiment Moore *et al.* [27] are reduced with the increase of ground clearance.

Fig. 13 Computed lift curve slope gradient as a function of altitude for NACA 0012

Figure 16 shows the simulation results of pressure coefficient at angles of attack 1° , 6° , 10° and $h/c=0.5$ with BEM method. The lower surface of the aerofoil obtained a higher pressure coefficient compared to upper surface. As the angle of attack increased the lower surface pressure distribution was much higher than the pressure distribution of the upper surface. That was awaited result. Figure 16 depicts ground effect on the aerofoil pressure distribution. It can be seen that the effect of the ground on aerodynamic performance is significant when the airfoil is located near the ground surface.

Fig. 14 Variation of C_L with altitude for NACA 0012 airfoil

Figure 17 illustrates the behaviour of the pressure along the upper and lower surface of the airfoil with ground clearance, for angles of attack of 1 and 8 deg, respectively. Figure 17 shows the variation in the pressure on the airfoil surface for $\alpha=1$ and $\alpha=8$ deg at different values of h/c at $y/c=0$ section. At $\alpha=1$ deg, the pressure is high on the lower surface at points very close to the leading edge. A suction effect can be seen from this figure on the lower surface at high ground proximity. The result of this suction effect is an important loss of lift. The suction effect on the lower surface disappears when angle of attack is reached to 4 deg. It can be seen from both Figure 17 and 12. Another observation from this Figure is a reducing suction on upper surface with increasing h . At $\alpha=1$ deg, the values of C_p are very high on lower surface. With increasing h/c , pressure decreases, as depicted Figure 17.

Table 2 NACA 0012 parameters

NACA 0012	
Span	0.96 m
Aspect Ratio	3.020
Chord	0.317 m
Area	0.303 m ²

Table 3 Comparisons lift curve slope gradient as a function of altitude for NACA 0012

Altitude (h/c)	$\partial C_L / \partial \alpha$ per radian		Difference %
	Experiment	Present study	
	N. Moore et al.	BEM	
1.0	3.7	3.57	3.5
0.9	3.7	3.63	1.9
0.8	3.9	3.70	5.1
0.7	4.0	3.81	4.8
0.6	4.2	3.95	6.0
0.5	4.2	4.16	1.0
0.4	4.8	4.50	6.3
0.3	5.2	5.09	2.1
0.2	6.9	6.36	7.8
0.15	8.8	7.77	11.7
0.1	9.3	11.10	19.4

Fig. 15 Variation of surface pressure coefficient for flow past 3D NACA 0012 airfoil, $\alpha=1^\circ$, $\alpha=6^\circ$, $\alpha=10^\circ$ and $h/c=0.5$

Fig. 16 Variation of surface pressure coefficient for flow past 3D NACA 0012 airfoil, $h/c=0.1$, $h/c=0.3$, $h/c=1$ and $\alpha=1^\circ$

(a) Pressure lower surface of section $\text{AoA} = 1^\circ$

(b) Pressure upper surface of section $\text{AoA} = 1^\circ$

(c) Pressure lower surface of section $\text{AoA} = 8^\circ$

(d) Pressure upper surface of section $\text{AoA} = 8^\circ$

Fig. 17 Pressure distribution on the NACA 0012 airfoil surface for $\alpha=1^\circ$ at different values of h/c at $y/c=0$ section

6. Conclusion

The paper deals with the source and dipole based panel methods for computing the potential flow around 2D and 3D NACA 0012 airfoil moving with uniform speed in an unbounded fluid. This paper shows the importance of employing BEM method technique for the solution of flow around 2D and 3D bodies. The following conclusions can be obtained from the present study:

- Author's BEM code is a low order panel method, panels do not have to represent exactly boundaries,
- This code does not taking into account viscosity effect,
- Results cannot estimate airfoil stall angle,
- The given simulation is limited to both single panel structure and panel number,
- Present 2D BEM simulations are in good agreement with XFOIL results,
- Author's BEM code fairly simulates the flow around the wing in ground effect,
- Wing in ground effect with clear wing becomes lift coefficient greater than in free stream flow,
- The BEM method has been showed to be computational efficient in analyzing the airfoil geometry,
- Although the accurate estimation of the lift coefficient cannot be awaited with based on potential theory, numerical solutions are generally good agreement with experimental results,
- The method is restricted to the evaluation of lift coefficient,
- Potential theory can simulate the suction effect on the lower surface at small ground clearance,
- This paper shows validation of BEM method for only one model NACA 0012 symmetrical hydrofoil,
- Our BEM code is a relatively basic program, with restrictions. As compared with engineering CFD codes. In this study viscosity effect is not taken into account,
- The method needs less computer power and small analyzing times,
- Potential theory is very practical method compared to RANSE techniques.

It may be concluded that the proposed method can evaluate lift coefficients with sufficient accuracy by using potential flow theory and can be used as a design tool for airfoils. For the future work, examination of the effect of panel structure and different panel numbers is intended. Moreover, it is planned to investigate the usefulness of the proposed method on different airfoil types.

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Nomenclature

AR	Aspect ratio
α	angle of attack [degrees]
U	free stream velocity [m/s]
c	chord of airfoil [m]
C_L	lift coefficient [$C_L = L/0.5\rho cU^2$]
C_p	pressure coefficient
g	acceleration of gravity [m/s ²]
h	altitude of airfoil [m]
h/c	altitude ratio
L	aerodynamic lift force [N]
S	span
$\vec{t}$	unit vector direction of mean velocity

Greek letters

Γ	circulation around hydrofoil
Φ	total potential
$\Delta\phi$	potential jump
ϕ	perturbation potential
ρ	density of fluid [kg/m ³]

Subscripts

A	airfoil surface
U	Upper surface
L	Lower surface

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Figures

Figure 1

NACA 0012 symmetric airfoil geometry

Figure 2

Definition of notations for airfoil in unbounded fluid

Figure 3

Discretization of 2D NACA 0012 airfoil by 34 constant lines

Figure 4

Discretization of 3D NACA 0012 airfoil by 34x10 panels (S=5)

Figure 5

Change of drag coefficient with number of panels

Figure 6

Change of lift coefficient with number of panels

Figure 7

Comparison between experimental data from Abbott & von Doenhoff [24] and Xfoil and present simulation of lift coefficient for 2D airfoil

Figure 8

Comparison between potential flow simulation data from Tarafder et al. [15] and present study of lift coefficient for 3D airfoil

Figure 9

Variation of surface pressure coefficient for flow past 2D NACA 0012 airfoil, $\alpha = 4^\circ$, $\alpha = 8^\circ$

Figure 10

Variation of surface pressure coefficient for flow past 3D NACA 0012 airfoil, $\alpha = 4^\circ$, $\alpha = 8^\circ$ and $S = 10$

Figure 11

Image method

Figure 12

Variation of C_L with altitude for NACA 0012 airfoil

Figure 13

Computed lift curve slope gradient as a function of altitude for NACA 0012

Figure 14

Variation of C_L with altitude for NACA 0012 airfoil

Figure 15

Variation of surface pressure coefficient for flow past 3D NACA 0012 airfoil, $\alpha=1^\circ$, $\alpha=6^\circ$, $\alpha=10^\circ$ and $h/c=0.5$

Figure 16

Variation of surface pressure coefficient for flow past 3D NACA 0012 airfoil, $h/c=0.1$, $h/c=0.3$, $h/c=1$ and $\alpha=1^\circ$

(a) Pressure lower surface of section AoA= 1 deg

(b) Pressure upper surface of section AoA= 1 deg

(c) Pressure lower surface of section AoA= 8 deg

(d) Pressure upper surface of section AoA= 8 deg

Figure 17

Pressure distrubution on the NACA 0012 airfoil surface for $\alpha=1$ deg at different values of h/c at $y/c=0$ section