The stability of multi-component plasmas subjected to sheared flows parallel to the background magnetic field

K. Shahzad (shahzad.khurram2010@gmail.com)
Pakistan Institute of Engineering and Applied Sciences (PIEAS)
Aman-ur-Rehman Aman
Pakistan Institute of Engineering and Applied Sciences (PIEAS)
Muhammad Yousaf Hamza
Pakistan Institute of Engineering and Applied Sciences (PIEAS)

Research Article

Keywords: Parallel electron current, Ions shear flow, Ion acoustic waves

Posted Date: April 26th, 2022

DOI: https://doi.org/10.21203/rs.3.rs-1518100/v1

The stability of multi-component plasmas subjected to sheared flows parallel to the background magnetic field

K. Shahzad[1, a)], Aman-ur-Rehman[1, 2, b)], Muhammad Yousaf Hamza[3)]

[1]Department of Physics and Applied Mathematics, Pakistan Institute of Engineering and Applied Sciences (PIEAS),P. O. Nilore, Islamabad 45650, Pakistan.

[2]Department of Nuclear Engineering, Pakistan Institute of Engineering and Applied Sciences (PIEAS), P. O. Nilore, Islamabad 45650, Pakistan.

[3]Department of Nuclear Engineering, Pakistan Institute of Engineering and Applied Sciences (PIEAS), P. O. Nilore, Islamabad 45650, Pakistan.

April 03, 2022

a) Email address: shahzad.khurram2010@gmail.com
b) Email address: amansadiq@gmail.com

ABSTRACT

Shear flow instability is studied in multi-component plasma. The excitation of low frequency electrostatic instabilities in a collision less plasmas of two system of species (H+, O+, e−) and (H−, O+, e−) is studied. Inertialess electrons follow the Boltzmenian distribution. The massive ions (oxygen) and the light species (hydrogen) provide the inertia. Light ions may be both positive hydrogen as well as negative hydrogen ions. Many authors have generally neglected the low concentration of hydrogen ions in oxygen plasma of ionosphere, but we have pointed out that the hydrogen ions as impurity plays a significant role to increase the growth rate of ion acoustic wave (IAW). We study the effect of shear on the following two systems of species separately and then compare the results of two systems numerically. We observed that in presence of shear the growth rate of the system with negative hydrogen ion is larger than with positive hydrogen ion. Fluid theory is used to study the three cases of ion-acoustic perturbations with same shear flow, different shear flow and zero shear flow. The kinetic model is used to study the effects of electron current in presence of negative shear flow. Ion-acoustic waves (IAWs) can easily be excited and overflow the heavy ion Landau damping either by electron parallel current or sheared flows of ion species. Our results may helpful to solve possible applications of low frequency space and laboratory plasmas.

Keywords: Parallel electron current, Ions shear flow, Ion acoustic waves

I. INTRODUCTION

Space plasmas with single ionic species are found less frequently as compare to the space plasmas with multi-ion species. Multi-ion species plasma consists of mixture of ions having different charges, masses and degree of ionization in addition to the background mobile electrons. One case of existence of multi-ion (hydrogen-oxygen) plasma naturally occurs in space and astrophysical regions. Such plasmas, consisting of both positive and negative ion species with electrons can also be created in laboratory. The effect of presence of negative ions has been observed on the plasma potentials. In presence of negative ion species, plasma shielding is affected [1]. The shielding effect is reduced because the electron density reduces by increasing the negative ion species, because charge neutrality condition is $n_e = n^+ - n^-,$ is basic technique to study the plasma behavior. The negative ions are formed by attachment of electrons with neutral atoms of gas. The negative hydrogen ions in hydrogen-oxygen plasma can be formed due to large polarizing ability of hydrogen atoms [2].

The negative ions can be produced in plasma processing reactors [3]. It also occurs in D and F region of ionosphere [4], cometary plasma, etc. [5]. The negative ion plasma has number of applications in electronics. There are many ways to produce the negative ions in electronic industries. The electrical discharging process in a gas and there are many other external sources to produce the negative ions [6-8]. Number of articles has published on ion-acoustic modes in multi-ion plasmas with and without the shear flow. Mamun and Shukla, studied the effect of negative ions charge density on the surface potential of the dust grains [9]. They investigated the charging of dust particles with the negative ions, and discussed the effects of concentration of negative to positive ion density ratio on nonlinear Kdv equation. They studied the magnetosonic waves with dust grains in pair-ion plasmas and observed the effect on amplitude of solitary waves due to the number density of negative ions. Recently, Hussain, Manhaz, Haseeb and Hasnain [10] studied the damped magnetosonic modes in hydrogen-oxygen plasma with inertial electrons.

Low-frequency ion-acoustic waves are one of the most elemental oscillations in sheared multi-ion plasma. In long-wavelength limit, the ions impart inertia with inertial less electrons provides restoring force [11]. Ion-acoustic waves have sturdy nonlinear effects and are exceedingly Landau damped unless $T_i \ll T_e$, where T_e and T_i are the electron and ion temperatures respectively [12–14]. On contrary, the condition $T_i \,/\, T_e \approx 1$ holds in many laboratory and space plasmas and observed low frequency ion-acoustic waves [15–16]. These waves have been investigated thoroughly in numerous kinds of high temperature laboratory plasmas [13]. Ion-acoustic wave characteristics are observed in Earth's ionosphere [17] and transport in solar wind, corona, chromosphere [18] and comets [19].

Shear flow, in across and along the magnetic-field has observed in fusion [20–22] and space [23] plasmas. Data acquired from number of sounding rocket experiments show a remarkable association between small scale structures and strong shear flow, plasma component energization [23] and broadband electrostatic fluctuations. A strong link between velocity shear of the field aligned flow, transverse ion heating, and broadband electrostatic ion cyclotron waves has been found [24].

The function of CDEIA instabilities in the ionosphere was discussed particularly by Kindel and Kennel [25]. It is shown that for a typical ionospheric ion and electron temperature ratio $T_i \,/\, T_e \approx 1$, that in current driven electrostatic ion acoustic (CDEIA) mode, the critical field aligned current is usually significantly larger than that of the observed values. However, Ref. [26] reports the results of the low frequency ion-acoustic like waves in the ionosphere. Ion-acoustic like fluctuation is also observed in association with interplanetary shocks and the Earth's bow shock for condition unfavorable for current driven ion acoustic instability [27].

D'Angelo investigated the setup of ion flow and purely growing electrostatic instability arises parallel to the external constant magnetic field having heterogeneous velocity relative to the electron [28]. It is stated that for D'Angelo instability, density inhomogeneity is not required. Dobrowolny exhibited that the same growing instability by movement of plasma as a whole with $V_{e0}(x) \approx V_{i0}(x)$ [29].

Magnetic field-aligned flows have long been observed to be inhomogeneous with spatial structures of different scales [30]. These low-frequency waves associated with sheared parallel flows has been observed in space [31]. The existence of low-frequency electrostatic($\omega \ll \Omega_i$)and long wavelength ($k\rho_i \ll 1$, $k\lambda_D \ll 1$) ion mode excited by a transverse shear in the parallel flow was proposed by D'Angelo in a fluid formulation and later D'Angelo and Goeler [32] confirmed it in laboratory experiments.

We want to study the excitation of IAWs by field-aligned and sheared flows in multi-ion plasma. Gavrishchaka, S. B. Ganguli, and G. I. Ganguli [33] investigated the electrostatic ion acoustic wave (IAW) instability in pure oxygen ion plasma of ionosphere, driven by field aligned flow of electrons. The field aligned shear flow of oxygen ions similar to the D' Angelo case, with the addition of field aligned flow of electrons. However, due to smaller value of parallel electron current, the magnetic field was assumed to be constant. The flow of ions leads to the non-resonant D' Angelo mode with instability condition on shear flow. However, if the following condition is not satisfied then it leads to the appearance of sheared driven modified ion-acoustic wave instead of purely growing mode. In ionospheric plasma, even for the condition, ($T_i / T_e \geq 1$), the electrostatic ion-acoustic wave (IAW) becomes unstable due to parallel flow of electron. It happens because growth rate overthrow the ion Landau damping.

Spacecraft observations have shown that sometimes the heavy ions of ionospheric origin can dominate the composition of the plasma sheet region and the outer magnetosphere [34]. Recently, Singh, Reddy and Lakhina [35] have reported that energetic oxygen O+ ions in the ring current region can be used to excite low frequency quasi-electrostatic waves. Lu Li et al. [36] studied the velocity shear instability by considering the H+ and O+ ions to have the same flow velocity and the same velocity shear. H. Saleem, S. Ali, and Aman-ur Rehman [37] used the kinetic model to study electron current driven instabilities in sheared plasma by considering the system of two ion species (H+ and O+).

But in our paper, we study the linear instability of low frequency electrostatic ion-acoustic wave (IAW) driven by the magnetic field-aligned flow of electrons in two system of species (H+, O+, e−) and (H−, O+, e−). We study these two systems with both fluid and kinetic models. We numerically compare and discuss the shear effects of positive and negative hydrogen ions in oxygen ion plasma separately. The behavior of light (positive and negative hydrogen) and heavy (positive oxygen) ions by using the fluid equations is discussed in section II. In the following section, three cases are discussed that is ions have same shear flow, different shear flow and zero shear flow. In section III, kinetic model is used to study the numerical and analytical effects of electron current with negative ions shear flow. Positive and negative ions with opposite sheared flow are also studied. Finally In section IV, Numerical and analytical results are derived by using the laboratory and astrophysical plasmas. The results derived from kinetic and fluid models are compared and discussed.

II. ANALYTICAL ANALYSIS WITH FLUID MODEL

Let us consider uniform magnetized collision less hydrogen-oxygen plasma. Plasma has a shear flow parallel to the constant magnetic field with shear velocity $\mathbf{v_{j0}} = v_0(x)\hat{\boldsymbol{e}}_{\parallel}$. We model the plasma as, the ion species have sheared flow while e− has no shear but they have field-aligned flow. The external magnetic field is $\boldsymbol{B_0} = B_0\hat{\boldsymbol{e}}_{\parallel}$ constant and zero order current is small. The momentum equation for jth species is as follows:

$$m_j n_j(\partial_t - \mathbf{v_j}\,.\boldsymbol{\nabla})\mathbf{v_j} = q_j n_j\left(\mathbf{E} + \frac{1}{c}\left(\mathbf{v_j} \times B_0\hat{\boldsymbol{e}}_{\parallel}\right)\right). \quad \#(1)$$

The pressure term has been dropped for all species. The equation of continuity is

$$\partial_t n_j + \boldsymbol{\nabla}.\left(n_j \mathbf{v}_j\right) = 0. \quad \#(2)$$

By using the perturbations, we linearize the above equations by removing the zero order, second order and higher order terms, and $\boldsymbol{\nabla}.\mathbf{v_0} = 0$. The electrons are assumed to be inertial less, and the steady state is satisfied. For the electrostatic instabilities, with condition $\omega \ll \Omega_{cj}$, the perpendicular component of ions perturbed velocity may be written as

$$\mathbf{v}_{j\perp 1} = \frac{c}{B_0}(\widehat{\boldsymbol{e}}_{\parallel} \times \boldsymbol{\nabla}\phi_1) - \frac{1}{\Omega_{cj}}\left(\partial_t + \mathbf{v}_{j\perp 1}.\boldsymbol{\nabla}_{\perp} + v_{0j}(x)\partial_z\right)\left(\mathbf{v}_{j\perp 1} \times \widehat{\boldsymbol{e}}_{\parallel}\right). \#(3)$$

Or

$$\mathbf{v}_{j\perp 1} = \mathbf{v}_{\boldsymbol{E}} + \mathbf{v}_{\boldsymbol{p}j}. \quad \#(4)$$

Since $|\mathbf{v}_{\boldsymbol{E}}| \gg |\mathbf{v}_{\boldsymbol{p}j}|$, so we have

$$\left(\mathbf{v}_{j\perp 1} \times \widehat{\boldsymbol{e}}_{\parallel}\right) = \frac{c}{B_0}\boldsymbol{\nabla}_{\perp}\phi_1. \quad \#(5)$$

Remove the $\left(\mathbf{v}_{j\perp 1}.\boldsymbol{\nabla}_{\perp}\right)\boldsymbol{\nabla}_{\perp}\phi_1$ second order term, and using equation (5) into equation (3) we get the linearized perpendicular velocity of ions as

$$\mathbf{v}_{j\perp 1} = \frac{c}{B_0}(\widehat{\boldsymbol{e}}_{\parallel} \times \boldsymbol{\nabla}\phi_1) - \frac{c}{B_0\Omega_{cj}}\left(\partial_t + v_{0j}(x)\partial_z\right)\boldsymbol{\nabla}_{\perp}\phi_1. \quad \#(6)$$

The linearized continuity equation become as

$$\left(\partial_t + v_{0j}(x)\partial_z\right)n_{j1} + n_{j0}\left(\boldsymbol{\nabla}_{\perp}.\,\mathbf{v}_{j\perp}\right) + n_{j0}\left(\partial_z v_{j\parallel}\right) = 0. \quad \#(7)$$

The linearized parallel component of velocity is obtained from equation (1) as

$$\left(\partial_t - v_{0j}(x)\partial_z\right)\mathrm{v}_{j\parallel} = -\frac{q_j}{m_j}\left(\partial_z - \frac{1}{\Omega_{cj}}\frac{\partial v_{0j}(x)}{\partial x}\partial_y\right)\phi_1. \quad \#(8)$$

The electrons are assumed to be inertialess which follow the Maxwellian's distribution as

$$n_{e1} = n_{e0}\exp\left(\frac{e\phi_1}{T_e}\right). \quad \#(9)$$

For $T_e \gg e\phi_1$, therefore $\exp\left(\frac{e\phi_1}{T_e}\right) \approx \frac{e\phi_1}{T_e}$ and use $\frac{e\phi_1}{T_e} = \Phi$, above equation reduces as

$$n_{e1} \simeq n_{e0}\Phi. \quad \#(10)$$

Let us assume the electrostatic perturbation of any physical quantity φ to be the form of $\varphi = \varphi(x)exp[i(\boldsymbol{k}_{\perp}.\boldsymbol{r}_{\perp} + \boldsymbol{k}_{\parallel}.\boldsymbol{r}_{\parallel} - \omega t)]$, then parallel velocity obtained from equation (8) becomes as

$$\mathrm{v}_{j\parallel} = \frac{c_{sj}^2 k_\parallel}{\Omega_{0j}} \left(1 - \frac{k_\perp}{k_\parallel} s_j\right) \Phi. \quad \#(11)$$

Where $c_{sj}^2 = \frac{T_e}{m_j}$, $s_j = \frac{1}{\Omega_{cj}} \frac{\partial v_{0j}(x)}{\partial x}$ is a shear flow of ions, and $\Omega_{0j} = \left(\omega - k_\parallel v_{0j}(x)\right)$.

Using the value of parallel and perpendicular velocities into equation (7), and put $\boldsymbol{\nabla} . \mathbf{v}_{\boldsymbol{E}} = 0$, the continuity equation for ions becomes as

$$\left(\partial_t + v_{0j}(x)\partial_z\right) n_{j1} + n_{j0} \left\{ \frac{c_{sj}^2 k_\parallel \sigma_j^2}{\Omega_{0j}} \partial z - \left(\partial_t + v_{0j}(x)\partial_z\right) \rho_{tj}^2 \boldsymbol{\nabla}_\perp^2 \right\} \Phi = 0. \quad \#(12)$$

Simplifying above equation with perturbed distributed wave function, the number density of ions becomes as

$$n_{j1} = n_{j0} \left(\frac{c_{sj}^2 k_\parallel^2 \sigma_j^2}{\Omega_{0j}^2} - k_\perp^2 \ \rho_{tj}^2 \right) \Phi. \quad \#(13)$$

Here $\rho_{tj} = \frac{c_{sj}}{\Omega_{cj}}$, and $\sigma_j^2 = 1 - \frac{k_\perp}{k_\parallel} s_j$. The quasineutrality for singly charged ions written as,

$$n_{H1} + n_{O^+1} = n_{e1}. \quad \#(14)$$

After a simple calculation, a general linear dispersion relation of ion-acoustic wave (IAW) for (H+, O+, e−) plasma becomes as,

$$\Omega_{0H^+}^2 \Omega_{0O^+}^2 - \frac{n_{H^+0}}{n_{e0}} \left(c_{sH^+}^2 k_\parallel^2 \sigma_{H^+}^2 - k_\perp^2 \ \rho_{tH^+}^2 \Omega_{0H^+}^2 \right) \Omega_{0O^+}^2 - \frac{n_{O^+0}}{n_{e0}} \left(c_{sO^+}^2 k_\parallel^2 \sigma_{O^+}^2 - k_\perp^2 \ \rho_{tO^+}^2 \Omega_{0O^+}^2 \right) \Omega_{0H^+}^2 = 0. \quad \#(15)$$

But in case of (H−, O+, e−) plasma, it becomes as,

$$\Omega_{0H^-}^2 \Omega_{0O^+}^2 - \frac{n_{H^-0}}{n_{e0}} \left(c_{sH^-}^2 k_\parallel^2 \sigma_{H^-}^2 - k_\perp^2 \ \rho_{tH^-}^2 \Omega_{0H^-}^2 \right) \Omega_{0O^+}^2 - \frac{n_{O^+0}}{n_{e0}} \left(c_{sO^+}^2 k_\parallel^2 \sigma_{O^+}^2 - k_\perp^2 \ \rho_{tO^+}^2 \Omega_{0O^+}^2 \right) \Omega_{0H^-}^2 = 0. \quad \#(16)$$

We assume that all ion species have same kinetic energy i.e. $\frac{1}{2} m_{H^+} v_{H^+}^2 = \frac{1}{2} m_{O^+} v_{O^+}^2$, In case of stationary plasma, there is no parallel flow of ions, $v_{H^+} = v_{O^+} = 0$, and $\sigma_{H^+}^2 = \sigma_{O^+}^2 = 1$, above dispersion relation (15) becomes as

$$\left(1 + \frac{n_{H^+0}}{n_{e0}} \ k_\perp^2 \ \rho_{tH^+}^2 + \frac{n_{O^+0}}{n_{e0}} \ k_\perp^2 \ \rho_{tO^+}^2 \right) \omega^2 - \left(\frac{n_{H^+0}}{n_{e0}} c_{sH^+}^2 k_\parallel^2 - \frac{n_{O^+0}}{n_{e0}} c_{sO^+}^2 k_\parallel^2 \right)$$

$$= 0, \quad \#(17)$$

and similarly with $\frac{1}{2} m_{H^-} v_{H^-}^2 = \frac{1}{2} m_{O^+} v_{O^+}^2$, in case of $v_{H^-} = v_{O^+} = 0$, and $\sigma_{H^-}^2 = \sigma_{O^+}^2 = 1$, the dispersion relation (16) becomes as:

$$\left(1+\frac{n_{H^-0}}{n_{e0}}\ k_\perp^2\ \rho_{tH^-}^2 + \frac{n_{O^+0}}{n_{e0}}\ k_\perp^2\ \rho_{tO^+}^2\right)\omega^2 - \left(\frac{n_{H^-0}}{n_{e0}} c_{sH^-}^2 k_\parallel^2 - \frac{n_{O^+0}}{n_{e0}} c_{sO^+}^2 k_\parallel^2\right) = 0. \quad \#(18)$$

In space plasmas, these ions may have field-aligned flow with same speed in same direction, $v_{H^+} = v_{O^+}$, then equation (15) reduces to

$$\left(1 + \frac{n_{H^+0}}{n_{e0}}\ k_\perp^2\ \rho_{tH^+}^2 + \frac{n_{O^+0}}{n_{e0}}\ k_\perp^2\ \rho_{tO^+}^2\right)\Omega_0^2 - \left(\frac{n_{H^+0}}{n_{e0}} c_{sH^+}^2 k_\parallel^2 \sigma_{H^+}^2 + \frac{n_{O^+0}}{n_{e0}} c_{sO^+}^2 k_\parallel^2 \sigma_{O^+}^2\right) = 0, \quad \#(19)$$

with $v_{H^-} = v_{O^+}$, equation (16) reduces to

$$\left(1+\frac{n_{H^-0}}{n_{e0}}\ k_\perp^2\ \rho_{tH^-}^2 + \frac{n_{O^+0}}{n_{e0}}\ k_\perp^2\ \rho_{tO^+}^2\right)\Omega_0^2 - \left(\frac{n_{H^-0}}{n_{e0}} c_{sH^-}^2 k_\parallel^2 \sigma_{H^-}^2 + \frac{n_{O^+0}}{n_{e0}} c_{sO^+}^2 k_\parallel^2 \sigma_{O^+}^2\right) = 0. \quad \#(20)$$

In case of single ion plasma equation (19) reduces to:

$$\left(1 + \ k_\perp^2\ \rho_i^2\right)\Omega_{i0}^2 - c_{si}^2 k_\parallel^2 \sigma_i^2 = 0, \qquad \#(21)$$

and for

$$0 < S_i \frac{k_\perp}{k_\parallel} < 1 \quad or \quad S_i \frac{k_\perp}{k_\parallel} < 0 \quad i.e \quad \sigma_i^2 > 0;$$

The equation (21) is yields a real frequency that is primarily IA in magnetized plasma just modified by parallel ion shear flow. But in case of

$$1 < S_i \frac{k_\perp}{k_\parallel} \qquad and \qquad \sigma_i^2 < 0$$

Then ion acoustic mode disappears and purely growing nonresonant D'Angelo instability takes place.

We have discussed three different limiting cases about the flow speed of species. Case (I) ions have same flow speed $v_{a0}(x) = v_{b0}(x)$, case (II) ions have different flow speed $v_{a0}(x) \neq v_{b0}(x)$, case (III) $v_{a0}(x) = v_{b0}(x) = 0$.

Since oxygen and hydrogen mass ratio $\frac{m_b}{m_a} = 16$, therefore two different IAW frequencies $c_{sb}k_z = \omega_{sb}$ *and* $c_{sa}k_z = \omega_{sa}$ exist in the system. Since $\frac{n_{a0}}{n_{b0}} < 1$ and $c_{sb}^2 << c_{sa}^2$ for $T_a = T_b$. Therefor the density factor n_{a0} and n_{b0} bring 2nd and 4th terms of equation (15) close to each other. Therefor we can assume that ions have same flow speed $v_{a0}(x) = v_{b0}(x)$. In case (II), we assume that the ions have same kinetic energies then we can consider that the ions have different flow speed $v_{a0}(x) \neq v_{b0}(x)$. While in case (III) IAW is studied without parallel flow of ions $v_{a0}(x) = v_{b0}(x) = 0$.

III. ANALYTICAL ANALYSIS WITH KINETIC MODEL

Since for a typical ionospheric plasma, ion to electron temperature ratio ($T_i\ /\ T_e \approx 1$), we can study the kinetic effects on low frequency linear ion-acoustic perturbations. Many years ago, the ion-acoustic perturbations have been studied in hydrogen pair-ion plasma and electron positron ion plasma with field-aligned shear flow. The excitation of electrostatic IAW in upper ionosphere by parallel electron current was investigated in the pure oxygen plasma with field-aligned shear flow of ions. Some years ago, Saleem, Batool, and Poedts [38] studded the sheared driven low frequency electrostatic instability in hydrogen PI plasma. In present case, both kinetic and fluid model approach is used to investigate the linear low frequency electrostatic IAW instability in two system of species (H+, O+, e−) and (H−, O+, e−). Positive and negative ions are assumed to flow either in same direction or in opposite direction.

The Vlasov and Poisson system of equations with $k^2\rho_i^2 \ll 1$ can be used to drive the relation of kinetic plasma dispersion as

$$\begin{aligned}\varepsilon(\omega, k) = 1 + \frac{1}{k_z^2\lambda_{De}^2}\left[1 + \xi_e^* Z(\xi_{e0}) - \frac{k_y}{k_z} s_e\{1 + \xi_{e0}Z(\xi_{e0})\}\right] \\ + \sum_j \frac{1}{k_z^2\lambda_{Dj}^2}\Bigg[1 \\ - \sum_{n=-\propto}^{n=+\propto} \Lambda_{jn}(\beta_j)\left\{\frac{k_y}{k_z}s_j - Z(\xi_{jn})\left(\xi_j^* - \xi_{jn}\frac{k_y}{k_z}s_j\right)\right\}\Bigg]. \quad \#(22)\end{aligned}$$

Where $\lambda_D$is the Debye length. We assume the case of zero order. For simplicity we take $s_e = 0$ and $\xi_e^* = \xi_{e0} = \frac{\omega - k_z v_{de}}{\sqrt{2} k_z v_{te}}, and\ \xi_j^* = \xi_{j0} = \frac{\omega - k_z v_{dj}}{\sqrt{2} k_z v_{t\alpha}}$, where $j \rightarrow$ hydrogen and oxygen ions. Plasma dispersion function has been expanded for electrons in the limit of $\xi_{e0} \ll 1$ and for ions in the limit of $\xi_{j0} \gg 1$ such as:

$$Z(\xi_{e0}) = i\sqrt{\pi} e^{-\xi_{e0}^2} - 2\xi_{e0} + \cdots,\ \ and$$

$$Z(\xi_{j0}) = i\sqrt{\pi} e^{-\xi_{j0}^2} - \frac{1}{\xi_{j0}}\left(1 + \frac{1}{2\xi_{j0}^2} + \cdots\right). \#(23)$$

In case of classical CDEIA (i.e., σ =1) expansions of equation (23) are generally Valid for τ << 1 [39]. However, for SMIA modes these expansions are valid even for τ ≥1, if σ > 1. For negative shear we have σ > 1, then real frequency increases. This enables the argument (ξ_{j0}) of the ions dispersion function $Z(\xi_{j0})$ to be sufficiently large ($i.e.,\ \xi_{j0} \gg 1$) **[33].**

The dispersion relation (22) in its real and imaginary parts can be expressed as,

$$\varepsilon(\omega, k) = \varepsilon_r(\omega, k) + i\varepsilon_i(\omega, k)\,.$$

Where $\varepsilon_r(\omega, k)\ and\ \varepsilon_i(\omega, k)$ are real and imaginary parts given as:

$$\varepsilon_r(\omega, k) = 1 + \frac{1}{k_z^2 \lambda_{De}^2} + \sum_j \frac{1}{k_z^2 \lambda_{Dj}^2}\left[1 - \Lambda_{j0} - \frac{\Lambda_{j0}\sigma_j^2}{2\xi_{j0}^2}\right], \#(24)$$

and

$$\varepsilon_i(\omega, k) = \frac{\sqrt{\pi}}{k_z^2 \lambda_{De}^2}\ \xi_{e0} e^{-\xi_{e0}^2} + \sum_j \frac{\sqrt{\pi}}{k_z^2 \lambda_{Dj}^2} \Lambda_{j0}\sigma_j^2 \xi_{j0} e^{-\xi_{j0}^2}. \#(25)$$

Here $\sigma_j^2 = 1 - s_j \frac{k_y}{k_z}$. The real frequency and the growth rate of the ion acoustic wave can be determined by expressing $\omega = \omega_r + i\gamma$, the real part becomes as,

$$\varepsilon_r(\omega,k) = 1 + \frac{1}{k_z^2\lambda_{De}^2} + \frac{1}{k_z^2\lambda_{DH}^2}\left(1 - \Lambda_{H_0} - \Lambda_{H_0}\sigma_{H}^2\left(\frac{k_z v_{tH}}{\omega - \omega_{H_0}}\right)^2\right)$$
$$+ \frac{1}{k_z^2\lambda_{DO^+}^2}\left(1 - \Lambda_{O_0^+} - \Lambda_{O_0^+}\sigma_{O^+}^2\left(\frac{k_z v_{tO^+}}{\omega - \omega_{O^+0}}\right)^2\right). \quad \#(26)$$

Where the imaginary part can be written as,

$$\varepsilon_i(\omega,k) = \frac{\sqrt{\pi}}{k_z^2\lambda_{De}^2}\left(\frac{\omega - \omega_{e0}}{\sqrt{2}k_z v_{te}}\right)e^{-\left(\frac{\omega-\omega_{e0}}{\sqrt{2}k_z v_{te}}\right)^2}$$
$$+ \frac{\sqrt{\pi}}{k_z^2\lambda_{DH}^2}\Lambda_{H_0}\sigma_H^2\left(\frac{\omega - \omega_{H_0}}{\sqrt{2}k_z v_{tH}}\right)e^{-\left(\frac{\omega-\omega_{H_0}}{\sqrt{2}k_z v_{tH}}\right)^2}$$
$$+ \frac{\sqrt{\pi}}{k_z^2\lambda_{DO^+}^2}\Lambda_{O^+0}\sigma_{O^+}^2\left(\frac{\omega - \omega_{O^+0}}{\sqrt{2}k_z v_{tO^+}}\right)e^{-\left(\frac{\omega-\omega_{O^+0}}{\sqrt{2}k_z v_{tO^+}}\right)^2}. \quad \#(27)$$

Using the limit $k_z^2\lambda_{DO^+}^2 \ll 1$, making assumption that $(\omega,k) \simeq 0$, the linear dispersion relation from equation (26) can be obtained as,

$$\left(1 + \frac{n_{H_0}}{n_{e0}}k_y^2\rho_{tsH}^2 + \frac{n_{O_0^+}}{n_{e0}}k_y^2\rho_{tsO^+}^2\right)\Omega_{H_0}^2\Omega_{O^+0}^2$$
$$- \left(\frac{n_{H_0}}{n_{e0}}\Lambda_{H_0}\sigma_H^2 k_z^2 c_{sH}^2\Omega_{O^+0}^2 + \frac{n_{O_0^+}}{n_{e0}}\Lambda_{O_0^+}\sigma_{O^+}^2 k_z^2 c_{sO^+}^2\Omega_{H_0}^2\right). \quad \#(28)$$

Above equation reduces to the equation (16) in the limit $k_y^2\rho_{tsj}^2 \ll 1$ i.e., $\Lambda_j \approx 1$. Growth rate can be written as

$$\gamma = -\frac{\varepsilon_i(\omega,k)}{\frac{\partial\varepsilon_r(\omega,k)}{\partial\omega_r}}. \quad \#(29)$$

Differentiating equation (26) with respect to ω_r to obtain,

$$\frac{\partial\varepsilon_r(\omega,k)}{\partial\omega_r} = \frac{1}{k_z^2\lambda_{DH}^2}\left(\Lambda_{H_0}\sigma_H^2\frac{2k_z^2 v_{tH}^2}{(\omega - \omega_{H_0})^3}\right)$$
$$+ \frac{1}{k_z^2\lambda_{DO^+}^2}\left(\Lambda_{O_0^+}\sigma_{O^+}^2\frac{2k_z^2 v_{tO^+}^2}{(\omega - \omega_{O^+0})^3}\right). \quad \#(30)$$

The normalized growth rate is obtained by using equations (27) and (30) into equation (29).

$$\frac{\omega_i}{\Omega_{O^+}} = \frac{\sqrt{\frac{\pi}{8}}(\omega - \omega_{O^+0})^3}{D}\Bigg[\left(\frac{\omega_{e0} - \omega}{\omega - \omega_{O^+}}\right) - \frac{n_{O_0^+}}{n_{e0}}\left(\frac{m_{O^+}}{m_e}\right)^{\frac{1}{2}}\left(\frac{T_e}{T_{O^+}}\right)^{\frac{3}{2}} \Lambda_{O_0^+}\sigma_{O^+}^2\, e^{-\left(\frac{\omega-\omega_{O^+0}}{\sqrt{2}k_z v_{tO^+}}\right)^2} - \frac{n_{H_0}}{n_{e0}}\left(\frac{m_H}{m_e}\right)^{\frac{1}{2}}\left(\frac{T_e}{T_H}\right)^{\frac{3}{2}}\left(\frac{\omega - \eta\omega_{O^+0}}{\omega - \omega_{O^+0}}\right)\Lambda_{H_0}\sigma_H^2\, e^{-\left(\frac{\omega-\eta\omega_{O^+0}}{\sqrt{2}k_z v_{tH}}\right)^2}\Bigg], \quad \#(31)$$

where

$$D = \left\{\frac{n_{O_0^+}}{n_{e0}}\left(\frac{m_{O^+}}{m_e}\right)^{1/2} \Lambda_{O_0^+}\sigma_{O^+}^2 k_z^3 c_{sO^+}^3 + \frac{n_{H_0}}{n_{e0}}\left(\frac{m_H}{m_e}\right)^{1/2}\left(\frac{\omega - \omega_{O^+0}}{\omega - \eta\omega_{O^+0}}\right)^3 \Lambda_{H_0}\sigma_H^2 k_z^3 c_{sH}^3\right\}. \quad \#(32)$$

Here $\eta = \pm\sqrt{\frac{m_{O^+}}{m_{H^\pm}}}, c_{sj} = \frac{T_e}{m_j}$, $\omega_{O^+0} = k_z v_{O^+0}$ *and* $\omega_{e0} = k_z v_{e0}$.

Now we study the growth rate of current driven ion-acoustic waves (CDIAWs) in multi-ion plasma by assuming that ion species have same field-aligned shear flow ($v_{H^+} = v_{O^+0}$). In this case we have $\Omega_0 = k_z v_0$, above equation becomes as,

$$\frac{\omega_i}{\Omega_{O^+}} = \frac{\sqrt{\frac{\pi}{8}}{\Omega_0}^3}{D}\Bigg[\left(\frac{k_z v_{De0}}{\Omega_0} - 1\right) - \frac{n_{O_0^+}}{n_{e0}}\left(\frac{m_{O^+}}{m_e}\right)^{\frac{1}{2}}\left(\frac{T_e}{T_{O^+}}\right)^{\frac{3}{2}} \Lambda_{O_0^+}\sigma_{O^+}^2\, e^{-\left(\frac{\Omega_0}{\sqrt{2}k_z v_{tO^+}}\right)^2} - \frac{n_{H_0^+}}{n_{e0}}\left(\frac{m_{H^+}}{m_e}\right)^{\frac{1}{2}}\left(\frac{T_e}{T_{H^+}}\right)^{\frac{3}{2}} \Lambda_{H_0^+}\sigma_{H^+}^2\, e^{-\left(\frac{\Omega_0}{\sqrt{2}k_z v_{tH^+}}\right)^2}\Bigg], \quad \#(33)$$

where

$$D = \left\{ \frac{n_{\mathrm{O}_0^+}}{n_{e0}} \left(\frac{\mathrm{m}_{\mathrm{O}^+}}{\mathrm{m}_\mathrm{e}} \right)^{1/2} \Lambda_{\mathrm{O}_0^+} \sigma_{\mathrm{O}^+}^2 k_z^3 c_{sO^+}^3 + \frac{n_{\mathrm{H}_0^+}}{n_{e0}} \left(\frac{\mathrm{m}_{\mathrm{H}^+}}{\mathrm{m}_\mathrm{e}} \right)^{1/2} \Lambda_{\mathrm{H}_0^+} \sigma_{\mathrm{H}^+}^2 k_z^3 c_{sH^+}^3 \right\}. \quad \#(34)$$

Similarly for the system of negative light ion with same shear velocity i.e. $\mathrm{v}_{\mathrm{H}^-} = \mathrm{v}_{O^+0}$ we have,

$$\frac{\omega_i}{\Omega_{O^+}} = \frac{\sqrt{\frac{\pi}{8}} \Omega_0{}^3}{D} \left[\left(\frac{\mathrm{k}_z \mathrm{v}_{De0}}{\Omega_0} - 1 \right) - \frac{n_{\mathrm{O}_0^+}}{n_{e0}} \left(\frac{\mathrm{m}_{\mathrm{O}^+}}{\mathrm{m}_\mathrm{e}} \right)^{\frac{1}{2}} \left(\frac{\mathrm{T}_\mathrm{e}}{\mathrm{T}_{\mathrm{O}^+}} \right)^{\frac{3}{2}} \Lambda_{\mathrm{O}_0^+} \sigma_{\mathrm{O}^+}^2 \, e^{-\left(\frac{\Omega_0}{\sqrt{2} k_z v_{tO^+}} \right)^2} \right.$$
$$\left. - \frac{n_{\mathrm{H}_0^-}}{n_{e0}} \left(\frac{\mathrm{m}_{\mathrm{H}^-}}{\mathrm{m}_\mathrm{e}} \right)^{\frac{1}{2}} \left(\frac{\mathrm{T}_\mathrm{e}}{\mathrm{T}_{\mathrm{H}^-}} \right)^{\frac{3}{2}} \Lambda_{\mathrm{H}_0^-} \sigma_{\mathrm{H}^-}^2 \, e^{-\left(\frac{\Omega_0}{\sqrt{2} k_z v_{t\mathrm{H}^-}} \right)^2} \right], \quad \#(35)$$

and

$$D = \left\{ \frac{n_{\mathrm{O}_0^+}}{n_{e0}} \left(\frac{\mathrm{m}_{\mathrm{O}^+}}{\mathrm{m}_\mathrm{e}} \right)^{1/2} \Lambda_{\mathrm{O}_0^+} \sigma_{\mathrm{O}^+}^2 k_z^3 c_{sO^+}^3 + \frac{n_{\mathrm{H}_0^-}}{n_{e0}} \left(\frac{\mathrm{m}_{\mathrm{H}^-}}{\mathrm{m}_\mathrm{e}} \right)^{1/2} \Lambda_{\mathrm{H}_0^-} \sigma_{\mathrm{H}^-}^2 k_z^3 c_{sH^-}^3 \right\}. \quad \#(36)$$

The first term on right hand side of equation (33) has the dominant effect, due to which growth rate becomes positive for $\frac{\mathrm{k}_z \mathrm{v}_{De0}}{\Omega_0} > 1$ corresponding to the unstable roots. As near the resonance $\omega = \mathrm{k}_z \mathrm{v}_{ti}$ large damping appears. But, growth rate becomes positive due to the electron current. Thus wave can appear and propagate through plasma with $\Omega_{ci} > \omega$, when it dominates over the Landau damping. For $k_y^2 \rho_i^2 << 1$ *and* $\omega > \mathrm{k}_z \mathrm{v}_{ti}$, in presence of shear flow IAW frequency shifts away from heavy ion Landau damping even for $\mathrm{T}_i \leq \mathrm{T}_e$. If $\mathrm{k}_y = 0$ or $\sigma_j \to 1$ then this effect is not observed. But IAW will disappear for $\mathrm{k}_z = 0$. According to the limits $\Omega_{ci} \gg \omega$ and $k_y^2 \rho_i^2 << 1$, we cannot suppose the case of $k \to \propto$.

For a classical current driven ion acoustic (CDEIA) instability (i.e., $\sigma_j = 1$) the above dispersion relations are valid for $\tau << 1$ [39]. However, for shear modified ion acoustic (SMIA) modes following expansions are valid even for $\tau \geq 1$ if $\sigma_{O^+} > 0$. Numerical analysis of dispersion relation shows that the destabilization of the CDEIA instability with negative shear occurs only for $\sigma_{O^+} > 0$. This implies that there is certain relationship between phase velocities of the mode in the "y" and “z” directions, for the given value of shear. Namely, the ratio of phase velocities in the z and y directions($V_{\varphi z}/V_{\varphi y}$)is positive even for positive shear velocity. This condition is exactly opposite the D'Angelo mode. Since the two instabilities are separated in the physical parameter space both can exist at the same time without competing with each other. Either one or both the conditions (i.e.,$s_{O^+}/u <\ or > 0$) may hold, it depends on the boundary conditions. Since the shear modified ion acoustic (SMIA) mode is driven by both shear and current (i.e., relative ion-electron drift V_{de}) the sign of $V_{\varphi z}$, is determined by V_{de}. The wave propagates along V_{de}. But on the other hand, D'Angelo mode can propagate parallel or antiparallel with respect to V_{de}. The ion-acoustic mode can excite without any current ($V_{de} = 0$) if shear is strong. In that situation the sign of $V_{\varphi z}$, is independentof V_{de}, but the condition between phase velocities $V_{\varphi y}$ and $V_{\varphi z}$, remainsthe same.

IV. Discussion and Summary:

In the following paper, we have studied electrostatic ion-acoustic waves in multi-ion plasma. Low frequency electrostatic IAWs in the upper ionosphere have been observed by Freja satellite [40] and many others. Ionosphere consists of several kinds of ions at different altitudes.

Ionosphere is normally assumed to be oxygen plasma with very small concentration of protons, and other heavy ions. Precipitation of lighter particles like, electrons, protons, and heavier ions such as N^+ (nitrogen) and O_2^+ (molecular oxygen) has been detected [41], but their density is very low. At altitude of $(1400\text{–}1700)\ km$ or less, the presence of concentration of hydrogen ions as compared to oxygen ions is nearly $4\%\ or\ less$. We have used the data of different research articles which imply that concentration of hydrogen ions in auroral latitudes between $800\ and\ 1700\ km$ is smaller than concentration of oxygen ions [40, 42]. It is an important aspect that many other authors have generally neglected such low concentration of hydrogen ions in oxygen plasma of ionosphere, but we have pointed out that in oxygen plasma, hydrogen ions as impurity plays a significant role to increase the growth rate of the linear ion acoustic wave (IAW) instabilities. These results may be helpful to investigate the many other wave modes in ionosphere.

We have narrated in this paper that the nonresonant D'Angelo mode becomes resonant ion acoustic mode in the multi-ion plasma. The D'Angelo mode has Re(ω_r)=0, while, for the ion-acoustic mode, $\mathrm{Re}(\omega_r) = k_z c_{sa} \sigma > 0$ or $\mathrm{Re}(\omega_r) = k_z c_{sb} \sigma > 0$. This implies that for the D'Angelo mode there will be a narrow frequency spectrum around the zero frequency and a wide range spectrum for the ion acoustic mode. The velocity shear plays a central role in either case. We have discussed this difference in more detail. The D'Angelo mode, in case of single ion is a purely growing mode for $\sigma^2 = (1 - s_i \frac{k_y}{k_z}) < 0$, given from equation (21). While for the condition of $\sigma^2 = \left(1 - s_i \frac{k_y}{k_z}\right) > 0$, then the real frequency is just ion acoustic but modified by shear.

Oxygen hydrogen plasmas along with electrons are present in the D and F regions of the ionosphere. Under the high pressure, negative hydrogen ions are formed in these regions by the attachment of electrons with neutral atoms or molecules. Such plasma can be created in laboratory. [43]. In this paper two systems with positive and negative hydrogen ions in oxygen plasma are solved in parallel. We have compared their results and found that growth rate of negative hydrogen ion system is larger than positive hydrogen ion system under the parallel shear flow of ions and electron current. It was proposed that ion heating effect in solar wind, solar corona and heat transfer in ionosphere is due to the ion landau damping of IAW [28]. The sources of free energy discussed in this paper to excite these IA modes are electron current and parallel shear flow of ions.

In the magnetized plasma of ionosphere, the main sources of free energy for the waves to grow are the field-aligned and cross-field flows [44]. Using plasma fluid and kinetic approach, theoretical model has been presented to explain the existence of current driven and shear modified ion-acoustic waves (IAWs) in the ionosphere. It shows that the parallel electron current due to parallel flow of electrons gives rise to unstable IAWs in hydrogen-oxygen ion plasma of ionosphere with ions shear flow. Growth is also observed in a strong shear even without parallel flow of electrons.

In both systems of species it is observed that purely growing shear driven instability takes the form of oscillatory instability in multi-ion component plasma and hence it gives rise to ion-acoustic (IA) mode in the framework of multi-fluid theory. For negative values of ion shear flow, ion acoustic wave (IAW) dispersion relation is modified and its phase velocity overthrow from the zone of heavy ion Landau damping even for plasma which has $\tau \geq 1$.

Both kinetic and fluid approaches have been used to study the relative shear flow of species in (H−, O+, e−) and (H+, O+, e−) two different systems, while in Ref. [37], only kinetic approach was used for the system of species (H+, O+, e−). We have plotted and discussed the results for these two systems of species with different cases. Results are compared and explored the effect of species concentrations on growth rate of ion-acoustic waves in multi-element plasma. We have considered the resonant mode, the following condition must satisfy the, $\sigma_j^2 > 1$ for which we choose negative sheared flow, i.e., $\frac{S_j}{u} < 0$. If this condition holds then we find $\sigma_j^2 > 1$ and wave frequency becomes larger, i.e., $k_z v_{tj} < \omega$ corresponding to the ions $j = H^+, H^-, O^+$, so the phase velocity of the wave overflows the heavy ion Landau damping region. We used the set of possible experimental parameters, as $v_{de}/v_{to^+} = 60$, $\tau = T_{O^+}/T_e = 0.3$, $T_{O^+}/T_{H^+} = 1.0$, $m_{O^+}/m_H = 16$, $m_{O^+}/m_e = 29392$, $s_{O^+} = |V_d'|/\Omega_{cO^+} = 0.3$, $\left(k_y \rho_{tO^+}\right)^2 = 0.0675$.

Figure (1) illustrates the numerical results of dispersion relation. In this figure, the density effect of positive hydrogen ions is discussed. Growth rate increases by increasing the concentration of hydrogen ions, when all species have field align flow. Peaks of curves shift towards the smaller value of kz and longer the value of wavelength, because light ions have large velocity as compare to heavy oxygen ions. Growth rates are discussed for different density ratios $\frac{n_{H^+}}{n_{e0}}$ (0.0, black dotted line; 0.02, blue dashed line; 0.03, green dashed line; 0.04, red dashed line, and 0.05, magenta dashed line, respectively). Figure (2) shows the numerical results. The current driven electrostatic mode due to parallel flow of electrons only is discussed here. In this case ions velocity is assumed to be zero ($v_{H^+0} = v_{O^+0} = 0$), but ions have shear flow. Growth rate reduces by increasing the number density of positive hydrogen ions because lighter ions provide the small shear as compared to the heavy ions. According to charge neutrality condition, the number density of oxygen ion reduces by increasing the number density of positive hydrogen ions. Growth rates observed with different density ratios $\frac{n_{H^+}}{n_{e0}}$ (0.0, black dotted line; 0.001, blue dashed line; 0.005, green dashed line; 0.01, red dashed line, 0.02, magenta dashed line, and 0.03, cyan dashed line, at the ratios of kz/ky= 0.16, 0.165, 0.15, 0.14, 0.09, 0.08 respectively).

In Figure (3) numerical results is discussed for (H−, O+, e−) plasma. Here we study the field aligned flow of electrons with ($v_{H^-0} = v_{O^+0} = 0$). Growth rate reduces by increasing the number density of negative hydrogen ions. According to the charge neutrality condition, the concentration of electrons reduces by increasing the negative hydrogen ions. The field align flow of electrons reduces, therefor growth rate decreases. Growth rate observed with density ratios $\frac{n_{H^-}}{n_{e0}}$ (0.0, black dotted line; 0.001, blue dashed line; 0.005, green dashed line; 0.01, red dashed line, 0.02, magenta dashed line, at the ratios of kz/ky= 0.16, 0.155, 0.15, 0.14, 0.12, respectively) with sheared parameters $\frac{|V_d'|}{\Omega_{cO^+}} = 0.3, \tau = 0.3\ and\ v_{de}/v_{tO^+} = 60$. In Figure (4) we see the comparison of results of two system of species (H+, O+, e−) and (H−, O+, e−) with sheared flow of plasma species. We observe that at the same value of shear the growth rate will be larger with negative hydrogen ions than the positive hydrogen ions, because oxygen ions concentration reduces by increasing positive hydrogen ions. Heavier ions provides the large shear than the lighter ions ($s_{H^+} \ll s_{O^+}$), therefore growth rate is smaller with the system of positive hydrogen ions. Figure (5) shows the comparison of same line streaming and counter streaming of plasma species. The growth rate with field align streaming flow of species is larger than counter streaming. Figure (6) shows the effect of shear. The growth rate and width of the curves increases by increasing the value of shear. The variation of growth rate with different values of ion to electron temperature ratios are shown in Figure (7). Growth rate increases and wave shifts towards the shorter value of kz for large value of τ. Numerically solved shear dependent growth rates versus normalized real frequency for the system of species (H−, O+, e−) with different shear values $|V_d'|/\omega_{cO^+} = 0.3$, 0.5, and 0.7, are shown in figure (8). The growth rate increases by increasing the shear value of the species.

Summary:

In this work we have studies ion acoustic waves (IAWs) in the hydrogen-oxygen plasma with two system of species (H+, O+, e−) and (H−, O+, e−). When shear flow is present in both the ionic species. Following are the main findings of this research work.

- It was pointed out that the growth rate increases large with field aligned flow of light species than heavier.

- The frequency of acoustic waves is larger with negative hydrogen ion then the positive hydrogen ion.
- The frequency of IAW is larger in same streaming than counter streaming of the species.
- The frequency of the IAWs increases and Peak of the frequency curves shift towards the shorter wavelength by increasing the value of shear flow.

Figure Captions:

Figure (1): Growth rate with v_{H^+0}, $v_{O^+0} \neq 0$, $\frac{|V_d'|}{\omega_{cO^+}} = 0.3$, $\frac{v_{de}}{v_{to^+}} = 60$, $\tau = 0.3, \frac{T_{O^+}}{T_{H^+}} = 1.0$, $\frac{m_{O^+}}{m_{H^+}} = 16, \frac{m_{O^+}}{m_e} = 29392$ and $\frac{n_{H^+}}{n_{e0}}$ (0.0, black dotted line; 0.001, blue dashed line; 0.005, green dashed line; 0.01, red dashed line, and 0.02, magenta dashed line, respectively).

Figure (2): Growth rate with v_{H^+0}, $v_{O^+0} = 0$, $\frac{|V_d'|}{\omega_{cO^+}} = 0.3$, $\frac{v_{de}}{v_{to^+}} = 60$, $\tau = 0.3, \frac{T_{O^+}}{T_{H^+}} = 1.0$, $\frac{m_{O^+}}{m_{H^+}} = 16, \frac{m_{O^+}}{m_e} = 29392$ and $\frac{n_{H^+}}{n_{e0}}$ (0.0, black dotted line; 0.001, blue dashed line; 0.005, green dashed line; 0.01, red dashed line, 0.02, magenta dashed line, and 0.03, cyan dashed line, at the ratios of kz/ky= 0.16, 0.165, 0.15, 0.14, 0.09 and 0.08 respectively).

Figure (3): Growth rate with v_{H^-0}, $v_{O^+0} = 0$, $\frac{|V_d'|}{\omega_{cO^+}} = 0.3$, $\frac{v_{de}}{v_{to^+}} = 60$, $\tau = 0.3, \frac{T_{O^+}}{T_{H^-}} = 1.0$, $\frac{m_{O^+}}{m_{H^-}} = 16, \frac{m_{O^+}}{m_e} = 29392$ and $\frac{n_{H^-}}{n_{e0}}$ (0.0, black dotted line; 0.001, blue dashed line; 0.005, green dashed line; 0.01, red dashed line, and 0.02, magenta dashed line, at the ratios of kz/ky= 0.16, 0.155, 0.15, 0.145, and 0.125 respectively).

Figure (4): v_{H^+0}, $v_{O^+0} \neq 0$, $\frac{|V_d'|}{\omega_{cO^+}} = 0.3$, $\frac{v_{de}}{v_{to^+}} = 60$, $\tau = 0.3, \frac{T_{O^+}}{T_{H^+}} = 1.0$, $\frac{m_{O^+}}{m_{H^+}} = 16, \frac{m_{O^+}}{m_e} = 29392$. For $\mathrm{nH}^+ = \mathrm{nH}^- = 0(\textit{black dashed line})$, $\mathrm{nH}^-/\mathrm{ne0} = 0.05(\textit{blue dashed line})$, $\mathrm{nH}^+/\mathrm{ne0} = 0.05(\textit{green dashed line})$ corresponding to the $\mathrm{kz}/\mathrm{ky} = 0.23,\ 0.14,\ \textit{and}\ 0.1$ respectively.

Figure (5): Comparison of same line streaming and counter streaming with v_{H^+0}, $v_{O^+0} \neq 0$, $\frac{|V_d'|}{\omega_{cO^+}} = 0.3$, $\frac{v_{de}}{v_{to^+}} = 60$, $\tau = 0.3, \frac{T_{O^+}}{T_{H^+}} = 1.0$, $\frac{m_{O^+}}{m_{H^+}} = 16, \frac{m_{O^+}}{m_e} = 29392$

Figure (6): Normalized growth rate with velocity different shears, black, blue, green, red, and magenta dashed lines corresponding the $\frac{|V_d'|}{\omega_{cO^+}} = 0.2,\ 0.3,$ $0.4,\ and$ 0.5 respectively. v_{de}/v_{to^+}=60.0, $\mathrm{nH}^+/\mathrm{ne0} = 0.01$ and $\tau = 0.3$

Figure (7): Temperature dependence growth rates are shown for $\tau = 0.1, 0.5,$ 1.0, 1.5, and 2.0, black, blue, green, red, and magenta curves respectively. Here $|V_d'|/\omega_{cO^+}$ equal to 0.3, $\mathrm{nH}^+/\mathrm{ne0} = 0.01$ and $v_{de}/v_{to^+} = 60$.

Figure (8): Growth rates are shown for $|V_d'|/\omega_{cO^+} = 0.3, 0.5,$ and 0.7, black, blue, and red curves respectively. Here τ equal to 0.3, $\mathrm{nH}^-/\mathrm{ne0} = 0.008$ and $v_{de}/v_{to^+} = 60$.

Data Availability Statement:

Data that support findings of this study are available from corresponding author upon sensible request.

Author Contributions Statement:

First author K. Shahzad has contributed 60%, second author Aman-ur-Rehman 25% and third author Muhammad Yousaf Hamza 15% to write the following manuscript. Shahzad designed this model, derived the mathematical equations and plotted the results. Aman-ur-Rehman helped to derive the numerical results. Muhammad Yousaf Hamza removed the typo mistakes and help to sentense correction.

Author Consent Statement:

It is stated that the following article is submitted with consent of all authors.

***References*:**

1: R. Sabry, W. M. Moslem, and P. K. Shukla, Phys. Plasmas **16**, 032302 (2009).

2: T. Ross, E. J. Baker, T. P. Snow, J. D. Destree, B. L. Rachford, M. M. Drosback, and A. G. Jensen, APJ **684**, 358 (2008).

3: S. Hussain and S. Mahmood, Phys. Plasmas **24**, 102106 (2017).

4: Yu. I. Portnyagin, O. F. Klyuev, A. A. Shidlovsky, A. N. Evdokimov, T. W. Buzdigar, P. G. Matukhin, S. G. Pasynkov, K. N. Shamshev, V. V. Sokolov, and N. D. Semkin, Adv. Space Res. **11**, 89 (1991).

5: P. H. Chaizy, H. Re`me, J. A. Sauvaud, C. d'uston, R. P. Lin, D. E. Larson, D. L. Mitchell, K. A. Anderson, C. W. Carlson, A. Korth, and D. A. Mendis, Nature (London) **349**, 393 (1991).
6: S. V. Vladimirov, K. Ostrikov, M. Y. Yu, and G. E. Morfill, Phys. Rev. E **67**, 036406 (2003).
7: M. Djebli, Phys. Plasmas **10**, 4910 (2003).
8: S. Hussain, N. Akhtar, and Saeed-ur-Rehman, Chin. Phys. Lett. **28**, 045202 (2011).
9: A. A. Mamun and P. K. Shukla, Phys. Plasmas **10**, 1518 (2003); Q.-Z. Luo, N. D'Angelo, and R. L. Merlino, Phys. Plasmas **6**, 3455 (1999).
10: S. Hussain, Manhaz, Q. Haseeb and H. Hasnain; Phys. Plasmas, **24**, 102126 (2017).

11: J. Castro, P.McQuillen, and T. C. Killian, Phys. Rev. Lett. **105**, 065004 (2010); L. Tonks and I. Langmuir, Phys. Rev. Lett.**33**, 195 (1929).
12: T.H. Stix, Waves in Plasmas,American Institute of Physics, New York, (1992).
13: Y.Nakamura, H. Bailung, and P.K. Shukla, Phys. Rev. Lett.**83**, 1602 (1999).
14: Z. Liu, L. Liu, and J. Du, Phys. Plasmas, **16**, 072111 (2009).
15: J. E. Wahlund, P. Louarn, T. Chust, H. Feraudy, A. Roux, B. Holback, P. O. Dovner, and G. Holmgren, Geophys. Rev. Lett. **21**, 1831 (1994).
16: S. Sen, R. A. Cairns, R. G. Storer, and D. R. MeCarthy, Phys. Plasmas, **7**, 1192 (2000).
17: M. E. Koepke, Phys. Plasmas, **9**, 2420 (2002).
18: S. R. Cranmer, A. A. Van Ballegooijen, and R. J. Edgar, Astrophys.**171**, 520 (2007).
19: F. L. Scarf, F. V. Coroniti, C. F. Kennel, D. A. Gurnett, W.-H. Ip,and E. J. Smith, Science **232**, 377 (1986).
20: F. Wagner, G. Becker, K. Behringer, D. Campbell, A. Eberhagen, W. Engelhardt, G. Fussmann, O. Gehre, J. Gernhardt, G. V. Gierke, G. Haas, *et al.*, Phys. Rev. Lett. **49**, 1408 (1982).
21: Ch. P. Ritz, Roger D. Bengtson, S. J. Levinson, and E. J. Powers, Phys. Fluids **27**, 2956 (1984).
22: K. H. Burrell, T. N. Carlstrom, E. J. Doyle, P. Gohil, R. J. Groebner, T. Lehechka, N. C. Luhmann, H. Matsumoto, T. H. Osborne, W. A. Peebles, and R. Philipona, Phys. Fluids **2**, 1406 (1991).
23: W. E. Amatucci, J. Geophys. Res. **104**, 14481 (1999).
24: J. Bonnell, P. Kintner, J. E. Wahlund, K. Lynch, and R. Arnoldy, Geophys.

Lett. **23**, 3297 (1996).
25: J. M. Kindel, and C.F. Kennel, J. Geophys. **76**, 3055 (1971).
26: J. E. Wahlund, A. I. Eriksson, B. Holback, M. H. Boehm, J. Bonnel, P. M. Kintner, C. E. Seyler, J. H. Clemmons, L. Eliasson, D. J. Knudsen, P. Norqvist, L. J. Zanetti; J. Geophys. **103**, 4343(1998).
27: D. A. Gumeft, and L.A. Frank, J. Geophys. **83**, 58 (1978).
28: N. D'Anglo, Phys. Fluids **8**, 1748 (1965).
29: M. Dobrowolny, Phys. Fluids**15**, 2263 (1972).
30: N.D'Angelo,J. Geophys. **78**, 1206 (1973).
31: N. D'Angelo, A. Bahnsen, and H. Rosenbauer, J. Geophys. **79**, 3129 (1974).
32: N. D'Angelo, and S.V. Goeler, Phys. Fluids **9**,309(1966).
33: V. V. Gavrishchaka, S.B. Ganguli, and G. I. Ganguli, Phys. Rev. Lett. **80**, 728 (1998); and V. V. Gavrishchaka, S. B. Ganguli, Geophysical Research, **104**, 683(1999).
34: W.K. Peterson, R. D. Sharp, E.G. Shelley, R.G. Johnson, H.Balsiger, J. Geophys. **86**, 761(1981).
35: S.V. Singh, R.V. Reddy, G.S. Lakhina, J. Atmos. Solar Terres. Phys., (2002).
36: Lu Li, Liu Zhen-Xing, Li Zhong-Yuan, Phys. Plasmas 7, 1014 (2000).
37: Hamid Saleem, Shaukat Ali Shan and Aman-ur Rehman, Phys. Of Plasma, 24, 122901 (2017)
38: H. Saleem, N. Batool, and S. Poedts Phys. Plasmas **18**, 052108 (2011).
39: Goldston, R.J., and P.H. Rutherford, Introduction to plasma physics, Inst. of Phys., Philadelphia, (1995).
40: J.E. Wahlund, P. Louarn, T. Chust, H. de Feraudy,A. Roux, B. Holback, B. Cabrit, A.I. Eriksson, P.M.Kinruer, M.C. Kelley, J. Bonnel and S. Chesney, Geophys. Res. Lett. **21**, 1835 (1994).
41: G.T. Marklund, Plasma Phys. Control. Fusion **39**, 195 (1997).
42: M.C. Kelley, The earth ionosphere: plasma physics and electrodynamics 2nd edn. Academic Press, Elsevier, Oxford, (2009).
43: M. Rosenborg and R. L. Merlino; J. Plasma Phys. **4**, 1017 (20013).
44: W.E. Amatucci, J. Geophys. Res. **104**, 14481(1999).

Figure 1:

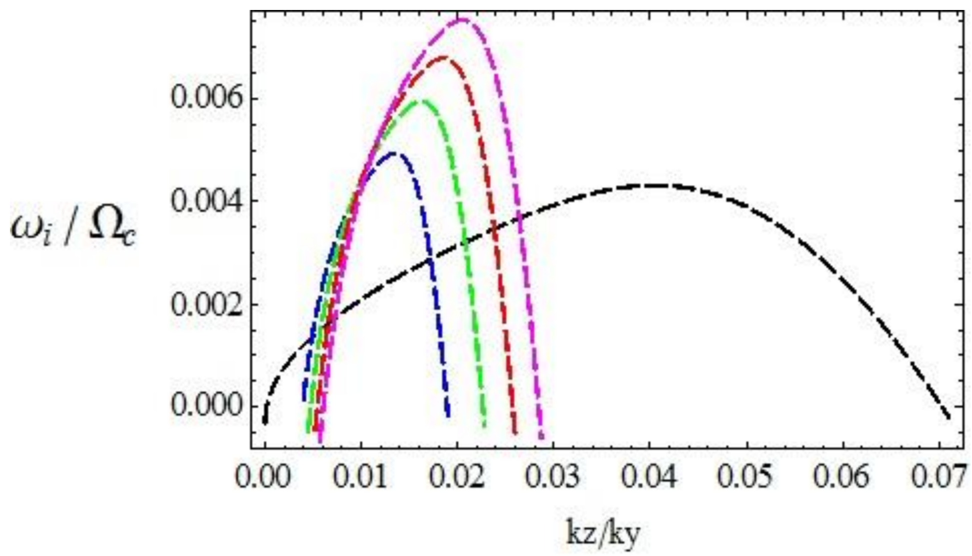


Figure 2:

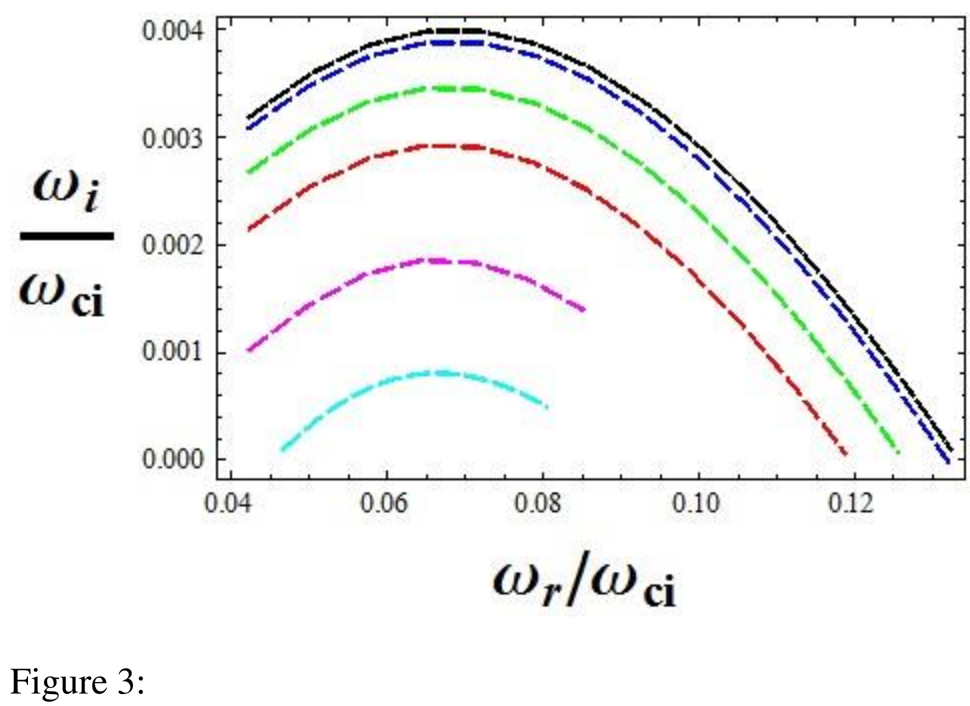


Figure 3:

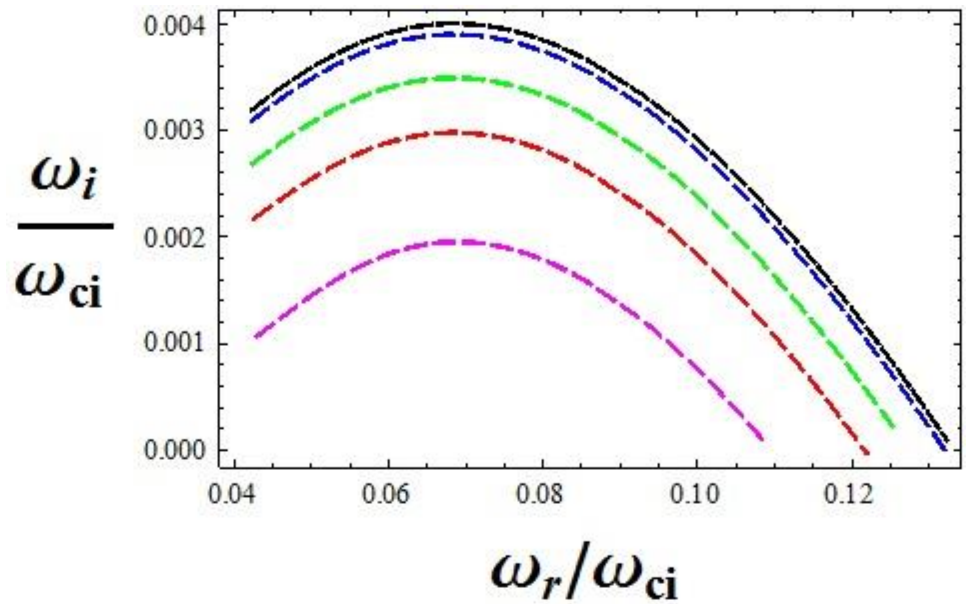


Figure 4:

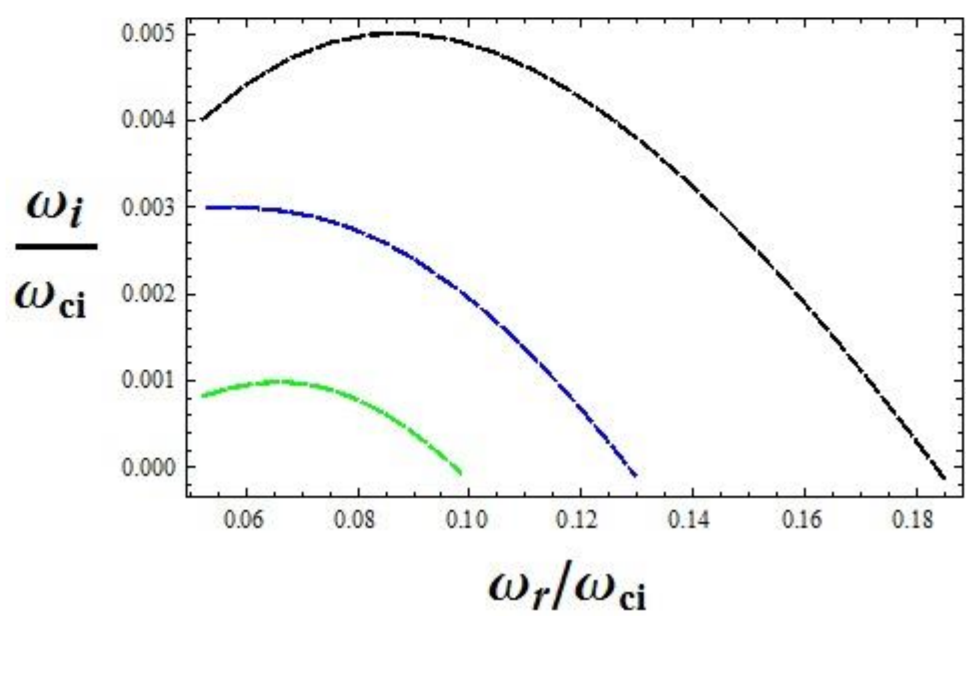


Figure 5:

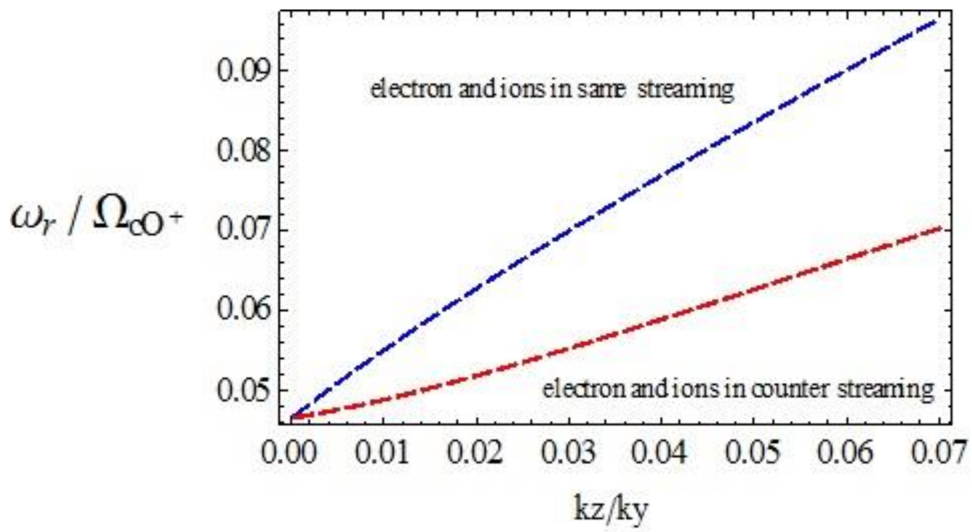


Figure 6:

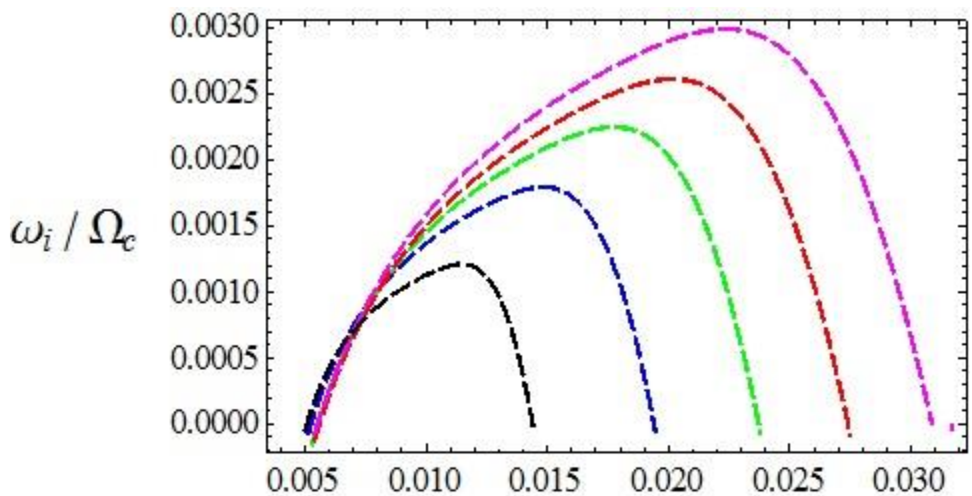


Figure 7:

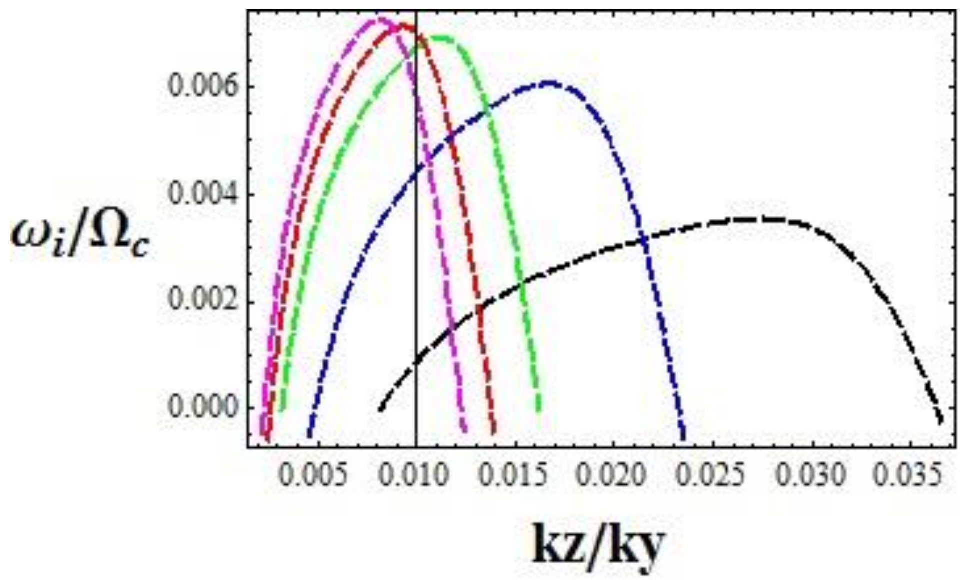
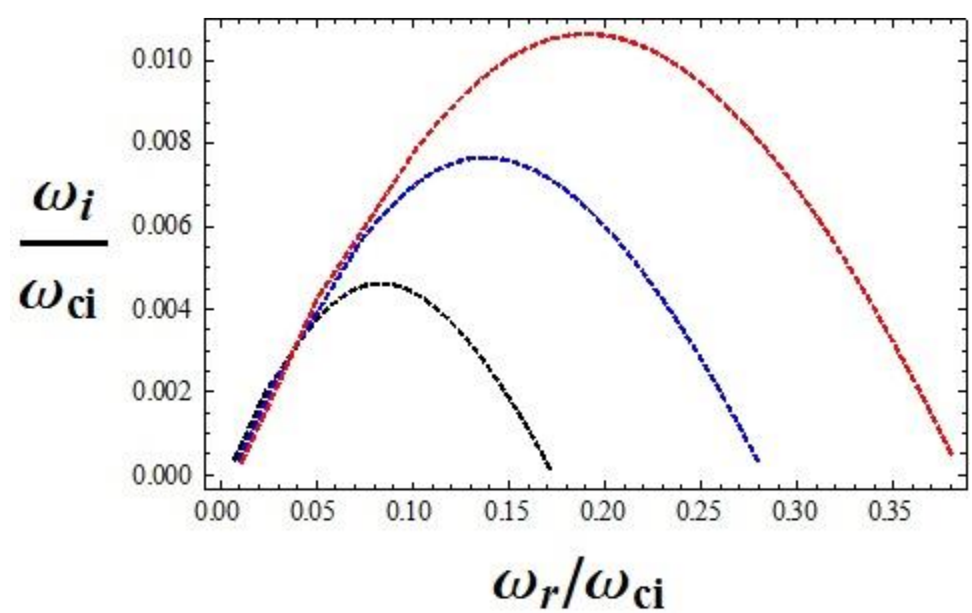

0.006
0.004
0.002
0.000
ω_i/Ω_c
0.005 0.010 0.015 0.020 0.025 0.030 0.035
kz/ky

Figure 8:


0.010
0.008
0.006
0.004
0.002
0.000
$\frac{\omega_i}{\omega_{ci}}$
0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35
ω_r/ω_{ci}