

# A first-principles model of curling stone dynamics

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# A first-principles model of curling stone dynamics

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## Abstract

An asymmetric force arising from grit and ice debris migration gives rise to a model of curling stone trajectories that is compatible with observations. There are (almost) no free parameters in this model.

## 1 Introduction and background

There has been a marked increase over the past decade or so in the number of papers published in general physics, dynamics, sports engineering, tribology and glaciology journals attempting to explain the observed behaviour of curling stones. [1–6] As a result, some progress has been made in the understanding of a curious phenomenon pertaining to the stone trajectories:

1. Two stones that travel the same distance, which come naturally to rest without contacting other stones, will curl the same amount whatever the initial angular speed of the stones.

Let us say that the initial velocity of a stone is directed along the  $+y$ -axis. If the initial angular velocity is counterclockwise then this stone will develop a transverse component of velocity that is along the  $-x$ -axis ( $+x$  for a clockwise initial rotation). This transverse motion is called curl within the sport, and the curl distance is typically  $1 - 1\frac{1}{2}$  m for a trajectory length of 30 m. Linear and angular motion cease at the same time.

We will denote the total distance travelled along the two axes as  $X, Y$  and the initial linear and angular speeds as  $v_0, \omega_0$ . The observation that  $X$  is independent of  $\omega_0$  has been known to investigating physicists for decades [6–9] and to practitioners of the game for longer. Over the last fifteen years data have been published confirming this strange result many times [6, 10] but an explanation has proved elusive. Several models have been proposed, but these are not first principles models in that they typically have at least two free parameters that are fixed only by comparison with observations: a friction coefficient determines the acceleration along the  $y$ -axis, while a second parameter is needed to describe the transverse motion along the  $x$ -axis.

The progress that has been made recently consists of the discovery, via both experimental and theoretical investigations, of two plausible mechanisms giving rise to an asymmetry in the forces acting on the stone. The asymmetry generates transverse motion and might possibly lead to  $X$  that depends on  $\omega_0$  only weakly. Note that in the game of curling the initial angular speed imparted to a stone is such as to provide  $N = 1 - 5$  stone rotations over the course of a trajectory (which lasts 20-25 s, typically). For  $N < 1$  the curl distance increases markedly with  $N$  (it must be zero for  $N = 0$ , by symmetry) but for  $N > 1$  the dependence is weak. This experimental fact is hard to explain. The natural assumption is that friction between stone and ice is responsible: stone rotation means that there is a left-right asymmetry in the local speed over the ice of a point on the contact annulus (the *running band*, RB – see fig.1). Thus if friction depends on speed then the net transverse force is nonzero. Unfortunately this simple idea does not work because all speed-dependent friction models give rise to a curl distance  $X$  that increase with  $\omega_0$ , contrary to observations. [8, 11]

The first plausible mechanism that might lead to curl distance compatible with observations is that of Nyberg *et al* [4] who reported the existence of scratches left on the ice over which the stone travels. The trailing half of the RB passes over scratches left by the leading half in such a way as to generate a transverse force. Models based on this mechanism have been proposed, and have been criticized or supported by subsequent investigations. The second plausible mechanism, due originally to Lozowski and Shegelski, [1, 12] proposes that the stone pivots about the underlying pebbles in an asymmetric manner that leads to observed behaviour.

In this paper we will present a variant of the pivot model based upon the retarding force produced by small particles of grit and ice debris that are distributed asymmetrically under the RB (which Lozowski’s pebbles are not). The resultant stone trajectories are functions of RB parameters; there are (almost!) no free parameters such as friction coefficient that can be chosen to be compatible with

observations. The model is simple (perhaps too simple to be the whole story) and it makes testable predictions so that future experiments can decide if it is a valid description of the curious phenomenon it seeks to explain.

One final introductory comment is necessary. A key feature of the ice over which curling stones travel is the stippled nature of its surface. Unlike the sheets of ice used in other winter sports, curling ice sheets are *pebbled* by technicians prior to a game. A sprayer spreads water droplets that freeze upon contact with the underlying ice, resulting in the stone RB riding over the underlying ice and making contact only with the tops of the pebbles. Typically there are about  $10^4$  pebbles per square meter of ice, and only about 2% of the RB surface is in contact with the ice beneath it at any given instant. [11, 13]

## 2 Asymmetric drag force

In an earlier paper [11] we calculated the effect of an asymmetric force of the type shown in figure 2a. This is the type of force that is exerted on a stone RB by particles of grit or ice debris that are trapped beneath it. Let us briefly summarize this calculation before developing it further. Consider a constant acceleration  $a$  of the leftmost point on a counterclockwise-rotating RB generating transverse motion. It is small (because  $X \ll Y$ ) compared with the acceleration  $\mu g$ , which is the net force due to friction acting on the stone centre of mass (CM). Both act in a direction opposing stone CM velocity  $\mathbf{v}$ . (Strictly, we should expect the direction of the asymmetric acceleration  $a$  and of  $\mu g$  to oppose local velocity at every point on the RB, but because transverse speed is much less than CM speed these directions are almost the same.) The resulting stone trajectory is sketched in figure 2b. The angles  $\alpha, \beta$  are given by

$$\tan \alpha = \frac{x}{y}, \quad \tan \beta = \frac{\dot{x}}{\dot{y}}. \quad (1)$$

The frame  $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$  is fixed in the ice, with origin at the point of release of the stone ( $\hat{\mathbf{z}}$  points out of the page, in fig. 2a).<sup>1</sup> The stone trajectory coordinates are  $(x(t), y(t))$  where  $x(t)$  is the transverse motion to be determined in this paper, and  $y(t)$  is the motion along the initial velocity direction, determined mostly by friction:  $y(t) \approx v_0 t - (\mu g + a)t^2/2$ . (We expect  $\mu g \gg a$ .) As pointed out above,

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<sup>1</sup>In this paper boldface indicates a vector quantity, and a caret indicates a unit vector. We employ Newton's dot notation for time derivative.

the local velocity is slightly different at different points on the RB due to stone rotation, but these differences are small and we will ignore them, so that friction here acts in the opposite direction to stone centre of mass (CM) velocity.

The torque about the origin due to the forces of magnitude  $ma, \mu mg$  is

$$\mathbf{G} = I_0 \ddot{\alpha} \hat{\mathbf{z}}. \quad (2)$$

Here  $I_0$  is the moment of inertia of the stone about the origin, given by  $I_0 = mr^2 + \varepsilon mR^2$ , where  $r$  is distance of the CM from the origin,  $R$  is RB radius, and  $\varepsilon$  is a dimensionless factor that depends on stone size and shape – for curling stones  $\varepsilon \approx 2.5$ . Torque can also be expressed as (see fig.2a)

$$\mathbf{G} = \mathbf{r} \times (-\mu mg \hat{\mathbf{v}}) + (\mathbf{r} + \mathbf{R}) \times (-ma \hat{\mathbf{v}}). \quad (3)$$

Equating these two expressions for torque we obtain the equation of motion:

$$\ddot{\alpha} = \frac{aR - (\mu g + a)r \sin(\beta - \alpha)}{r^2 + \varepsilon R^2}. \quad (4)$$

Equation(4) can be integrated numerically, and yields curl distances  $X$  that are compatible with observation for values of  $a$  in the range  $0.007 \text{ ms}^{-2} < a < 0.010 \text{ ms}^{-2}$ .

Equation(4) and its numerical solution (plotted in fig. 2b) is as far as we developed the calculation in [11]. We can take the calculation further by inserting eq.s(1) into eq.(4) and making the approximation  $\alpha, \beta \ll 1$  (again, this is valid because  $X \ll Y$ ), leading to

$$\ddot{\alpha} \approx \frac{aR}{y^2 + \varepsilon R^2} - \frac{(\mu g + a)y^2 \dot{\alpha}}{(y^2 + \varepsilon R^2)y}. \quad (5)$$

It is not difficult to show that an approximate solution to eq.(5) is

$$\alpha \approx \frac{ky^2}{\sqrt{y^2 + \pi^2 \varepsilon R^2}}, \quad k \equiv \frac{\pi}{2\sqrt{\varepsilon}} \frac{a}{v_0^2}. \quad (6)$$

For example, in figure 2c we plot  $\alpha(t)$  for  $\alpha$  obtained by numerically integrating eq.(5) and for  $\alpha$  given by eq.(6): note the good agreement. From eq.(6) we obtain the following equation for curl distance (noting that  $Y \gg R$ ):

$$X \approx kY^2. \quad (7)$$

We see that  $X$  is independent of  $\omega_0$ . For a typical curling trajectory we have  $Y = 30$  m,  $\mu = 0.01$  and so we obtain curl distances observed in practice ( $1.0 \text{ m} < X < 1.5 \text{ m}$ ) if the asymmetric acceleration is within the range  $0.007 \text{ ms}^{-2} < a < 0.010 \text{ ms}^{-2}$ . Furthermore, from eq.s(1),(5) we can show that  $\beta \approx 2\alpha$  and  $x(t) \approx ky^2(t)$ . This expression yields the right shape for the curling stone trajectory: most of the transverse motion occurs late on in the trajectory, as observed for real curling stones. (For completeness note that  $Y \approx v_0^2/2(\mu g + a)$  and the trajectory duration is  $\tau \approx v_0/(\mu g + a)$ . Thus for our typical curling trajectory  $v_0 = 2.55 \text{ ms}^{-1}$  and  $\tau = 23.6 \text{ s}$ .)

The calculations of this section show that if we can develop a model of curling leading to an asymmetric acceleration  $a$  (as shown in fig. 2a) in the range  $0.007 \text{ ms}^{-2} < a < 0.010 \text{ ms}^{-2}$ , then we have a physical description of curling that is compatible with observation, including the ‘curious phenomenon’ described in the Introduction. We now outline such a model.

### 3 Particle abrasion model geometry

This model is based on simple geometrical considerations of the mechanical abrasion effects of small particles of ice and grit.

The underside of the curling stone RB is roughened, as noted by several authors. [4, 13] We can assign length scales to this roughening based upon observation. Thus the RB surface varies horizontally over a length  $\delta_{xy} \approx 250\mu\text{m}$  and vertically over a length  $\delta_z \approx 10\mu\text{m}$  (Nyberg, 2013, fig.5). Sometimes curlers intentionally roughen the RB, and this action changes the curling behaviour of the stone, so it is clear that RB surface roughness is a relevant factor in determining stone trajectories. The rough RB surface abrades the tops of the pebbles over which it runs, wearing down the pebbles and creating ice debris – loose particles of ice that adhere to the pebble top surface or to the stone RB.

Our model presumes that some of this ice debris becomes attached to the RB for a short period of time, before being separated from it. The particles may be knocked off the RB when struck by a pebble, or they may be ground down by the RB and pebble top surface (like grains of wheat between millstones). Additionally particles of grit (harder than the ice) find their way onto the ice sheet, perhaps via curlers’ clothing. [14] Small grit particles are responsible for the gradual wearing (over a period of years) of the curling stone RB and, as every curler knows, large grit particles are responsible for the sudden veering to one side of a moving stone

as it 'picks up'.<sup>2</sup> Being harder than ice these grit particles will not be ground down like ice debris, but they may otherwise act like ice debris particles. We assume, then, that both ice debris and grit particles become attached to the RB, are moved over the ice as the stone slides and turns, eventually being melted or ground down to nothing or knocked off the RB. The size of the ice debris particles is determined by the RB roughness, as sketched in figure 3, because the ice is pressed into the harder RB surface by the weight of the stone, and so conforms to its shape. Small grit particles will be embedded in the debris ice. We expect a 'Goldilocks effect' to be at work here: agglomerations of grit and ice debris must be of just the right size ( $\delta_{xy}$  horizontally and  $\delta_z$  vertically) to stick to the RB and abrade the ice. If they are much larger then they will be easily knocked off the RB, and if they are smaller then they will be hidden in the RB surface and will not cause much abrasion. Thus the greatest contribution to the friction and asymmetric forces – because it dissipates the most energy – comes from protrusions of vertical extent matching that of the RB roughness. We elaborate on this effect in figure 3.

Our model expresses both the vertical and horizontal components of forces that act on a curling stone in terms of the projected area of ice that is deformed/crushed. While attached to the RB, the grit and debris particles exert a force proportional to the cross-sectional area they they present to the ice. Thus if there are  $n$  agglomerations of grit and ice in contact with the pebbles while stuck to the RB (let us refer to such agglomerations as *protrusions*, for ease of reference) then the stone weight can be expressed in terms of protrusion dimensions as

$$mg \approx n\bar{P}A_{xy}, \quad A_{xy} \approx \delta_{xy}^2. \quad (8)$$

Here  $A_{xy}$  is the area of a protrusion pressing down on the ice surface.  $\bar{P}$  is the dynamic ice hardness; it characterizes the deformation of ice that is subjected to a force, and is measured to be 18 MPa (i.e.  $18 \text{ MJm}^{-3}$ ) for curling ice at a temperature of -5 C.[15] (Eq.(8) is derived by balancing forces: the stone weight  $mg$  equals the reaction force  $\bar{P}\delta_{xy}^2$  exerted by the ice on which a protrusion rests multiplied by the number of protrusions  $n$ ). Balancing horizontal force we obtain

$$\mu mg \approx n\bar{P}A_{xyz}, \quad A_{xyz} \approx \frac{1}{4}\delta_{xy}\delta_z, \quad (9)$$

$A_{xyz}$  is the area of a protrusion seen from the side – it is the area of pebble ice that is abraded by the horizontal movement of the protrusion. Why in eq.(9) do we

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<sup>2</sup>One of the purposes of sweeping curling stones is to remove these large grit particles from the ice in front of a stone, to prevent the unwanted veering off course.

take the depth of a protrusion to be  $\frac{1}{4}\delta_z$ ? The ‘Goldilocks effect’ means that the maximum grit particle vertical extent is about  $\delta_z$ , the RB vertical scale, and so we expect an attached particle to extend at most  $\frac{1}{2}\delta_z$  below the bottom of the RB (see fig. 3c). The least extent is zero (an attached particle does not extend beyond the RB at all) and so our best estimate of average extent<sup>3</sup> is  $\frac{1}{4}\delta_z$ .

From eq.s(8),(9) we obtain:

$$\mu \approx \frac{\delta_z}{4\delta_{xy}}. \quad (10)$$

Thus friction coefficient is given in terms of RB roughness length scales, in our model. Substituting typical values we find from eq.(9)  $\mu \approx 0.01$  and from eq.(8)  $n \approx 150$ .

Consider now the energy expended by the stone over the course of its trajectory. This can be written as

$$\mu mgY = \bar{P}V_{tot} + Q, \quad (11)$$

where  $V_{tot}$  is the total volume of surface ice that is abraded–crushed–by the stone over the length  $Y$  of its trajectory. In words: the work done by the stone equals the energy needed to crush ice plus the heat generated. We see that

$$V_{tot} = 2\eta RY\delta_z \quad (12)$$

where  $\eta$  is the fraction of RB area that is in contact with the ice.  $\eta = 1$  for flat ice but is much smaller for pebbled ice:  $\eta = \sigma\pi r_{peb}^2$  where  $\sigma$  is the area density of pebbles on the curling ice sheet and  $r_{peb}$  is the mean radius of the pebble top surface. (See [11, 14] for details of pebble size, shape and density.) Substituting typical values<sup>4</sup> we obtain  $\eta = 0.031$  and so  $Q = 31$  J, which represents about 60% of the energy expended by the stone, from eq.(11). If all the heat generated is used to melt ice, then the mean depth  $\delta_m$  of ice melted due to a single pass of a stone is found from  $Q = 2\eta RY\rho c\delta_m$  where  $\rho = 917$  kgm<sup>-3</sup> is ice density and  $c = 334$  kJ kg<sup>-1</sup> is the latent heat of melting for ice. Thus  $\delta_m \approx 1\mu\text{m}$ . This value is smaller

<sup>3</sup>A more complete model of particle adhesion will require knowledge of the distribution of grit and ice debris particle sizes, so that a more detailed estimate of protrusion dimensions can be made. In the absence of such data, the best we can say is that the particles attached to the RB protrude about  $\frac{1}{4}\delta_z$  on average.

<sup>4</sup>We will assume for calculations in this paper the following parameter values, which are typical: pebble radius  $r_{peb} = 0.001$  m, trajectory length  $Y = 30$  m, RB radius  $R = 0.065$  m, RB thickness = 0.006 m, stone mass  $m = 18$  kg, pebble density  $\sigma = 10^4$  m<sup>-2</sup>.

than the roughness of the RB and so we do not expect the liquid water thus formed to provide lubrication for the stone passing over the ice, but it is sufficient to cause adhesion of debris and grit particles to each other and to the RB.

The mean number of pebbles supporting the stone at any one time is  $n_{peb} = \eta A_{RB} / \pi r_{peb}^2 \approx 25$ , so that the mean number of protrusions per pebble is  $n/n_{peb} \approx 6$ .

An important geometrical aspect of the debris abrasion model is illustrated in figure 4. Because of the finite width of the RB (5-8 mm, depending on wear) the length  $L_{abrade}$  of RB which runs over a pebble depends on the location of the pebble. Pebbles near the left and right edges are abraded by a much greater length than those near the centre. The depth to which a pebble is abraded will be proportional to  $L_{abrade}$  and so we expect pebbles that are under the trailing half of the RB to be abraded more than those under the leading half, and those near the edge to be abraded more than those near the center. As a consequence, pebbles at ‘mid-latitudes’ in the trailing half are not in such close contact with the RB as are pebbles at other locations. These mid-latitude pebbles will thus contribute less to the torque acting about the RB, which we calculate in the next section and which determines the transverse motion of the curling stone.

## 4 Particle abrasion model statistics

Equation(10) shows how our particle abrasion model predicts the friction coefficient in terms of more fundamental parameters (the length scales of the RB). Friction accounts for the longitudinal motion (along the  $y$ -axis) of the curling stone. To obtain model predictions for the transverse motion ( $x$ -axis) we need to derive an expression for the transverse acceleration  $a$  in terms of the same parameters. Because our model attributes transverse motion to an asymmetric force (as shown in fig.2a) arising from an asymmetric distribution of myriad microscopic grit and ice debris particles around the RB, our description is necessarily statistical.

The curling stone RB encounters particles of ice debris and grit that lie on the tops of pebbles with which it makes contact; these particles exist on the ice before the RB reaches them. Other ice debris particles are created as the RB passes over them, due to abrasion. Let us henceforth differentiate between these two types of particle: we label as *Type I* those particles that existed on the ice before the RB encounters them, and as *Type II* those particles newly created by the RB. Type I may include ice debris particles from an earlier trajectory, as well as non-ice grit particles. Type II consists of ice particles created during the current trajectory.

We begin our statistical analysis by considering a small particle of either type that becomes trapped under the RB at an angle  $\phi_0$  (see fig. 5a). It remains trapped under the same part of the RB until it reaches angle  $\phi_0 + \gamma$ , at which point it falls off or has been ground down to insignificance. While at angle  $\phi$  (where  $\phi_0 < \phi < \phi_0 + \gamma$ ) it exerts a torque about the stone CM of magnitude  $G_1(\phi) = ma_1 R \cos \phi$ , where  $ma_1$  is the force exerted by the particle due to its abrading of the ice beneath it:

$$ma_1 \approx \bar{P} \delta_{xy} \delta_z. \quad (13)$$

The mean torque exerted by this trapped particle is found by averaging over  $\phi$ :

$$G_1(\phi_0, \gamma) = \frac{1}{\gamma} \int_{\phi_0}^{\phi_0 + \gamma} d\phi G_1(\phi). \quad (14)$$

For  $n$  such particles becoming trapped at different initial angles the mean torque is

$$G(\gamma) = \frac{1}{\pi} \int_0^\pi d\phi_0 n p(\phi_0) G_1(\phi_0, \gamma). \quad (15)$$

Here  $p(\phi_0)$  is the distribution of particles with initial angle  $\phi_0$ . (In the Appendix we show why  $0 < \phi_0 < \pi$ .) We calculate it by noting that the distribution of pebbles (from which the ice debris particles arise) is uniform over the ice, in particular the distribution along the  $x$ -direction (see fig.5b) is  $p(x) = \frac{1}{2R}$  for  $-R < x < R$ . Conservation of probability requires  $p(x)dx = p(\phi_0)d\phi_0$  and so

$$p(\phi_0) = \frac{1}{2} \sin \phi_0 \quad (16)$$

(because, from fig.5b,  $dx = R d\phi_0 \sin \phi_0$ ). Substituting and evaluating the integrals we obtain for the mean torque:  $G(\gamma) = -\frac{1}{4} ma_1 R n \frac{(1 - \cos \gamma)}{\gamma}$ . We can also write this torque in terms of the asymmetric acceleration  $a$  of figure 2a:  $G(\gamma) = -maR$ . Equating these two expressions for torque yields

$$a(\gamma) = \frac{1}{4} na_1 \frac{(1 - \cos \gamma)}{\gamma}. \quad (17)$$

In eq.(17) we show the asymmetric acceleration as a function of  $\gamma$ . We need to average over this angle, which is a measure of the particle *persistence* under the RB. In the Appendix we show that the probability distribution function for a particle persisting for angle  $\gamma$  is  $p_\lambda(\gamma) = \lambda \exp(-\lambda \gamma)$  so that the expected asymmetric acceleration is

$$a = \int_0^\infty d\gamma p_\lambda(\gamma) a(\gamma) = \frac{1}{4} na_1 \lambda \ln \left( \frac{\sqrt{\lambda^2 + 1}}{\lambda} \right) \quad (18)$$

We note from eq.s(9),(13) that  $na_1 = \mu g$  so that, finally

$$\frac{a}{\mu g} \approx \frac{1}{4} \lambda \ln \left( \frac{\sqrt{\lambda^2 + 1}}{\lambda} \right) = \frac{1}{8} \frac{\ln(\bar{\gamma}^2 + 1)}{\bar{\gamma}}, \quad \bar{\gamma} = \frac{1}{\lambda}. \quad (19)$$

Equation(10) predicts a value for the symmetric friction coefficient  $\mu$  in terms of more fundamental model parameters – constants that are known before the curling stone trajectory begins. But eq.(19) predicts a value for the asymmetric transverse acceleration  $a$  in terms of known parameters plus an unknown parameter, the decay constant  $\lambda$  that describes the persistence of a Type I or II particle trapped under the RB. ( $\bar{\gamma} = 1/\lambda$  is the *mean lifetime* of the particle, expressed as an angle.) Thus in pursuing our statistical calculation we seem to have just traded one unknown parameter for another. In fact, however, the transverse acceleration is pretty much constant over a significant range of  $\bar{\gamma}$  values. We plot  $a(\bar{\gamma})$  in figure 5c; note that the acceleration rises quickly from zero at  $\bar{\gamma} = 0$  (which corresponds to no persistence – the particle adhesion model does not apply at all) to a value that is within 10% of  $0.09\mu g$  for  $\frac{1}{6} < \bar{\gamma} < \frac{5}{6}$  rotations. For higher values of  $\bar{\gamma}$  the transverse acceleration slowly decreases. Thus if the mean lifetime of a particle under the RB (that is, the number of stone rotations for which it remains trapped and influencing the stone trajectory) is between  $\frac{1}{6}$  and  $\frac{5}{6}$  then from eq.s(5),(6) the curl distance is  $X \approx 1.3$  m, independent of the number of rotations of the stone. The particle adhesion model is thus compatible with observations of curling stone trajectories (including the ‘curious phenomenon’ discussed in the Introduction) if particles of grit and ice are trapped under the RB for a significant fraction of a rotation.

How likely is this? Is there any reason to suppose that trapped particles will persist for half a rotation or so, and not much more or much less than this amount? In fact, there are two threads of indirect evidence to suppose so. First, anecdotal evidence from practitioners of the sport tell us that there is a small amount of grit and ice debris under a RB, which needs to be brushed off prior to making a shot. If particle persistence was large (say  $\bar{\gamma} > 1$ ) then we would expect a lot more debris accumulating under the RB – most of the particles gathered during the course of a trajectory would still be under the RB at the end of the trajectory. On the other hand *some* particles are present ( $\bar{\gamma}$  cannot be negligibly small) otherwise cleaning the RB before each shot would not be necessary.

Second, we saw in figure 4b that the length of RB that passes over a pebble varies with the angle  $\phi_0$  at which the pebble first makes contact with the RB. For Type I particles we see from eq.(16) that the most likely angle for them to first

encounter the RB is  $\phi_0 = \frac{\pi}{2}$ . If it becomes embedded within a protrusion then the most likely angle for it to encounter another pebble and be knocked off the RB is at  $\phi_0 + \gamma = \frac{3\pi}{2}$ , by the same geometrical argument as led to eq.(16). For Type I particles, then, we might expect  $\bar{\gamma} = \pi$ . Type II particles are likely created and destroyed/ removed from underneath the RB at  $\phi_0 = \frac{\pi}{2}, \frac{3\pi}{2}$  for the same reason, but are also created at  $\phi_0 = 0, \pi$  because at these locations  $L_{abrade}$  is greatest, as we have seen. Thus for Type II particles we might expect  $\bar{\gamma} = \frac{\pi}{2}$ . The mean value of particle persistence would thus be somewhere in the range  $\frac{\pi}{2} < \bar{\gamma} < \pi$ .

## 5 Summary and further work

There are two significant forces that determine the trajectory of a curling stone as it travels along a sheet of ice. A left-right symmetric friction force acts approximately in a direction opposing stone CM velocity; this force is characterized by a friction coefficient  $\mu$ . A left-right asymmetric force characterized by an acceleration  $a$  acting at one edge of the stone RB (a single force representing the sum of many forces acting on the RB) determines the transverse motion, i.e. the stone movement perpendicular to its initial direction of motion. The total transverse distance  $X$  covered over the whole trajectory (the curl distance) is much less than the distance  $Y$  traveled along the ice, and this fact is reflected in the relative magnitudes of the forces:  $ma \ll \mu mg$ . The curious phenomenon observed by curlers and by investigating physicists alike is that  $X$  is independent of the number of rotations  $N$  made by the stone over the course of its trajectory:  $X = 1 - 1\frac{1}{2}$  m for  $N = 1 - 5$  rotations (typical for rotations of stones in the game of curling).

Our particle abrasion model is a simple geometrical and statistical model that reproduces the observed behaviour of curling stones, and which expresses the accelerations  $\mu g$  and  $a$  in terms of fundamental parameters. Thus in eq.(10) we expressed  $\mu$  as a function of RB horizontal and vertical roughness length scales. In eq.(19) we expressed  $a$  in terms of  $\mu$  and a statistical parameter  $\bar{\gamma}$  that represents the angular persistence of particles that adhere to the RB before being worn down or knocked off. We expect the sensitivity of  $a$  to  $\bar{\gamma}$  to be small; the resultant magnitudes for  $X$  and  $Y$  are compatible with measured values.

The model is perhaps too simple in the sense that it predicts the effects of RB protrusions to be proportional to the projected area presented to the ice through which they plow. No doubt a detailed microscopic analysis of such abrasion interactions would be more nuanced than our model suggests. It is important to note, however, that even our model, simple as it is, exhibits subtleties. Here are three

examples, two of which have arisen in the text.

1. Protrusion length scales match RB roughness length scales.
2. Pebble wear due to RB abrasion depends upon location on the RB (fig.4a).
3. The amount of wear of an ice debris particle trapped between a pebble and the RB depends on whether the particle is attached to the pebble or to the RB.

We have seen that protrusions of the RB length scale require more force to dislodge or wear down than do larger or smaller protrusions (and so the contributions of such protrusions to  $a$  and  $\mu g$  is the most significant): larger ones require little effort to dislodge while smaller ones do not contribute, as they are hidden within the RB undulations. As for the second subtlety, we have seen that the length of RB that runs over a given pebble (and hence the pebble wear) varies significantly with pebble location due to RB thickness. The last subtlety has not been mentioned so far as it did not figure in our calculation of stone trajectory – we mention it simply to emphasize a non-obvious consequence of our basic model. If our debris particle is attached to a pebble then the length of RB that passes over it is  $L_{abrade}$  as shown in fig.4b; this length is much greater than the length of pebble that passes beneath our debris particle if it is attached to the RB – this length is at most the pebble diameter.

The fact of a very simple model of curling stone dynamics exhibiting, and in some ways depending upon, such subtle effects points to the complexity of the microscopic dynamics. Consider just the length parameters that we have needed to take into account: in decreasing size these are trajectory length, curl distance, stone radius, RB radius, RB thickness, pebble radius, RB roughness length scales (horizontal and vertical) –  $6\frac{1}{2}$  orders of magnitude from 30 m to 10  $\mu$ m. The first two have been derived in terms of the other six. These six length parameters (and other relevant parameters such as pebble area density, stone mass, dynamic ice hardness) are fixed before a game of curling begins. One other parameter, grit persistence angle, is probably not important in determining curling stone trajectories because for a wide range of values the model predictions are not sensitive to it. Nevertheless it remains to be proven that curl distance is insensitive to persistence angle. The latter should be derivable in terms of more fundamental parameters but such a derivation will require knowledge of unknowns such as the distribution of grit particle sizes.

More experimental work is needed to test our model, as well as to answer questions about its microscopic components, such as:

1. Do grit and ice debris particles get transported around the RB during a trajectory?
2. Do such particles exist in the numbers predicted and at the sizes required for the model to work?
3. Do they form protrusions as supposed herein?

If the answer is ‘no’ to any of these questions then our model is invalid. If ‘yes’ then the particle abrasion model is a first-principles description of curling stone dynamics.

## 6 Appendix

First, we show why we can assume that grit and debris particles initially encounter the RB (at angle  $\phi_0$  in the calculations of sect. 4) only for  $0 < \phi_0 < \pi$ . This angular limitation is geometrically obvious for Type I particles that are distributed over the ice prior to the arrival of the stone: of course such pebbles can only make initial contact with the leading half of the advancing RB. However some of our particles are Type II ice debris made as the stone passes over – the RB abrades any pebble it makes contact with, and these abrasions can be at any initial angle  $0 < \phi_0 < 2\pi$ . Even for debris particles that arise from such abrasions, however, we can neglect the contribution they make to the torque if they are located in the trailing half  $\pi < \phi_0 < 2\pi$ . The reasons are twofold. We showed in figure 4a that pebbles in the ‘mid-latitudes’ of the trailing half are more severely worn by the stone passing over them than are other pebbles and so are in less close contact with the RB. We expect that the contribution of these pebbles to the CM torque is reduced compared to other pebbles because the abrasion force will be less than for the other pebbles. At the very back of the RB are some pebbles that are indeed in close contact, as we see in figure 4a, but these contribute very little to the torque because they are close to the  $y$ -axis. Thus in our torque calculation it is reasonable for us to make the approximation  $0 < \phi_0 < \pi$ .

Second, we derive the equation for  $p_\lambda(\gamma)$ , the probability density function describing the persistence of ice debris particle adhering to the curling stone RB. A debris particle is attached to the RB for an angle  $\gamma$ , as the stone rotates. We will assume that the probability for this particle to detach from the RB over the

interval  $d\gamma$  is proportional to  $d\gamma$ .<sup>5</sup> That is, we assume a particle attached at  $\gamma$  becomes detached by the time it reaches  $\gamma + d\gamma$  with probability  $\lambda d\gamma$ , for some constant  $\lambda$ . Thus if  $n(\gamma)$  is the number of particles that have been attached for an angle  $\gamma$  then the number that detach between  $\gamma$  and  $\gamma + d\gamma$  is  $dn = -n\lambda d\gamma$ . Thus  $n = n_0 \exp(-\lambda\gamma)$  where  $n_0$  is the number of particles that were attached to the RB at  $\gamma = 0$ , and so the probability that a particle persists for angle  $\gamma$  is  $P \equiv \frac{n}{n_0} = \exp(-\lambda\gamma)$ . The probability that a particle detaches at  $\gamma$  or some larger angle is thus

$$P = 1 - \int_{\gamma}^{\infty} ds p_{\lambda}(s) \quad (20)$$

and so  $p_{\lambda}(\gamma) = -\frac{dP}{d\gamma} = \lambda e^{-\lambda\gamma}$ , as claimed in sect. 4.

## References

- [1] Lozowski, E., Shegelski, M.R.A.: Pivot–slide model of the motion of a curling rock. *Can. J. Phys.* **94**, 1305-1309 (2016).
- [2] Ivanov, A.P., Shuvalov, N.D.: One the motion of a heavy body with a circular base on a horizontal plane and riddles of curling. *Regular and Chaotic Dynamics* **17**, 97-104 (2012).
- [3] Maeno, N.: Assignments and progress of curling stone dynamics. *Proc. IMechE Part P: J. Sports Eng. and Tech.* (2016). <https://doi.org/10.1177/1754337116647241>.
- [4] Nyberg, H., Alfredson, S., Hogmark, S et al: The asymmetrical friction mechanism that puts the curl in the curling stone. *Wear*, **301**, 583-589 (2013).
- [5] Penner, A.R.: A Scratch-Guide Model for the Motion of a Curling Rock. *Tribol. Lett.* **67**, 35 (2019). <https://doi.org/10.1007/s11249-019-1144-0>.
- [6] Maeno, N.: Curl mechanism of a curling stone on ice pebbles. *Bull. Glaciological Research* **28**, 1-6 (2010).
- [7] Johnston, G.W.: The dynamics of a curling stone. *Can. Aeronaut. Space J.*, **27**, 144-161 (1981).

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<sup>5</sup>This assumption is reasonable: it amounts to saying that the longer a particle is trapped under the RB, the greater the chance of it wearing down or being knocked off the RB.

- [8] Denny, M.: Curling rock dynamics. *Can. J. Phys.* **76**, 295-304 (1998).
- [9] Lozowski, E. *et al*: Comparison of IMU measurements of curling stone dynamics with a numerical model. *Procedia Engineering* **147**, 596-601 (2016).
- [10] Jensen, E.T. and Shegelski, M.R.A.: The motion of curling rocks: Experimental investigation and semi-phenomenological description. *Can. J. Phys.* **82**, 791-809 (2004).
- [11] Denny, M.: Ice deformation explains curling stone trajectories. *Tribol. Lett.* **70**, 41 (2022). <https://doi.org/10.1007/s11249-022-01582-7>.
- [12] Lozowski, E., Shegelski, M.R.A.: First principles pivot-slide model of the motion of a curling rock: Qualitative and quantitative predictions. *Cold Regions Sci. and Tech.* **146**, 182-186 (2018).
- [13] Kameda, T., Shikano, D., Harada, Y. *et al*: The importance of the surface roughness and running band area on the bottom of a stone for the curling phenomenon. *Sci Rep.* (2020). doi: 10.1038/s41598-020-76660-8.
- [14] Minnaar, J.: Scottish Ice Curling Group (Reports) <https://scottishcurlingicegroup.org/reports.html> (2007). Accessed 4 January 2022.
- [15] Poirier, L. Lozowski, E.P., Thompson, R.I.: Ice hardness in winter sports. *Cold Regions Sci. and Tech.* **67**, 129-134 (2011).

## 7 Statements and Declarations

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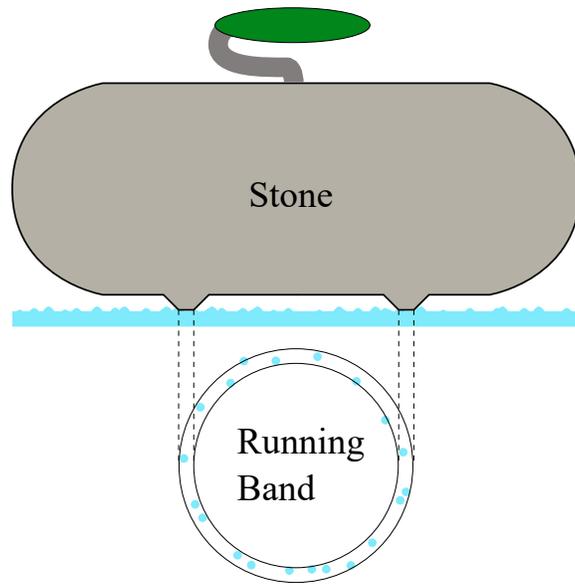


Figure 1: Curling stone on pebbled ice (viewed from the side) and running band (viewed from above). The only contact between stone and ice is where the running band touches the pebbles that support the stone's weight

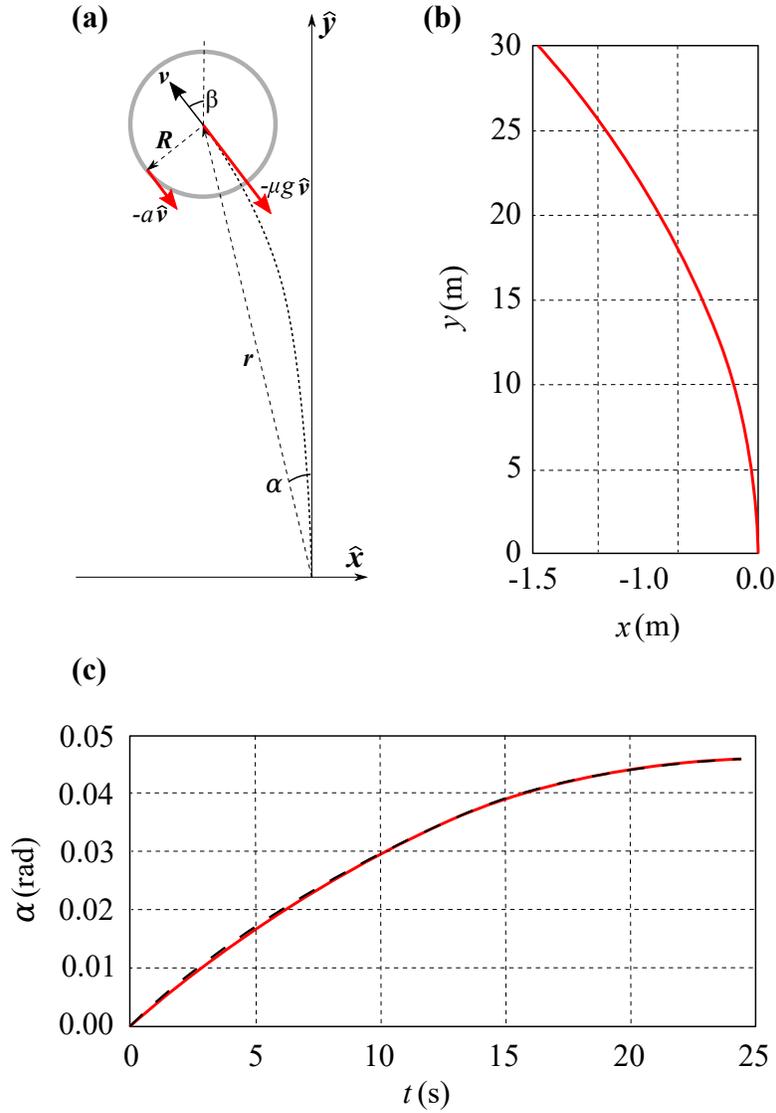


Figure 2: Forces and their effects on stone trajectory. **a** An acceleration  $\mu g$  acts at the (counterclockwise rotating) stone CM, opposing stone velocity  $\mathbf{v}$ . This acceleration is the net effect of friction forces between RB and ice, and is here assumed to be symmetric (independent of stone angular speed  $\omega$ ). A small acceleration  $a$  acts at the left edge of the RB, in the same direction as friction. This acceleration is (in our model) the net effect of asymmetrically-distributed grit and ice debris particles that are trapped beneath the RB. **b** The stone trajectory for  $v_0 = 2.55 \text{ m}^{-1}$ ,  $\mu = 0.01$ ,  $a = 0.001 \text{ ms}^{-2}$ ,  $\varepsilon = 2.5$ ,  $R = 0.065 \text{ m}$ . **c** Stone position vector rotation angle  $\alpha(t)$ , for the same parameter values

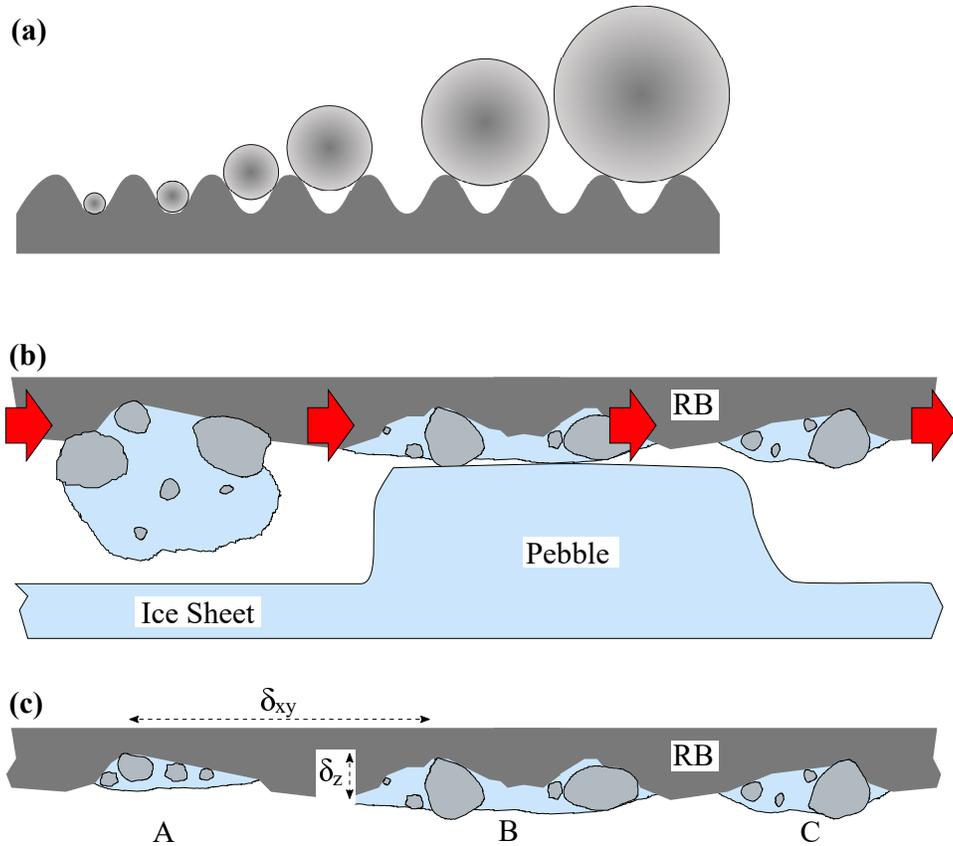


Figure 3: Ice debris model ‘Goldilocks effect’. **a** A simple analogy: balloons blown across a plowed field. The larger balloons on the right are more easily moved; the small ones on the left are not. It takes more force to dislodge embedded balloons. **b** Similarly, it takes less energy to detach grit and ice protrusions from the RB if they are large – the protrusion on the left will be knocked off, and not abraded. **c** The lower surface of the running band is roughened with horizontal and vertical length scales  $\delta_{xy} \approx 250\mu\text{m}$ ,  $\delta_z = 10\mu\text{m}$ . Protrusion of similar size (B,C) will be hard to detach and will abrade the ice pebble surface; smaller protrusions (A) will not cause much abrasion. Abrasion gives rise to the asymmetric acceleration  $a$  of fig. 2a due to the asymmetric distribution of grit and debris particles. In the abrasion model these protrusions get worn down and reform repeatedly as the stones moves over the ice

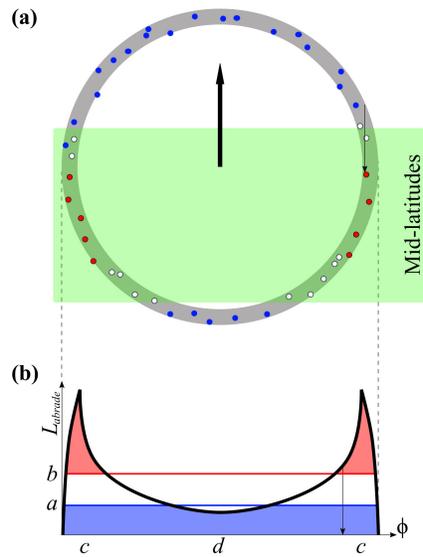


Figure 4: Pebble wear. **a** Pebbles under a RB moving in the direction shown (arrow) are worn down differently. Those in the ‘mid-latitude’ region are worn least by the action of the RB. **b** The length  $L_{abrade}$  of RB that is in contact with a pebble depends on  $\phi$ , as shown. Pebbles with  $L_{abrade} > b$  are worn the most; those with  $L_{abrade} < a$  are least worn

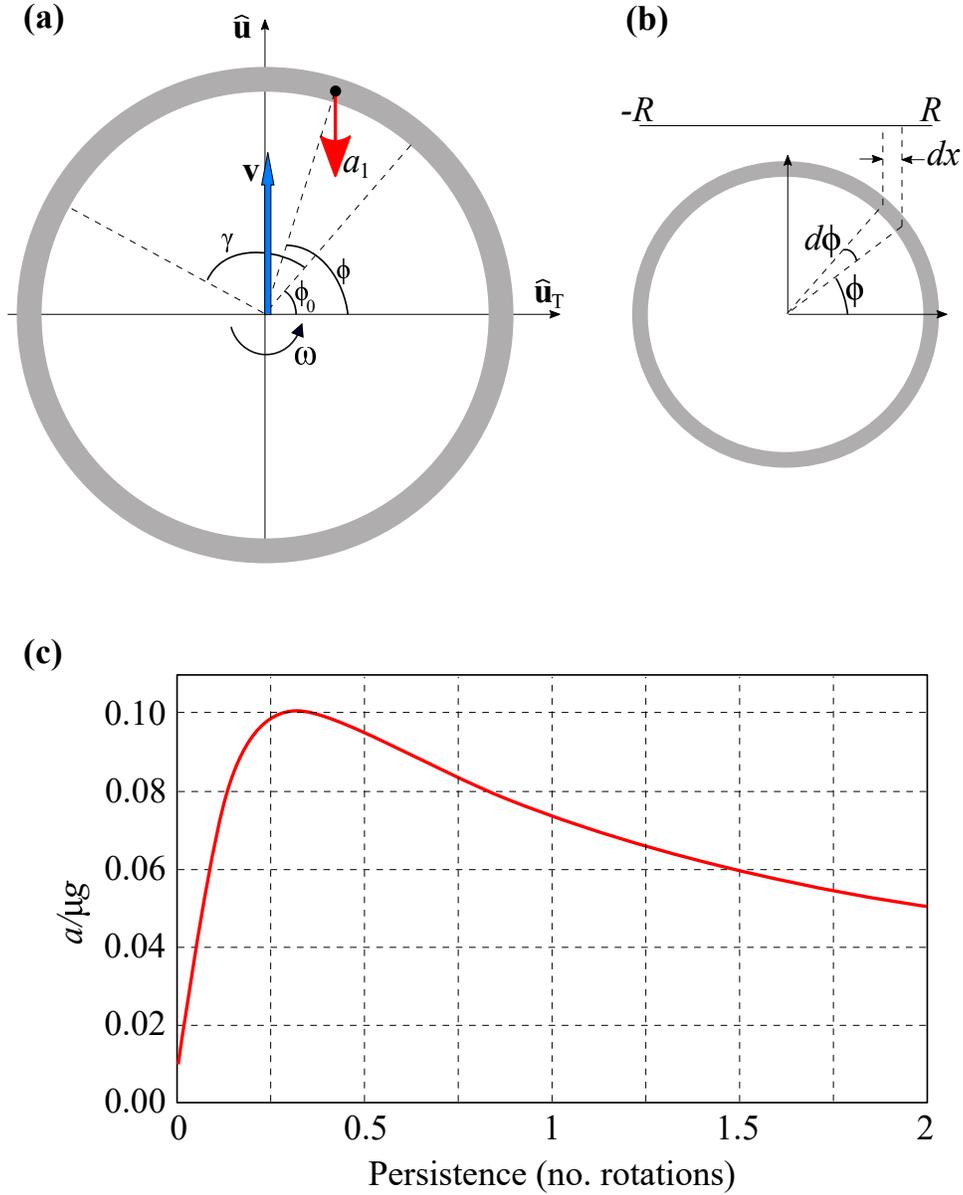


Figure 5: Debris model statistics. **a** A single debris particle exerts a torque about the CM at a point  $\phi$  on the RB, where  $\phi_0 < \phi < \phi_0 + \gamma$ . **b** A uniform distribution of debris particles along  $x$  transforms to a non-uniform distribution (eq.(16)) along  $\phi$ . **c** Mean transverse acceleration versus mean angular lifetime  $\bar{\gamma}$