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Medical Image Reconstruction Method Based on Smooth L0 Norm

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Abstract

Compressed sensing (CS) has a wide range of applications in sparse signal reconstruction. Aiming at the problems of low recovery accuracy and long reconstruction time of existing reconstruction algorithms in medical imaging, this paper proposes a corrected smoothing L0 algorithm based on compressed sensing (CSL0). Firstly, an approximate hyperbolic tangent function (AHTF) which is more similar to the L0 norm is proposed to approximate the L0 norm. Secondly, aiming at the problem that the steepest descent method has a "sawtooth phenomenon" and the modified Newton method is sensitive to the initial value selection, the steepest descent method and the modified Newton method are jointly optimized to improve the reconstruction accuracy. In addition, the CCSL0 algorithm is proposed by introducing a composite inverse proportional function, and the reconstruction time of the CSL0 algorithm is further optimized. Finally, the CSL0 algorithm and CCSL0 algorithm are simulated on medical images. The results show that the proposed algorithm improves the reconstruction accuracy of the test image by 0 – 0.96 dB.

Keywords: Smoothed L0, Compressed sensing (CS), Image processing, Sparse reconstruction

1 Introduction

With the development of computer technology, medical imaging technology has become increasingly mature, and image reconstruction algorithms have become increasingly prominent in clinical and industrial applications, such as Computed Tomography (CT) [1], Magnetic Resonance Imaging (MRI) [2], and X-ray scattering [3]. However, with the increase of image information, to a certain extent, it will cause problems such as poor image reconstruction accuracy and long reconstruction time, which will bring a series of challenges for subsequent processing. In recent years, sparse representation and sparse signal recovery have received extensive attention in the field of compressed sensing (CS). It can perceive high-dimensional signals through non-correlated low-dimensional space, that is, the original image information can be quickly and effectively recovered through a small amount of sample information [4].

Compressed sensing theory is mainly divided into three parts: signal sparse representation, measurement matrix selection, and signal reconstruction. The mathematical model is as follows:

$$\mathrm{y} = \Phi x \tag{1}$$

Formula (1): y is a known $m \times 1$ vector representing the observed signal. Φ is a matrix of $m \times n$, $m < n$,Φ is called the observation matrix.x is an unknown $n \times 1$ vector, representing the sparse signal to be recovered. Since the number m of equations in Equation (1) is less than the number n of unknowns, $y = \Phi x$ is a system of underdetermined equations. If the traditional least square method is used for the underdetermined equations, the accuracy of the solution of the equation group is low and the robustness is poor, and it is easy to fall into the local optimal solution or even no solution. Therefore, how to efficiently recover raw data from low-dimensional data space is the core content of the reconstruction algorithm, and it is also one of the hot issues of domestic and foreign scholars.

At present, a widely used reconstruction algorithm is a greedy algorithm, such as the orthogonal matching pursuit algorithm (OMP) [5], compressed sampling matching pursuit algorithm (CoSaMP) [6], and generalized orthogonal matching pursuit algorithm (GOMP) [7]. The disadvantage of a greedy algorithm is that the reconstruction accuracy of the high-dimensional signal is poor, and most of these algorithms need to be done in the case of known sparsity. Another is convex optimization algorithms, such as base pursuit algorithm (BP) [8], and subspace pursuit algorithm (SP) [9]. However, this kind of algorithm has a large amount of calculation, so the reconstruction time is long. In 2009, Mohimani et al. proposed in [10] the SL0 algorithm of the smooth norm to obtain the sparse solution of underdetermined equations. Specifically:

$$\min_{x} \|x\|_0 \qquad s.t. \;\; y = \Phi x \tag{2}$$

$\|x\|_0$ is the l_0 norm of x , representing the number of non-0 elements in x. $y = \Phi x$ is the constraint condition. The key of the SL0 algorithm is to approximate the l_0 norm by Gaussian function and to minimize the l_0 norm by solving the minimum solution of Gaussian continuous function. On this basis, subsequent scholars have carried out explorations.

In 2012, Zhao et al. introduced the hyperbolic tangent function to approximate the L0 norm in [11] and used a modified Newton method to solve it, and proposed the NSL0 algorithm. Although the NSL0 algorithm improves the convergence speed and reconstruction accuracy, searching for a better function sequence to improve the reconstruction accuracy is still the direction that needs further research. In 2013, Zheng et al. proposed to increase the iteration error based on the SL0 algorithm, and optimize the OSL0 algorithm of iteration number [12]. Although the reconstruction accuracy has been improved to some extent, the reconstruction time is long. In 2018, Wang et al. proposed the MSL0 algorithm by constructing a set of composite inverse proportional functions that approximate the l_0 norm and combining it with the residual measurement mechanism [13]. The algorithm is fast, but the reconstruction accuracy is in general. In 2019, Zeng et al. extended the smooth L0 norm method to radar signal processing and introduced the Newton method into the LMS algorithm [14]. However, this algorithm still uses the initial Gaussian function, and the reconstruction effect is not ideal.In 2020, Li et al. proposed a regularization model combining the L0 norm and L1 norm and used the monotone alternating direction algorithm and the hard threshold method to solve the optimization problem [15]. However, this algorithm has a large amount of calculation and high complexity.

Given the shortcomings of the above algorithms, the contributions of this paper are as follows:

1. An approximate hyperbolic tangent function (AHTF) which is closer to the L0 norm is constructed. Compared with the existing function, it has stronger contraction in the definition domain, thereby enhancing the sparsity of the signal, and thus improving the accuracy of reconstruction.

2. CSL0 algorithm is proposed. The approximate hyperbolic tangent function is jointly optimized by the steepest descent method and the modified Newton method, which can not only avoid the problem that the steepest descent method affects the convergence speed in the later

stage, but also avoid the problem that the modified Newton method is sensitive to the initial value selection and improve the reconstruction accuracy.

3. CCSL0 algorithm is proposed. The combined optimization of composite inverse proportion function and approximate hyperbolic tangent function with a simpler form is introduced to further shorten the running time of the CSL0 algorithm and improve the reconstruction speed.

4. Simulation experiments were conducted on medical images and conventional images. The experimental simulation analysis of the CSL0 algorithm, CCSL0 algorithm, and several existing mainstream algorithms shows that the proposed algorithm improves PSNR and time.

2 CSL0 algorithm

2.1 AHTF function

L0 norm represents the number of non-zero elements in the vector. The idea of the SL0 algorithm is to use a smooth Gaussian function to achieve the approximation of the L0 norm, and its expression is as follows:

$$\varphi_\sigma(x_i) = e^{-\frac{{x_i}^2}{2\sigma^2}} \tag{3}$$

x_i represents the component of signal x and σ represents the smooth parameter. It can be obtained that:

$$\lim_{\sigma\to 0} \varphi_\sigma(x_i) = \begin{cases} 1, & x_i = 0 \\ 0, & x_i \neq 0 \end{cases} \tag{4}$$

Can be deduced:

$\|x\|_0 = \lim_{\sigma\to 0} F_\sigma(x_i) = \lim_{\sigma\to 0} \sum_{i=1}^{N} (1 - \varphi_\sigma(x_i))$

In order to further improve the function approximation performance, this paper constructs the following approximate hyperbolic tangent function (AHTF):

$$\mathrm{f}_\sigma(x_i) = \frac{1}{8}\left(\frac{e^{\frac{k{x_i}^2}{\sigma^2}} - 1}{e^{\frac{k{x_i}^2}{\sigma^2}} + 1}\right) + \frac{7}{8}\left(1 - e^{-\frac{8k{x_i}^2}{\sigma^2}}\right) \tag{5}$$

$k > 0$ represents the shape parameter that controls the contraction of $f_\sigma(x_i)$. σ represents the smooth parameter. The smaller σ is, the steeper the function is, but it is not smoother. In fact, for any $\sigma > 0$ and x_i , there is $f_\sigma(x_i) \geq 1 - \varphi_\sigma(x_i)$. The following is strict mathematical proof.

Hypothesis $k = 1$, $g_\sigma(x_i) = f_\sigma(x_i) - (1 - \varphi_\sigma(x_i))$.The definition of formula (3) and formula (5) shows that when $x_i = 0$ or $x_i = \pm\infty$,$g_\sigma(x_i) = 0$. The following discussion is $0 < |x_i| < +\infty$:

$$\begin{aligned} &g_\sigma(x_i) = f_\sigma(x_i) - (1 - \varphi_\sigma(x_i)) \\ &= \frac{1}{8}\left(\frac{e^{\frac{{x_i}^2}{\sigma^2}} - 1}{e^{\frac{{x_i}^2}{\sigma^2}} + 1}\right) + \frac{7}{8}\left(1 - e^{-\frac{8{x_i}^2}{\sigma^2}}\right) - 1 + e^{-\frac{{x_i}^2}{2\sigma^2}} \end{aligned} \tag{6}$$

Let $\frac{{x_i}^2}{2\sigma^2} = a$, After simplification:

$$\mathrm{g}_\sigma(x_i) = e^{-a} - \frac{1}{4}\left(\frac{1}{e^{2a}+1}\right) - \frac{7}{8}e^{-16a} \tag{7}$$

From formula (7), when $0 < |x_i| < +\infty$, $g_\sigma(x_i) \geq 0. In summary, for any \mathrm{x}_i$, $f_\sigma(x_i) - (1 - \varphi_\sigma(x_i)) \geq 0$ holds. This shows that the approximate hyperbolic tangent function is closer to the L0 norm than the Gaussian function. In order to intuitively represent the difference between the approximate hyperbolic tangent function and its function, this paper compares the distribution of four functions in the interval [-1,1] when $\sigma = 0.1$, as shown in Figure 1.

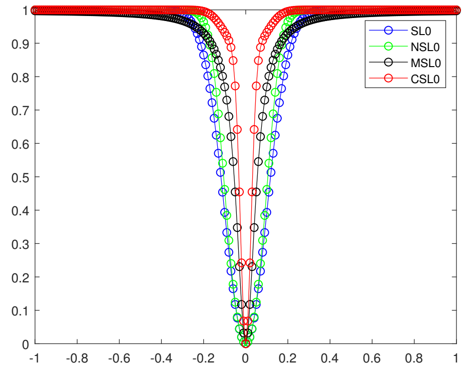


Fig. 1 Comparison of approximate hyperbolic tangent function (AHTF) with other functions.

It can be seen from Figure 1 that in the interval [-0.2-0.2], the AHTF function is better than other functions ' steepness, so the AHTF function proposed in this paper is better than other functions in an approximation of the L0 norm.

2.2 Optimistic method

The SL0 algorithm uses the steepest descent method to solve the L0 norm, which has a large step at the beginning of the iteration and a low requirement for the initial point. However, the steepest descent method is generally applicable to the prophase process of solving the optimization problem but is not ideal for the anaphase process. This is because the "sawtooth" shape appears in the iterative direction of the steepest descent method. Although the steepest descent method can quickly approach the adjacent area of the optimal solution in the initial stage, this result is slow globally, which greatly affects the convergence rate of the algorithm. Therefore, in [11], Zhao et al. proposed to use the modified Newton method instead of the steepest descent method to solve the problem. Although the problem of the "sawtooth" shape was solved, the modified Newton method itself had shortcomings. This method was sensitive to the selection of initial values. If the initial point was selected near the minimum point, the modified Newton method could quickly converge to the optimal point. If the initial point is not appropriate, far from the minimum point, the Newton method is likely to not converge, or the convergence results are not ideal. In order to solve the shortcomings of the above methods, the CSL0 algorithm proposed in this paper uses the steepest descent method and the modified Newton method to jointly optimize the method, which can not only avoid the problem that the steepest descent method affects the convergence speed in the later stage, but also avoid the problem that the modified Newton method is sensitive to the initial value selection, and improve the reconstruction effect. The first half of the optimization algorithm is solved by the steepest descent method, and the second half is solved by the modified Newton method. Next, calculate the gradient $d_R = -\nabla F_\sigma(x)^T$ and Newton direction $d_N = -\nabla^2 F_\sigma(x)^{-1} \nabla F_\sigma(x)$ of the AHTF function.

$$\nabla F_\sigma(x) = \left[\frac{\partial f_\sigma(x_1)}{\partial x_1}, \cdots, \frac{\partial f_\sigma(x_N)}{\partial x_N}\right]^T \tag{8}$$

$$\frac{\partial f_\sigma(x_i)}{\partial x_i} = \frac{1}{8}\left(\frac{\frac{4kx_i}{\sigma^2} e^{\frac{kx_i^2}{\sigma^2}}}{\left(e^{\frac{kx_i^2}{\sigma^2}}+1\right)^2}\right) + \frac{7}{8}\cdot\frac{16kx_i}{\sigma^2}\cdot e^{-\frac{8kx_i^2}{\sigma^2}},$$

$$\nabla^2 F_\sigma(x) = \begin{bmatrix} \frac{\partial^2 f_\sigma(x_1)}{\partial {x_1}^2} & 0 & \cdots & 0 \\ 0 & \frac{\partial^2 f_\sigma(x_2)}{\partial {x_2}^2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\partial^2 f_\sigma(x_N)}{\partial {x_N}^2} \end{bmatrix} \tag{9}$$

$$\frac{\partial^2 f_\sigma(x_i)}{\partial {x_i}^2} = \frac{1}{8}\left(\frac{\frac{4k}{\sigma^2} e^{\frac{k{x_i}^2}{\sigma^2}}\left(1+\frac{2k{x_i}^2}{\sigma^2}\right)\left(e^{\frac{k{x_i}^2}{\sigma^2}}+1\right) - \frac{16k^2{x_i}^2}{\sigma^4} e^{\frac{2k{x_i}^2}{\sigma^2}}}{\left(e^{\frac{k{x_i}^2}{\sigma^2}}+1\right)^3}\right) + \frac{7}{8}\left(\frac{16k}{\sigma^2} e^{\frac{-8k{x_i}^2}{\sigma^2}} - \frac{16^2k^2{x_i}^2}{\sigma^4} e^{\frac{-8k{x_i}^2}{\sigma^2}}\right)$$

From the above derivation process, it can be found that the eigenvalue of Hesse matrix $\nabla^2 F_\sigma(x)$ may not be positive, so the Hesse matrix may not be a positive definite matrix, and it cannot guarantee that the Newton direction d is the downward direction. Therefore, this paper will get a modified Newton direction by modifying the above Hesse matrix, as follows: Construct a new matrix: $G = \nabla^2 F_\sigma(x) + \xi_k I$. I represent the unit matrix, ξ_k is a set of proper positive numbers, if you want the matrix G to be a positive definite matrix, then all eigenvalues of the matrix G are positive. Through calculation, it can be found that when $\xi_k = \frac{1}{8}\left(\frac{\frac{16k^2{x_i}^2}{\sigma^4} e^{\frac{2k{x_i}^2}{\sigma^2}}\left(e^{\frac{k{x_i}^2}{\sigma^2}}+1\right)}{\left(e^{\frac{k{x_i}^2}{\sigma^2}}+1\right)^4}\right) + \frac{7}{8}\left(\frac{16^2k^2{x_i}^2}{\sigma^4} e^{\frac{-8k{x_i}^2}{\sigma^2}}\right)$, matrix G is a positive definite matrix. The final modified Newton direction is:

$$d_N = -G^{-1}\nabla F_\sigma(x) = -\frac{\sigma^2 x + 112\sigma^2 x \cdot e^{-\frac{9kx_i^2}{\sigma^2}}}{\sigma^2 + 2kx + 112 e^{-\frac{9kx_i^2}{\sigma^2}}}.$$

In summary, the specific steps of the CSL0 algorithm are as follows:

CSL0 algorithm:
Step 1: Initialize :
Reconstructed vector: $x = \Phi^T\left(\Phi\Phi^T\right)^{-1}y$, Shape parameter : k,Smooth parameter decreasing sequence : $[\sigma_1, \sigma_2, \cdots, \sigma_j]$, $\sigma_1 = 2\max\lvert x\rvert$, Smooth parameter threshold: $\sigma_{\min}$, Number of internal iterations:L, Error threshold: ε, Initial $r_0 = 0$.
Step 2: While $\sigma_j > \sigma_{\min}$
Let $\sigma_j = \beta\sigma_{j-1}$,$\sigma = \sigma_j$.
Step 3: for $l = 1, \cdots, p, \cdots, L$.
$if \ \ l \le p$:$\hat{x} \leftarrow \hat{x} - \mu\sigma^2 d_R$.
$\hat{x} \leftarrow \hat{x} - \Phi^T\left(\Phi\Phi^T\right)^{-1}(\Phi\hat{x} - y)$.
else:$\hat{x} \leftarrow \hat{x} + d_N$.
$\hat{x} \leftarrow \hat{x} - \Phi^T\left(\Phi\Phi^T\right)^{-1}(\Phi\hat{x} - y)$.
Step 4:Calculation error:$r = y - \Phi x$.
Step 5:If $\lVert r - r_0\rVert < \varepsilon$ or $l \ge L$, Stop iteration, otherwise, return to step 2.

3 CCSL0 algorithm

In order to further improve the convergence rate and shorten the running time of the algorithm, this paper introduces the composite inverse proportion (CIPF) function: $\omega_\sigma(x_i) = \frac{{x_i}^2}{{x_i}^2 + a\sigma^2}$. $a > 0$, is a shape parameter that adjusts the steepness of $\omega_\sigma(x_i)$. As shown in Figure 2, it can be seen that the contraction of the CIPF function is similar to that of the AHTF function. It is found that the CIPF function is not suitable to be solved by the modified Newton method, but the optimization time of the steepest descent method is better than the AHTF function due to its simple form. Therefore, this paper selects the CIPF function and AHTF function to reconstruct the image by the joint optimization of the steepest descent method and the modified Newton method.

$$\lim_{\sigma\to 0}\omega_\sigma(x_i) = \begin{cases} 0, & x_i = 0 \\ 1, & x_i \ne 0 \end{cases} \tag{10}$$

Compound inverse proportion function gradient $d_\omega = \frac{2a\sigma^2 x}{(x^2 + a\sigma^2)^2}$.

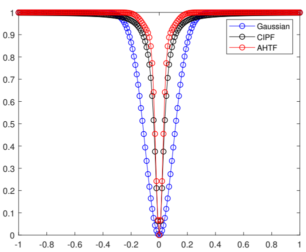


Fig. 2 Gaussian function, CIPF function and AHTF function.

In summary, the specific steps of the CCSL0 algorithm are as follows:

CCSL0 algorithm:
Step 1: Initialize :
Reconstructed vector: $x = \Phi^T\left(\Phi\Phi^T\right)^{-1}y$, Shape parameter : k, a,Smooth parameter decreasing sequence : $[\sigma_1, \sigma_2, \cdots, \sigma_j]$, $\sigma_1 = 2\max\lvert x\rvert$, Smooth parameter threshold: $\sigma_{\min}$, Number of internal iterations:L, Error threshold: ε, Initial $r_0 = 0$.
Step 2: While $\sigma_j > \sigma_{\min}$
Let $\sigma_j = \beta\sigma_{j-1}$,$\sigma = \sigma_j$.
Step 3: for $l = 1, \cdots, p, \cdots, L$.
$if \ \ l \le p$:$\hat{x} \leftarrow \hat{x} - \mu\sigma^2 d_\omega$.
$\hat{x} \leftarrow \hat{x} - \Phi^T\left(\Phi\Phi^T\right)^{-1}(\Phi\hat{x} - y)$.
else:$\hat{x} \leftarrow \hat{x} + d_N$.
$\hat{x} \leftarrow \hat{x} - \Phi^T\left(\Phi\Phi^T\right)^{-1}(\Phi\hat{x} - y)$.
Step 4:Calculation error:$r = y - \Phi x$.
Step 5:If $\lVert r - r_0\rVert < \varepsilon$ or $l \ge L$, Stop iteration, otherwise, return to step 2.

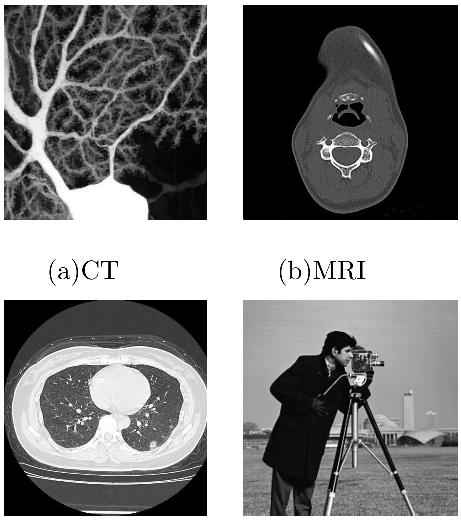
4 Numerical experiment

In order to verify the performance of the proposed algorithm, we compared the performance of the popular SL0, NSL0, and MSL0 algorithms with the proposed CSL0 algorithm and CCSL0 algorithm in the reconstruction of two-dimensional images on the simulation platform. Each algorithm selects $L = 3, \beta = 0.7$, μ takes the optimal value of each algorithm and $a = 0.2$ in CCSL0. In this experiment, the wavelet transform is used to sparsely represent the image, and the random Gaussian matrix is selected as the measurement matrix to compress the signal. The reconstruction performance of different algorithms for CT, MRI, X-ray and Cameraman images was tested when the compression ratios (m / n) were 0.3, 0.4, and 0.5. The average value of each index after multiple tests was taken. The original test images are shown in Figure 3. The peak signal to noise ratio (PSNR) and the running time (t) of the reconstruction algorithm are selected as the performance indexes, and the running time is obtained by the tic and toc functions. The peak signal-to-noise ratio of two-dimensional images is defined as:

$$\mathrm{PSNR} = 10\log_{10}\left(\frac{MAX_I^2}{MSE}\right) \tag{11}$$

$$MSE = \frac{1}{mn \cdot \sum_{i=1}^{m} \sum_{j=1}^{n} \left[x(i,j) - \hat{x}(i,j) \right]^2} \tag{12}$$

MAX_I represents the maximum value of pixels, MSE represents the mean square error, x is the original image, and $\hat{x}$ is the restored image.



(a)CT (b)MRI

(c)X-ray (d)Cameraman

Fig. 3 original test image, size (512 * 512).

Under the condition of compression ratio (m / n = 0.3,0.4,0.5), this paper compares the PSNR and time of four images reconstructed by SL0, NSL0, MSL0, CSL0, and CCSL0 algorithms in Figure 3. In Table 1 and Table 2, the two data with the best PSNR and time are selected for roughening. It can be seen that for different reconstructed images, the CSL0 and CCSL0 algorithms proposed in this paper improve the PSNR of the reconstructed image by 0-0.96 dB compared with other algorithms. The running time of the CSL0 algorithm is similar to that of the NSL0 algorithm, which is better than that of the SL0 algorithm. Based on the CSL0 algorithm, the running time of the CCSL0 algorithm after introducing the composite inverse proportional function is only behind the MSL0 algorithm and is ahead of its three algorithms.

In order to save space, the following shows only the reconstructed results when m / n = 0.4.As shown in figure 4.

Table 1 PSNR(dB) of each algorithm for four images

Test images	Algorithm	m/n=0.3	m/n=0.4	m/n=0.5
CT	SL0	21.01	24.82	27.81
	NSL0	21.55	25.19	28.01
	MSL0	21.33	25.07	27.87
	CSL0	**22.45**	**25.45**	**28.05**
	CCSL0	**22.51**	**25.49**	**28.13**
MRI	SL0	27.95	31.10	34.02
	NSL0	28.28	31.37	**34.23**
	MSL0	28.11	31.29	34.08
	CSL0	**28.67**	**31.45**	34.15
	CCSL0	**28.68**	**31.67**	**34.46**
X-ray	SL0	20.41	23.98	27.15
	NSL0	20.89	24.39	27.36
	MSL0	20.68	24.23	27.21
	CSL0	**21.66**	**24.64**	**27.43**
	CCSL0	**21.68**	**24.71**	**27.53**
Cameraman	SL0	24.67	29.10	32.91
	NSL0	25.64	29.57	33.24
	MSL0	25.17	29.35	33.01
	CSL0	**26.29**	**29.67**	**33.30**
	CCSL0	**26.33**	**29.76**	**33.44**

Table 2 time(s) of each algorithm for four images

Test images	Algorithm	m/n=0.3	m/n=0.4	m/n=0.5
CT	SL0	1.48	1.74	1.90
	NSL0	0.98	1.15	1.32
	CSL0	1.04	1.24	1.42
	MSL0	**0.54**	**0.61**	**0.73**
	CCSL0	**0.77**	**0.93**	**1.09**
MRI	SL0	1.29	1.49	1.68
	NSL0	0.86	1.01	1.18
	CSL0	0.82	0.96	1.07
	MSL0	**0.48**	**0.55**	**0.62**
	CCSL0	**0.57**	**0.66**	**0.77**
X-ray	SL0	1.59	1.85	2.13
	NSL0	1.04	1.22	1.44
	CSL0	1.04	1.27	1.48
	MSL0	**0.57**	**0.68**	**0.76**
	CCSL0	**0.74**	**0.91**	**1.10**
Cameraman	SL0	1.44	1.67	1.93
	NSL0	0.97	1.12	1.31
	CSL0	0.99	1.17	1.40
	MSL0	**0.53**	**0.60**	**0.70**
	CCSL0	**0.71**	**0.86**	**1.04**

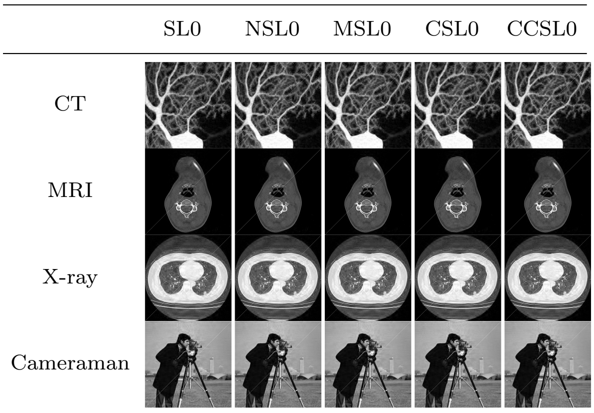


Fig. 4 Reconstruction effect diagram..

Figure 4 shows the four images reconstructed by SL0, NSL0, MSL0, CSL0, and CCSL0 algorithms. It can be seen that the images reconstructed by CSL0 and CCSL0 algorithms are clearer, the images contain fewer impurities and the whole image is smoother.

5 Conclusion

Aiming at the problem of low recovery accuracy and long reconstruction time of existing reconstruction algorithms in medical imaging, based on compressed sensing theory, this paper first proposes an approximate hyperbolic tangent function and proposes the CSL0 algorithm by combining the steepest descent method and the modified Newton method. Secondly, the CCSL0 algorithm is proposed by introducing a composite inverse proportion function and approximate hyperbolic tangent function in the way of joint optimization. The simulation results show that CSL0 and CCSL0 not only improve the PSNR value of the reconstructed image but also reduce the running time of the algorithm to some extent. In future research, we will improve the reconstruction algorithm under the condition of noise interference and apply it to a wider range of fields.

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