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A signal enhancement method based on the reverberation statistical information

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Abstracts:

Reverberation is the main background interference in underwater active sonar target detection that seriously interferes with the extraction of target echo. In this paper, a signal enhancement algorithm utilizing fractional lower-order moments based on reverberation statistical distribution information is proposed. As fractional lower-order moments can only be applied on symmetric α-stable random variables, energy redistribution method is proposed to make reverberation signal satisfy the symmetric α-stable distribution. To evaluate the proposed algorithm, an experiment involving simulated linear frequency modulation reverberation with the proposed method and comparison methods is performed and discussed. Moreover, an experiment is conducted using reverberation as measured from the lake. The results show that the proposed algorithm can achieve better reverberation suppression and signal enhancement performance compared with other methods.

Keywords:

Signal enhancement, reverberation statistical distribution information, fractional lower-order moments, fractional Fourier transform, energy redistribution.

1 Introduction

In shallow water, reverberation caused by the scattering of the transmitted signal during propagation is the main interference in underwater signal processing. Due to reverberation generation principle, high correlation with target echo greatly degrades some traditional methods such as matched filter performance. Obviously, it is easily to see that the signal with high signal to reverberation ratio

(SRR) can not be obtained by increasing the source level. In generally, there are two aspects to suppress reverberation suppression: transmission signal design at transmitter and signal processing at the receiver[1].

For the former, Hague[2] and Touati[3] designed waveform based on developing CW waveform to detect the target. Dosiy[4], Collin[5] and Guan[6] designed specific type of pulses such as geometric comb, phase-coded sequence and so on to suppress reverberation. As some designed transmitted pulse can only mitigate the effects of reverberation, signal processing method should be proposed to suppress reverberation. Reverberation is a kind of nonstationary colored background noise, Kay proposed a prewhitener which is based on an autoregressive model to make the target more easily detected than current FFT processor[7]. Li gave a novel space time adaptive prewhitener for reverberation based on a two-dimensional autoregressive model to obtain better detection performance[8]. In recent years, non-negative matrix factorization (NMF) theory is widely used in reverberation suppression. Jia proposed a method that non-negative matrix factorisation is applied to express the time-frequency matrix as a low-rank matrix[9]. Lee[1] proposed an algorithm for the reverberation suppression of continuous wave signals using non-negative matrix factorization. As NMF-based reverberation suppression algorithms are only applicable to continuous wave reverberation, Kim[10] proposed two pre-processing methods, namely dechirping transformation and modulo operation, to facilitate application of the NMF method to LFM reverberation. Array processing as an effective method to improve the signal-to-noise ratio is also used for reverberation suppression. Zhu proposed an approach of sparse spatial spectral estimation based on singular value decomposition[11]. High order cumulates are also widely used to enhance underwater acoustic signals. However, when characteristic index α value of considering signal is between 0 and 2, the second-order and higher-order statistics of considering signal do not exist anymore. Therefore, the traditional signal processing methods based on second-order statistics (such as power spectrum) will lead to performance degradation or even failure when processing signals subject to alpha stable distribution.

The classical Gaussian model is generally used to explain the reverberation process when the scatterer distribution approximately satisfies the central limit theorem (CLT)[12]. However, in some marine environments when the scatterer distribution cannot meet the CLT, the reverberation amplitude distribution deviates from Gaussian. As the tail widens in the statistical distribution of seabed reverberation and there are strong non-Gaussian characteristics for the amplitude fluctuations of certain impact characteristics caused by the strong instability of shallow water reverberation, the α-stable distribution is used to build reverberation in shallow waters. In this condition, the fractional lower-order moments (FLOM), which is an effective tool to suppress the symmetric α-stable (SαS) distribution interference, is applied to enhance target echoes. However, considering the non-zero skewness of reverberation statistics that result in its non-compliance with SαS, in this paper, an energy redistribution method based on adaptive fractional Fourier transform (FrFT) is proposed to make signals to satisfy SαS distribution.

This paper is organized as follows. Section 2 provides a short description on the reverberation of α-stable mode and gives the FLOM theory. The proposed energy redistribution in fractional domain is also given in detail in this section. Section 3 give the simulation and experimental results of the proposed method in shallow waters. Some conclusions follow in Section 4.

2. Methods

2.1 Reverberation model which followed α-stable distribution

As α-stable distribution can maintain the limit distribution of the generation mechanism and propagation conditions of natural noise process, it has been used extensively to model impulsive phenomenon in many systems. Reverberation probability density function has the same characteristics of a single peak, bell shape, and thick tail as the α-stable distribution Therefore, intuitively, some statistical characteristics of α-stable distribution are applied to describe reverberation in this section.

In 1925, Levy proposed α stable distribution which also be called non-Gaussian stable distribution based on generalized central limit theorem. Except for a few special cases, there is no unified and closed analytical expression for the probability density function of the distribution, characteristic function is generally used to describe its distribution characteristics. If the random variable obeys α Distribution law, if and only if the characteristic function is

$$\varphi(t) = \exp\left\{ \mathrm{i}\delta t - |\gamma t|^{\alpha} \left[1 + \mathrm{i}\beta \operatorname{sgn}(t)\omega(t,\alpha)\right]\right\} \tag{1}$$

$$\omega(t,\alpha) = \begin{cases} \tan\left(\dfrac{\pi\alpha}{2}\right), & \alpha \neq 1 \\ \dfrac{2}{\pi}\log|t|, & \alpha = 1 \end{cases} \tag{2}$$

$$\operatorname{sgn}(t) = \begin{cases} 1, \ t > 0 \\ 0, \ t = 0 \\ -1, t < 0 \end{cases} \tag{3}$$

where, $0 < \alpha \leq 2, -1 \leq \beta \leq 1, \gamma > 0, -\infty < \delta < +\infty$, i is the imaginary unit. $\operatorname{sgn}(t)$ is a symbolic function. It can be seen that α-stable distribution is uniquely determined by the four parameters. α is the characteristic index which determines the trailing thickness of the probability density function. Different from the Gaussian distribution, α-stable distribution decays in algebraic form and the attenuation velocity is related with α. Parameter β determines the degree of distribution symmetry. When $\beta = 0$, the distribution also be called symmetric α stable distribution (SαS). Υ is the scale parameter which measured the dispersion of the distribution. The distribution can be regarded as Gaussian distribution $S_2(\Upsilon,1,\delta) = N(\Upsilon,1,\delta)$, Cauchy distribution $S_1(\Upsilon,0,\delta)$ and Levy distribution

($S_{0.5}(\Upsilon,1,\delta)$).

The time domain expression of active sonar emission waveform is $s(t)$. Assuming that the number of discrete scatterers satisfying the i.i.d. condition in the sonar resolution unit is n, the signal received at the receiving can be expressed as

$$r(t) = a_0 s(t-\tau_0) + \sum_{i=1}^{N} a_i g_i ss(t-\tau_i) + n_w(t) \tag{4}$$

In equation 4, the first term on the right of the equation represents the received target signal, where a_0 is the attenuation coefficient of the target signal which is related with the propagation loss of the acoustic signal of the excitation target in the back-and-forth sound path and the absorption loss of the medium to the sound energy. τ_0 is the time delay of target echo signal. In this paper, the Doppler effect caused by target movement does not be discussed. The second term represents the sum of the scattering signals of the scatterers, a_i is the attenuation coefficient of the scattering signal of the i^{th} scatterer which is related with the propagation loss and absorption loss before and after the scattering of the sound wave; g_i represents the scattering intensity coefficient of the i^{th} scatterer which is related with the variation coefficient of scattering intensity caused by different scattering coefficient and incident grazing angle when acoustic wave enters the i^{th} scatterer. $ss(t-\tau_i)$ is the waveform expression of sound wave scattered by the i^{th} scatterer. The third term represents marine environmental noise.

2.2 Fractional lower order moments for signal enhancement in reverberation environment

In this part, the basic theory of FLOM is introduced firstly. As it can only be used to process the signal which satisfy SαS distribution, an energy redistribution method which is applied to redistribute the received signal energy in fractional domain is proposed in detail. Then, the target echo broadening and compression in fractional domain is analyzed.

2.2.1 Basic knowledge of fractional lower order moments

Assume random variable $x: S\alpha S$, $0<\alpha\le 2$, the fractional order moment of the SαS random variable $x: S_\alpha(\beta, \Upsilon, 0)$ with zero location parameter $(\delta=0)$ is given by

$$\mathrm{E}\left[|X|^p\right]=\frac{2^{p+1}\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(-\frac{p}{\alpha}\right)}{\alpha\sqrt{\pi}\Gamma\left(-\frac{p}{2}\right)}\Upsilon^{\alpha/p}, p<\alpha \tag{5}$$

Where $\Gamma(\bullet)$ is the Gamma function,

$$\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}\mathrm{d}t \tag{6}$$

If the random variables X and Y obey SαS distribution, and the range of α is from 1 to 2. The fractional lower order correlation (FLOC) can be expressed as

$$R_{XY}^p=E\left[XY^{<p-1>}\right],\ \ 1\le p<\alpha \tag{7}$$

Based on the Wigner-Ville distribution (WVD) and FLOM theories, if the random variables obey SαS distribution, the fractional lower order of WVD (FLOM-WVD) expression can be written as

$$WVD_{FLOS}(t,f)=\int_{-\infty}^{\infty}x^{\langle P\rangle}(t+\tau/2)x^{-\langle P\rangle}(t-\tau/2)e^{-j2\pi f\tau}d\tau \tag{8}$$

where $x^{\langle P\rangle}=|x|^p\operatorname{sign}(x), p<\alpha/2$

2.2.2 Received signal redistributed in fractional domain

The fractional lower order moment theory could only be used on the received signal which satisfy SαS distribution (β=0). However, due to the influence of the number of scatters, the observation duration and other reasons, the distribution of received signal is difficult completely satisfy SαS distribution. In this part, in order to make the reverberation greater conformity with the SαS distribution. an energy redistribution method based on fractional Fourier transform(FrFT) is proposed.

FrFT is a type of linear integral transformation, performed on LFM signals, that is used to transform the signal from one domain to another, which is known as the fractional domain and is represented by u. Then, the p^{th} order FrFT on the function $s(t)$ can be denoted by $F^p(u)$ and defined as

$$F^p(u)=\int_{-\infty}^{+\infty}\dot{K}_p(u,t)s(t)\mathrm{d}t \tag{9}$$

where $\dot{K}_p(u,t)=\sqrt{1-i\cot\left(\frac{p\pi}{2}\right)}\exp\left[i\pi\left(u^2\cot\left(\frac{p\pi}{2}\right)-2u\csc\left(\frac{p\pi}{2}\right)+t^2\cot\left(\frac{p\pi}{2}\right)\right)\right]$, $p\ne 2n$, $n\in Z$. With some simplification, the above equation can be written as

$$F^p(u)=\begin{cases} B_p\int_{-\infty}^{+\infty}\exp\left[i\left(\frac{t^2+u^2}{2}\cot\left(\frac{p\pi}{2}\right)-\frac{tu}{\sin(p\pi/2)}\right)\right]s(t)\mathrm{d}t, & p\ne 4n\\ s(t), & p=4n\\ s(-t), & p\ne 2(2n+1)\end{cases} \tag{10}$$

Where $B_p = \sqrt{\frac{1-\cot(p\pi/2)}{2\pi}}$, and $f(t)$ in equation is expressed as

$$f(t) = \exp\left[i\pi\left(2f_0t + \mu t^2\right)\right], -\frac{T}{2} < t < \frac{T}{2} \tag{11}$$

Where f , μ and T are the initial frequency, frequency modulation rate and duration. In particular, when $p = 1$, the FrFT on $f(t)$ can be regarded as Fourier transform.

We applied (10) to equation (4), the received signal in fractional domain with transform p will be

$$\begin{aligned} F^p(u) &= \int_{-\infty}^{+\infty} K_p(u,t)s(t)\mathrm{dt} \\ &= \int_{-\infty}^{+\infty} K_p(u,t)\left[a_0 s(t-\tau_0) + \sum_{i=1}^{N} a_i g_i ss(t-\tau_i) + n_w(t)\right]\mathrm{dt} \\ &= S(u_p) + SS(u_p) + N(u_p) \end{aligned} \tag{12}$$

$$r(t) = a_0 s(t-\tau_0) + \sum_{i=1}^{N} a_i g_i ss(t-\tau_i) + n_w(t) \tag{12}$$

where

$$S(u_p) = \int_{-\infty}^{+\infty} K_p(u,t)a_0 s(t-\tau_0)\mathrm{dt} \tag{13}$$

$$SS(u_p) = \int_{-\infty}^{+\infty} K_p(u,t)\sum_{i=1}^{N} a_i g_i ss(t-\tau_i)\mathrm{dt} \tag{14}$$

$$R(u_p) = \int_{-\infty}^{+\infty} K_p(u,t)\sum_{i=1}^{N} n_w(t)\mathrm{dt} \tag{15}$$

Here we give a reverberation model example which satisfy α-stable distribution (α=1.2176, β=0.6,γ=0.0420,δ=0.0004). With different transform p value, the four parameters of redistributed signal are shown in the following box

P value	-0.54	-0.5	-0.02	0.02	0.12	0.22	0.3	0.36	0.4
α	1.503	1.349	1.251	1.163	0.952	0.917	0.893	0.969	1.104
β	0.093	0.10	0.032	0.003	-0.03	-0.132	0.029	0.120	0.056
γ	0.47	0.43	0.112	0.11	0.18	0.22	0.24	0.28	0.34
δ	0.01	0.05	0.01	0	0.05	0.255	0.037	0.741	0.146

The distributions of α and β with different transform p value is given in Fig.1. It is easily to see that the redistributed signal follows different statistical distribution as Fig.1 shows.

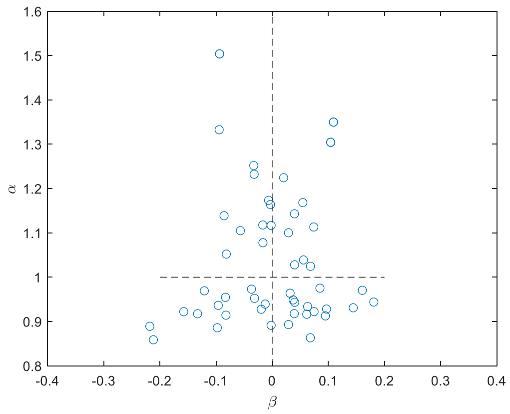
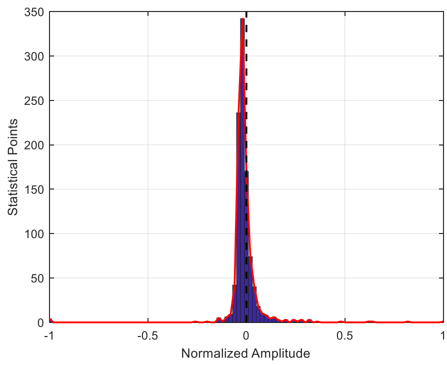
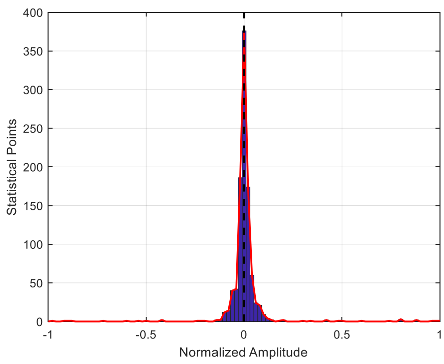


Fig.1 α and β values of redistributed signal with various transform p values

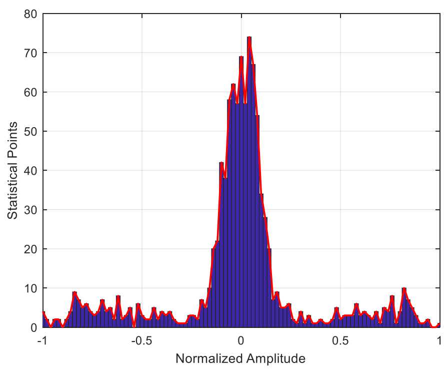
Fig.2 (a),（b）and (c) give the probability density functions of three cases (p=0, p=0.02 and p=0.22) respectively. When p equals to 0, there is obvious offset around zero as Fig.2 (a) shows. For p=0.22 condition, there is a thicker and less smooth tail than p=0.02 condition (Fig.2 (b)). Furthermore, the α=1.163 and β=0.003 of redistributed signal (p=0.02) is most followed SαS distribution. More specifically, β is closest to 0 while α is between 0 and 1 is the optimal P selection principle which is totally different from the traditional fractional Fourier transform.



(a) Probability density functions(p=0)



(b) Probability density functions(p=0.02)



(c) Probability density functions(p=0.22)

Fig.2 The probability density distribution of three cases.

2.2.3 The target echo broadening and compression in fractional domain

In this part, we discuss the influence of the redistributed signal method on the target echo. Fig.3 gives the transform relation between in time domain and fractional domain of target echo.

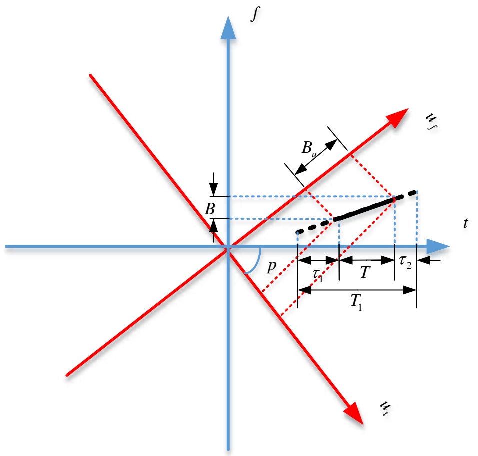


Fig.3 The corresponding relation between echo signal fractional domain

According to Fig.3, there are some changes in the bandwidth of received signal in fractional domain due to the fractional Fourier transform. We assumed the frequency bandwidth of transmitted signal is B. The duration of transmitted signal is T .The observation duration is T_1. The transform angle of fractional Fourier transform is p. With the geometric relation, the bandwidth in fractional domain can be written as

$$B_u = (B - \chi T)\sin(p\pi/2) \tag{16}$$

Where the $\chi = -\frac{1}{\tan(2p/\pi)}$ is related with the chirp rate of transmitted signal in fractional domain. As the equation, the target echo broadening and compression in fractional domain is related with the observation signal length T and the transform angle p. In addition, it can be seen that the information and energy of the echo signal are not changed after FrFT.

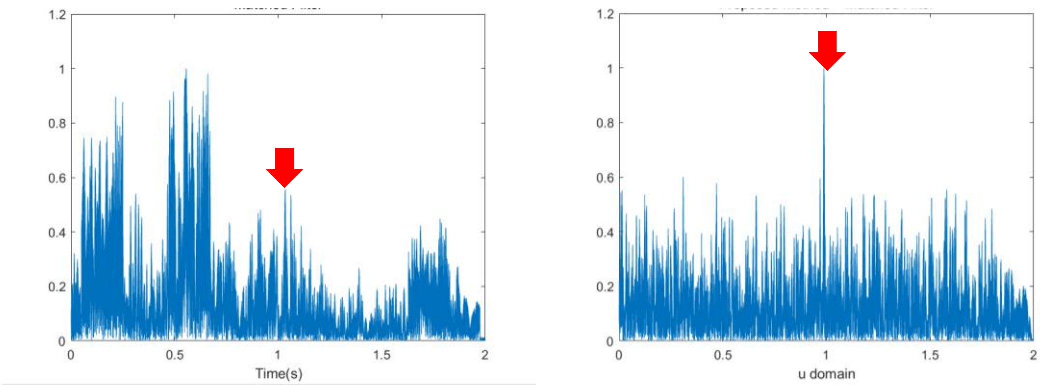
3 Results and discussion

3.1 The performance of the proposed method in shallow water environment

This section presents the proposed method's simulation results and experimental results. The performance of echo target enhancement in the reverberation environment will be discussed using peak-signal-to-reverberation (PSRR) at different condition (ref1). The expression of PSRR is defined by

$$PSRR = 20\log 10\left(\frac{M_{\mathrm{sig}}}{M_{\mathrm{rbn}}}\right) \tag{17}$$

where M_{sig} is the signal peak value and M_{rbn} is the reverberation peak value.



(a) Matched filter (b) Proposed method

Fig.4 The processing results by proposed method and comparison methods.

Fig.4 shows the results by the proposed method and matched filter in reverberation environment（SRR=10dB）, respectively. In Fig.4 (a), it is hard to observe the target component is located at the true echo time delay (1 s). However, this component is easy to locate the target in Fig.4(b). According to Fig.4, the proposed algorithm effectively removes reverberation, confirming that the target is clearly detected. It illustrates that the proposed algorithm suppresses the reverberation component effectively in server reverberation environments.

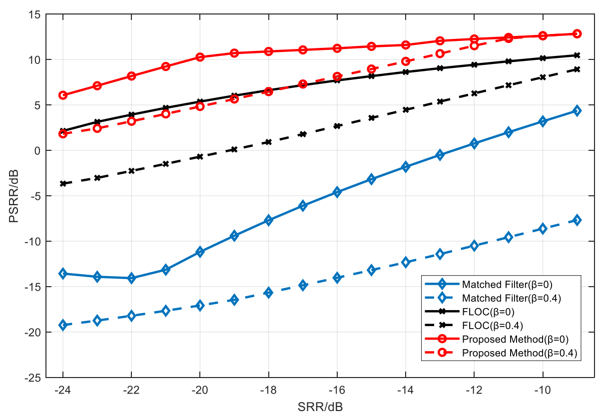


Fig.5 Enhancement performance of proposed method and comparison method in various condition

Fig.5 shows the PSRR of the proposed method, FLOC and the matched filter results. The proposed algorithm (red solid line) achieves an PSRR gain of 6 to 12 dB under the input SRR values is from -24 dB to -9 dB condition. Compared with the other two methods (blue and black solid lines), the proposed method could get most outstanding performance. Consider the case of β=0.4， the performance of proposed method (red dotted line) is similar to the result of β=0（red solid line） which suggests that the energy redistribution could make received signal obey SαS distribution when SRR is above -12dB.

3.2 Experimental results and discussion

To further verify the method, lake trial data of a suspended object echo signal are processed. The slope of the lake bottom is about 8.8 degrees. A monostatic sonar is used to transmit a wide-band linear frequency modulated (LFM) signal, the frequency range is from 25kHz to 50kHz and the time duration is 2ms. The receiving system is an 18-element uniform line array.

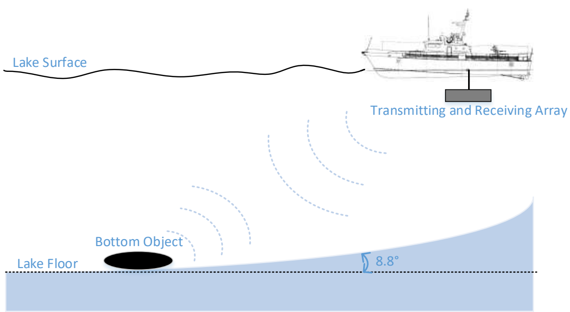
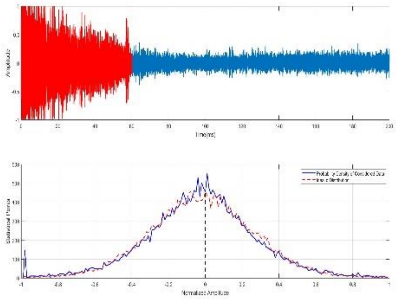
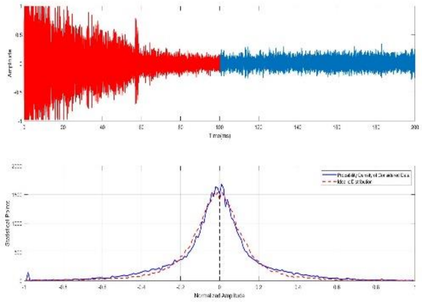


Fig.6 Schematic diagram of lake experiment.

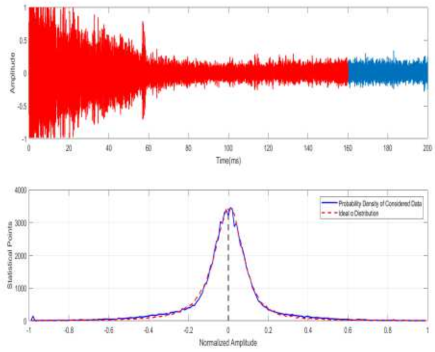
The probability density characteristics of instantaneous values are analyzed by intercepting the data in different observation times. We consider the time duration T is 60ms, 100ms， 160ms and 200ms respectively. According to [13] [14], the α stable distribution（α，β，γ，μ) parameter estimation of observation duration could be calculated.



(a) T=60ms



(b）T=100ms



(c) T=160ms

Fig.7 Time domain waveform and probability density curve of reverberation in different observation duration

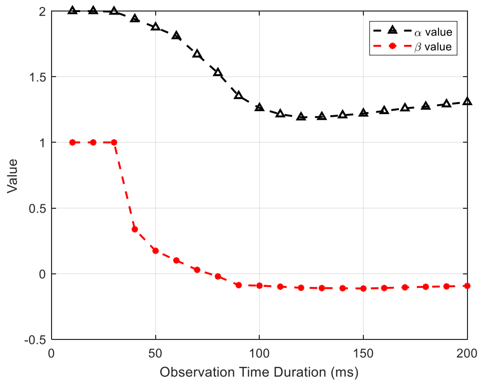


Fig.8 α and β values with different observation time duration

According to the Fig.7 (a), the probability density curve of the considered time duration (T=60ms) roughly fits the ideal α distribution (α=1.8093，β=0.1010 and γ=0.1977). The fitting performance of Fig.7 (b) is slightly better than Fig.7 (a). Especially, compared with Fig.7 （a) and （b), there is less fluctuation at the tail of probability density curve in Fig.6 (c). Therefore, with time duration increasing, probability density curve of reverberation has a better agreement with ideal SαS distribution.

Fig.8 gives the relationship between parameters (α and β) of α distribution and observation duration. It is easy to observe that α and β tend to be stable with the increase of observation duration. In particular, β gradually closes to zero.

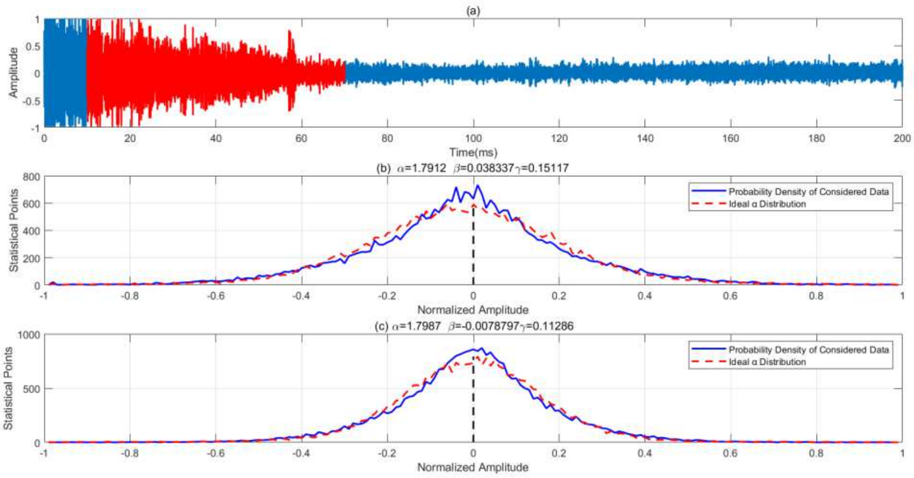


Fig.9 Probability density curve of time domain waveform and fractional domain with optimal transform angle

According to Fig.9 (b) and (c), with the proposed method, the better fitting result between ideal SαS distribution. As the signal is redistributed in the fractional domain, the probability density curve of processed signal is much smoother than the received signal.

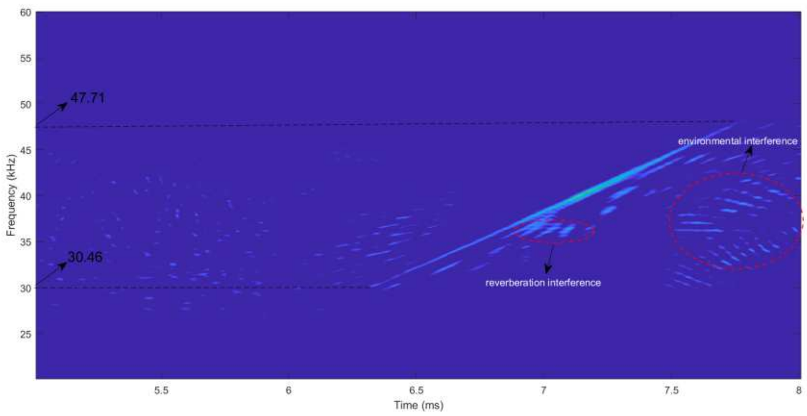


Fig.10 Time-frequency distribution of observed reverberation signal with traditional WVD.

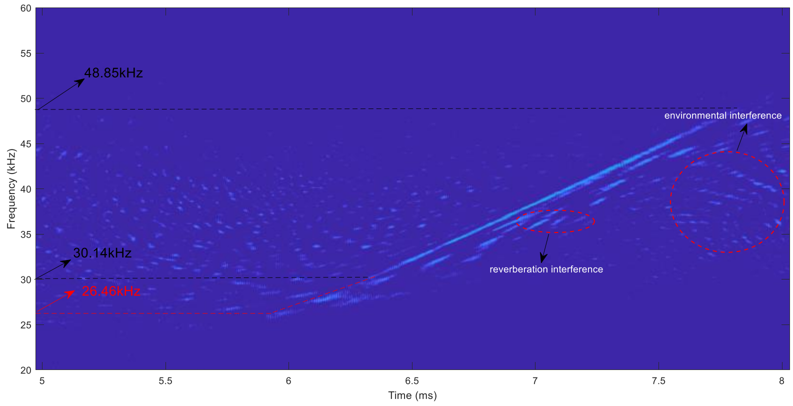


Fig.11 Time-frequency distribution of observed reverberation signal with proposed method.

The observed bandwidth in Fig.10 is from 30.46 kHz to 47.71 kHz. As the bandwidth of transmitted signal is 25kHz, 69% bandwidth can only be reflected in Fig.10. The result by proposed method is shown in Fig.11. Obviously, the bandwidth of observed signal is from 48.85kHz to 26.46 kHz which could reach 89.56% of transmitted signal bandwidth. The target echo enhancement performance (marked in the ellipse) is much better than Fig.10 shows. In addition, the reverberation and environment interference suppression performance is much better than the traditional WVD method (marked in the ellipse).

4. Conclusions

Reverberation suppression is an essential issue in active sonar system. In this paper, reverberation statistical distribution information is proposed to enhance the target signal. In detail, a suppression method is proposed using FLOM theory in condition of α stable distribution reverberation. Considering limitation of FLOM theory, energy redistribution method is applied to make reverberation signal obey SαS distribution. By applying the proposed pre-processing method to received signal, the FLOM-WVD method can get better performance than the traditional WVD method. The simulation and experimental results show that the proposed algorithm can effectively suppress underwater reverberation and improve the SRR of underwater target echoes.

Abbreviations

FrFT: fractional Fourier transform; FLOM: fractional lower-order moments; FLOC: fractional lower order correlation; WVD: Wigner-Ville distribution; SαS：symmetric α-stable.

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Authors' contributions

GY proposed the framework of the whole algorithm; JJ performed the simulations, analysis on the results. XKL and GY have participated in design of this research. All authors read and approved the final manuscript

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Availability of data and materials

Please contact author for data requests.

Declarations

Ethics approval and consent to participate

Not applicable.

Consent for publication

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Competing interests

The authors declare that they have no competing interests.

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