# Rheological behavior of oil- silicon dioxide- multi walled carbon nanotube hybrid nanofluid: Experimental study and neural network prediction

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# Abstract

Hybrid nanofluids have great potential for use in thermal systems due to their improved thermal properties. In this paper, the rheological behavior of oil (5w30)- 10% multi walled carbon nanotubes (MWCNT)- 90% silicon dioxide ($SiO_2$) is experimentally examined in the temperature range of 5°C to 65°C. The volume fractions (VFs) are in the range of 0.05 to 1 vol.% and the shear rate (SR) range is 665.5-13330 1/s. Measuring viscosity at different SRs indicated a pseudoplastic rheological behavior of the nanofluids in all VFs and temperatures. Measurement results show that the dynamic viscosity in different volume fractions is reduced when the temperature is increased from 5°C to 65°C. In addition, when the VF is increased from zero to 1%, the dynamic viscosity is augmented between 30.43% and 70.55%. Based on obtained data, a novel three-variable correlation for relative viscosity is proposed which estimates experimental results with a good accuracy. Then, the correlation results are compared to available correlations for hybrid nanofluids in the literature. Finally, a GMDH-type neural network model based on experimental data is developed to predict the relative viscosity of oil (5W30)/$SiO_2$- MWCNT hybrid nanofluids which reveals the predictability of studied hybrid nanofluid using GMDH-type neural network.


# Nomenclature

| | |
|---|---|
| Dynamic Viscosity (kg/m.s) | μ |
| Shear rate (SR)(1/s) | γ |
| Shear stress (SS)(Pa) | τ |
| Volume fraction (VF) | ϕ |
| Mass(kg) | M |
| Consistency index | m |
| Power law index | n |
| Density(kg/$m^3$) | ρ |

# 1. Introduction

Hybrid nanofluids are a novel type of nanofluid with improved properties than conventional nanofluids and so they have better performance characteristics than mono nanofluids [1]. Hybrid nanofluids can be used in a variety of equipment such as cooling systems [2–5], fuel cells [6], heat pipes [7], heat sinks [8–11], and different types of heat exchangers [12]. Determining the hybrid nanofluids rheological behavior and developing the relationship between viscosity, volume fraction (VF) and temperature and also determining the changes in shear stress (SS) and the applied shear rate (SR) are very important and are studied by many researchers [13–15].

## 1.1. Oil based hybrid nanofluids

Asadi and Asadi [16] investigated the MWCNT/ZnO–10W40 hybrid nanofluid. They found that increasing temperature, decreases the dynamic viscosity of nanofluids up to 85% and when the VF is increased, it is increased by 45%. In addition, according to experimental data, a novel model for predicting the dynamic viscosity of the intended nanofluid is presented. Esfe et al. [17] examined the rheological behavior of the Multi-walled carbon nanotubes MWCNT-$Al_2O_3$ (30:70)/ 5w50 nanofluid. Viscosity measurement exhibited that the behavior of nonofluids is non-Newtonian in all temperatures and VFs which behave like a pseudoplastic fluid. Results show that the nanofluid viscosity is reduced when the temperature is augmented, and it is increased when the VF is augmented. The experimental results of Esfe et al. [18] showed the non-Newtonian behavior of MWCNT (30%)–TiO2 (70%)/SAE50 hybrid nanofluid at all VFs. To predict viscosity, three experimental correlations were suggested, and two different independent variables were taken into account for all of them. The results showed that the suggested correlation that includes temperature as an independent variable has better accuracy than other correlations that do not include temperature as an independent variable. Goodarzi et al. [19] performed an experimental research about the temperature and nanoparticles concentration effects on ZnO–MWCNTs/SAE 10W40 hybrid nanofluid and measured the viscosity of hybrid nanofluid. Results revealed that this nanofluid behaves Newtonian at all temperatures and VFs. Esfe et al. [20] presented an artificial neural network (ANN) design and experimental assessment of MWCNTs-ZnO(10:90)/ 5W50 hybrid nanofluid. They also developed a mathematical correlation to calculate the viscosity and performed optimization of nanofluids viscosity considering temperature, VF, and SR. Finally, they found that the ANN data have less than 7% of error.

## 1.2. Aqueous hybrid nanofluids

Esfe et al. [21], examined the effect of the VF of nanoparticles on the dynamic viscosity of Ag–MgO (50:50)/water hybrid nanofluid with a particle size of 40 nm (MgO) and 20 nm (Ag) and the VF ranges from 0–2%. In addition, new correlations are developed which have significantly good accuracy in comparison to the predicted values by experimental data. The rheological behavior of the Fe-CuO/ water-EG (20:80) hybrid nanofluid is experimentally studied by Bahrami et al. [22]. Experimental results indicated that the samples with low concentration show Newtonian behavior, whereas the samples with high concentration have non-Newtonian behavior like a shear thinning fluid and follow the power law model. The results of curve fitting showed that the power law index decreases to 0.36. Ruhani et al. [23] experimentally established a new model for the rheological behavior of the Silica–EG/Water (30:70) hybrid nanofluid. They found that the developed correlation is a proper model to estimate the viscosity of the intended nanofluid. Results show that as the VF is increased, the hybrid nanofluid viscosity is increased and SS and SR have a linear relation and thus the intended fluid is Newtonian. In highest VF, the viscosity loss percentage from the lowest temperature to the highest temperature is 89%. Moreover, in the highest working temperature, the relative viscosity increase percentage in the highest VF compared to the lowest VF is 48%. Asadi et al. [24] experimentally examined the CuO-$TiO_2$/water hybrid nanofluid. Their results showed that the prepared nanofluid was a Newtonian fluid. Additionally, the maximum dynamic viscosity stands at a VF of 1% and a temperature of 25°C. Giwa et al. [25] experimentally investigated the viscosity of deionized water (DIW)-based MWCNT-$Fe_2O_3$ (20:80) nanofluids. In comparison to the base fluid, the viscosity of the hybrid nanofluid increased by 35.7%. Moreover, a correlation was developed to calculate the viscosity on the basis of the results of the experiments.

## 1.3. Hybrid nanofluids including MWCNTs and $SiO_2$ nanoparticles

In recent years, many studies have been done on dynamic viscosity of mono nanofluids, some of which include $SiO_2$ [26–28] and some MWCNTs [29–31]. Table 1 presents the studies conducted between 2016 to 2022 on hybrid nanofluids including $SiO_2$ and MWCNT nanoparticles. As can be seen, in the study by Esfe et al. [32], hybrid nanofluid has a Newtonian behavior in volume fractions up to 1% and a non-Newtonian behavior in higher volume fractions. Therefore, changing the volume fraction can change the rheological behavior. According to Table 1, in both studies of Motahari et al. [33] and Haldar et al. [34], the nanoparticles mass fraction is similar (MWCNTs-$SiO_2$ (20:80)), but the base fluid is different (the base fluid in study by Motahari et al. [33] is 20W50 and in Haldar et al. [34] is hydraulic oil (SAE68)). The results show that the hybrid nanofluid in the study of Motahhari et al. [33] is Newtonian and in the study of Haldar et al. [34] is non-Newtonian. Consequently, the base fluid can play an important role in the rheological behavior of the hybrid nanofluid. In addition to, based on Table 1, in both studies of Afrand et al. [35] and Esfe et al. [36], the base fluid is SAE40, but the nanoparticles mass fraction is different, so that in the study of Afrand et al. [35], the mass fraction of $SiO_2$-MWCNT nanoparticles is (50:50) and in the study of Esfe et al. [36], this ratio is (30:70). The results show that the nanofluid is Newtonian in the study by Afrand et al. [35], while it is non-Newtonian in the study of Esfe et al. [36]. Also, the nanoparticle characteristics including purity, size, specific surface area and true density can affect the behavior of rheology.

Table 1
Studies on hybrid nanofluids including $SiO_2$ and MWCNT nanoparticles (2016–2022)

| Authors | Year | Base fluid | Nano particles | Rheological behavior | Methodology | Proposed correlations |
|---|---|---|---|---|---|---|
| Esfe et al. [32] | 2016 | SAE40 | MWCNTs/$SiO_2$ (20:80) | Newtonian at the VF up to 1% and non-Newtonian for VFs of 1.5% and 2%. | Experimental | At T = constant; $\mu = f(\varphi)$ |
| Afrand et al. [35] | 2016 | SAE40 | $SiO_2$-MWCNTs (50:50) | Newtonian | Experimental | At T = constant; $\mu = f(\varphi)$ |
| Nadoosha et al. [37] | 2018 | 10W40 | $SiO_2$-MWCNTs (90:10) | - | Experimental<br>ANN | - |
| Esfe and Abbasian Arani [38] | 2018 | 5W50 | MWCNTs-$SiO_2$ (40:60) | Non-Newtonian | Experimental<br>ANN | $\mu = f(T,\varphi,\dot{\gamma})$ |
| Motahai et al. [33] | 2018 | 20W50 | MWCNTs-$SiO_2$ (20:80) | Newtonian | Experimental | $\mu = f(T,\varphi)$ |
| Amini et al. [39] | 2019 | glycerol | $SiO_2$/MWCNTs (90:10)<br>&<br>$SiO_2$/MWCNTs (95:5) | - | Experimental | $\mu = f(T,\varphi)$ |
| Haldar et al. [34] | 2020 | hydraulic oil(SAE68) | MWCNTs/$SiO_2$<br>(20:80) | Non-Newtonian | Experimental<br>ANN | At T = constant; $\mu = f(\varphi)$ |
| Esfe et al. [40] | 2021 | SAE40 | $SiO_2$-MWCNT (90:10) | Non-Newtonian | Experimental<br>RSM | $\mu = f(T,\varphi)$<br>$\mu = f(\dot{\gamma},\varphi)$<br>$\mu = f(T,\varphi,\dot{\gamma})$ |
| Esfe and Saedodin [41] | 2021 | SAE50 | $SiO_2$-MWCNT (90:10) | Non-Newtonian | Experimental<br>RSM | $\mu = f(T,\varphi)$<br>$\mu = f(\dot{\gamma},\varphi)$<br>$\mu = f(\dot{\gamma},T)$<br>$\mu = f(T,\varphi,\dot{\gamma})$ |

| Authors | Year | Base fluid | Nano particles | Rheological behavior | Methodology | Proposed correlations |
|---|---|---|---|---|---|---|
| Esfe et al. [36] | 2022 | SAE40 | $SiO_2$-MWCNT (30:70) | Non-Newtonian | Experimental RSM | $\mu = f(T, \varphi, \dot{\gamma})$ |
| Present work | 2022 | 5w30 | $SiO_2$-MWCNT (90:10) | Non-Newtonian | Experimental ANN | $\mu = f(T, \varphi, \dot{\gamma})$ |

According to the comprehensive literature review, the behavior of hybrid nanofluids with new compounds, both in terms of base fluid and nanoparticles, is not well studied and cannot be predicted by available correlations. Thus, in the present study, an experimental investigation is done on viscosity of the new hybrid nanofluid 5w30/$SiO_2$-MWCNT (90:10) within the VF range of 0.05%-1%, the temperature range of 5 °C-55 °C and the shear rate (SR) range of 665.5-13330 1/s. Moreover, a novel three-variable correlation are proposed based on experimental data and its results are compared to available correlations in the literature associated with hybrid nanofluids. In addition, a GMDH-type neural network model is developed on the basis of experimentation to predict the relative viscosity of oil (5W30)/$SiO_2$- MWCNT (90:10) hybrid nanofluid.

## 2. Experiment

In this section, properties of the nanoparticles and the used oil is described. Also the preparation method of the lubricant engine oil (5W30)/$SiO_2$-MWCNT (90:10) hybrid nanofluid as well as the nanofluid viscosity measurement method are explained.

# 2.1. Nanoparticles and oil preparation

The used oil is the 5w30 engine oil made by Behran company. The oil density is 853 kg/$m^3$ at the temperature of 15°C and its viscosity index is 160. Pour point and ignition point for this oil are − 42°C and 218°C respectively. The nanoparticles silicon dioxide ($SiO_2$)-90% and MWCNTs-10% are provided from an American nanomaterials research company. Figures 1 and 2 show the image of these nanoparticles. Also, in Fig. 3 nanofluid samples are shown in the order of VF. Also, the used nanoparticles properties are presented in Table 2.

Table 2
$SiO_2$ and MWCNTs properties

| **Property** | **Silicon dioxide** | **Multi-walled carbon nanotubes** |
|---|---|---|
| Nanoparticles purity | 99% | 95% |
| Color | White | Black |
| Size | 11–13 nm | Outer diameter<br>20–30 nm<br>Inner diameter 3–5 nm |
| Density (ρ) | 2.4 g/cm³ | 2.1 g/cm² |
| Specific surface area | 200 m²/g | 223 m²/g |

In order to assure nanoparticles structure and size, dry nanoparticle samples of $SiO_2$ and MWCNTs are examined using XRD (X-ray diffraction). The size and structure of Nanoparticles are proven by XRD charts. The sharp and narrow diffraction peak in XRD results declared that MWCNT and $SiO_2$ nanoparticles have an excellent crystal phase structure. The picture of the X-ray diffraction related to nanoparticles are shown in Fig. 4.

## 2.2. Hybrid nanofluid preparation

The hybrid nanofluids in VFs of 0, 0.05, 0.1, 0.2, 0.5, 0.75 and 1 percent of nanoparticles are prepared.

The total VF of the nanoparticles within the hybrid nanofluid is computed using Eq. (1).

$$\phi = \frac{\left(\frac{m}{\rho}\right) SiO_2 + \left(\frac{m}{\rho}\right) MWCNT}{\left(\frac{m}{\rho}\right) oil + \left(\frac{m}{\rho}\right) SiO_2 + \left(\frac{m}{\rho}\right) MWCNT} \times 100 \tag{1}$$

In Eq. (1), $\varphi$ is nanoparticles VF, $m$ stand for mass and ρ refers to density. The nanoparticles mass of $SiO_2$ and MWCNT is measured through laboratory electronic balance.

In order to prepare the nanofluid, the two-step technique is used. In this technique, first, the needed values of the nanoparticles to achieve the desired VF are accurately measured and after that, using proper methods, it is poured into the water-oil hybrid fluid. The nanoparticles size and their dissemination in the base fluid plays a main role in the Hydraulic characteristics of the nanofluid. For this reason, the most essential matter to consider in the two-step technique is to troubleshoot accumulation and adhesion of the nanoparticles. In order to do so, the nanofluid is rotated using a magnetic agitator for three hours to do the suspension process and the nanofluid to become stable. The nanofluid is exposed to the waves of the supersonic device by a device with a power of 400 watts and the frequency of 24 kHz which is shown in Fig. 5.

## 2.3. Measuring the dynamic viscosity

For measuring the nanofluids viscosity, the CAP2000 + viscometer made by Brookfield company in America is used which is depicted in Fig. 6. The nanofluid viscosity in various VFs of the nanoparticles and for temperature range of 5 to 65°C (with the step of every 10°C) is measured. First, the viscometer is calibrated with oil at ambient temperature. The experiment is done for SRs of 665.5-13330 1/s. The viscometer precision is ± 2 percent of the maximum value that could be measured using the viscometer. All experiments for each VF and certain temperatures are repeated in different SRs.

## 3. Results And Discussion

In this part, first, the nanofluid rheological behavior in terms of being Newtonian is studied. Then, the measured values of the viscosity of hybrid nanofluid in various VFs and temperatures are reported. Next, the values of experimental viscosity are compared against predictions of available theoretical and experimental models and the abilities of such models are evaluated. Eventually, an equation is represented to estimate hybrid nanofluid viscosity in different temperatures and VFs, which is developed for different applications such as simulations.

## 3.1. Rheological behavior of nanofluid

In order to assess the rheological properties of hybrid nanofluid, the nanofluid viscosity is measured in the range of 50-1000 rpm (665.5-13330 1/s) and thus it is measured in various SRs. Figures 7 and 8 show the changes in SS and dynamic viscosity in terms of SR for hybrid nanofluid in various temperatures and VFs, respectively.

According to Fig. 7, the changes in SS are non-linear in terms of SR which indicates the behavior of the nanofluids is non-Newtonian. Based on Fig. 8, the most viscosity reduction is observed at 5°C. These changes indicate the dependency of viscosity to SR. Therefore, the 5W30/$SiO_2$- MWCNT(10–90) nanofluid can be considered as non-Newtonian which it behaves like a pseudoplastic fluid in all VFs and temperatures. Also, it is concluded that when the SR is augmented in low temperatures, the viscosity changes is increased. That means that in lower temperatures, the fluid behavior is close to non-Newtonian behavior. Contrary to the results of present study, the experimental results of Afrand et al. [35] on $SiO_2$-MWCNTs/SAE40 hybrid nanofluid, show that the base fluid and nanofluid have a Newtonian behavior. Therefore, the results of Fig. 8, show that the base oil also plays an important role in the rheological behavior of nanofluid. Also, the experimental study of Motahari et al. [33] on MWCNT-$SiO_2$ (20–80)/20W50, show that the base fluid and nanofluid have a Newtonian behavior. So, in addition to the importance of the role of base oil, the mass fraction of each nanoparticles affects the rheological behavior of nanofluid.

To further investigate the rheological behavior of hybrid nanofluid, the consistency index (m) and power law index (n) of the well-known power law model (Eq. 2) are calculated at a shear rate of 800 RPM in Fig. 9.

$$\tau = m\gamma^{n} \qquad (2)$$

Figure 9-(a) shows that with increasing temperature from 5°C to 45°C, the consistency index decreases, which corresponds to the trend of decreasing viscosity with increasing temperature, in Fig. 8. Also, from 45°C to 65°C, *m* values do not change significantly, which indicates that the slope of viscosity changes in terms of VF is almost constant in this range; In other words, a sharp decrease in viscosity is prevented by increasing the temperature.

In addition to, as can be seen in the Fig. 9-(b), the power law index is less than one at all temperatures and VFs. Therefore, it can be concluded that the viscosity decreases with increasing shear rate and the hybrid nanofluid has the characteristics of a pseudoplastic (shear-thinning) fluid.

# 3.2. Changes of hybrid nanofluid viscosity by temperature

As it is observed in Fig. 10, when the temperature is increased at constant VF, the nanofluid viscosity is decreased. In fact, as the temperature is increased, the base fluid and nanoparticles will have free molecular motion and molecules will collide less. In addition, when the temperature is increased, the intermolecular distance is increased in the base fluid and nanoparticles. Therefore, the resistance against the flow and consequently the viscosity is decreased. To put it another way, the viscosity in fluids is a result of molecules cohesive force, therefore, the viscosity of fluids is decreased when the temperature is increased. In fluids, molecules with more energy in higher temperatures, dominate the forces of cohesion and consequently, molecules move with more energy. In the SR of 600 RPM for the base fluid of 5w30, when the temperature changes from 5°C to 65°C, the dynamic viscosity is reduced. Also, for the VFs of 0.05, 0.1 and 0.2, it is reduced by 93.22, 93.23, and 93.01 percent, respectively.

# 3.3. Changes of hybrid nanofluid viscosity by the VF

In Figs. 11, the changes in dynamic viscosity of hybrid nanofluid in terms of the nanoparticles VF are illustrated in different temperatures for various SRs. As it could be observed, as the VF of nanoparticles is increased, the nanofluid viscosity is increased in all temperatures. When the nanoparticles $SiO_2$ and MWCNT are incorporated into the oil, the contact and intermolecular forces of oil particles and nanoparticles are increased which results in increased fluid resistance against flow, which means that the viscosity increases. Also, as the nanoparticles VF is increased, due to the increase in molecular forces between nanoparticles, the probability of addition to their branches increases and also, more resistance is created between the layers of the fluid against motion from this point of view.

At the temperature of 5°C, when the VF is increased from zero to 0.1%, the dynamic viscosity is increased from 34.38–40.88%. Also, at the temperatures of 15, 25, 35, 45, 55 and 65°C, it increases by 38.88–

44.23%, 30.43–53.90%, 31.73–36.27%, 34.27–38.35%, 43.05–48.12%, 56.38–70.55%, respectively.

Figure 12 shows the relative viscosity in terms of VF at various temperatures at a SR of 800 RPM. As shown, the maximum increase in viscosity occurs at temperature of 65°C and a VF of 1, which is about 58.8% in relation to the base fluid. Also, in only four cases, a reduction in viscosity in relation to the base fluid occurs. both of which are related to the VF of 0.05 and 0.1 and at temperatures of 55°C and 65°C. The maximum decrease in viscosity is associated to temperature of 65°C and VF of 0.05, which is about 5%.

In this section, the experimental data of present paper are compared with predictions from several conventional models. These models include the Einstein model, the Brinkman model, the Batchelor model, and the Wang and Mujumdar model. Einstein [42] proposed Eq. (3) for calculating relative viscosity. Brinkman [43] developed Einstein model for higher concentrations of fine particles as Eq. (4). Batchelor [44] assumed the base fluid to be uniform and also assumed that the nanoparticles were evenly distributed in the fluid and had a spherical shape. In this case, taking into account the Brownian motion of the particles, he presented Eq. (5) for the nanofluid relative viscosity. Wang and Mujumdar [45] also presented Eq. (6) to predict relative viscosity.

$$\frac{\mu_{nf}}{\mu_{bf}} = 1 + 2.5\phi \qquad (3)$$

$$\frac{\mu_{nf}}{\mu_{bf}} = \frac{1}{(1-\phi)^{2.5}} \qquad (4)$$

$$\frac{\mu_{nf}}{\mu_{bf}} = 1 + 2.5\phi + 6.2\phi^2 \qquad (5)$$

$$\frac{\mu_{nf}}{\mu_{bf}} = 1 + 7.3\phi + 123\phi^2 \qquad (6)$$

The measured relative viscosity of the present study at temperature of 45°C and SR of 800 RPM is compared with mentioned conventional models in Fig. 13. It is worth noting that since relationships (3) to (6) are not a function of temperature and SR, they will produce the same result at other temperatures and SRs.

As illustrated by Fig. 13, the models of Einstein, Brinkman and Batchelor give almost the same results, and Wang model is different from these three models, and its results are closer to the results of the present experimental work, but nevertheless, the difference between our results and old previous models is significant. The smallest difference belongs to the VF of 0.05, which is about 0.63%, and the largest one occurs at the VF of 1, which is about 62.49%. These differences can be attributed to the utilizing multi-walled carbon nanotubes that have a larger contact area with other molecules and their molecular interaction is different from spherical nanoparticles. The mentioned conventional models are presented for the presence of spherical nanoparticles. While, in the present study, both spherical nanoparticles ($SiO_2$) and cylindrical nanoparticles (MWCNTs) are used. Therefore, conventional models are not suitable to predict the behavior of prepared hybrid nanofluid, and therefore a novel model is presented in the next section.

As it is observed, relationships (3) to (6), cannot correctly predict the viscosity changes of the nanofluid studied in this paper in terms of VF and are weak in this regard. Nanofluid of present research is hybrid and non-Newtonian and so its viscosity is a function of VF, temperature and SR. While in relationships (3) to (6), the effects of temperature and SR are not considered. Therefore, a novel correlation (7) is developed for the studied nanofluid. The constant coefficients *a* to *s* in this proposed correlation are presented in Table 3.

$$\frac{\mu_{nf}}{\mu_{bf}} = a + bT + c\gamma + d\phi + eT^2 + f\gamma^2 + g\phi^2 + hT^3 + i\gamma^3 + j\phi^3 + kT\gamma + lT\phi + m\gamma\phi + nT^2\gamma + oT^2\phi + p\gamma^2 T + q\gamma^2\phi + r\phi^2 T + s\phi^2\gamma \quad (7)$$

**Table 3.** Constant coefficients in the proposed correlation

| Coefficient | Value | Coefficient | Value |
|---|---|---|---|
| *a* | $9.7872\times10^{-1}$ | *k* | $-3.9462\times10^{-6}$ |
| *b* | $3.5488\times10^{-3}$ | *l* | $-4.5068\times10^{-3}$ |
| *c* | $8.9778\times10^{-5}$ | *m* | $-1.5642\times10^{-4}$ |
| *d* | $8.2047\times10^{-2}$ | *n* | $-3.3825\times10^{-8}$ |
| *e* | $-1.0124\times10^{-4}$ | *o* | $1.5964\times10^{-4}$ |
| *f* | $-7.2729\times10^{-8}$ | *p* | $5.4565\times10^{-9}$ |
| *g* | $6.1913\times10^{-1}$ | *q* | $2.0629\times10^{-7}$ |
| *h* | $1.0452\times10^{-6}$ | *r* | $-1.6534\times10^{-3}$ |
| *i* | $-4.5686\times10^{-11}$ | *s* | $-1.6420\times10^{-4}$ |
| *j* | $-2.2656\times10^{-1}$ | | |

This proposed correlation (7) predicts the relative viscosity with accuracy parameters which is presented in Table 4. Figure 14 shows the results predicted by the developed correlation compared to the experimental data. As shown, the proposed correlation predicts the experimental results with a relative error of less than 8.48%.

To further examine the accuracy of the proposed correlation, the predicted results by the proposed correlation are compared with recent correlations (Esfe and Arani [38] and Esfe et al. [40]) in which hybrid nanofluids including $SiO_2$ and MWCNT nanoparticles have non-Newtonian behavior.

A comparison of the results in Figure 15 as well as the accuracy parameters of recent correlations in Table 4 demonstrate the inability of recent correlations to accurately predict the viscosity of oil (5W30)/SiO2- MWCNT hybrid nanofluid.

# 3.4. GMDH-type neural network (GMDH-NN)

Combination of linear regression and artificial neural network algorithms establishes the polynomial neural networks, which group method of data handling (GMDH) method as a self-organizing system is the well-known and extensively-used algorithm among other polynomial neural networks [46, 47]. The aim of this algorithm is proposing a correlation in a feed-forward network employing regression procedure based on a quadratic node transfer function [48]. The network introduces the estimation function $\hat{f}$ for prediction of output $\hat{y}$ in terms of a set of inputs $x = (x_1, x_2, \ldots, x_n)$ so that the minimum relative difference to the real output could be obtained. The definition of system is as below.

$$y_i = f\left(x_{i1}, x_{i2}, x_{i3}, \ldots, x_{in}\right), for i = 1,2, \ldots, M$$

8

The GMDH-type neural network is trained for prediction of output data $\hat{y}$ based on input variables in such a manner that the squares of the difference between the predicted and actual output values are minimized as follow.

$$\hat{y}_i = \hat{f}\left(x_{i1}, x_{i2}, x_{i3}, \ldots, x_{in}\right)$$

9

$$\sum_{i=1}^{M} \left[\hat{y}_i - y_i\right]^2 \rightarrow Minimum$$

10

The achieved nonlinear relationship of input/output variables is represented in the form of Kolmogorov–Gabor function as follows [48].

$$y = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_i x_j + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} a_{ijk} x_i x_j x_k + \ldots$$

11

In order to examine the accuracy of our proposed GMDH-type neural network, root mean square error (RMSE), mean absolute error (MAE), and absolute fraction of variance ($R^2$) are used, which is calculated as follow.

$$RMSE = \sqrt{\frac{1}{M}\sum_{i=1}^{M}\left(y_{exp} - y_{pred}\right)^2}$$

12

$$MAE = \frac{1}{M}\sum_{i=1}^{M}\left(y_{exp} - y_{pred}\right)$$

13

$$R^2 = 1 - \sum_{i=1}^{n}\frac{\left(y_{exp} - y_{pred}\right)^2}{{y_{exp}}^2}$$

14

In the present work, the GMDH-type neural network is employed to predict the polynomial models of relative viscosity of oil (5W30)/$SiO_2$-MWCNT hybrid nanofluid associated with their effective input variables. A series of 376 experimental data consists of temperature, SR, and VF of nanofluid as input variables and relative viscosity of nanofluid as the single output parameter is considered. For producing the network, 90% of the data is dedicated for GMDH-type neural network training and the rest 10% is dedicated for testing the network. The developed model is shown in Appendix A.

Figure 16 illustrates the predictive ability and values of residuals of trained network in order to estimate the unforeseen relative viscosity of our hybrid nanofluid obtained from experimental analysis. It is obviously seen that there is an excellent agreement between the experimental data and those predicted by the GMDH-type neural network. RMSE, MAE, $R^2$ and maximum relative error values of the neural network are listed in Table 4, demonstrating the high-precision performance of the GMDH-type neural network.

Table 4
Accuracy parameters of the GMDH-NN model and the proposed correlation.

| Parameters | Training | Testing | Proposed correlation | Esfe and Arani correlation [38] | Esfe et al. correlation [40] |
| --- | --- | --- | --- | --- | --- |
| RMSE | 0.01811 | 0.02133 | 0.02207 | 0.07911 | 0.09629 |
| MAE | 0.01245 | 0.01436 | 0.01530 | 0.04816 | 0.08803 |
| $R^2$ | 0.999747 | 0.999656 | 0.978966 | 0.832476 | 0.924291 |
| Maximum Relative Error (%) | 8.31 | 6.60 | 8.48 | 24.75 | 18.02 |

The experimental, correlation and GMDH-NN results are compared in Fig. 17. As it can be found that, there is closer agreement between the experimental results and GMDH-NN outputs especially at high relative viscosities, than that was established between the experimental data and the proposed correlation. This result can be also deduced from Table 3 and indicates the ability of the GMDH-NN model.

# 4. Conclusion

In this research, the viscosity of oil (5W30)/$SiO_2$-MWCNT hybrid nanofluid is measured at nanoparticle VFs of 0, 0.05, 0.1, 0.2, 0.5, 0.75 and 1% at temperatures of 5, 15, 25, 35, 45, 55 and 65°C and at various SRs. The main obtained results are as follows:

- The viscosity measurement of base fluid at various SRs demonstrated that the base fluid has a non-Newtonian behavior.
- When the VF is increased by 0 to 1% and temperature of 5 to 65°C, the dynamic viscosity of the nanofluid is increased and the percentage of its changes is between 30.43–70.55%.
- Experimental data indicate that as the temperature is increased, the dynamic viscosity of nanofluid is decreased.
- A correlation is developed for relative viscosity in terms of temperature, VF and SR which predicts experimental results with less than 8.48% relative error.
- A GMDH-NN considering temperature, VF and SR as input variables is built which predicts the relative viscosity of oil (5W30)/SiO2- MWCNT hybrid nanofluid more precisely than proposed correlation.

# Declarations

## Ethics approval and consent to participate

Not applicable.

## Consent for publication

Not applicable.

## Availability of data and material

All data analyzed during this study are included in this published article.

## Competing interests

The authors declare that they have no competing interests.


## Funding

Not applicable.


## Authors' contributions

M. Sepehrnia performed experiment, data analysis, conceptualization and project administration. K. Mohammadzadeh conducted data analysis and he was a major contributor in writing the manuscript. M.H. Rouzbahani and M.J. Ghiasi did the research and investigation process, specifically performed the experiments, or data collection. M. Amani performed the programming and applied the machine learning techniques to analyze the data. All authors read and approved the final manuscript.


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## Figures

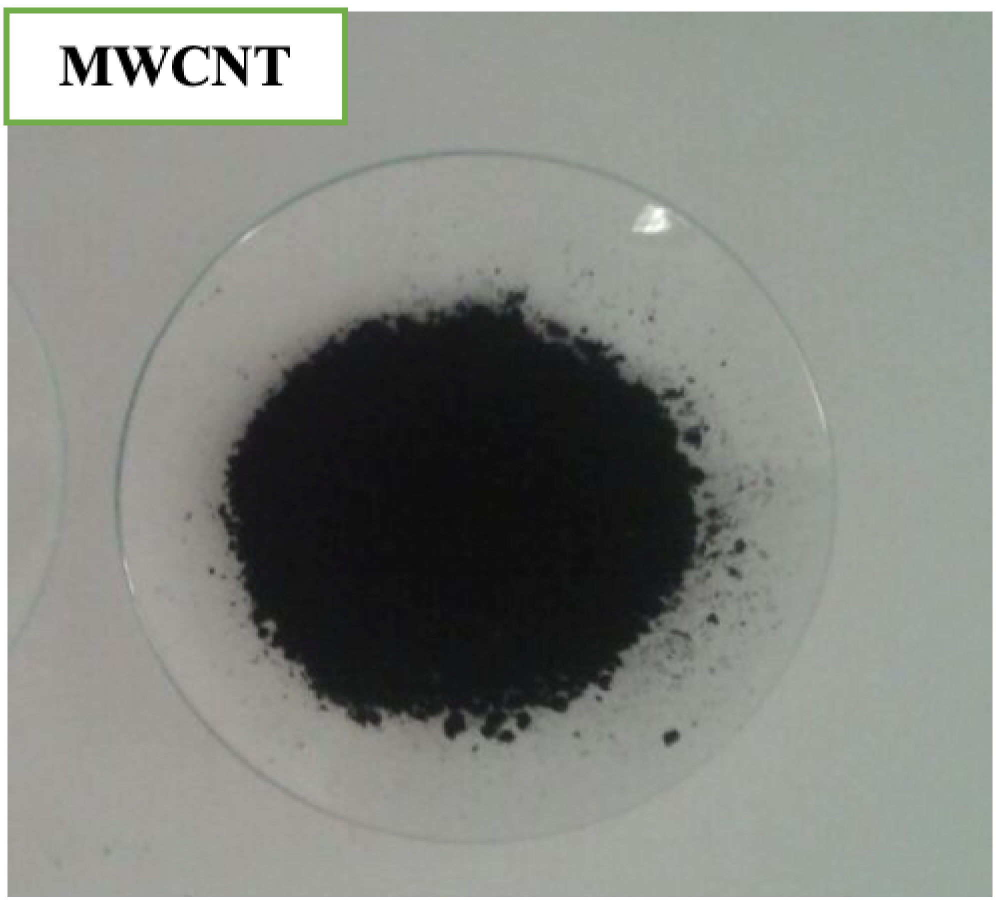



**Figure 1**

The MWCNTs

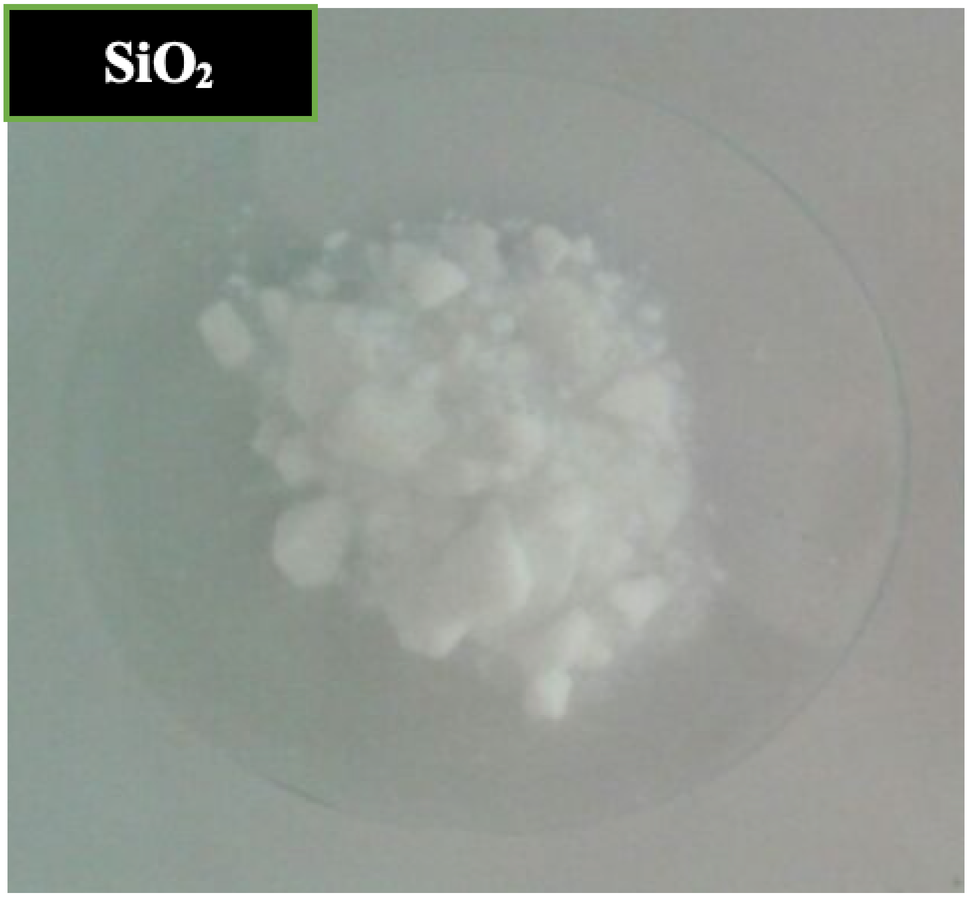



**Figure 2**

The $SiO_2$

**Figure 3**

Nanofluid samples in the order of VF

## Figure 4

XRD chart: (a) the MWCNTs; (b) $SiO_2$


## Figure 5

The supersonic device


**Figure 6**

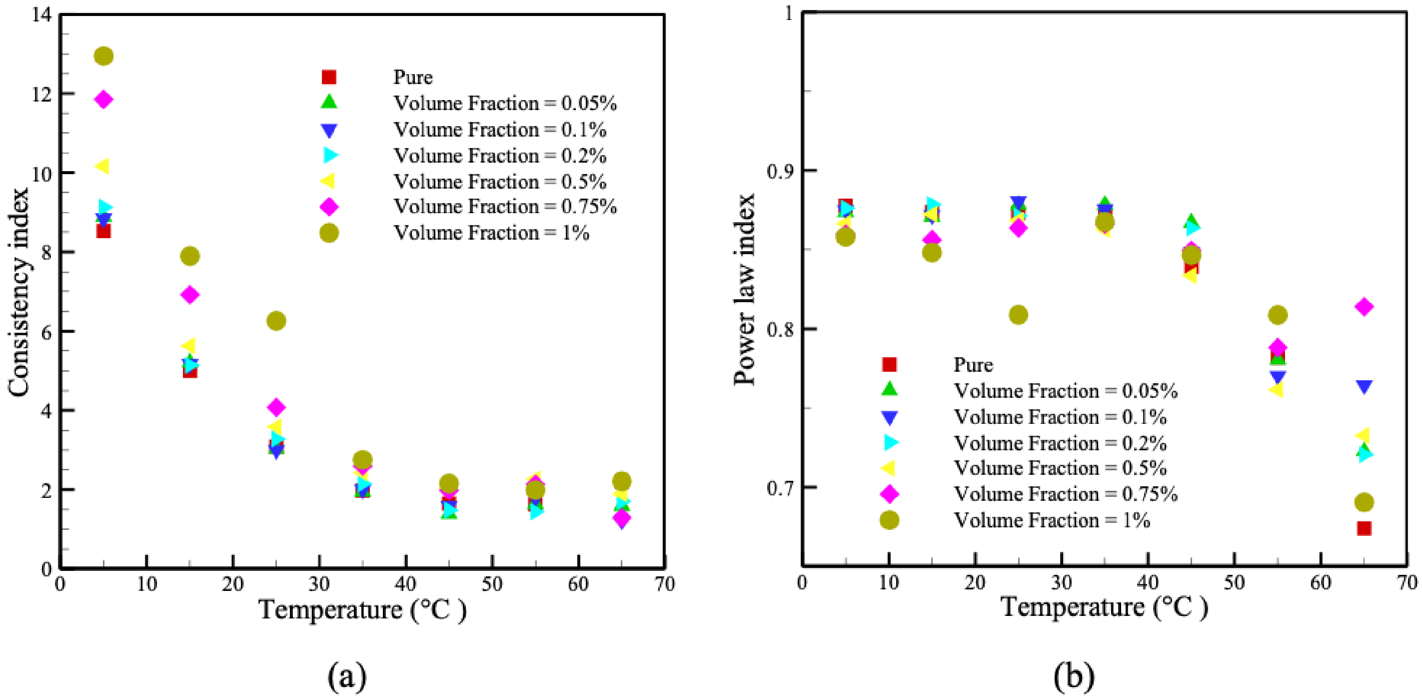
The CAP2000+ viscometer

**Figure 7**

SS Vs SR

**Figure 8**

Viscosity Vs SR




## Figure 9

Non-newtonian behavior of hybrid nanofluid based on power law model: (a) consistency index; (b) power law index

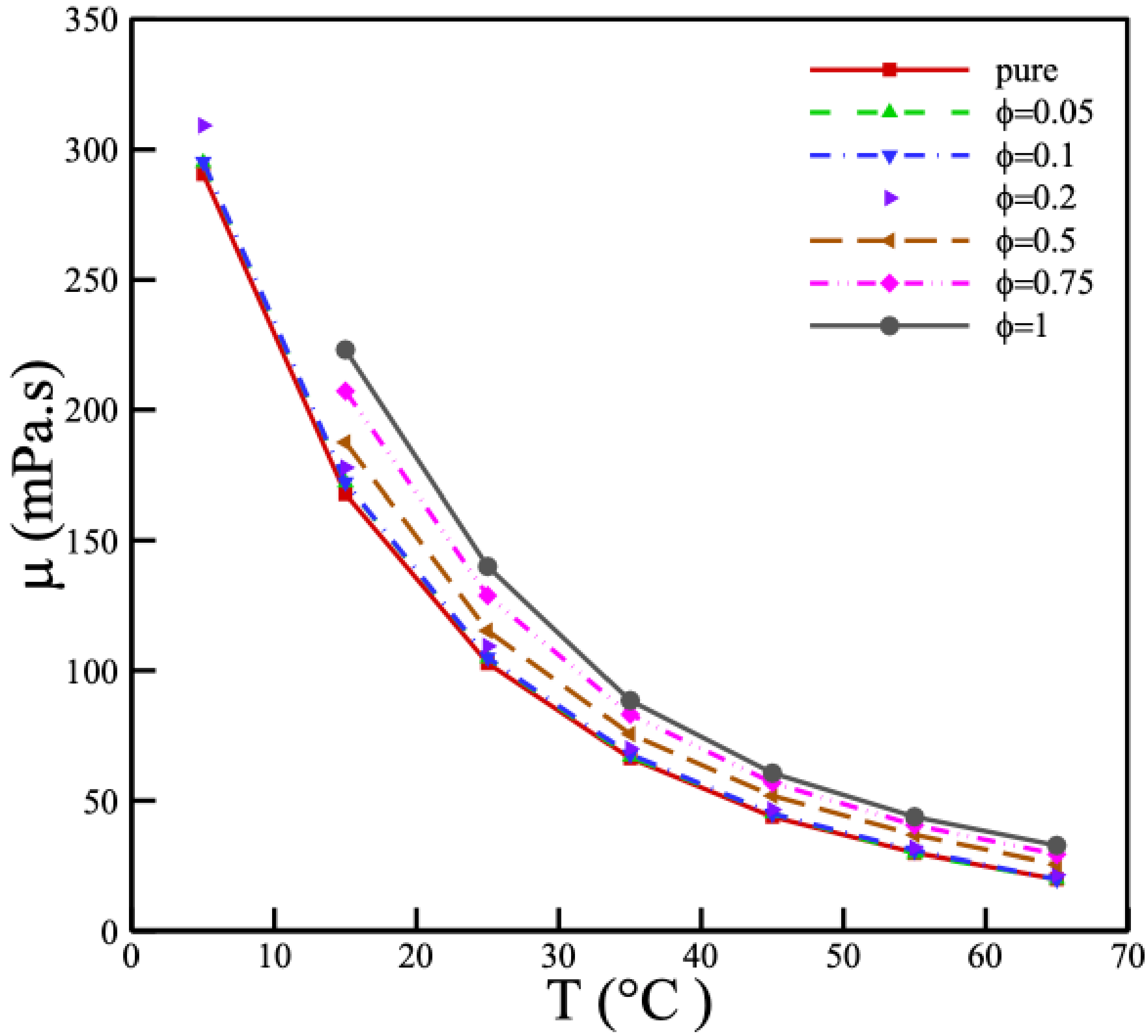
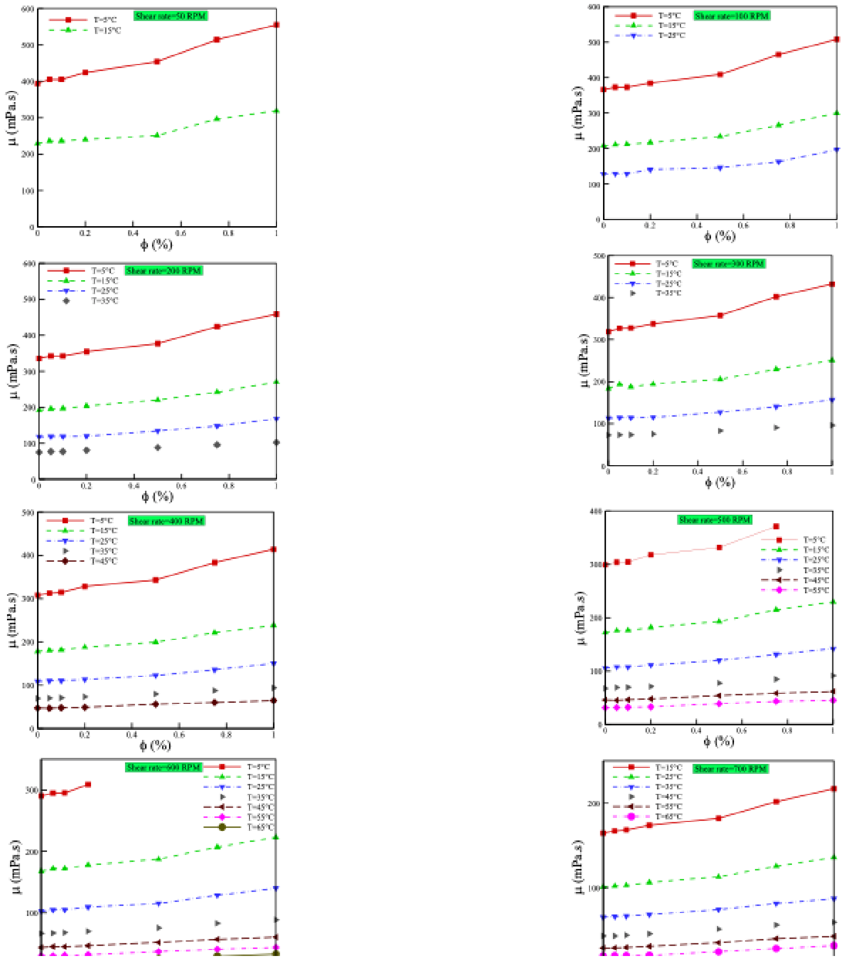



**Figure 10**

Variations in dynamic viscosity by the temperature at different VFs



## Figure 11

Viscosity Vs VF at various temperatures for different SRs

## Figure 12

Relative viscosity Vs VF at various temperatures at a SR of 800 RPM

**Figure 13**

Comparison of the measured relative viscosity at temperature of 45 °C with conventional models

**Figure 14**

Comparison of the measured relative viscosity with proposed correlation (6)

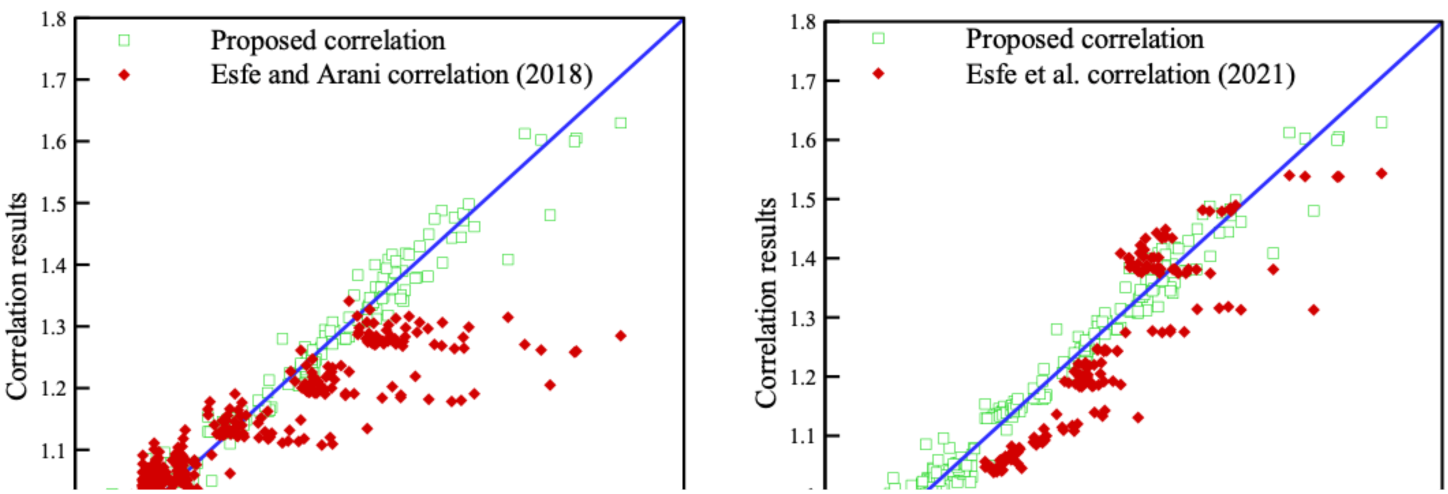



**Figure 15**

Comparison of the predicted results by proposed correlation with recent correlations: (a) Esfe and Arani [38]; (b) Esfe et al. [40]

**Figure 16**

(a) Comparison of experimental data and predicted data and (b) values of residuals of neural network model.

### Figure 17

Comparison of experimental, correlation and GMDH-NN results

## Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- AppendixA.docx