Research on Multi-source Sparse Optimization Method and Its Application On Polymorphic-oscillatory signal separation

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Research Article

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Research on Multi-source Sparse Optimization Method and Its Application On Polymorphic-oscillatory signal separation

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Abstract: In general, during the whole life cycle operation of mechanical system, its corresponding condition signal often presents multi-component polymorphic-oscillatory characteristic and is accompanied with strong interference noise. In order to identify system operating status, how to achieve polymorphic signal decomposition is an unavoidable focus. Weak signal features corrupted by heavy background noise can be effectively extracted through sparse decomposition. In order to solve the problems of classical sparse decomposition method, such as the lack of signal fidelity, the local optimal solution caused by the non-convex objective function, and the poor universality of the model, a novel multi-source sparse optimization objective function with convexity is constructed based on the generalized mini-max concave penalty function. Then the sparse coefficients of unilateral attenuation transient component, bilateral attenuation transient component and harmonic component are calculated respectively based on Laplace wavelet dictionary, Morlet wavelet dictionary and DFT dictionary using forward backward splitting algorithm. Ultimately, each distinct component can be extracted based on these sparse coefficients. Comparison with the classical resonance sparse signal decomposition (RSSD) based on L1-norm, signal adaptive decomposition and spectral kurtosis show that the proposed method can accurately preserve the amplitude of morphological components under the low SNR premise. Experimental case infers that the proposed method compared with double tree complex wavelet (DTCW) possesses potential value of application on mechanical system fault detection without the prior knowledge of specific number of faults.

Keywords: Polymorphic-oscillatory property, Multi-dictionary, Sparse representation, Convex optimization, Mechanical system signal

1.Introduction

As for mechanical system, effective condition monitoring is the premise to ensure safe operation. In general, affected by signal transmission path, interference from other components, on the process of full life cycle, the corresponding signal characteristic is usually weak, and mechanical system signal often include frequency-doubling modulation component and multiple transient impulse components, which presents significant morphological diversity. Especially when mechanical system occurs abnormal state in the process of operation, the impact characteristic in condition signal will be unusually significant, and the harmonic component also presents modulation feature simultaneously[1]. So machinery status information is difficult to be identified consequently[2]. Therefore, many scholars play an attention to achieve effective separation of condition signal to extract specific component by using signal processing methods, promoting the application and development of mechanical system signal. For example, time-frequency analysis represented by wavelet transform (WT)[3,4], time-domain adaptive decomposition with empirical mode decomposition (EMD)[5] and local mean decomposition (LMD)[6] as the core, which are widely used in mechanical system signal information extraction . As for wavelet transform, basis function is the key factor for application, while it is usually determined manually that affects the extraction effect in practice. On the other hand, although time-domain adaptive decomposition overcomes the constraint of Fourier transform, however, it is greatly affected by noise, and its boundary effect and distortion phenomenon remain to be solved[7] . Compared with above signal processing methods, sparse decomposition is a novel signal processing theory that give the signal sparsest expression under over-complete dictionary combined with optimization algorithm, this advantage infers that it has good anti-noise ability and weak feature extraction, and can effectively solve under-determined problem on mechanical system status monitoring[8]. At present, researches on sparse decomposition mainly focus on three aspects, namely, penalty function, over-complete dictionary, sparse solution, and a series of research results have been applied on mechanical signal processing. For example, Li[9] develops an over-complete dictionary based on Laplace wavelet, L1 norm acting as penalty item and using Split Augmented Lagrangian Shrinkage algorithm (SALSA) to get sparse coefficients, this method has been successfully applied on rolling bearing fault signal weak feature identification. SELESNICK[10] proposes an over-complete dictionary based on Tunable Q-Factor Wavelet Transform (TQWT), and gives penalty item using arc-tangent non-convex penalty function, applying SALSA algorithm combination with majorization–minimization algorithm (MM) to solve sparse coefficients, and corresponding impact component and harmonic component in mechanical monitoring signal are separated successfully. Wang[11] utilizes Discrete Fourier Transform (DFT) in incorporation with Short-Time Fourier Transform (STFT) to design a transform dictionary, using non-convex penalty function as the penalty item, optimizing iteratively by SALSA algorithm, and the transient and harmonic component of mechanical signal are extracted respectively.

Although above sparse decomposition methods have achieved good effect in mechanical system signal decomposition and status condition, while the existing methods are directly applied to deal with mechanical polymorphic signal, it will lead to a series of unavoidable problems. For example, classical sparse decomposition methods based on L1 norm regularization have the problem on reducing the accuracy of signal separation and is not conducive to further extract signal feature. Besides, the existing sparse decomposition methods usually contain non-convex penalty function which leads to the corresponding objective function is also non-convex, as a result, sparse decomposition is often affected by interference and can not be guaranteed to converge to the global optimum, which is not conducive to accurately extract each component. Finally, the extant sparse identification models have poor generality, as current researches mainly concentrate on single component extraction, once a complex state occurs during operation, considering the significant morphological differences between multiple signal components, they can not be effectively separated in consequence.

In this paper, a novel multi-source sparse optimization method is proposed to resolve signal decomposition distortion, local optimal solution caused by non-convex objective function, model poor universality and so on. Firstly, based on morphological analysis of mechanical system signal, an over-complete dictionary is designed according to the oscillation characteristic in signal component, a signal fidelity item is established to comprehensively solve mechanical system polymorphic signal separation. Secondly, a multi-source non-convex penalty function containing good amplitude preserving capability is presented combination with sparse penalty item. Furthermore, aiming at engineering requirement, a multi-source non-convex penalty function with good amplitude preserving capability is studied to construct the sparse penalty term. Finally, giving an optimal objective function and its convexity preserving criteria, a multi-dictionary sparse decomposition method is estimated ultimately. Simulation and engineering experiment verify that the proposed model can accurately achieve mechanical poly-morphic signal decomposition and apply for machinery intelligent operation and maintenance potentially.

2.Proposed method

2.1 Signal sparse decomposition model

As for mechanical system, given signal $\boldsymbol{y}(\boldsymbol{t}) \in \boldsymbol{R}^N$and background Gaussian noise, the traditional single-component sparse decomposition model can be expressed as following. $\boldsymbol{TC}$ represents specific target component, and $\boldsymbol{n}(\mathbf{t})$represents the residual independent component respectively.

$$\boldsymbol{y} = \boldsymbol{TC} + \boldsymbol{n}(\boldsymbol{t}) \tag{1}$$

Sparse decomposition is the processing of extracting target component $\boldsymbol{TC}$ from signal $\boldsymbol{y}(\boldsymbol{t})$ in essence. on the premise that the target component $\boldsymbol{TC}$ can be sparsely represented in relevantly over-complete dictionary $\boldsymbol{A} \in R^{\mathrm{N\times M}}$, conforming to following mathematical model, $\boldsymbol{c}$ symbolizes sparse representation coefficient corresponding to target component $\boldsymbol{TC}$.

$$\boldsymbol{TC} = \boldsymbol{Ac} \tag{2}$$

Spontaneously, sparse decomposition can be simplified into a mathematical model optimization problem in nature. $||\boldsymbol{c}||_0$symbolizes the number of not-zero elements in sparse representation coefficient $\boldsymbol{c}$, which is sparsity.

$$\min_{c} ||\boldsymbol{c}||_0 \quad s.t. \quad ||\boldsymbol{Ac} - \boldsymbol{y}||_2^2 \le \varepsilon \tag{3}$$

In order to calculate conveniently, above optimal model can be further expressed as an unconstrained sparse regularized linear inverse issue.

$$\boldsymbol{c}^* = arg \min_{c\in R^N} \left\{\frac{1}{2}\|\boldsymbol{y} - \boldsymbol{Ac}\|_2^2 + P(c)\right\} \tag{4}$$

$\boldsymbol{c}^*$is the estimator of sparse representation coefficient $\boldsymbol{c}$ obtained by optimization solution, $P(c)$ represents sparse penalty function used to constrain the sparsity of sparse representation coefficient $\boldsymbol{c}$. As a result, synthesizing formula (1-4) , it can obtain the estimated value $\boldsymbol{TC}^*$ of target component as shown below.

$$\boldsymbol{TC}^* = \boldsymbol{A}c^* \tag{5}$$

However, considering the research object in this paper is mechanical system polymorphic signal and there are multiple target components with prominent oscillation diversity, the above single-component modeling method can not effectively achieve polymorphic identification. Therefore, the generalized idea is introduced to divide signal $\boldsymbol{y}(\boldsymbol{t})$ into several target components $\boldsymbol{TC}_i$ based on polymorphic-oscillatory property, such as transient impact component, harmonic component and noise interference component.

$$\boldsymbol{y}(t) = \sum_{i=1}^{m} \boldsymbol{TC}_i(t) + \boldsymbol{n}(t) \tag{6}$$

In above formula, $\boldsymbol{m}$ represents the number of target components, and the corresponding generalized multi-source sparse decomposition objective function can be expressed consequently as follows, $\boldsymbol{A_i} \in \mathrm{R}^{\mathrm{N\times M}}$ represents a series of sparse dictionaries corresponding to target component $\boldsymbol{TC}_i$, and $\boldsymbol{P}(\boldsymbol{c_1}, \cdots \boldsymbol{c_m})$ represents the multi-source generalized sparse penalty function.

$$\boldsymbol{J}(\boldsymbol{c_1}, \cdots, \boldsymbol{c_m}) = \frac{1}{2}\|\boldsymbol{y} - \sum_{i=1}^{m} \boldsymbol{A}_i\boldsymbol{c}_i\|_2^2 + \boldsymbol{P}(\boldsymbol{c_1}, \cdots \boldsymbol{c_m}) \tag{7}$$

It can be seen intuitively that each sparse representation coefficient $\boldsymbol{c_i}$ corresponding to target component $\boldsymbol{TC_i}$ can be obtained by calculating above objective function, so as to effectively obtain each signal component and realize multi-source sparse decomposition of mechanical system signal.

However, compared with single-component sparse decomposition, multi-component sparse decomposition confronts a large number of target components $\boldsymbol{TC_i}$, complex oscillation characteristics and possible mutual interference among each target component, significantly increasing the difficulty of sparsely optimal decomposition. It raises a higher standard for sparse sub-dictionary$\boldsymbol{A_i}$ construction of and multi-source sparse penalty function $\boldsymbol{P(c_1,\cdots c_m)}$ design. Around above content, this paper puts forward a series of innovative solutions consequently.

2.2 Sparse dictionary construction

In generally, sparse dictionary $\boldsymbol{A_i}$ determines decomposition effect in a large extent, which affects the precision of fault component extraction. According to the sparse dictionary source, it can be divided into two categories: analytic dictionary [12] and learning dictionary [13]. As for analytic dictionary, it has a high matching accuracy, while it need to comprehensively analyze signal oscillation characteristic, obtaining sufficient prior knowledge. On the other hand, learning dictionary does not require complex prior knowledge and is highly adaptive, but it takes a long time to train dictionary iteratively and the constructed dictionary lack distinctly physical meaning [14]. On account of multiply and complex characteristic in mechanical system signal, higher requirement is put forward for dictionary construction. In order to ensure signal accurate separation, it is necessary to construct an appropriate over-complete dictionary according to the oscillation characteristic in each target component $\boldsymbol{TC_i}$,

Based on analyzing time-domain waveform of multiple target components $\boldsymbol{TC_i}$ in signal as shown in Figure 1, the oscillating difference between steady-state harmonic component and impact component are quietly obvious, so it is convenient to design corresponding sparse dictionaries to match impact component and harmonic component. Figure 2 shows the sparse characteristic of shock target component and harmonic target component respectively by DFT dictionary. It can be seen that the DFT dictionary can effectively extract harmonic target component from the signal, so the DFT dictionary can be acted as the sparse sub-dictionary $\boldsymbol{A_0}$ corresponding to harmonic target component.

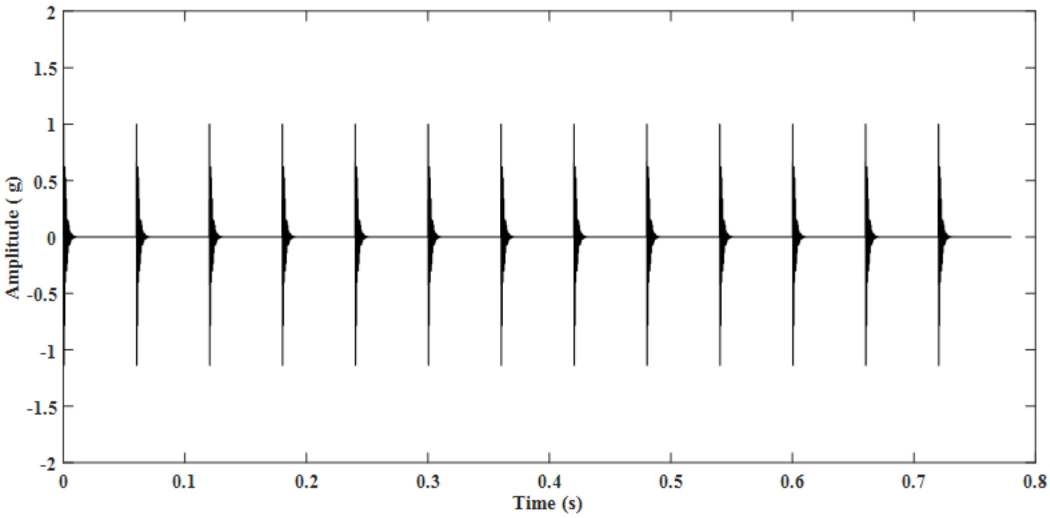

Figure 1(a) Impact component

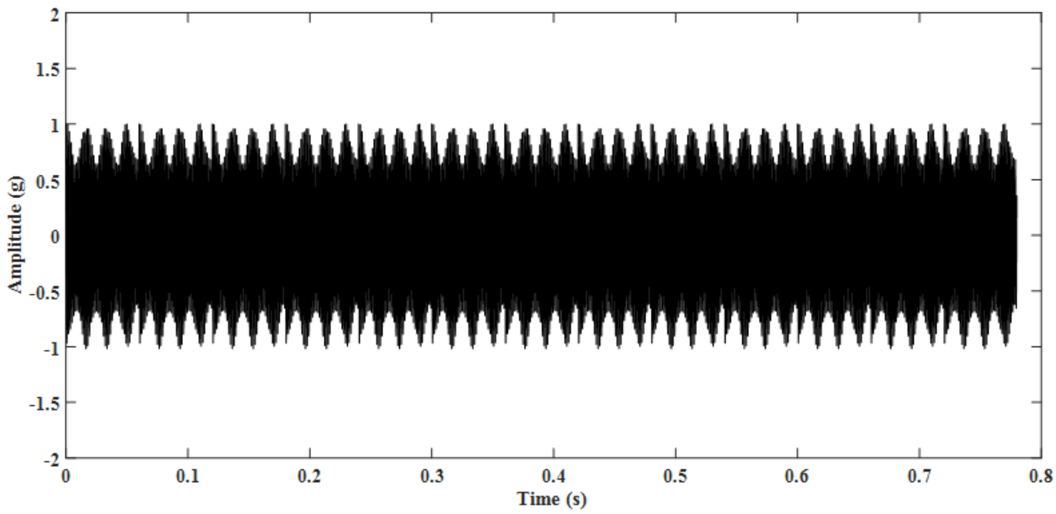

Figure 1(b) Steady-state harmonic component

Figure 1 Polymorphic-oscillatory signal

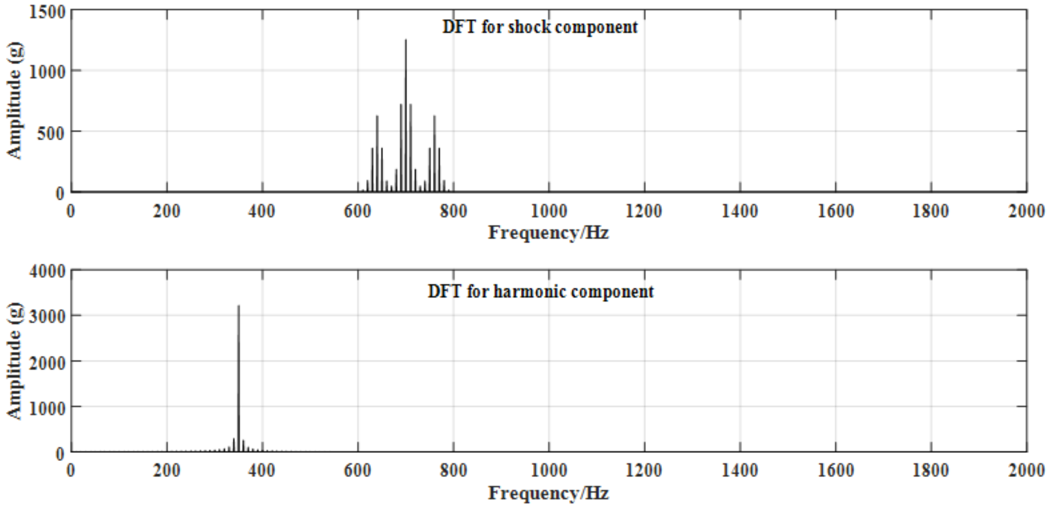

Figure 2 Sparse feature of polymorphic-oscillatory signal by DFT dictionary

As for various impact target components $\boldsymbol{TC_i}$ in mechanical system signal, due to their similar oscillation characteristics, it is difficult to separate and extract them directly through transform domain. Therefore, it is necessary to further construct the corresponding time domain dictionary based on impact target component characteristic in time domain, achieving impact target component extraction. Ultimately, the corresponding sparse sub-dictionary $\boldsymbol{A_1}$ can be constructed by Laplace wavelet function[15], as shown in following. $f_1$andB_1 respectively represent natural frequency and amplitude normalized parameter, τ_1 and ξ_1 respectively represent delay parameter and damping ratio.

$$A_1(t,\tau_1) = B_1 exp\left(\frac{-\xi_1}{\sqrt{1-\xi_1^2}} 2\pi f_1(t-\tau_1)\right) \times sin2\pi f_1(t-\tau_1) \tag{8}$$

At the same time, considering mechanical system signal contains bilateral attenuation characteristic, the corresponding sparse sub-dictionary $\boldsymbol{A_2}$ can be constructed by Morlet wavelet function identically[16], as shown in following. $f_2$andB_2 respectively represent natural frequency and amplitude normalized parameter, τ_2 and ξ_2 respectively represent delay parameter and damping ratio.

$$A_2(t,\tau_2) = B_2 exp\left(\frac{-\xi_2}{\sqrt{1-\xi_2^2}}[2\pi f_2(t-\tau_2)]^2\right) \times cos2\pi f_2(t-\tau_2) \tag{9}$$

Finally, Figure 3 presents that the established sparse dictionary is accord with transient shock target component in waveform.

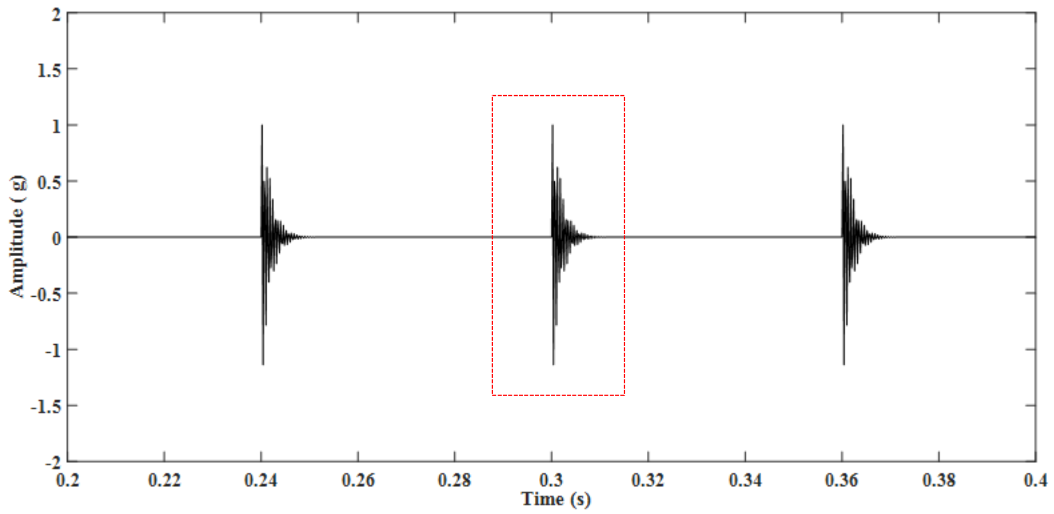

Figure 3(a) Unilateral impact attenuation component

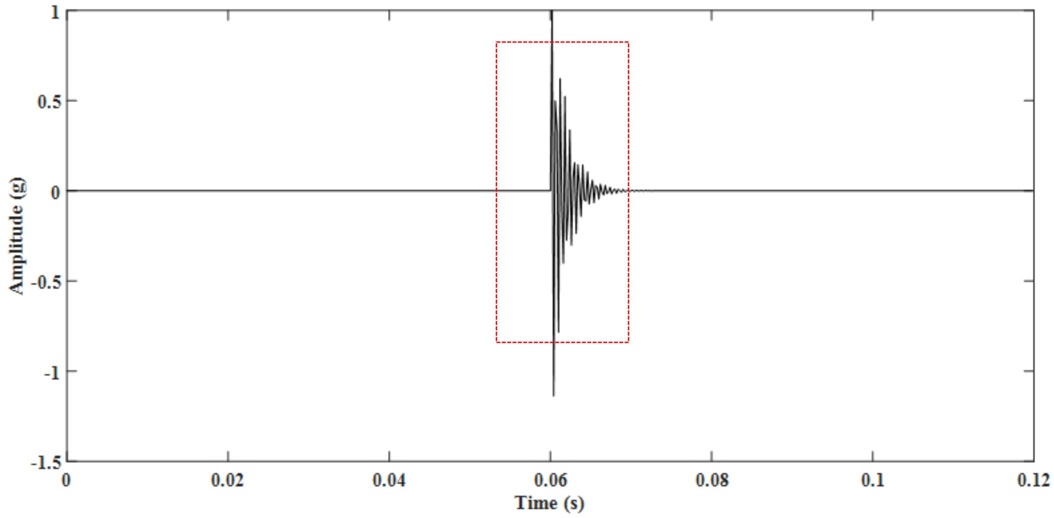

Figure 3(b) Laplace wavelet dictionary

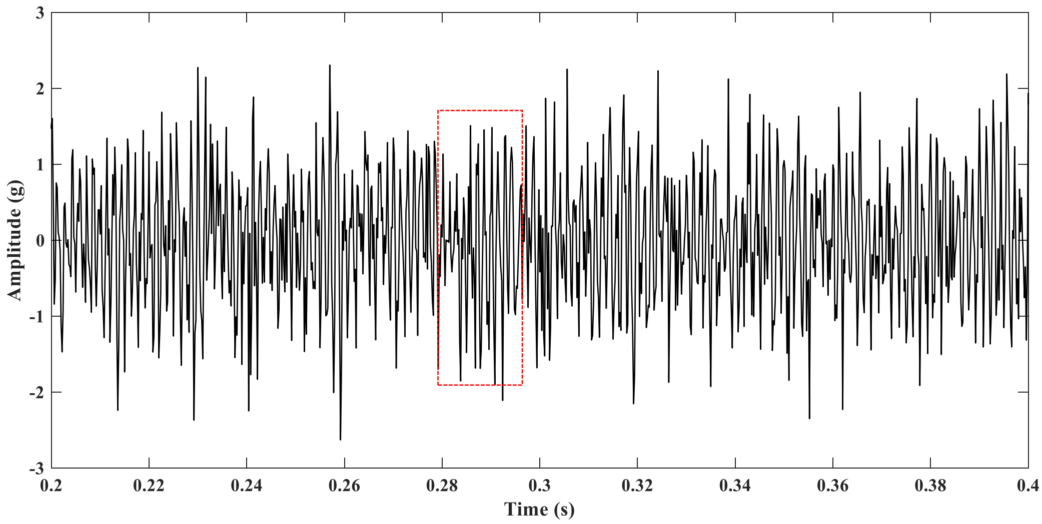

Figure 3(c) Bilateral attenuation impact component

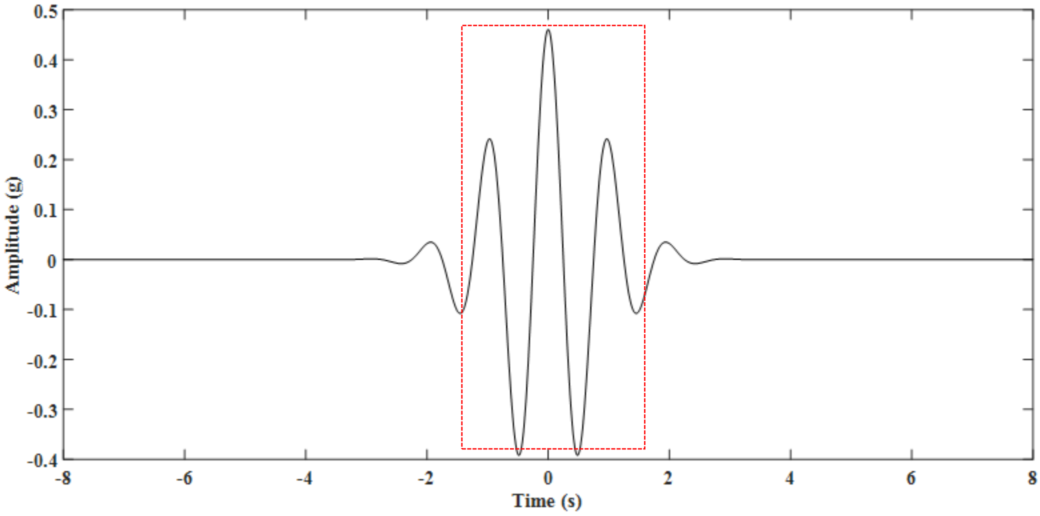

Figure 3(d) Morlet wavelet dictionary

Figure 3 Transient shock component and corresponding sparse sub-dictionary

2.3 Multi-source sparse penalty function design

On the other hand, another critical content of sparse decomposition is how to establish an effectively multi-component sparse penalty function $\boldsymbol{P}(\boldsymbol{c_1},\cdots \boldsymbol{c_m})$. As a key element of objective function $\boldsymbol{J}(\boldsymbol{c_1},\cdots,\boldsymbol{c_m})$, its amplitude preserving ability is directly concerned to the sparsity of sparse representation coefficient $\boldsymbol{c_i}$, thus obviously affecting the mechanical system morphological signal separation fidelity[17].

Table 1 exhibits the explicit function expression of common penalty functions (a=0.5), besides, Figure 4 further presents the mathematical law of these penalty functions and their corresponding threshold function, where soft threshold is corresponding to L1 norm threshold. Analyzing function characteristic, it can be inferred that when amplitude is significant, all threshold functions have a fixed distance withe y=x except the threshold function corresponding to MC penalty function. As a result, compared with L1 norm, parameterized non-convex penalty function has certain ability of preserving amplitude, especially, MC penalty function has better effect on preserving amplitude. However, non-convex penalty functions often introduce a series of optimization problems, for example, input disturbance has a great influence on the final solution, pseudo local minimum, no guaranteed global convergence and so on. In practical application, it is necessary to further carry out convexity derivation on parameterized non-convex penalty function to constrain parameters and ensure that the objective function is convex, so as to ensure that the optimization solution of objective function is not affected by initialization condition, and can finally converge to the global optimal value.

Table 1 Common sparse penalty function and its function expression

Sparse penalty function	Function expression
Logarithm	$\frac{1}{a}\log(1+a\lvert x\rvert)$
Arctan	$\frac{2}{a\sqrt{3}}\left(\tan^{-1}\left(\frac{1+2a\lvert x\rvert}{\sqrt{3}}\right)-\frac{\pi}{6}\right)$
MC	$\begin{cases} \lvert x\rvert-\frac{1}{2a}x^2 & \lvert x\rvert\le a \\ \frac{a}{2} & \lvert x\rvert\ge a \end{cases}$

L1 norm	$\lvert x\rvert$

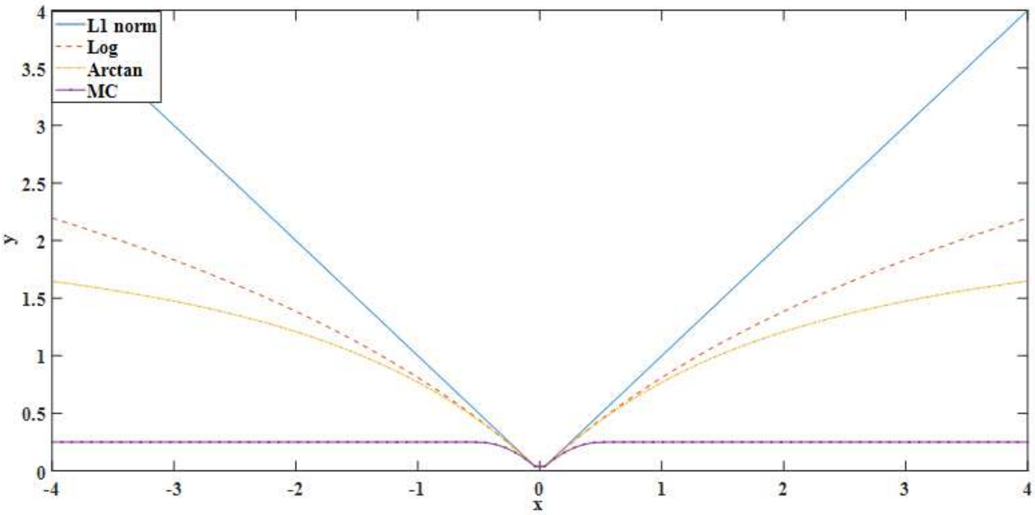

Figure 4(a) Sparse penalty functions

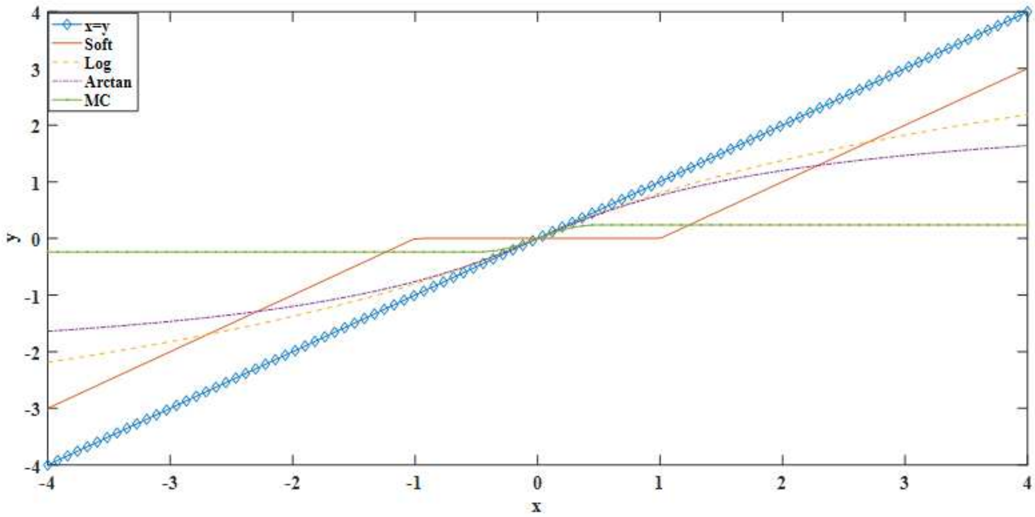

Figure 4(b) Threshold function

Figure 4 Common sparse penalty functions and its threshold functions

SELESNICK proposed GMC non-convex penalty function based on MC non-convex penalty function, which can effectively induce sparsity while maintaining the convexity of the objective function [18]. GMC non-convex penalty function possesses good signal fidelity and can effectively enhance mechanical system morphological signal sparsity, improving the efficiency of signal feature extraction. As a result, on account of comparison and analysis on various penalty functions fidelity, a multi-source parameterized non-convex penalty function $\boldsymbol{P}(\boldsymbol{c_1},\cdots \boldsymbol{c_m})$ for mechanical system signal feature extraction is constructed based on GMC penalty function.

GMC non-convex penalty function is derived from the generalization of unitary Huber function $s_b(x)$. Assuming that v is a point in the domain of x, the corresponding unitary Huber function can be expressed as the following mathematical model.

$$s_b(x) = \min_{v\epsilon R}\left\{|v| + \tfrac{1}{2}b^2(x-v)^2\right\} \tag{10}$$

Assuming the existence of matrix $\boldsymbol{B}\epsilon\mathrm{R}^{\mathrm{M\times N}}$, the correspondingly generalized Huber function can be expressed furthermore.

$$s_B(x) = \min_{B\epsilon R^{M\times N}}\left\{\|v\|_1 + \tfrac{1}{2}\|\boldsymbol{B}(x-v)\|_2^1\right\} \tag{11}$$

On this basis, the corresponding GMC non-convex penalty function is deduced as follows. Where, λ represents the sparse regularization parameter. Finally , they are shown in following.

$$P_B(x) = \lambda\|x\|_1 - \lambda s_B(x) \tag{12}$$

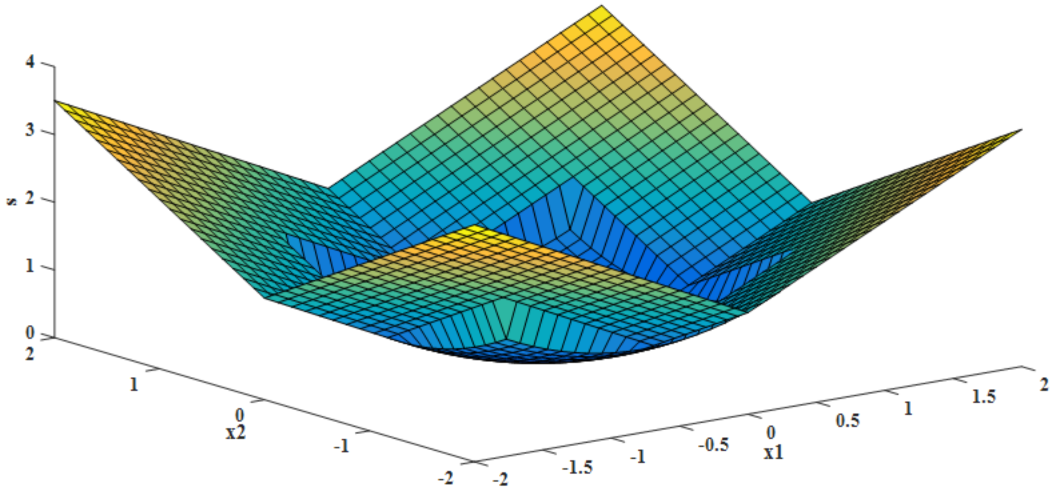

Figure 4 Generalized Huber function

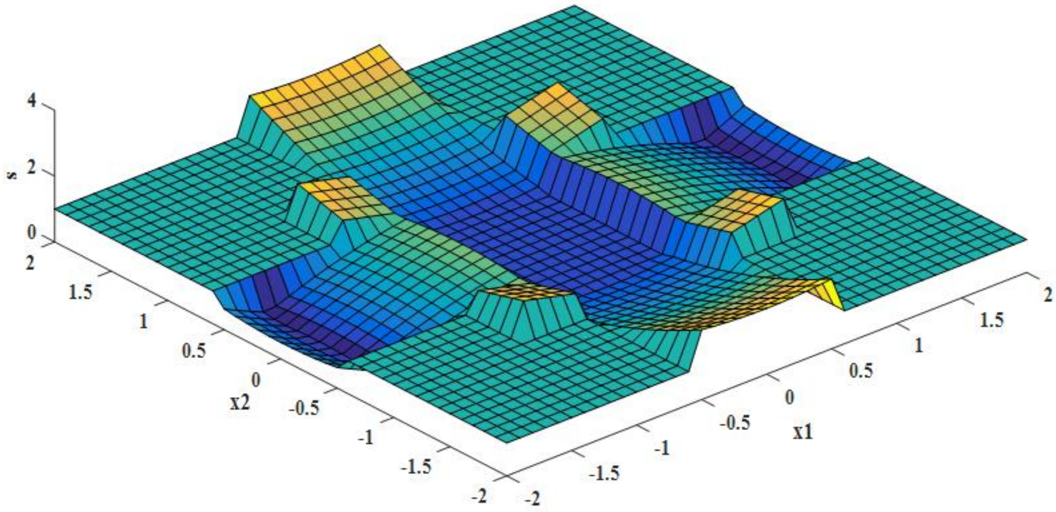

Figure 5 GMC non-convex penalty function

The above GMC non-convex penalty function can remarkable ensure the sparsity of sparse representation coefficient $\boldsymbol{c_i}$ under the corresponding over-complete sparse sub-dictionary $\boldsymbol{A_i}$, and avoid amplitude underestimation caused by L1 norm simultaneously, thus achieving signal decomposition fidelity. However, considering the goal in this paper is to implement polymorphic-oscillatory mechanical system signal separation, which requires sparse induction of multiple target components $\boldsymbol{TC_i}$. In this paper, based on the structural characteristics of GMC penalty function, and mechanical system signal, convexity preservation play a role in principle, the following multi-source parameterized non-convex penalty function is derived. Where, m represents the total number of target component, and i represents the i-th target component, λ symbolizes regularization parameter, $\boldsymbol{B}$ and $\boldsymbol{v}$ are the parameterized matrix and vector.

$$\boldsymbol{P}(\boldsymbol{c_1}, \cdots, \boldsymbol{c_m}) = \sum_{i=1}^{m} \lambda_i \|c_i\|_1 - \min_{v_i} \left\{ \sum_{i=1}^{m} \lambda_i \|v_i\|_1 + \frac{1}{2} \left\| \sum_{i=1}^{m} \boldsymbol{B_i}(c_i - v_i) \right\|_2^2 \right\} \tag{13}$$

2.3.1 Convex condition of objective function

On the other hand, given that the convexity preserving condition of objective function $\boldsymbol{J}(\boldsymbol{c_1}, \cdots, \boldsymbol{c_m})$ is the premise to ensure that sparse decomposition can obtain global optimal solution that convex optimization theory can be applied to solve.

Combination with above formula, the generalized multi-source sparse decomposition objective function as shown in formula (7) can be further expressed in following form.

$$\boldsymbol{J}(\boldsymbol{c_1},\cdots,\boldsymbol{c_m}) = \frac{1}{2}\|y - \sum_{i=1}^{m} A_i c_i\|_2^2 + \sum_{i=1}^{m} \lambda_i \|c_i\|_1 - \min_{v_1\cdots v_i}\left\{\sum_{i=1}^{m} \lambda_i \|v_i\|_1 + \frac{1}{2}\|\sum_{i=1}^{m} \boldsymbol{B_i}(c_i - v_i)\|_2^2\right\} \quad (14)$$

In order to express the objective function conveniently, the following definition is introduced.

$$\boldsymbol{A} = [\boldsymbol{A_1} \quad \cdots \quad \boldsymbol{A_m}], \boldsymbol{B} = [\boldsymbol{B_1} \quad \cdots \quad \boldsymbol{B_m}], \lambda = \begin{bmatrix} \lambda_1 \boldsymbol{I_1} \\ \vdots \\ \lambda_m \boldsymbol{I_{1m}} \end{bmatrix}, \boldsymbol{c} = \begin{bmatrix} c_1 \\ \vdots \\ c_m \end{bmatrix}, \boldsymbol{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_m \end{bmatrix} \quad (15)$$

Accordingly, the generalized multi-source sparse decomposition objective function can be simplified into vector form as follows.

$$\begin{cases} J(\boldsymbol{c},\boldsymbol{v}) = \frac{1}{2}\boldsymbol{c}^{\boldsymbol{T}}(\boldsymbol{A}^{\boldsymbol{T}}\boldsymbol{A} - \boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{B})\boldsymbol{c} + \lambda\|\boldsymbol{c}\|_1 + \max_{v}\{g(\boldsymbol{c},\boldsymbol{v})\} \\ g(\boldsymbol{c},\boldsymbol{v}) = \frac{1}{2}y^T y - \lambda\|\boldsymbol{v}\|_1 + (\boldsymbol{v}^{\boldsymbol{T}}\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{B} - y^T\boldsymbol{A})\boldsymbol{c} - \frac{1}{2}\boldsymbol{v}^{\boldsymbol{T}}\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{B}\boldsymbol{v} \end{cases} \quad (16)$$

As seen in formula (16), $\max_{v}\{g(\boldsymbol{c},\boldsymbol{v})\}$ is the point-by-point maximum value of convex function of sparse representation coefficient $\boldsymbol{c}$, $\|\boldsymbol{c}\|_1$ is a L1 norm and also presents convex characteristic. As a result, on account of convex function property, if and only if $\frac{1}{2}\boldsymbol{c}^{\boldsymbol{T}}(\boldsymbol{A}^{\boldsymbol{T}}\boldsymbol{A} - \boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{B})\boldsymbol{c}$is a convex function, $J(\boldsymbol{c},\boldsymbol{v})$ is also a convex function naturally. In conclusion, the convex preserving criteria is shown in following.

$$\boldsymbol{A}^{\boldsymbol{T}}\boldsymbol{A} - \boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{B} \geq 0 \quad (17)$$

Expressing in matrix form as follows.

$$\begin{pmatrix} \boldsymbol{A_1^T A_1} - \boldsymbol{B_1^T B_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \boldsymbol{A_i^T A_i} - \boldsymbol{B_i^T B_i} \end{pmatrix} \geq 0 \quad (18)$$

Finally, the convex preserving parameter condition of objective function is shown in following.

$$\boldsymbol{B_i} = \sqrt{\gamma}\boldsymbol{A_i} \quad 0 \leq \gamma \leq 1 \quad (19)$$

In this paper, $\gamma =$ 0.5, therefore, the objective function as shown in formula (16) is already strictly convex, as long as the relationship between parameterized matrix $\boldsymbol{B_i}$and sparse dictionary $\boldsymbol{A_i}$conform to above criterion, the multi-source sparse decomposition problem can also be transformed into a convex optimization solution problem.

`Based on established convex preserving multi-source sparse optimization objective function, in order to achieve polymorphic-oscillatory mechanical system signal accurate separation, convex optimization algorithm is applied to solve objective function and obtain corresponding sparse representation coefficient $\boldsymbol{c_i}$, extracting target components $\boldsymbol{TC_i}$effectively, it can be expressed as a saddle point solution problem as shown in following.

$$(\boldsymbol{c}^{\boldsymbol{opt}},\boldsymbol{v}^{\boldsymbol{opt}}) = arg\min_{c}\max_{v}\left\{\frac{1}{2}\|y - \boldsymbol{A}\boldsymbol{c}\|_2^2 + \lambda\|\boldsymbol{c}\|_1 - \lambda\|\boldsymbol{v}\|_1 - \frac{\gamma}{2}\|\boldsymbol{A}(\boldsymbol{c} - \boldsymbol{v})\|_2^2\right\} \quad (20)$$

Besides, the Forward-Backward Splitting algorithm (FBS)[19] is applied to solving sparse representation coefficient c_i, which can quickly and accurately achieve mechanical system signal separation. Firstly, objective function $J(\boldsymbol{c},\boldsymbol{v})$ takes partial derivatives respectively as shown in following, *sgn* represents symbolic function.

$$\partial_c J(\boldsymbol{c},\boldsymbol{v}) = \boldsymbol{A}^T(\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}) - \gamma\boldsymbol{A}^T\boldsymbol{A}(\boldsymbol{x} - \boldsymbol{v}) + \lambda sgn(\boldsymbol{x}) \quad (21)$$

$$\partial_v J(\boldsymbol{c},\boldsymbol{v}) = \gamma\boldsymbol{A}^T\boldsymbol{A}(\boldsymbol{x} - \boldsymbol{v}) - \lambda sgn(\boldsymbol{v}) \quad (22)$$

$$sgn(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases} \quad (23)$$

Rewriting the above formula (21-22) in matrix form.

$$\begin{bmatrix}\partial_c J(\boldsymbol{c},\boldsymbol{v})\\ \partial_v J(\boldsymbol{c},\boldsymbol{v})\end{bmatrix}=\begin{bmatrix}\boldsymbol{A}^T(\boldsymbol{A}\boldsymbol{x}-\boldsymbol{y})-\gamma\boldsymbol{A}^T\boldsymbol{A}(\boldsymbol{x}-\boldsymbol{v})+\lambda sgn(\boldsymbol{x})\\ \gamma\boldsymbol{A}^T\boldsymbol{A}(\boldsymbol{x}-\boldsymbol{v})-\lambda sgn(\boldsymbol{v})\end{bmatrix}\tag{24}$$

Substituting it into FBS iterative optimization algorithm.

$$\begin{bmatrix}\boldsymbol{\omega}^{(k)}\\ \boldsymbol{\theta}^{(k)}\end{bmatrix}=\begin{bmatrix}\boldsymbol{c}^{(k)}-\mu\boldsymbol{A}^T\left(\boldsymbol{A}\boldsymbol{c}^{(k)}-\boldsymbol{y}-\gamma\boldsymbol{A}\left(\boldsymbol{c}^{(k)}-\boldsymbol{v}^{(k)}\right)\right)\\ \boldsymbol{v}^{(k)}-\mu\gamma\boldsymbol{A}^T\boldsymbol{A}\left(\boldsymbol{v}^{(k)}-\boldsymbol{c}^{(k)}\right)\end{bmatrix}\tag{25}$$

$$\begin{bmatrix}c^{(k+1)}\\ v^{(k+1)}\end{bmatrix}=\begin{bmatrix}\min\limits_{c}\left\{\frac{1}{2}\left\|\omega^{(k)}-c\right\|_2^2+\mu\lambda\|c\|_1\right\}\\ \min\limits_{c}\left\{\frac{1}{2}\left\|\theta^{(k)}-v\right\|_2^2+\mu\lambda\|v\|_1\right\}\end{bmatrix}\tag{26}$$

Above formula is equivalent to following form.

$$\begin{bmatrix}\boldsymbol{c}^{(k+1)}\\ \boldsymbol{v}^{(k+1)}\end{bmatrix}=\begin{bmatrix}soft\left(\boldsymbol{\omega}^{(k)},\mu\lambda\right)\\ soft\left(\boldsymbol{\theta}^{(k)},\mu\lambda\right)\end{bmatrix}\tag{27}$$

μ represents constant parameter, conforming to $0<\mu<2/\rho$, and the detailed expression is shown below.

$$\rho=max\left\{1,\frac{\gamma}{(1-\gamma)}\right\}\|\boldsymbol{A}^T\boldsymbol{A}\|_2\tag{28}$$

$$soft(x,T)=x\cdot max\left\{0,1-\frac{T}{|x|}\right\}\tag{29}$$

In conclusion, the optimization solution algorithm of multi-source fidelity sparse decomposition model can be summarized as shown in Table 2, and the proposed method in this paper is shown in Figure 6.

Table 2 Forward-Backward Splitting to extract conditional target component

① Given mechanical system signal $\boldsymbol{y}(\boldsymbol{t})$,sparse sub-dictionary $\boldsymbol{A_i}$, regularization parameter λ_i,constant parameter u,γ, iterative parameter N_{iter}
② Solving coefficient is converted to saddle point solution, as shown $(\boldsymbol{c^{opt}},\boldsymbol{v}^{opt})= arg\min\limits_{c}\max\limits_{v}\left\{\frac{1}{2}\|y-\boldsymbol{A}\boldsymbol{c}\|_2^2+\lambda\|\boldsymbol{c}\|_1-\lambda\|\boldsymbol{v}\|_1-\frac{\gamma}{2}\|\boldsymbol{A}(\boldsymbol{c}-\boldsymbol{v})\|_2^2\right\}$
③ Initialization $\boldsymbol{c}_i^{(0)}=\boldsymbol{c}_i^T\boldsymbol{y}$ $\boldsymbol{v}_i^{(0)}=0.$
④ While (N<N_{iter}) for i = 1, 2, …, m $$\boldsymbol{\omega}_i^{(k)}=\boldsymbol{c}_i^{(k)}-\mu\boldsymbol{A}_i^T\left(\sum_{i=1}^{m}\boldsymbol{A}_i\boldsymbol{c}_i^k-\boldsymbol{y}-\gamma\sum_{i=1}^{m}\boldsymbol{A}_i\left(\boldsymbol{c}_i^k-\boldsymbol{v}_i^k\right)\right)$$ $$\boldsymbol{\theta}_i^{(k)}=\boldsymbol{v}_i^k-\mu\gamma\left[\boldsymbol{A}_i^T\left(\sum_{i=1}^{m}\boldsymbol{A}_i\left(\boldsymbol{v}_i^{(k)}-\boldsymbol{c}_i^k\right)\right)\right]$$ $$\boldsymbol{c}_i^k=soft\left(\boldsymbol{\omega}_i^{(k)},\mu\lambda_i\right)$$ $$\boldsymbol{v}_i^{k+1}=soft\left(\boldsymbol{\theta}_i^{(k)},\mu\lambda_i\right)$$ Renewal rate $$\Delta=max\left(\frac{\sum_{i=1}^{m}\left\|\boldsymbol{c}_i^{k+1}-\boldsymbol{c}_i^k\right\|_2}{\sum_{i=1}^{m}\left\|\boldsymbol{c}_i^{k+1}\right\|_2},\frac{\sum_{i=1}^{m}\left\|\boldsymbol{v}_i^{k+1}-\boldsymbol{v}_i^k\right\|_2}{\sum_{i=1}^{m}\left\|\boldsymbol{v}_i^{k+1}\right\|_2}\right)$$
⑤ Extracting target component estimated value $\boldsymbol{TC_i^*}=\boldsymbol{A}_i\boldsymbol{c}_i^{opt}$.

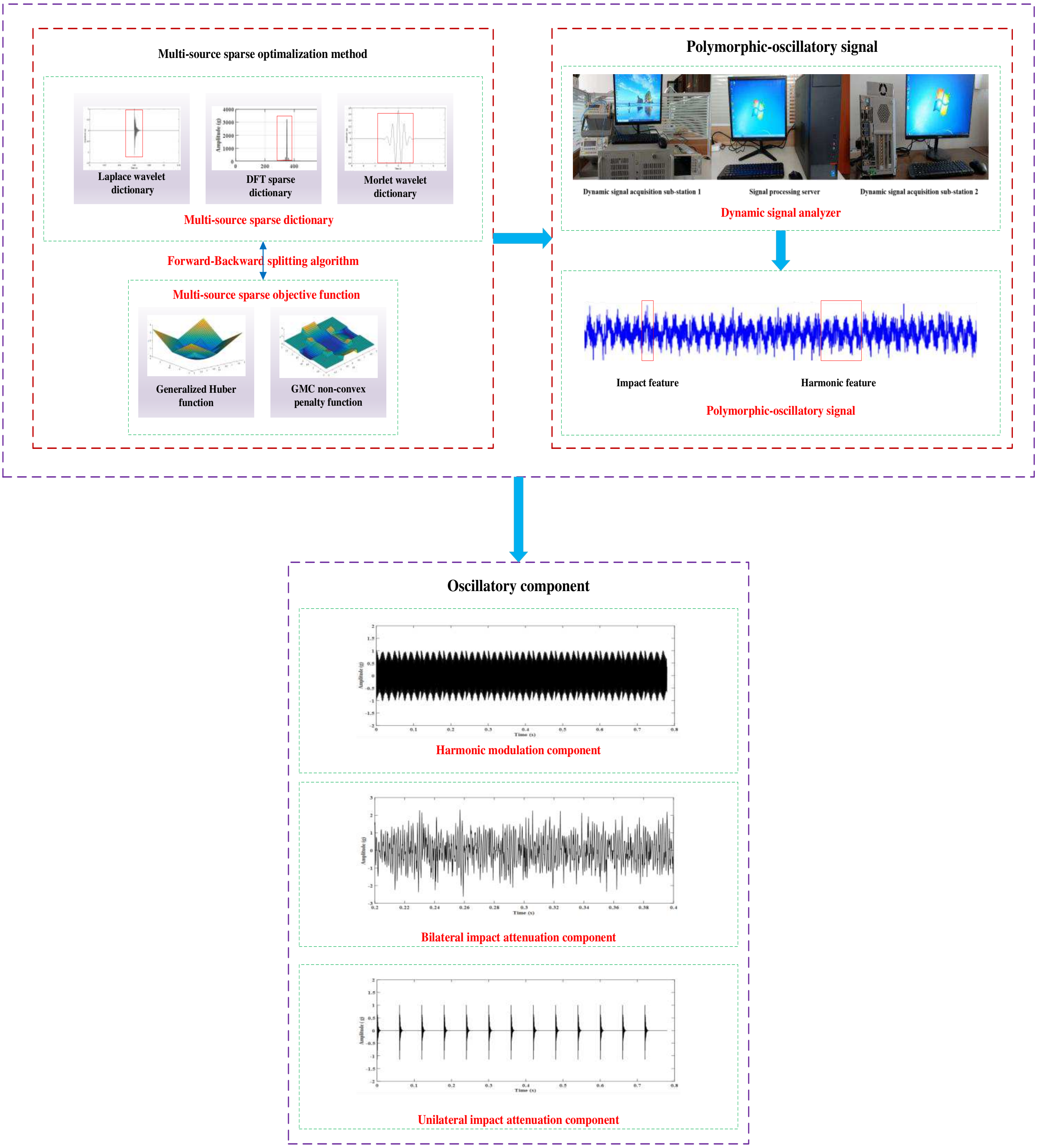

Figure 6 The proposed method in this paper

3. Comparison method

In order to verify the effectiveness of proposed method in this paper, three signal decomposition methods are adopted, resonant sparse signal decomposition (RSSD) based on L1 norm, signal time-domain adaptive decomposition and spectrum kurtosis, possessing own characteristics respectively. As for RSSD, it's sparse objective function constructed based on L1 norm penalty function, which is widely used in weak signal decomposition[20]. By comparing with the proposed method in this paper, the superiority of GMC non-convex penalty function can be verified. On the other hand, for signal time-domain adaptive decomposition, it can realize signal adaptive decomposition through specific iterative algorithm or model optimization algorithm in time domain according to the signal time-scale characteristic without setting up basis function in advance. This is the essential difference between it and sparse decomposition, which requires giving correspondingly sparse dictionary. Ultimately, spectrum kurtosis is very sensitive to shock component in signal[21], which is important to identify mechanical system working state. Therefore, several comparison signal decomposition algorithms selected are representative and can comprehensively evaluate the reliability of proposed method in this paper.

3.1 Resonance Sparse Signal Decomposition

In general, the theoretical basis of RSSD mainly includes two aspects: Tunable-Q wavelet transform (TQWT) and signal sparse decomposition. TQWT can directly specify Q-factor (any positive real value) which is primarily plays an important role in evaluting the oscillation characteristic of variable target component in given signal $\boldsymbol{y(t)}$, and redundancy factor r to design wavelet, which further increases the flexibility of quality factor selection and makes wavelet acquisition more convenient. Besides, a series of wavelet basis parameters, such as low-pass scale factor α and high-pass scale factor β can be obtained spontaneously as following.

$$\beta = \frac{2}{Q+1} \qquad \alpha = 1 - \frac{\beta}{r} \tag{30}$$

In TQWT, sub-wavelet contained in wavelet basis function library is generated by certain Q value and r value and has an uniform quality factor Q value. However, the sub-wavelet center frequency f_c and bandwidth B_W with different decomposition series J are various. Assuming the sampling frequency of given signal $\boldsymbol{y(t)} \in \boldsymbol{R}^N$ is F_S, then the correspondingly center frequency and bandwidth of each wavelet are calculated by the following formula.

$$f_c \approx \alpha^j \frac{2-\beta}{4\alpha} f_s \tag{31}$$

$$B_W = \frac{1}{2}\beta\alpha^{j-1}\pi \tag{32}$$

As a result, once the quality factor Q, redundancy r and decomposition series J has been determined, the quality factor tunable wavelet basis function library with the same signal characteristic structure is also constructed, besides, each tunable wavelet is a sub-band contained in the wavelet base function library substantially.

In summary, RSSD uses quality factor tunable wavelet to construct a series of basis functions, which is distinguish with the proposed method in this paper, it utilizes Laplace wavelet dictionary, Morlet wavelet dictionary and DFT dictionary.

On the other hand, tunable Q-factor wavelet is accuracy accouplement with target component $\boldsymbol{TC_i}$, and RSSD uses morphological component analysis to separate signal according to the oscillation characteristic, and establishes the optimal sparse representation of high resonance component and low resonance component respectively. Consequently, it can extract only two target components $\boldsymbol{TC_1}$, $\boldsymbol{TC_2}$ in given signal, in comparison, the parameter m which symbolizes the extracted number of target component $\boldsymbol{TC_i}$ can be set arbitrarily, for verification, the value $m = 3$.

Assuming that the target components $\boldsymbol{TC_1}$, $\boldsymbol{TC_2}$ correspond to the high and low resonance components respectively, and $S_H = (s_{H,1}, s_{H,2}, \cdots, s_{H,jH+1})$, $S_L = (s_{L,1}, s_{L,2}, \cdots, s_{L,jL+1})$ are high quality factor tunable wavelet basis function and low quality factor tunable wavelet basis function which are used to extract high and low resonance component respectively, each element corresponds to the sub-wavelet function in the library. j_H, j_L is the decomposition series corresponding to S_H, S_L.

$W_H = (w_{H,1}, w_{H,2}, \cdots, w_{H,jH+1})^T$ and $W_L = (w_{L,1}, w_{L,2}, \cdots, w_{L,jL+1})^T$ is the sparse decomposition coefficient. Finally, the sparse objective function is shown in following.

$$J(W_H, W_L) = \|x - S_H W_H - S_L W_L\|_2^2 + \sum_{m=1}^{j_H+1} \lambda_{H,m} \|w_{H,m}\|_1 + \sum_{n=1}^{j_L+1} \lambda_{L,n} \|w_{L,n}\|_1 \quad (33)$$

In above formula, $(x - S_H W_H - S_L W_L)$ represents residual component in the form of L2 norm, $\sum_{m=1}^{j_H+1} \lambda_{H,j} \|w_{H,m}\|_1$ and $\sum_{n=1}^{j_L+1} \lambda_{L,n} \|w_{L,n}\|_1$ represent the target components sparsity degree respectively based on L1 norm. However, the proposed method in this paper is based on MC penalty function. As L1 norm is non-differentiable, RSSD adopts splitting augmented Lagrange search algorithm to minimize the objective function $J(W_H, W_L)$ by updating the transform coefficients W_H and W_L by iteration, and achieves the effective separation of high resonance component and low resonance component finally.

3.2 Signal Time-Domain Adaptive Decomposition

Compared with classical frequency domain decomposition based on band division or sparse decomposition in advance of setting basis function, at present, most signal time-domain adaptive decomposition methods extract or obtain target components directly in the time domain. These methods realize signal decomposition through specific iterative algorithm or model optimization algorithm, such as local mean decomposition(LMD), intrinsic time-scale decomposition(ITD). Ultimately, this paper selects LMD as a representative method to compare with multi-source sparse optimization method.

LMD can self-adaptive divide a signal into a series of production function (PF) components and effectively process polymorphic-oscillatory mechanical system signal, each PF component is multiplied by an envelope element and a pure frequency modulation (FM) element. Finally, the decomposition procedures are as follows.

Step 1: Finding out all the extreme points n_i of signal $y(t)$, and then calculate the mean value $m_i = \frac{(n_i + n_{i+1})}{2}$ and local envelope value $a_i = \frac{|n_i - n_{i+1}|}{2}$ of all adjacent local extreme points.

Step 2: Smoothing local mean value m_i and local envelope value a_i by moving average function to obtain corresponding local mean function $m_{11}(t)$ and local envelope function $a_{11}(t)$.

Step 3: Separating local mean function $m_{11}(t)$from signal $y(t)$and obtaining $h_{11}(t) = x(t) - m_{11}(t)$. Furthermore, dividing by the local envelope function $a_{11}(t)$ for demodulation, obtaining $s_{11}(t) = \frac{h_{11}(t)}{a_{11}(t)}$, if $s_{11}(t)$ is a pure FM element, it's local envelope function $a_{12}(t) = 1$. Otherwise, if $a_{12}(t) \neq 1$, it indicates that $s_{11}(t)$ is not a pure FM element, and the iterative process of step (1) ~ (3) is repeated until $s_{1n}(t)$ is a pure FM element or $a_{1n}(t) = 1$.

Step 4: The envelope element $a_1(t) = a_{11}(t) a_{12}(t) \cdots a_{1n}(t)$ corresponding to first PF component can be obtained by multiplying a series of local envelope functions $a_{1n}(t)$ generated in above iteration process.

Step 5: By multiplying envelope element $a_1(t)$ and pure FM element $s_{1n}(t)$, the first PF component in signal $y(t)$ is obtained, as shown $PF_1(t) = a_1(t) s_{1n}(t)$.

Step 6: The residual signal $u_1(t)$ is obtained by subtracting first PF component $PF_1(t)$, and $u_1(t)$ repeats above steps as original signal. Through k cycles, all PF components are separated until $u_k(t)$ is a monotone function. Finally, the signal $y(t)$ is represented as the sum of a series of PF_i components, $y(t) = \sum_{i=1}^{k} PF_i + u_k$.

As shown above, mechanical system signal is initially a multi-component amplitude and frequency modulation signal, which showed polymorphic oscillation characteristic. LMD decomposition is applied on signal, a set of PF components and a residual component can be obtained which exhibit complete modulation property, or oscillation feature, highlighting signal local characteristic information. As a result, it is appropriate to select LMD as a comparison algorithm in this paper.

4.Discussion

4.1 Simulation signal study

In practical engineering application, mechanical systems often produce impact feature due to partial fault, and harmonic feature for distributed fault, consequently, for general mechanical system, it can be assumed that the number of oscillating target components in signal is $\boldsymbol{m}=3$, and corresponding objective function can be expressed as following.

$$\boldsymbol{J}(\boldsymbol{c_1},\boldsymbol{c_2},\boldsymbol{c_3})=\frac{1}{2}\left\|y-\sum_{i=1}^{3}A_ic_i\right\|_2^2+P(c_1,c_2,c_3) \tag{34}$$

In above formula, i represents the i-th target component, and A_i represents the corresponding sub-sparse dictionary, $P(c_1,c_2,c_3)$ symbolizes sparse penalty function. The multi-source fidelity sparse decomposition model can effectively realize sparse decomposition of mechanical system polymorphic signal contained oscillatory target components greater than 2. Due to mechanical system fault inevitability, the corresponding fault signal usually possesses unilateral attenuating impact fault component, bilateral attenuating impact fault component and harmonic interference component, that is, the number of fault types is at least 3. Therefore, in order to verify the effectiveness on mechanical multi-fault feature extraction and fault diagnosis by using proposed method with difficult decomposition and high computational complexity, this paper primarily constructs a composite signal with oscillatory morphological component $\boldsymbol{m}=3$, as shown below. Particularly, resonance frequency $f=1\ 800\ Hz$, damping ratio $\xi=0.050$, time delay $\tau=0.05\ s$, impulse period $T=0.15\ s$, amplitude adjustment coefficient $B=1$. On the other hand, $h(t)$ presents the harmonic modulation characteristic, $f_A=60\ Hz, f_F=10\ Hz, f_H=700Hz$. $\boldsymbol{n}(t)$ symbolizes noise interference component, and corresponding time domain waveform is shown in following ultimately.

$$\boldsymbol{y}(t)=x_1(t)+x_2(t)+x_3(t)+\boldsymbol{n}(t) \tag{35}$$

$$x_1(t)=\sum_k\left[Bexp\left(\frac{-\xi}{\sqrt{1-\xi^2}}2\pi f(t-\tau-kT)\right)\times\sin 2\pi f(t-\tau-kT)\right] \tag{36}$$

$$x_2(t)=(1.5+0.3\cos(2\pi f_At))\times\cos(2\pi f_Ht+0.2\cos(2\pi f_Ft)) \tag{37}$$

$$x_3(t)=\sum x_1(t)+x_2(t) \tag{38}$$

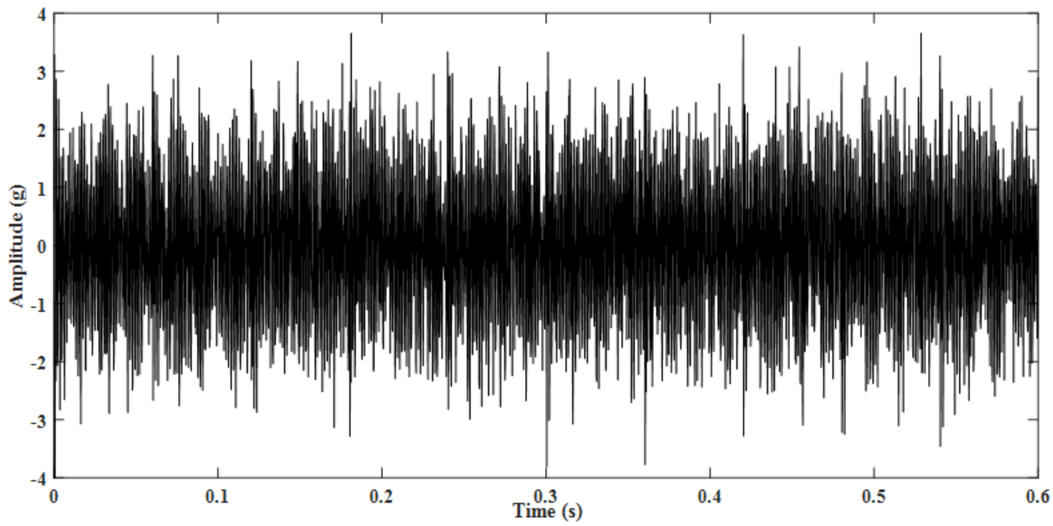

Figure 7 Simulated polymorphic-oscillatory signal

As can be seen intuitively from Figure 7, the impact component contained in simulation signal has been completely submerged by harmonic modulation component and noise interference, leading to decomposition prominently difficult.

Consequently, the proposed method is used to verify the separation reliability. In order to accurately evaluate the similarity between the extracted component $\hat{y}$ and the real component y, introducing correlation coefficient CC.

$$\boldsymbol{CC} = \frac{\langle \hat{y}, y \rangle}{\|\hat{y}\|_2 \|y\|_2} \tag{39}$$

According to the procedure illustrated in Figure 6, Laplace function, Morlet function and DFT transformation are suitable for the sparse representation of unilateral attenuating target component $x_1(\mathrm{t})$, bilateral attenuating target component $x_3(t)$ and harmonic target component $x_2(t)$. Subsequently, prior to signal decomposition, the regularization parameters λ_1, λ_3 ,λ_2 need to be set to appropriate values. In this paper, ergodic selection strategy is used to determine the optimal regularization parameters. During the searching process, the step size is set to 0.1, and the parameter variable range is from 1 to 5, and the overall extraction effect is described by the average correlation coefficient $CC = (CC_1 + CC_2 + CC_3)/3$. Ultimately, the optimal regularization parameters are calculated $\lambda_1 = 4.2, \lambda_2 = 3.1, \lambda_3 = 3.1$, and corresponding maximum correlation coefficient $\mathrm{CC} = 0.9574$ as shown in following.

The time-domain waveform of extracted target component $\boldsymbol{TC_i}$ is shown in Figure 8, by analyzing correlation coefficient $\boldsymbol{CC_i}$, it can be seen that the extracted target component and the original signal component has high similarity, indicating that the proposed method can effectively separate polymorphic-oscillatory signal. Besides, Figure 9 is a simulation signal partial magnification, which can be distinctly seen that the amplitude of each extracting sub-signal is basically consistent with the target component $\boldsymbol{TC_i}$, indicating the signal fidelity of proposed method. Furthermore, part of pulse amplitude is slightly higher than original target component, as extracting sub-signal still contains small background noise which increases signal amplitude. Considering that the simulation signal contains bilateral attenuating impact component $x_2(t)$, Morlet wavelet dictionary in this paper can effectively match corresponding sparse representation coefficients, so as to identify accurately transient impact component with different oscillation characteristics, as the fault information of mechanical system is mainly contained in impact envelope component of signal, it indicates that this method has potential value in the field of mechanical fault diagnosis.

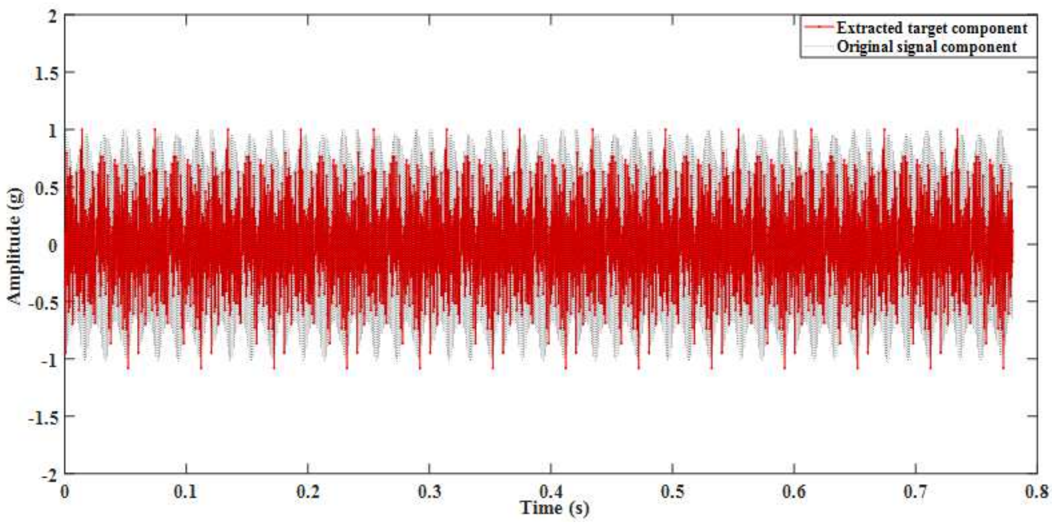

Figure 8(a) Harmonic target component

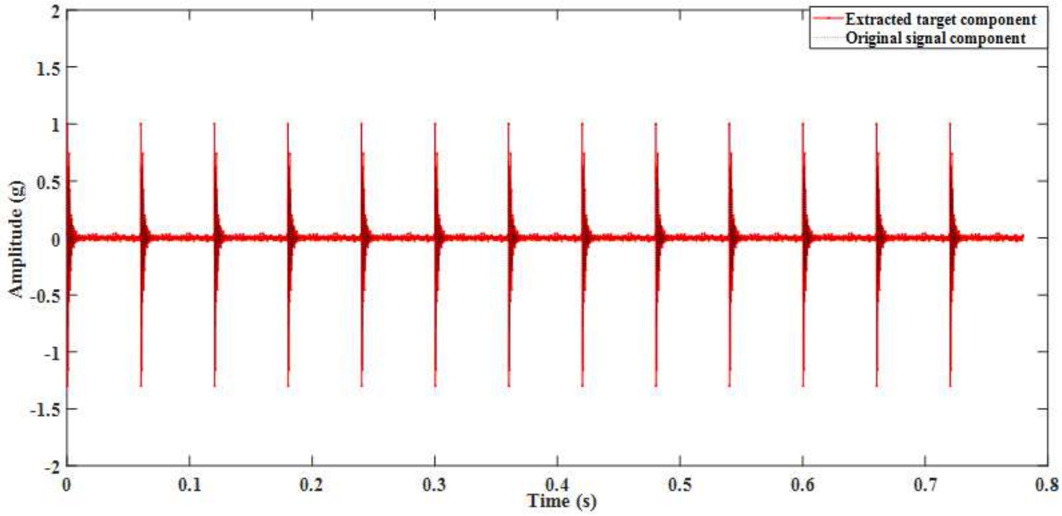

Figure 8(b) Unilateral attenuating impact component

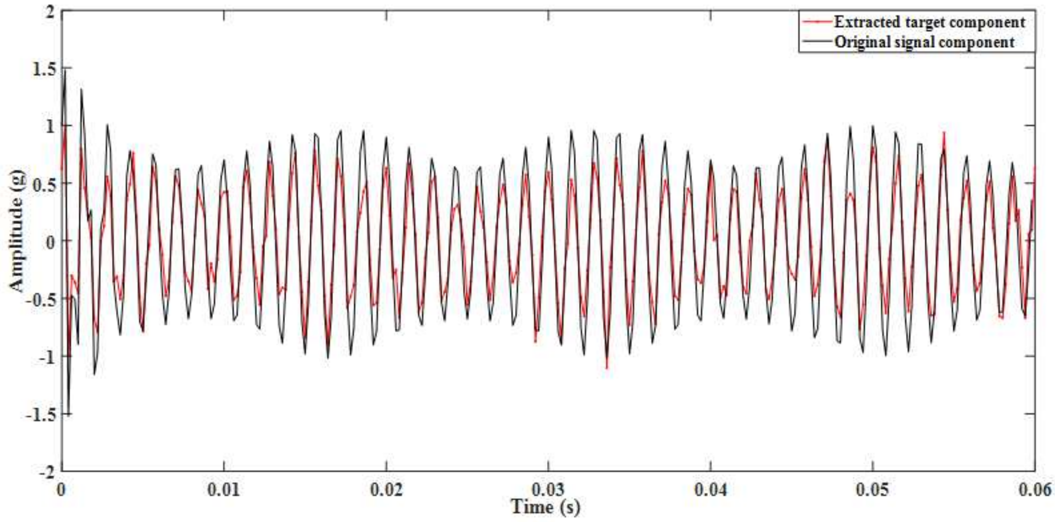

Figure 8(c) Bilateral attenuating impact component

Figure 8 Multi-source sparse optimization method is used to extract target component in time domain

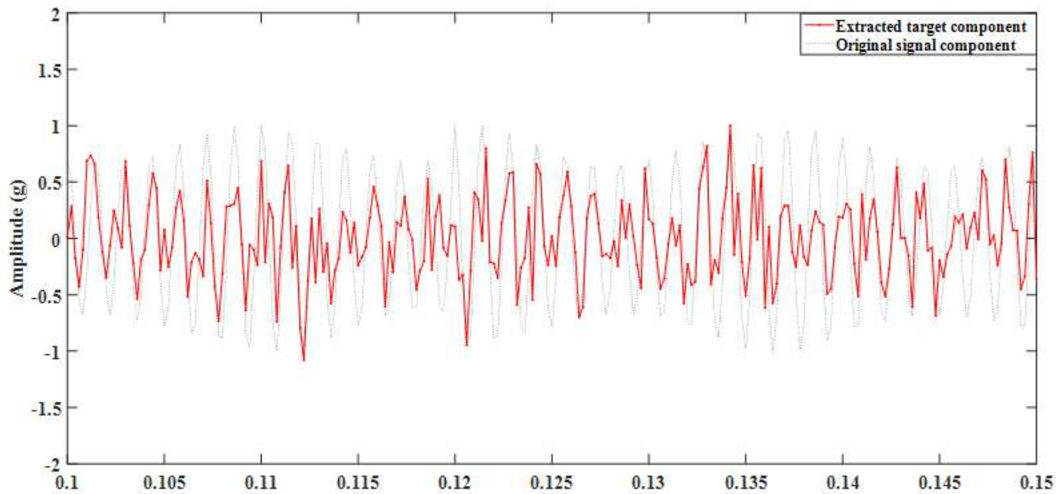

Figure 9(a) Partial enlarged view of harmonic target component

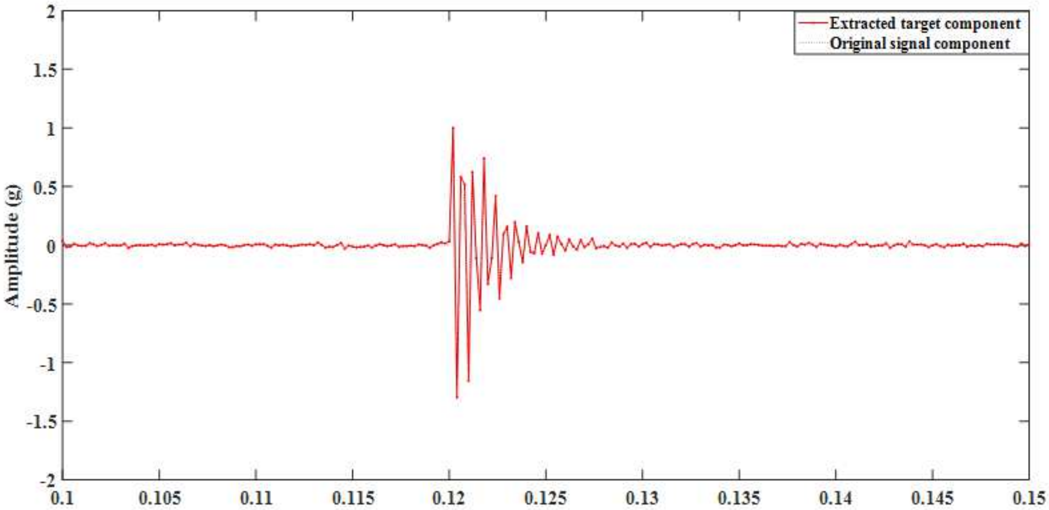

Figure 9(b) Partial enlarged view of unilateral attenuating impact component

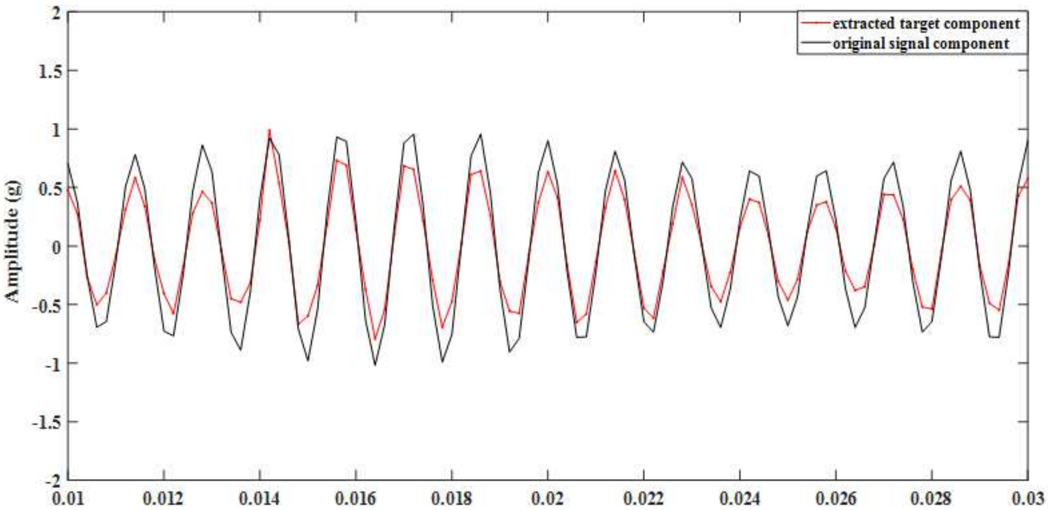

Figure 9(c) Partial enlarged view of bilateral attenuating impact component

Figure 9 Partial enlarged view of sparse decomposition

In order to verify the effectiveness of proposed method, the above simulation signal is also analyzed by RSSD method based on L1 norm and fast spectral kurtosis method respectively. Compared with multi-source sparse optimization method in this paper, the RSSD corresponding to L1 norm has no multi-source sparse dictionary and can only decompose simulation signal into harmonic component and impact component simply according to different Q-factors. In practical engineering application, impact component often presents different oscillation characteristics, so the decomposition results are prone to morphology aliasing. On the contrary, the multi-source sparse optimization method in this paper can achieve accurately polymorphic-oscillatory components identification. Figure 10 shows the signal components extracted by RSSD based on L1 norm, compared with Figure 8, the correlation coefficient by RSSD are significantly lower than proposed method in this paper, indicating multi-source sparse optimization method has good signal separation accuracy. Besides, for the transient impact component, the low resonance component based on RSSD cannot identify unilateral attenuation impact characteristic and bilateral attenuation impact characteristic. As a result, it can be seen that the residual component also contains the target component $x_1(t)$, $x_2(t)$, which presents the amplitude underestimation defect by L1 norm.

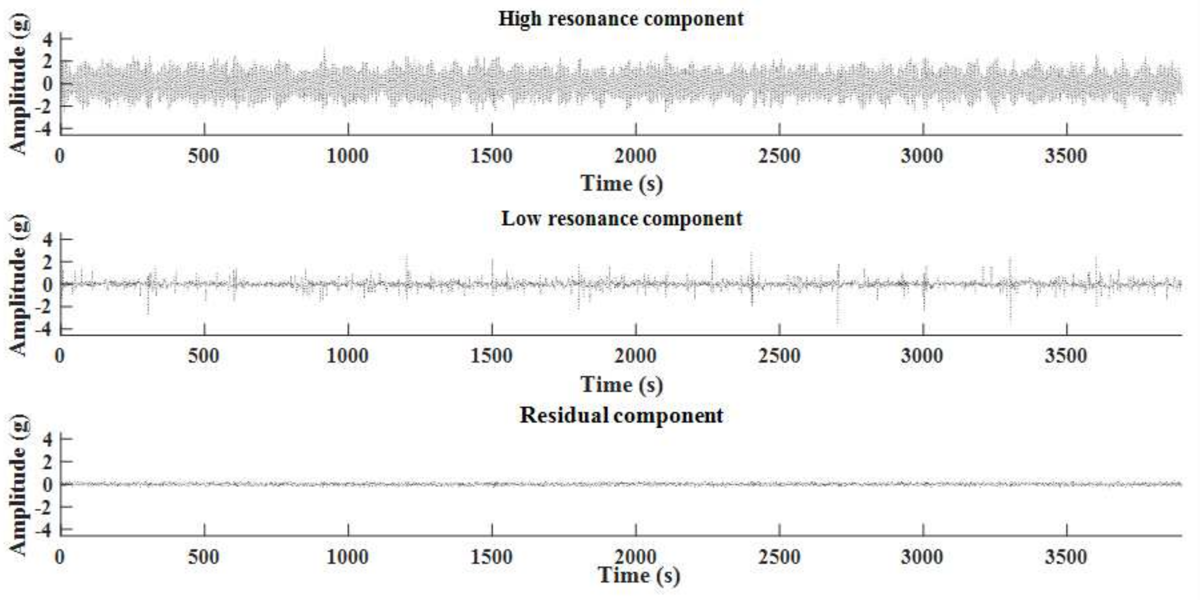

Figure 10 RSSD decomposition method based on L1 norm

LMD is used to decompose the simulation signal, and $PF_1 \sim PF_2$ is obtained, which is impossible to separate signal components with different oscillation characteristics. As a result, the time-domain adaptive decomposition can be appropriately applied for signal pre-processing.

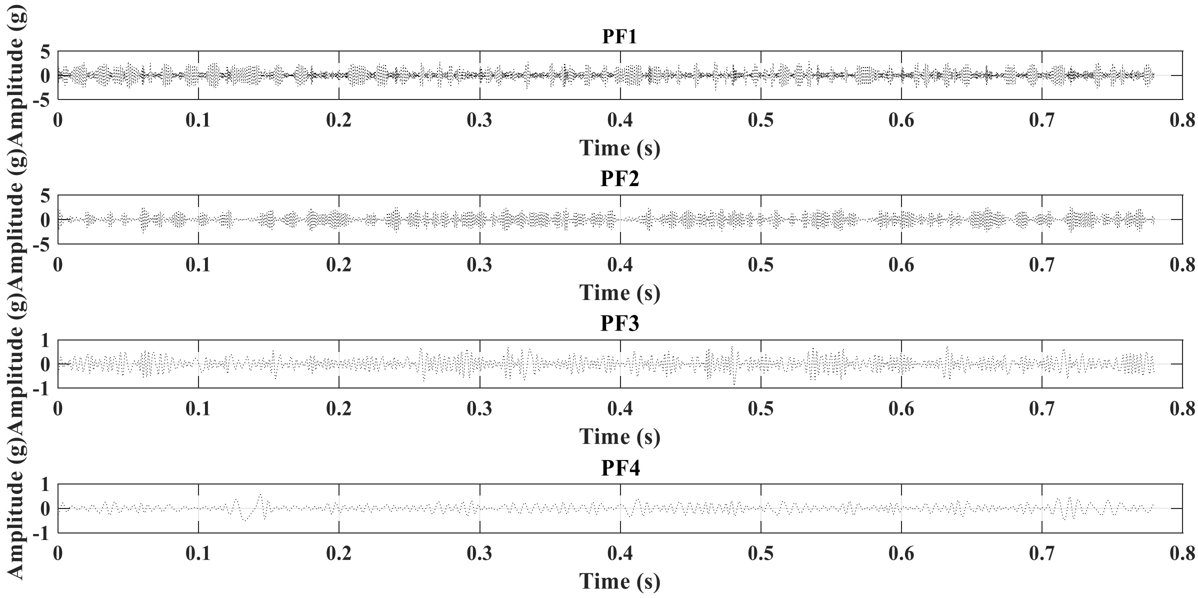

Figure 11 LMD adaptive decomposition

Similarly, VMD is also applied to simulation signal processing and corresponding result is shown in following.

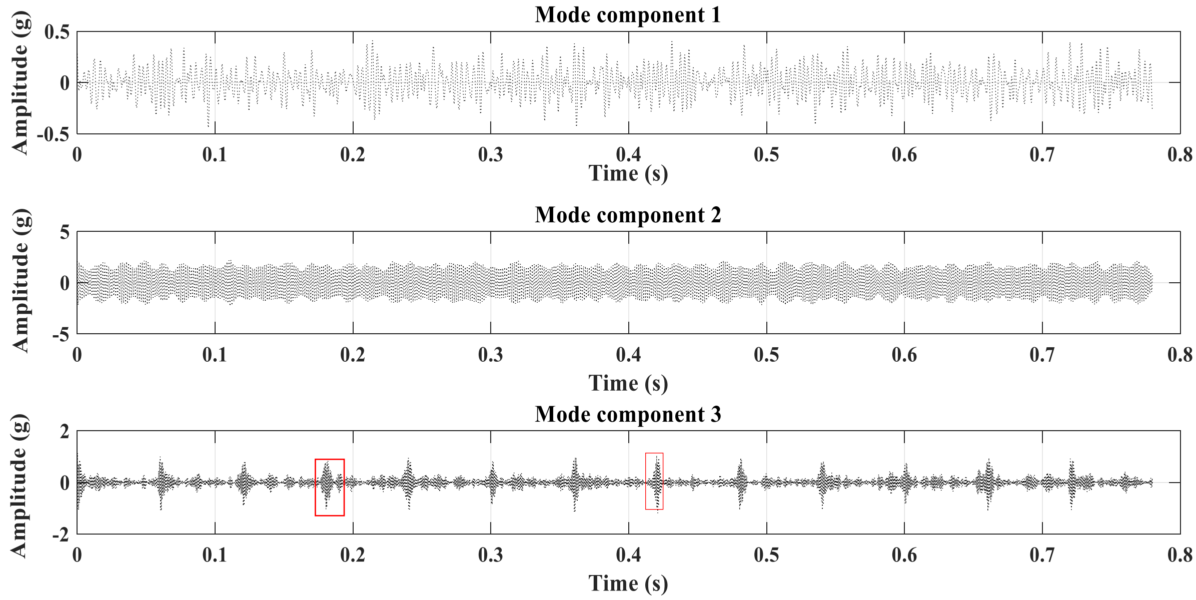

Figure 12 VMD adaptive decomposition

By analyzing the above decomposition results, it can be clearly seen that when the existing signal time-domain adaptive decomposition is applied to mechanical system signal, the decomposition results lack reliability due to the similarity of oscillation morphology between different signal components. The number of components obtained by LMD is obviously more than the real number of signal components. Although VMD can be used to obtain three signal components, the third modal component significantly reflects both bilateral attenuation and unilateral attenuation feature, which indicates that for polymorphic-oscillatory signal, an appropriate sparse dictionary must be designed to achieve to match signal components.

In comparison with sparse decomposition theory, fast spectral kurtosis can not extract each target component in the simulation signal, but only the impact component $x_1(t)$, $x_2(t)$ in simulation signal $y(t)$. In terms of extraction accuracy, it is obvious from Figure XXX that the extracted component also contains background noise $n(t)$, and the amplitude of extracted component is also significantly underestimated. Compared with the multi-source sparse decomposition optimization method proposed in this paper, this method has poor identification ability for polymorphic-oscillatory components.

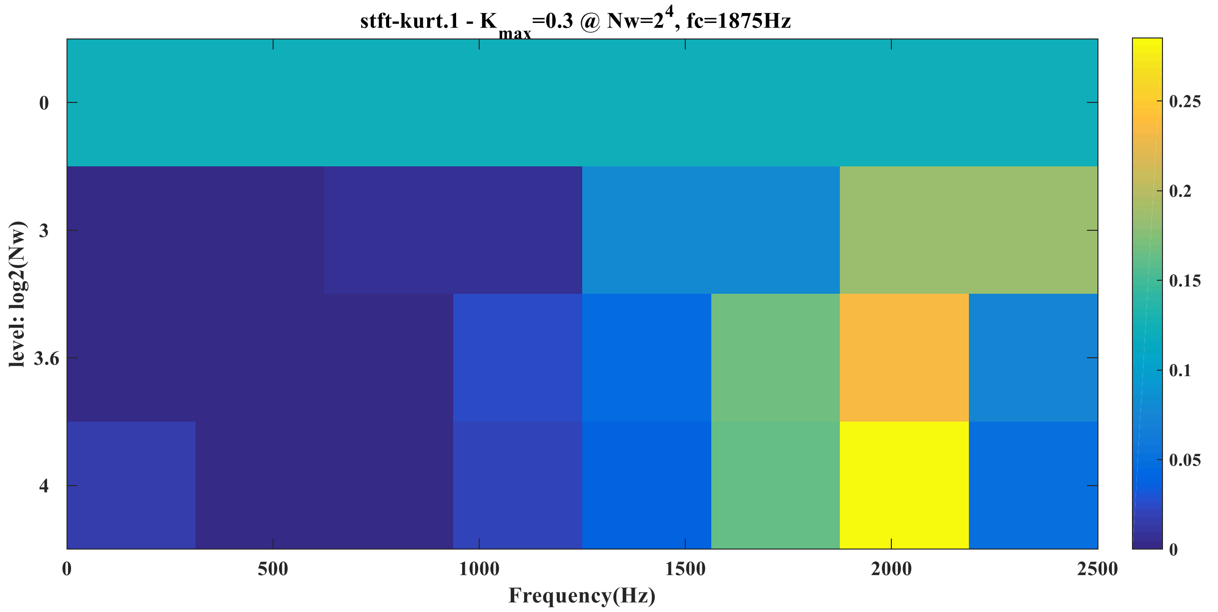

Figure 13(a) Spectral kurtosis

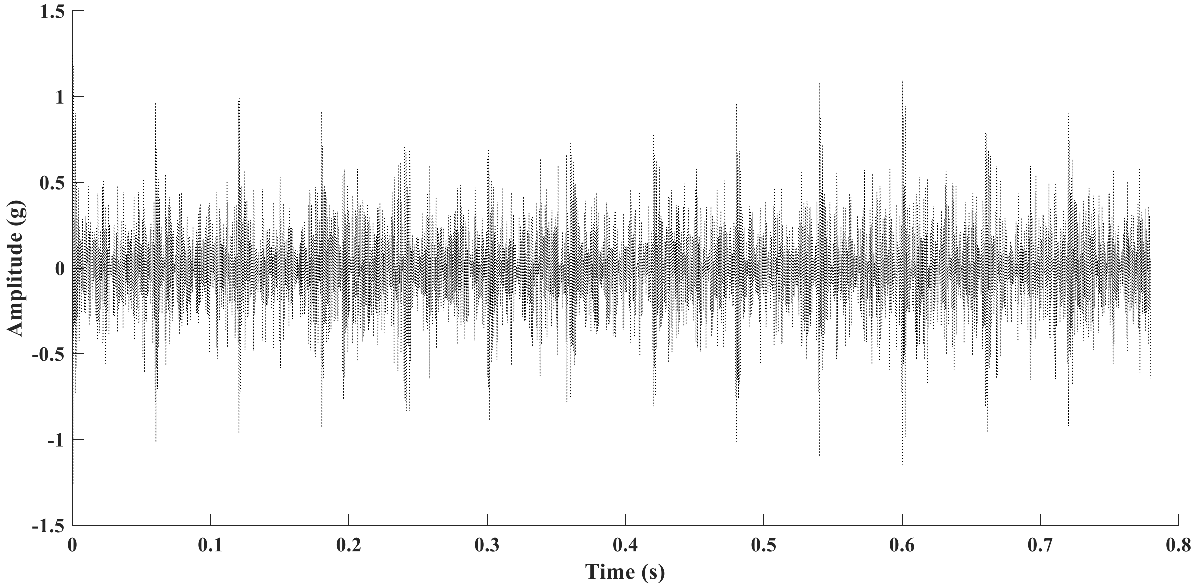

Figure 13(b) Extracted component

Figure 13 Decomposition result by spectral kurtosis

Compared with the proposed method in this paper, due to its own limitation, the signal component extracted by spectrum kurtosis presents significantly oscillatory impact characteristic, but is powerless to other signal component. Meanwhile, spectral kurtosis method is very sensitive to noise, so it is difficult to ensure the extraction accuracy. It infers that spectrum kurtosis can be acted as a back-end processing link of the proposed method. Specifically, the proposed method is used to separate shock component, and then combined with spectrum kurtosis to realize the polymorphic-oscillatory signal impact characteristic perfectly.

4.2 Engineering experiment verification

By analyzing simulation signal, it demonstrates preliminary multi-source sparse optimization method in dealing with polymorphic-oscillatory signal has fidelity ability, and as the advantage of multi-source sparse dictionary, it can effectively extract the oscillation component, this suggests that the proposed method in this paper has wide application value in the field of mechanical fault diagnosis, the proposed method has obvious advantages compared with fast spectral kurtosis and sparse decomposition method based on L1 norm, signal adaptive time-domain decomposition.

Furthermore, this paper uses the mechanical system signal collected from experimental equipment to verify the practical application effectiveness and universality of proposed method.

Figure 14 Engineering application experimental system

In general, using the experimental device including gear fault and bearing fault respectively develops an engineering test signal, the former reflects unilateral attenuation impact feature, the latter reflects harmonic modulation characteristic, so the engineering test signal possesses polymorphic-oscillatory property, with the help of proposed method in this paper to separate two morphology signal components, on this basis, to extract feature information in signal component to realize fault identification.

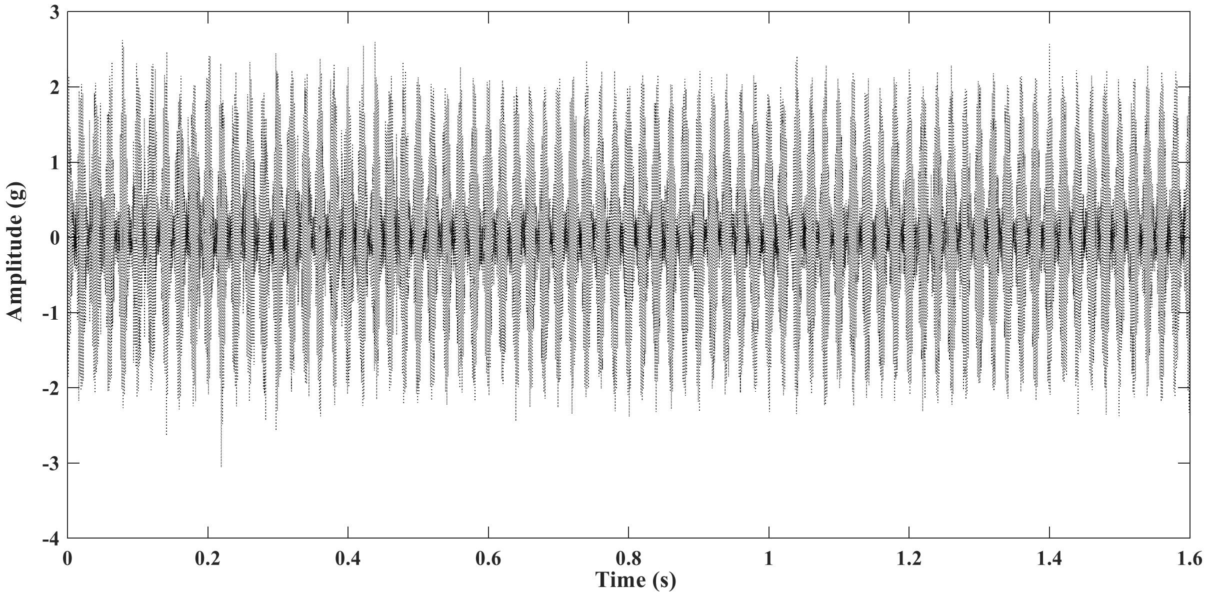

Figure 15 Engineering experimental signal

It can be seen from the above figure that the signal presents obvious harmonic modulation characteristic and impact characteristic, which are corresponding to working condition of different components in mechanical system.The proposed method in this paper can be used to achieve to match different oscillatory signal component respectively, extracted signal component is shown in following.

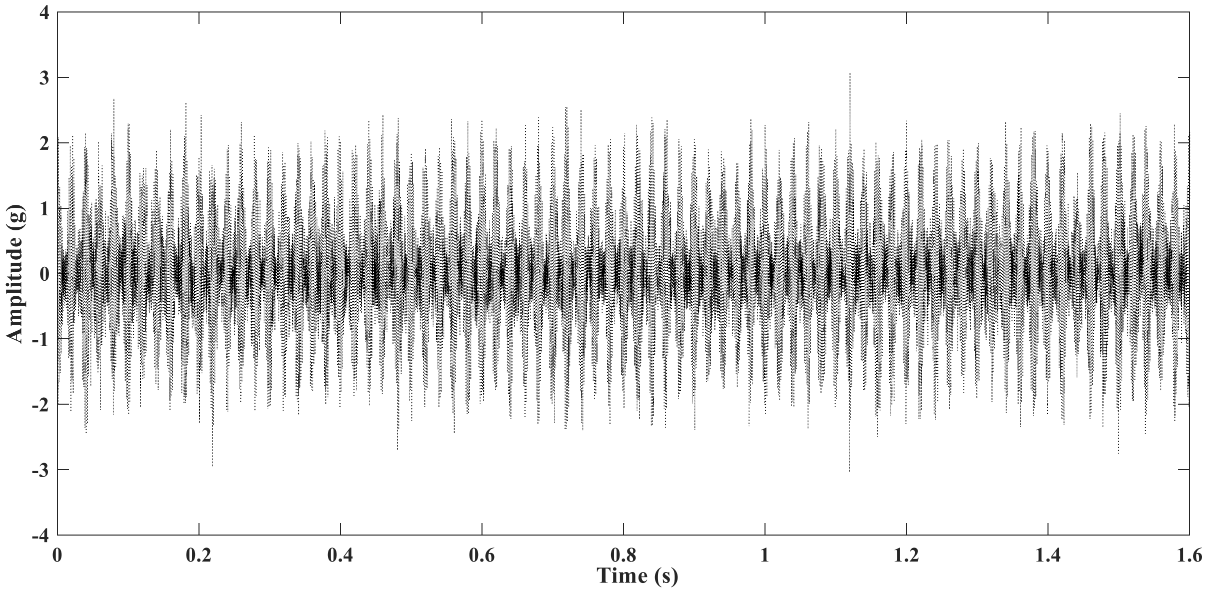

Figure 16(a) Extracted harmonic modulation component

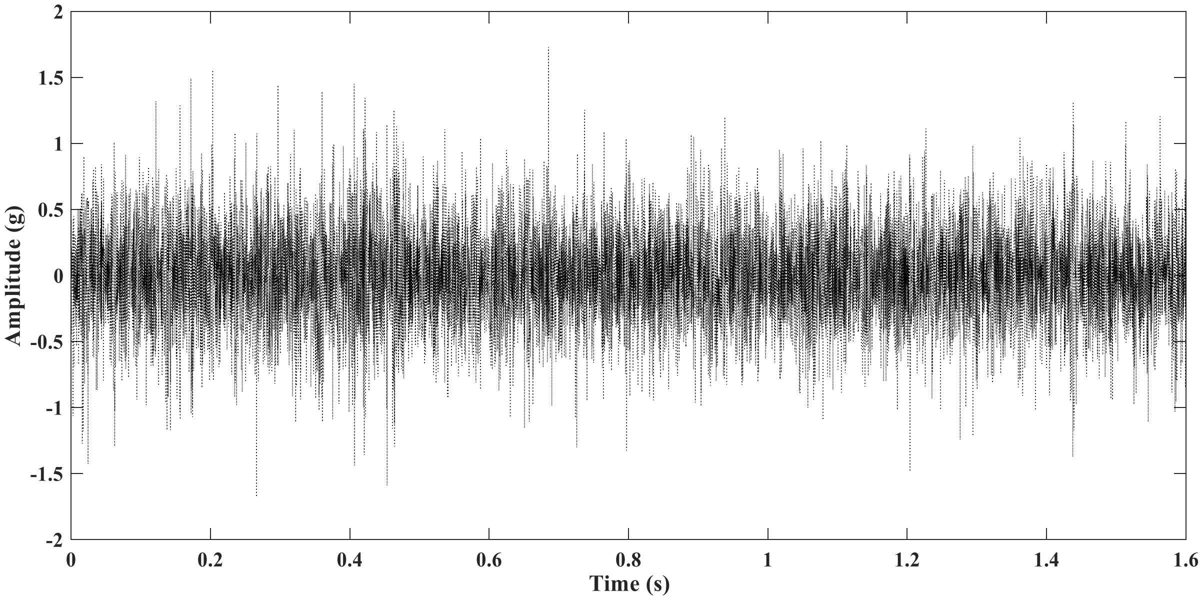

Figure 16(b) Extracted impact component

Figure 16 Extracted result by proposed method in this paper

In order to compare, we also introduces double tree complex wavelet (DTCW) to process engineering signal, and double tree complex wavelet has translation invariant property and reduces frequency band aliasing on each signal component consequently.

Set the decomposition level here to one, and the corresponding signal component is shown in following.

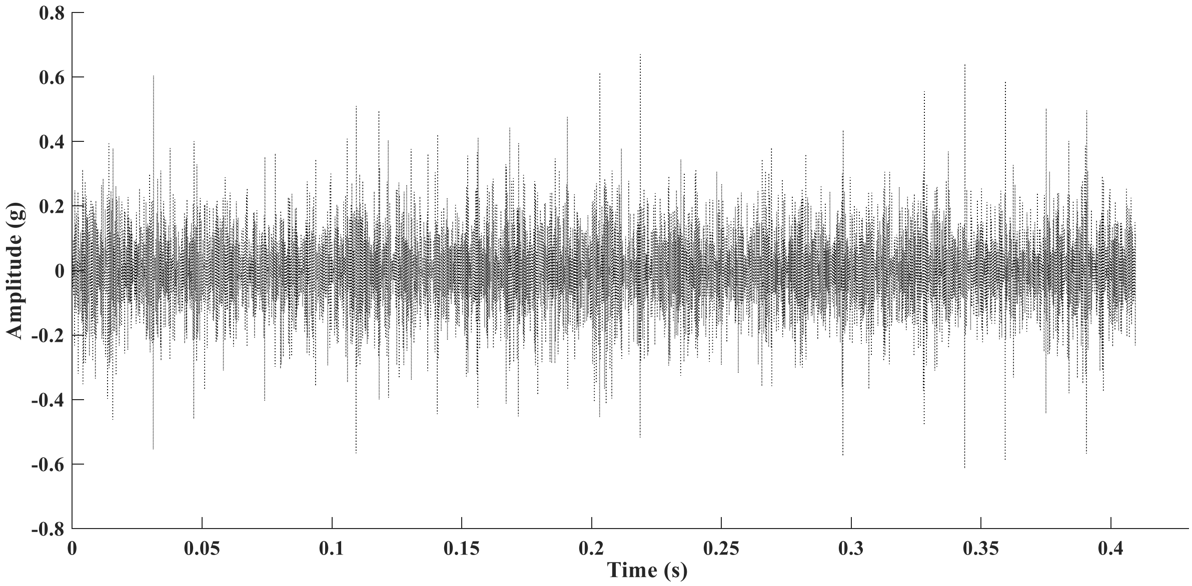

Figure 17(a) Extracted low-frequency signal component

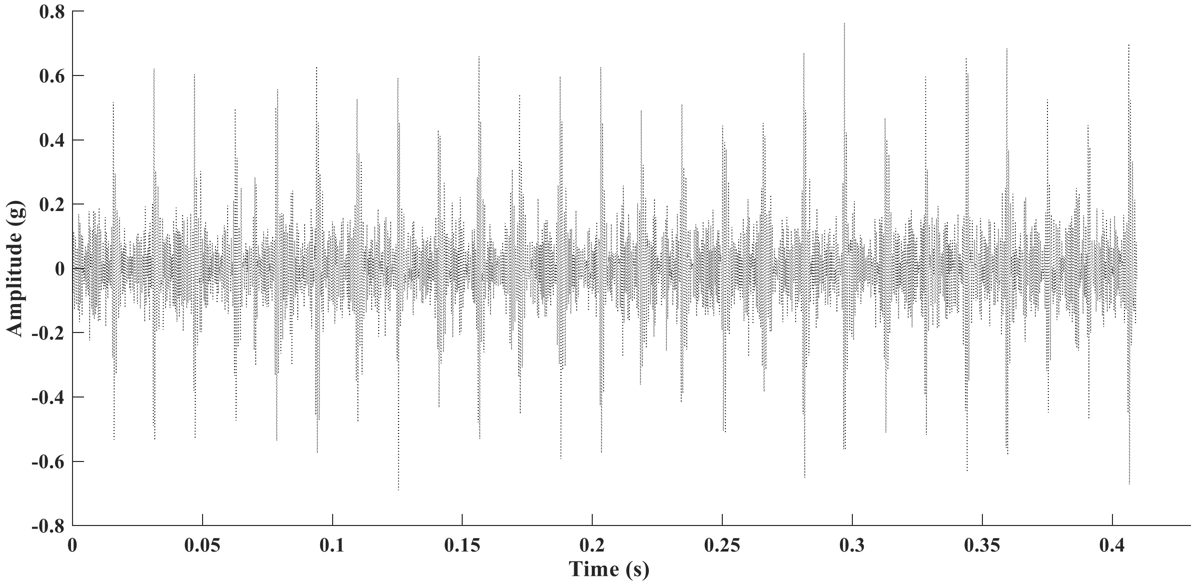

Figure 17(b) Extracted high-frequency signal component

Figure 17 Extracted signal component by DTCW

Figure 17 is the result of engineering signal decomposition by DTCW. It can be seen that the decomposition result, especially the high-frequency signal component, appears morphological aliasing phenomenon, showing harmonic characteristic and impact characteristic, resulting in significantly lower impact amplitude presented low-frequency signal component, which will affect the fault detection effect subsequently. In comparison, the proposed method in this paper can decouple impact component and harmonic component, so that the oscillatory components conform to different parts in mechanical equipment, which is convenient for fault identification consequently.

In conclusion, compared with RSSD based on L1 norm, time-domain adaptive decomposition, spectral kurtosis and wavelet decomposition, the proposed method in this paper has significant advantages in extraction of multi-morphological oscillatory signal separation.

5.Results

(1) In this paper, to conform the specific requirements of polymorphic-oscillatory signal separation, an over-complete dictionary matching the oscillation characteristic of each signal component is studied and constructed. Secondly, a multi-source sparse decomposition model is established based on GMC penalty function, which can separate polymorphic-oscillatory signal in a comprehensive fidelity. Thirdly, the convex condition of objective function is derived to determine parameters varying range, and an effective solution of objective function is also proposed, so as to establish the multi-source sparse optimization method, and the effectiveness is verified by mechanical system fault signal ultimately.

(2) Simulation experiment verify that compared with the RSSD based on L1 norm, the proposed multi-source sparse optimization method has better noise reduction and amplitude fidelity. Compared with fast spectral kurtosis, it presents better feature extraction ability and can separately extract target oscillatory signal component. Compared with the signal time-domain adaptive decomposition, its sparse dictionary can better match variable oscillating feature in signal to achieve accurate extraction.

(3) Through engineering experimental signal analysis, it is verified that the proposed method still has excellent signal reconstruction accuracy and potential fault diagnosis performance in practical application. Therefore, it can be concluded that

the multi-source sparse optimization method proposed in this paper has obvious advantages over classical sparse decomposition method, and can effectively apply on signal feature extraction and mechanical fault detection.

Availability of data and materials

The data comes from the experimental device. If need it, please contact us later

Abbreviations

RSSD: resonance sparse signal decomposition

DTCW:double tree complex wavelet

WT:wavelet transform

EMD:empirical mode decomposition

LMD:local mean decomposition

SALSA:Split Augmented Lagrangian Shrinkage algorithm

TQWT:Tunable Q-Factor Wavelet Transform

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Contributions

All authors read and approved the final manuscript.

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Ethics declarations

Competing interests

The authors declare that they have no competing interests.

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