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Method Article

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Probing the rainfall time series over northeast India through composite and binary fuzzy relation

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Abstract

The current work reports a binary fuzzy relation (BFR) based approach to have an insight into the behavior of rainfall over northeast India. A fuzzy set is a generalization of classical sets, where the characteristic function is generalized to membership function to get hold of the scenarios that cannot be explained within the confinement of dichotomy. A fuzzy relation deals with the degree of belongingness of an ordered pair to some given relation and the most convenient way of representing it through a membership matrix. In this work we have considered Pre monsoon (X_1), Summer monsoon (X_2), Post monsoon (X_3) and Annual (X_4)rainfall for the period 1871- 2016 over North-East India. We have considered BFR for the pairs (X_1,X_2), (X_2,X_3), (X_3,X_4), (X_2,X_4). It may be noted that in this work we have considered the conditional probabilities of the bivariate frequency distributions as the membership grades for the corresponding BFR. In the next phase we have generated composition of 2 BFR's through fuzzy max-min relationships. In this way we have compared the membership grades of the BFRs and the composite BFR's to understand the roles played by summer monsoon rainfall and post monsoon rainfall in the annual rainfall and the study has concluded through the fuzzy cardinalities associated with the composite BFRs.

Keywords: BFR; Composite BFR; Fuzzy cardinality; rainfall; northeast India

1.Introduction

In recent years Fuzzy Set Theory and fuzzy logic have emerged as a well-known soft computing technique that is more flexible than the classical set theory and propositional logic. It can be considered as a logical system that resembles the pattern of human reasoning and interpretation. Fuzzy set theory and fuzzy logic was introduced by LA Zadeh (Zadeh 1965) through his research work on "Fuzzy sets". Mamdani and Assilion (1974) practically applied the concept introduced by Zadeh to an automatic steam engine. In the last couple of decades Fuzzy logic has gained considerable attention in the field of hydrology. Some areas of fuzzy logic that have been explored include fuzzy-based regression Bogardi et al. (2004), hydrologic forecasting Kojiri (1988), hydrologic modelling Bogardi et al. (2004) and water resources risk assessment Feng and Luo (2011). Very recently an exhaustive review by Kambalimath and Deka (2020) has demonstrated the various applications of fuzzy logic in hydrology. In a very recent work Janarthanan et al. (2021) demonstrated a predictive model on rainfall using fuzzy logic. In the current work we have attempted Binary Fuzzy Relation (BFR) towards understanding of the pattern of rainfall over Northeast India. A relation is a fundamental concept in mathematics and represents the connection of elements in a given set. In various fields of artificial intelligence such as clustering analysis, classification and decision, relations have immense application. A fuzzy relation is a generalization of crisp relation where the membership grades represent the degree of belongingness of an ordered pair to the fuzzy relation under consideration. A binary fuzzy relation can be applied on two domains $U \times V$ or on a single domain by considering $U \times U$. A detailed account of fuzzy relations has been presented in Wang et al. (2019). In a very recent work Chattopadhyay et al. (2021) reported a study where BFR was implemented towards the study of Air quality index. Theoretically, we can consider fuzzy set as a pair (U, μ),where U is a crisp set called the domain or Universe of discourse and μ is a membership function defined as $\mu: U \longrightarrow [0,1]$. For a given domain different fuzzy set can be constructed by varying the form of μ. For every element of the domain the membership function generates a degree of membership of that element to the corresponding fuzzy set. All the operations of classical set theory have their generalization in fuzzy set theory Pękala (2018), Maués et al. (2020), Jasiulewicz-Kaczmarek et al. (2021), Zheng et al. (2021). Bukhari et al. (2020) reported a study on fuzzy logic supplemented by artificial neural network to study the prediction of summer precipitation over different meteorological stations in Pakistan. Fung et al. (2019) developed drought prediction using weighted wavelet–fuzzy–support vector regression (weighted W–F–SVR) model and root-mean-square-error (RMSE). Cai et al. (2020) employed fuzzy interpretive structural modeling (FISM) to describe the interrelationship between the evaluation factors with linguistic preferences. Giardina et al. (2019) approached new index called the fuzzy environmental analogy index (FEAI) in Italy to investigate similarities between neighbouring areas in terms of environmental pressures. Abdul-Wahab et al. (2019) used hierarchical fuzzy logic control system to predict optimum sampling rates of air quality monitoring stations. Adedeji et al. (2019) developed a midterm forecast using the Adaptive-Neuro-Fuzzy Inference System (ANFIS) for electricity consumption. Shiri (2018) applied a neuro-fuzzy technique in low altitude location to estimate daily pan evaporation values. Coceal et al. (2018) developed a method called fuzzy-logic algorithm to detect the sea breeze around London.Azad et al. (2019)applied different fuzzy models like genetic algorithm, root mean square error, mean absolute error in meteorological stations of Iran to predict air temperature.Bischokov et al. (2019) usedfuzzy logic to minimize the risk in the production of agricultural product. Kaiju et al. (2018) developed T-S fuzzy neural network model to predict short-term photovoltaic power.Topaloğlu et al. (2018) examined the effects of membership function and determined the optimal membership function for wind power plant installation.Tzimopoulos et al. (2018) established fuzzy linear regression to obtain the relationship between rainfall and altitude in different meteorological stations. In the region of North-West Malaysia Safar et al. (2019) approachedfuzzy logic in rainfall prediction.Sardooi et al. (2018) compared the neuro fuzzy models with best fitting time series models ARIMA (3,0,4) and ARIMA (2,0,1) for modelling of drought. Chen et al. (2021) proposed fault diagnosis method considering meteorological factors based on P system.

In view of the literature surveyed above we hereby developed a binary fuzzy relation(BFR) based model to demonstrate in the intrinsic patterns and inter dependence through time series. In this connection we have dealt with BFR, Composite BFR, domain and range of BFR and fuzzy cardinality. Rest of the paper is organised as follows: In Section 2 we have demonstrated the methodology involving BFR, domain and range and fuzzy cardinality. In Section 3 we have presented the results in details and also presented the outcomes. We have concluded in Section 4.

2. Methodology

2.1 Data

In this section we briefly describe the data explored in this study. The rainfall data over North-East India for the period 1871-2016 are explored and the data are obtained from the website of the Indian InstituteofTropicalMeteorology(IITM),Pune,India.Thewebsiteis https://www.tropmet.res.in/DataArchival-51-Page.The IITM, an Autonomous Institute under the Ministry of Earth Sciences, Government of India, has archive the data and associated details are available in Sontakke et al. (2008). We would like to further mention that the present study deals with the homogenised rainfall data for seasonal and annual scale and the data are archived in the 10th of millimetre (mm).Pal et al. (2020) has worked on the descriptive and inferential part of the study and Sharma and Chattopadhyay(2021) has further worked on the long-range correlational pattern. The homogenisation procedure was developed by Parthasarathy et al. (1993) and the methodology is also documented by IITM through a copyrighted material that is available at the IITM website.

2.2 Binary Fuzzy Relation

Crisp relation deals with the presence or absence of interaction or interconnectedness between the elements of more than one sets. We can generalize this concept to allow for various degrees or strengths of relation between elements. In fuzzy relation degree of association is represented by membership grades. Fuzzy Set Theory is an extension and generalization of basic concept of crisp set. Let us consider two crisp sets X and Y said to be Cartesian product, is the ordered pair such that first element belongs to the set X and second element belongs to the set Y.

$$X \times Y = \{(x, y) |\ x \in X \ and \ y \in Y\} \tag{1}$$

The generalization of the Cartesian product for a family of crisp sets $\{X_i | i \in \mathrm{N}_n\}$is denoted by

$$X_1 \times X_2 \times \ldots .. \times X_n. \tag{2}$$

Elements of the cartesian product of n crisp sets such that $x_i \in X_i \ \forall i \in \mathrm{N}_n$ is denoted by

$$X_{i \in \mathrm{N}_n} X_i = \{(x_1, x_2, \ldots ., x_n) |\ x_i \in X_i \ \forall \ i \in \mathrm{N}_n\} \tag{3}$$

A relation among crisp sets $X_1, X_2, \ldots .., X_n$ which is a subset of cartesian product$X_{i \in \mathrm{N}_n} X_i$, denoted by R$(X_1, X_2, \ldots .., X_n)$. (4)

Therefore, we can write R$(X_1, X_2, \ldots .., X_n) \subset X_1 \times X_2 \times \ldots .. \times X_n$ .So the cartesian product $X_1 \times X_2 \times \ldots .. \times X_n$ represents the universal set.

Crisp relation R defined by characteristic function is assigning value of 1 which belongs to the set and 0 which does not belong. Therefore,

$$\mu_R(x_1, x_2, \ldots ., x_n) = \begin{cases} 1 & if \ and \ only \ if \ (x_1, x_2, \ldots ., x_n) \in R\ , \\ 0 & otherwise \end{cases} \tag{5}$$

2.3 Domain and Range

The domain of a fuzzy binary relation ρ on the universe A $\times B$ is a fuzzy set defined on A and is denoted by dom(ρ) and characterised by the membership function

$$\mu_{dom(\rho)}: \mathrm{A} \rightarrow [0,1] \text{ s.t. } \forall \ (x, y) \in A \times B\ , \tag{6}$$

membership grade of x is given by$\mu_{dom(\rho)}$ (x) = $\max_{y \epsilon B} \mu_\rho$(x, y) (7)

The range of a fuzzy binary relation ρ on the universe $A \times B$ is a fuzzy set defined on B and is denoted by ran (ρ) and is characterised by the membership function

$\mu_{ran(\rho)}$: B→ [0,1] s.t. $\forall\ (x, y) \in A \times B$, (8)

membership grade of y is given by $\mu_{ran(\rho)}$ (y) = $\max_{x \epsilon A} \mu_\rho$(x, y) (9)

2.4 Fuzzy cardinality

A convex and normalized fuzzy set is said to be fuzzy number if its membership function is piecewise continuous. Therefore, fuzzy number contains the real numbers within some intervals to varying degrees.

On a finite universal set X of a fuzzy set A, the Scalar cardinality is defined as the summation of the membership grades of all the elements of X in A. Thus,

$|A| = \sum_{x \in X} \mu_A(x)$ (10)

Fuzzy Cardinality is defined as a fuzzy number rather than a real number. A fuzzy Cardinality |Å| is a fuzzy set, when fuzzy set A is a finite support whose membership function is defined as

$\mu_{|Å|}(|A_\alpha|) = \alpha$ (11)

3. Results and Discussion

In the present work we are considering the random variables X_1, X_2, X_3 and X_4that stands for pre monsoon, summer monsoon, post monsoon, and annual rainfall over North East India during the period 1871- 2016. The overview of BFR has already been presented in section 2. At the first phase of our study, we applied BFR to understand the influence of one random variable on the other. For example, we consider a composition of membership matrices for X_1-X_2 and X_2 - X_3 to derive the membership matrix for X_1-X_3. Also, we derive the membership matrix for X_1- X_3 and compare the two matrices to discern how X_2 has played its role as a connector between X_1andX_3. This approach has been adopted for the other pairs of random variables as well. The outcomes are presented below. Prior to that we derived the fuzzy cardinalities for the fuzzy relations represented by the membership matrices. In this connection we would like to mention that the membership matrices have been generated by constructing bivariate frequency distribution and considering the relative frequencies as the membership grades of the ordered pairs in the BFR. The fuzzy cardinalities come out to be the following:

Case I: Here we present the fuzzy cardinality for the case of the fuzzy relation generated due to the pair of random variables (X_1, X_2) that implies the association between pre-monsoon and summer monsoon:

$$\frac{0.06}{17}+\frac{0.21}{16}+\frac{0.22}{15}+\frac{0.28}{14}+\frac{0.33}{13}+\frac{0.34}{12}+\frac{0.42}{11}+\frac{0.48}{10}+\frac{0.50}{9}+\frac{0.51}{8}+\frac{0.57}{7}+\frac{0.65}{6}+\frac{0.66}{5}+\frac{0.77}{4}+\frac{0.78}{3}+\frac{0.93}{2}+\frac{1}{1} \tag{12}$$

Case II: Here we present the fuzzy cardinality for the case of the fuzzy relation generated due to the pair of random variables (X_2, X_3) that implies the association between summer monsoon and post-monsoon:

$$\frac{0.06}{15}+\frac{0.13}{14}+\frac{0.16}{13}+\frac{0.22}{12}+\frac{0.23}{11}+\frac{0.28}{10}+\frac{0.45}{9}+\frac{0.50}{8}+\frac{0.54}{7}+\frac{0.71}{6}+\frac{0.76}{5}+\frac{0.77}{4}+\frac{0.79}{3}+\frac{0.83}{2}+\frac{1}{1} \tag{13}$$

Case III: Here we present the fuzzy cardinality for the case of the fuzzy relation generated due to the pair of random variables (X_3, X_4) that implies the association between post-monsoon and annual:

$$\frac{0.05}{11}+\frac{0.09}{10}+\frac{0.16}{9}+\frac{0.26}{8}+\frac{0.33}{7}+\frac{0.50}{6}+\frac{0.66}{5}+\frac{0.67}{4}+\frac{0.73}{3}+\frac{0.83}{2}+\frac{1}{1} \tag{14}$$

Case IV: Here we present the fuzzy cardinality for the case of the fuzzy relation generated due to the pair of random variables (X_2, X_4) that implies the association between summer monsoon and annual:

$$\frac{0.08}{12}+\frac{0.13}{11}+\frac{0.16}{10}+\frac{0.33}{9}+\frac{0.4}{8}+\frac{0.5}{7}+\frac{0.6}{6}+\frac{0.83}{5}+\frac{0.86}{4}+\frac{0.91}{3}+\frac{0.92}{2}+\frac{1}{1} \tag{15}$$

Case V: Here we present the fuzzy cardinality for the case of the binary fuzzy relation generated due to the pair of random variables (X_1, X_3) that implies the association between pre-monsoon and post-monsoon:

$$\frac{0.19}{15}+\frac{0.25}{14}+\frac{0.28}{13}+\frac{0.4}{12}+\frac{0.41}{11}+\frac{0.42}{10}+\frac{0.43}{9}+\frac{0.56}{8}+\frac{0.57}{7}+\frac{0.58}{6}+\frac{0.59}{5}+\frac{0.71}{4}+\frac{0.75}{3}+\frac{0.8}{2}+\frac{1}{1} \tag{16}$$

Case VI: Here we present the fuzzy cardinality for the case of the composite binary fuzzy relation generated due to the pair of random variables (X_1, X_3) that implies the association between pre-monsoon and post-monsoon(composite):

$$\frac{0.06}{17}+\frac{0.13}{16}+\frac{0.21}{15}+\frac{0.22}{14}+\frac{0.23}{13}+\frac{0.28}{12}+\frac{0.33}{11}+\frac{0.34}{10}+\frac{0.42}{9}+\frac{0.48}{8}+\frac{0.5}{7}+\frac{0.51}{6}+\frac{0.57}{5}+\frac{0.65}{4}+\frac{0.77}{3}+\frac{0.78}{2}+\frac{1}{1} \tag{17}$$

Case VII: Here we present the fuzzy cardinality for the case of the binary fuzzy relation generated due to the pair of random variables (X_1, X_4) through X_2 that implies the association between pre-monsoon and annual through summer monsoon:

$$\frac{0.08}{15}+\frac{0.26}{14}+\frac{0.29}{13}+\frac{0.31}{12}+\frac{0.33}{11}+\frac{0.4}{10}+\frac{0.44}{9}+\frac{0.55}{8}+\frac{0.6}{7}+\frac{0.66}{6}+\frac{0.68}{5}+\frac{0.7}{4}+\frac{0.73}{3}+\frac{0.91}{2}+\frac{1}{1} \tag{18}$$

Case VIII: Here we present the fuzzy cardinality for the case of the composite binary fuzzy relation generated due to the pair of random variables (X_1, X_4) through X_2 that implies the association between pre-monsoon and annual throughsummer monsoon(composite):

$$\frac{0.06}{21}+\frac{0.08}{20}+\frac{0.13}{19}+\frac{0.16}{18}+\frac{0.21}{17}+\frac{0.28}{16}+\frac{0.33}{15}+\frac{0.34}{14}+\frac{0.4}{13}+\frac{0.42}{12}+\frac{0.48}{11}+\frac{0.5}{10}+\frac{0.51}{9}+\frac{0.57}{8}+\frac{0.6}{7}+\frac{0.65}{6}+\frac{0.66}{5}+\frac{0.77}{4}+\frac{0.78}{3}+\frac{0.93}{2}+\frac{1}{1} \tag{19}$$

Case IX: Here we present the fuzzy cardinality for the case of the binary fuzzy relation generated due to the pair of random variables (X_1, X_4) through X_3 that implies the association between pre-monsoon and annual through post-monsoon:

$$\frac{0.08}{15}+\frac{0.26}{14}+\frac{0.29}{13}+\frac{0.31}{12}+\frac{0.33}{11}+\frac{0.4}{10}+\frac{0.44}{9}+\frac{0.55}{8}+\frac{0.6}{7}+\frac{0.66}{6}+\frac{0.68}{5}+\frac{0.7}{4}+\frac{0.73}{3}+\frac{0.91}{2}+\frac{1}{1} \tag{20}$$

Case X: Here we present the fuzzy cardinality for the case of the composite binary fuzzy relation generated due to the pair of random variables (X_1, X_4) through X_3 that implies the association between pre-monsoon and annual through post-monsoon(composite):

$$\frac{0.09}{17}+\frac{0.16}{16}+\frac{0.25}{15}+\frac{0.26}{14}+\frac{0.33}{13}+\frac{0.4}{12}+\frac{0.41}{11}+\frac{0.42}{10}+\frac{0.43}{9}+\frac{0.5}{8}+\frac{0.56}{7}+\frac{0.59}{6}+\frac{0.66}{5}+\frac{0.67}{4}+\frac{0.73}{3}+\frac{0.75}{2}+\frac{1}{1} \tag{21}$$

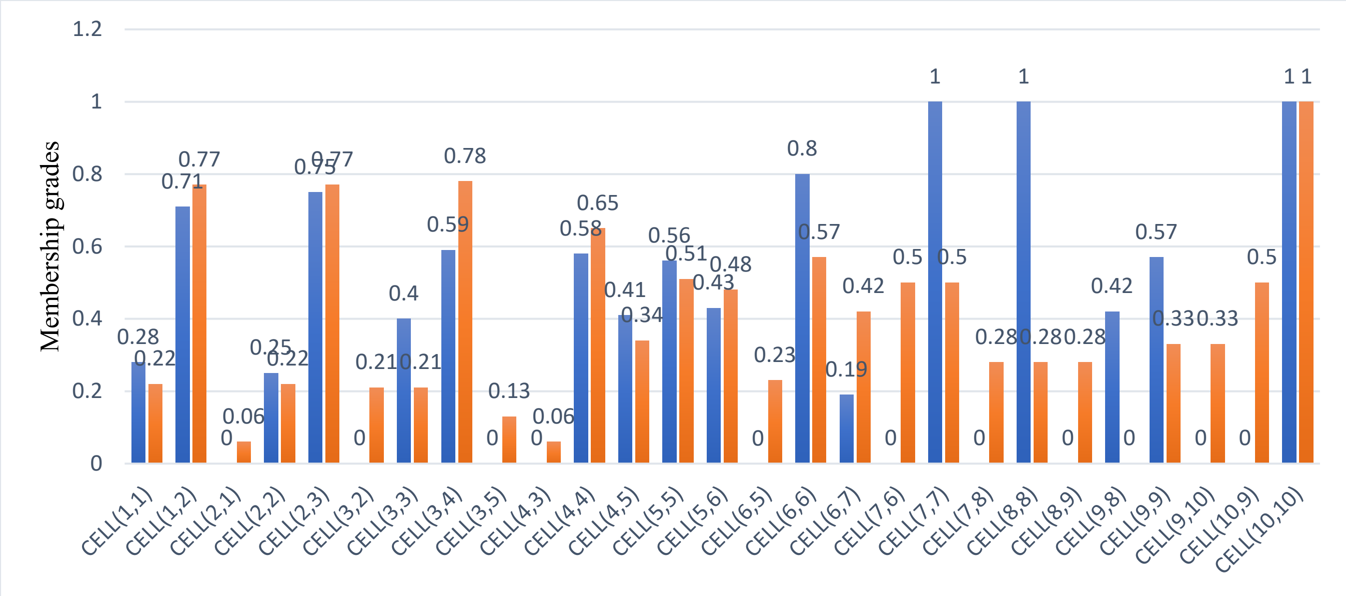

Fig. 1 Membership grades for BFR(Blue) and composite BFR(Orange)for pre monsoon(X_1)and post monsoon (X_3) rainfall.

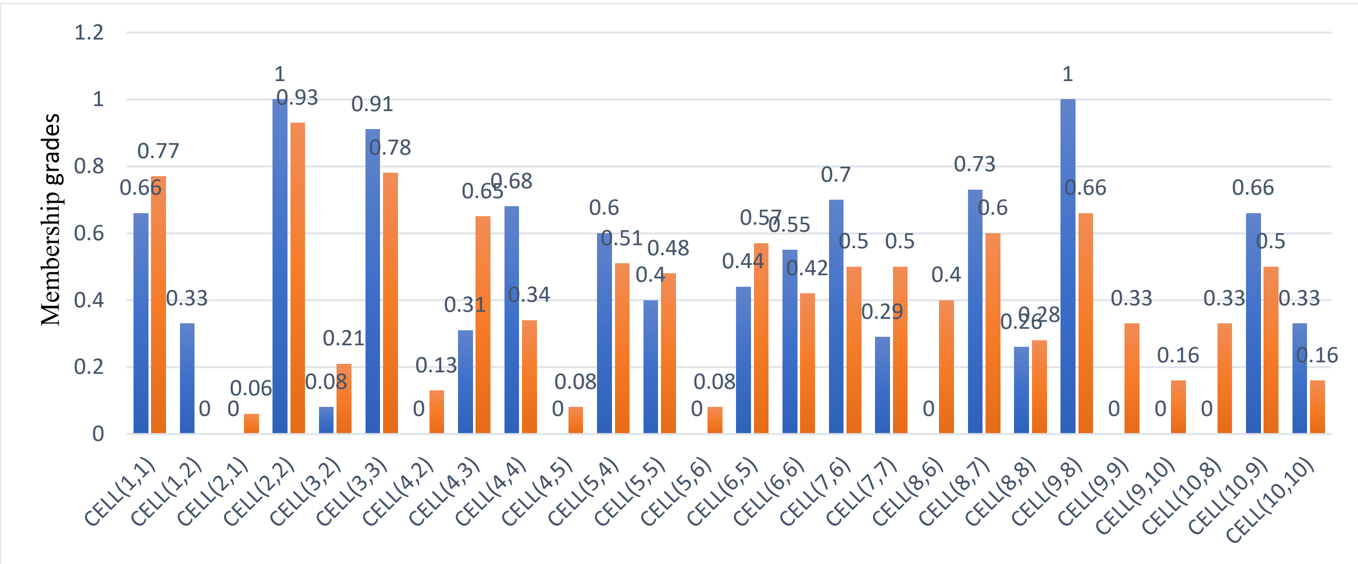

Fig. 2 Membership grades for BFR and composite BFR for pre monsoon(X_1) and annual(X_4) through summer monsoon(X_2) rainfall.

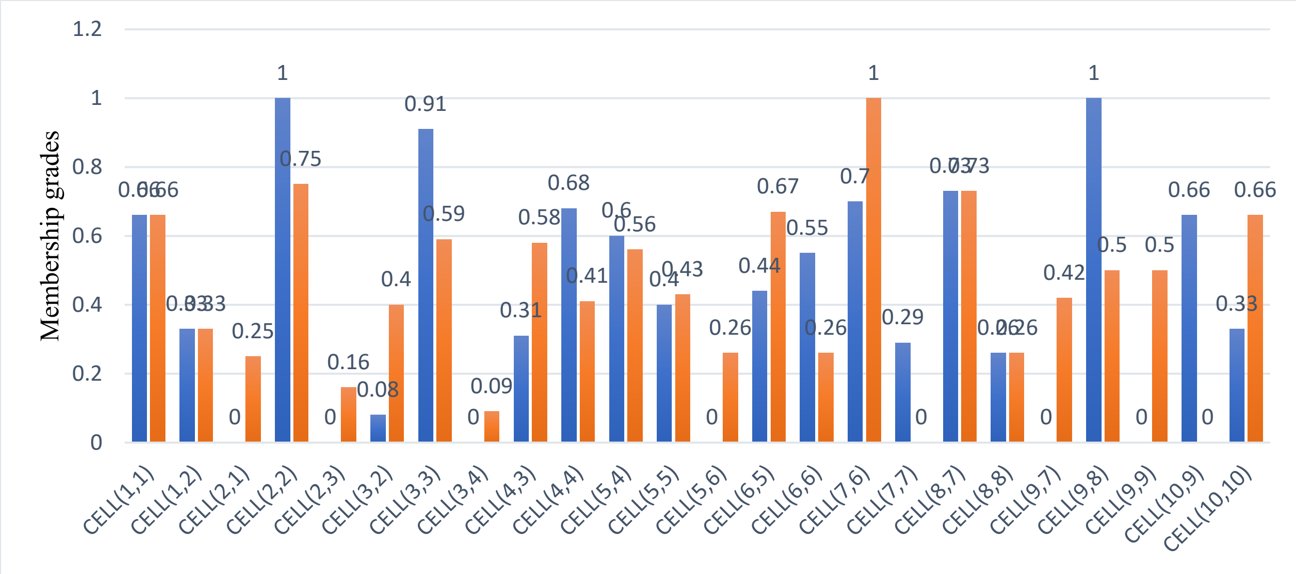

Fig 3. Membership grades for BFR and composite BFR for pre monsoon(X_1) and annual(X_4) through summer monsoon(X_3) rainfall.

At this juncture we demonstrate the impact of taking composition of 2 BFRs in the study of the ISMR in different scales. In cases VI, VIII, Xwe have derived the fuzzy cardinalities by making compositions of more than one BFRs. we have observed that the membership grades of composite BFRs have dominated membership grades of single BFRs. Moreover, for the composite BFRs the integers have higher membership grades than the single BFRs and this indicates the potential of composite BFR in assigning membership grades to the bivariate frequency distributions of ISMR differentiated through scales. We also observed through composite BFR that inclusion of one seasonal scale is of significant interest to study the inter relationship between two scales of ISMR. We also understand that the contribution of pre monsoon rainfall to the annual rainfall studied through summer monsoon in a composite manner leads to many cases with significant membership grades to 0.4, 0.42,0.48,0.5,0.51(case VIII). However, when composited through post monsoon the most significant membership grades 0.4, 0.41, 0.42, 0.43 (case X). Combining these two results it is understandable that over North-East India pre monsoon rainfall has moderate influence to the annual rainfall. However, when composited with summer monsoon it influences the annual rainfall in a more significant way than when it is composited with post monsoon rainfall. To further consolidate the outcomes, we have presented the non-zero membership grades in Figs. 1,2 and 3. We have observed that in many cases the composite BFRs have higher membership grades and when mingled with fuzzy cardinality the dominance of summer monsoon is apparent through composite BFR.These graphs shows that the membership grades for the composite BFR (Orange) are dominating the BFR (Blue) for the cases where the pre monsoon rainfall is of moderate amount. This implies that considering composite BFR is showing the impact of pre monsoon on the post monsoon through the membership grades.For higher rainfall amounts BFR and composite BFR are having more or less comparable membership grades.

4. Conclusion

In the work reported in the previous sections we have carried out Binary Fuzzy Relation (BFR)based approach to the rainfall data over North-East India. The data are collected from the website of IITM,Pune and Pre monsoon (March-May), Summer monsoon (June-August), Post monsoon (September-November) as well as annual data are taken into account. The rainfall amounts corresponding to the above four-time spans are considered to be 4 random variables X_1,X_2,$X_3$and X_4. All the random variables having same number of realizations in the study. At the first phase of this work, we have divided the range of all the four random variables into 10 classes and the corresponding frequencies are computed. Subsequently, the random variables are considered in pairs (X_1,X_2), (X_2,X_3), (X_3,X_4) and (X_2,X_4). From the bivariate frequency distribution, we have computed the probabilities corresponding to each cell of the bivariate frequency table. The probabilities are now regarded as membership grades corresponding to the BFR under consideration. This implies that the chance of occurrence is now being considered as degree of belongingness of the ordered pairs in the relations determined by the BFRs. In this way we have constructed membership matrices $M_{\rho_{12}}$, $M_{\rho_{23}}$, $M_{\rho_{34}}$, $M_{\rho_{24}}$. The membership matrix for ρ_{12} and ρ_{23} correspond to the BFR between pre monsoon and summer monsoon and summer monsoon and post monsoon respectively. Subsequently, we have used the composition of this 2 BFR that is $M_{(\rho_{12} o\, \rho_{23})}$ to obtain the membership matrix for $M_{\rho_{13}}$. By composition of 2 BFR we have obtained the BFR between pre monsoon and post monsoon through summer monsoon. Hence the entries to this membership matrix indicate the connectivity between pre monsoon and post monsoon taking the summer monsoon into account. On the other hand, we have obtained a direct BFR between pre monsoon and post monsoon without taking summer monsoon into account. Furthermore, we have carried out construction of BFR membership matrix for the relations ρ_{14} . While doing the same we have followed two paths. In the first case, we have considered ρ_{14} as a composite of two relations ρ_{12} and ρ_{24}. And in the second case we have considered ρ_{14} as a composite of ρ_{13} and ρ_{34}. The basic idea behind this construction is to understand the influence of summer monsoon and post monsoon in the annual rainfall over North-East India.Outcomes have been pictorially presented in Figs. 1,2 and 3, where we have presented the non-zero membership grades. We have observed that in many cases the composite BFRs have higher membership grades and when mingled with fuzzy cardinality the dominance of summer monsoon is apparent through composite BFR. As future study we propose fuzzy measure theory to understand the relative contribution of the three seasons to the annual rainfall through Belief measure and we apprehend that it would put further lights into the intrinsic complexity of the system.

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Conflict of interest

The authors hereby declare that they have no conflict of interest associated with this work.

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