Modeling and prediction of gear flank twist for generating gear grinding

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Modeling and prediction of gear flank twist for generating gear grinding

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ABSTRACT: Gear flank twist has a significant effect on the load-carrying capacity. However, most prediction models of gear flank twist can't be applicable for gears with profile modification. A new method for predicting gear flank twist for the generating grinding process is developed in this paper, taking into consideration the effect of lead modifications, profile modifications and position errors of CNC machine tools on gear flank twist. And the influences of four gear parameters on gear flank twist were studied using orthogonal experiments, which proves that the change of module has the greatest effect on gear flank twist. And the maximal profile modification value has minimal effect on gear flank twist. The experimental results proved that the model can accurately predict the gear flank twist. Besides, this paper described an gear parameters optimization study to reduce gear flank twist based on nine well-planned orthogonal experiments.

Key words: Gear flank twist; Gear generating grinding; Mathematical model; Gear modification

1.Introduction

Gears are widely used in ships, automobiles and other fields. Gear generating grinding is important finishing manufacturing processes for gears [1]. In order to reduce gear noise and mesh impact, it is necessary to modify gear profile and gear lead slope.

Gear modification includes profile modification and lead modification [2]. Gear profile modification can be realized by dressing the grinding wheel into a specific shape [3]. Lead modification can be realized by changing the center distance between the gear and the grinding wheel during axial feeding [3]. The gear flank twist occurs in this processing. Gear flank twist has a negative effect on gear, reduce transmission accuracy, increase mesh impact, and thus reduce gear life. Thus, it's important to eliminate gear flank twist.

Various methods have been proposed to reduce the flank twist and can be divided into three groups. Some researches proposed gear processing parameters modification method, some researchers proposed machining tool reshaping methods, others adopted both methods above. Tian X [4] proposed a method by establishing coordinate transformation matrix between gear blank and tool coordinate system, obtaining the relationship between coordinates of gear flank and master-slave axis movement amount, then reducing gear flank twist by compensating for master-slave axis movement amount. Tran V T [5] proposed an additional rotation angle for the gear blank during its manufacturing process. A nonlinear function was proposed and supplemented to this additional rotation angle of work gear. The twist of the helical tooth flank is reduced greatly. Shih Y P [6] proposes a tooth flank modification method in the five-axis gear profile grinding machine. Each axis of the grinding machine is formulated as a polynomial, and the tooth flank can be approximated to the theoretical tooth flank by adjusting the coefficients of the polynomials based on their sensitivity. Fong Z H [7] proposed a tooth flank crowning method for helical gears, which uses a diagonal feed on a grinding machine with a variable lead grinding worm, and this method can reduce gear flank twist. Hsu R H [8] proposed a methodology to reduce the tooth flank twist by applying a modified variable tooth thickness tool and having a diagonal feed without varying the center distance. Jiang W J [9] proposed a new mathematical model for the grinding wheel of ZC1 worm, which can reduce the tooth errors of ZC1 worm.

In order to improve the measuring efficiency of machined gear, it’s necessary to establish model of gear surface topology. So far various gears surface topology models have been proposed. Vedmar L [10] proposed a method to calculate the roughness of gear surface by establishing the model of the tool in arbitrary directions, obtaining the geometry of the three-dimensional surface of the manufactured gear, and comparing this surface with the surface of an ideal gear. Chen [11] proposed a new method for predicting gear surface roughness with the generating grinding process by an algorithm for geometrical analysis of the grooves on the gear surface left by idea conic grains. Zhou WH [12] proposed a new approach to modeling the tooth surface topology in continuous generating grinding by replicating the surface topography of the grinding worm, obtaining the tooth surface topography by the Boolean operation, and obtaining the gear surface topology by comparing this surface with the surface of an ideal gear. Yan and Yu [13-14] proposed a gear meshing line calculation method in a simple mathematical form, and the optimization method is proposed to reduce gear flank twist based on the model.

The research on modeling and predicting the gear flank twist considering the lead modification, profile modifications and position errors of CNC machine tools has not been touched yet. The modeling and prediction of the gear flank twist for generating gear grinding is investigated in this paper. First, the calculation method of gear grinding mesh track was introduced. Then the influence of position error of CNC machine tools on flank twist was discussed. Next the influences of four gear parameters on gear flank twist were discussed, and optimization method is proposed to reduce gear flank twist Finally, the simulation results are given and verified in part of characteristic in comparison with experimental data.

2 Gear flank twist prediction model via homogeneous transformation

2.1 Homogeneous transformation matrix between coordinate systems

Fig.1 shows the spatial meshing coordinate system of gear and worm grinding wheel. Four spatial coordinate systems S_1 (x_1,y_1,z_1), S_2 (x_2,y_2,z_2), S(x,y,z) and S_p (x_p,y_p,z_p) are created to describe the generation mechanism of helical gear with longitudinal and profile correction. S_1 and S_2 are rigidly connected to worm grinding wheel and gear. The coordinate system S and S_p are fixed in space.

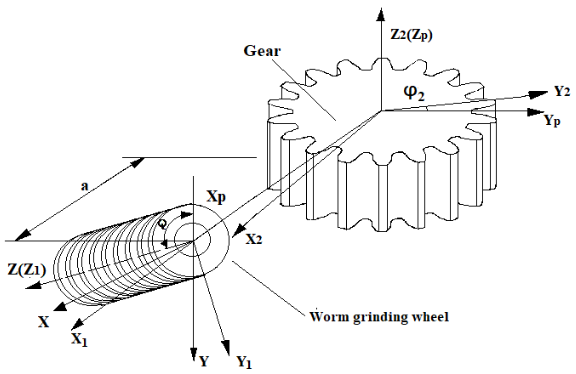


Fig.1 Homogeneous transformation between coordinate systems

The coordinates of an arbitrary point P in coordinate systems S_n, S_p, S_1 and S_2 are (x_n, y_n, z_n), (x_p, y_p, z_p), (x_1, y_1, z_1), and (x_2, y_2, z_2), respectively. When the coordinate system S_1 is translated to coordinate system S_2, the relationship between (x_1, y_1,z_1) and (x_2, y_2, z_2) of point P can be represented as follows.

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix} = M_{21} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} \tag{1}$$

$$M_{21} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \tag{2}$$

a_{11}=cos$\varphi_1$cosφ_2+sin$\varphi_1$sin$\varphi_2$cos$Q$; a_{12}=-sin$\varphi_1$cosφ_2+cos$\varphi_1$sin$\varphi_2$cos$Q$; a_{13}=-sin$\varphi_2$sin$Q$; a_{14}=a cosφ_2; a_{21}=-cos$\varphi_1$sinφ_2+sin$\varphi_1$cos$\varphi_2$sin$Q$; a_{22}=sin$\varphi_1$sinφ_2+cos$\varphi_1$cos$\varphi_2$cos$Q$; a_{23}=-cos$\varphi_2$sin$Q$; a_{24}=-a sinφ_2; a_{31}=sin$\varphi_1$sin$Q$; a_{32}=cos$\varphi_1$sin$Q$; a_{33}=cos$Q$; a_{34}=0; a_{41}=0; a_{42}=0; a_{43}=0; a_{44}=1; a is the center distance between the wheel and work gear centers. Q is the setting angle of the wheel. φ_1 is the worm grinding wheel's rotation angle. φ_2 is the work gear's rotation angle.

2.2 Surface equation of gear

The transverse profile of the involute helical gear in coordinate system S_2 can be expressed as follows:

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} r_b cosu + r_b usinu \\ r_b sinu - r_b ucosu \end{bmatrix} \tag{3}$$

where r_b is the radius of base circle, u is the sum of spread angle.

The surface equation of the involute helical gear in coordinate system S_2 can be expressed as follows:

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix} = \begin{bmatrix} r_b \cos(u+\theta) + r_b usin(u+\theta) \\ r_b sin(u+\theta) - r_b ucos(u+\theta) \\ p\theta \\ 1 \end{bmatrix} \tag{4}$$

where u is the generating angle, Θ is the rotation angle of gear transverse profile rotating around gear axis, and $p=r_b/\tan\beta_b$ is spiral parameter. Fig.2(a) shows the transverse profile of the involute helical gear. Fig.2(b) shows the flank surface of the involute helical gear.

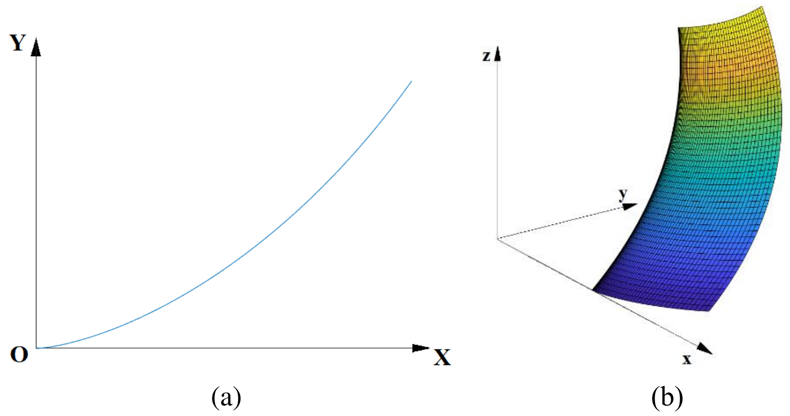


Fig.2 Tooth surface topology

The surface equation of the involute helical gear in coordinate system S_p can be expressed as follows:

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} r_b \cos(u+\theta+\varphi_2) + r_b usin(u+\theta+\varphi_2) \\ r_b sin(u+\theta+\varphi_2) - r_b ucos(u+\theta+\varphi_2) \\ p\theta \\ 1 \end{bmatrix} \tag{5}$$

Assuming profile modification curve equation is as follows

$$H(u) = \lambda(\frac{r_b}{2}u^2 - \frac{l_2}{2})^2 + h_2 \tag{6}$$

where l_2 is the involute rolling length, h_2 is the maximal modification value of the gear tooth profile. $\lambda = \frac{-4h_2}{l_2^2}$.

Because modified curve whose normal distance to involute is H, gear profile modified flank equation in coordinate system S_p can be obtained as follows:

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} r_b \cos(u+\theta+\varphi_2) + (r_b u + H)sin(u+\theta+\varphi_2) \\ r_b sin(u+\theta+\varphi_2) - (r_b u + H)cos(u+\theta+\varphi_2) \\ p\theta \\ 1 \end{bmatrix} \tag{7}$$

The normal vector of crowning lead modified flank can be obtained as follows:

$$\begin{bmatrix} n_{\mathrm{xp}} \\ n_{\mathrm{yp}} \\ n_{\mathrm{zp}} \\ 1 \end{bmatrix} = \begin{bmatrix} \mathrm{p}[r_{\mathrm{b}}u + H(u)]\sin(u + \Theta + \varphi_1) - \mathrm{p}H'(u)\cos(u + \Theta + \varphi_1) \\ -\mathrm{p}[r_{\mathrm{b}}u + H(u)]\cos(u + \Theta + \varphi_1) - \mathrm{p}H'(u)\sin(u + \Theta + \varphi_1) \\ [r_{\mathrm{b}}u + H(u)][r_{\mathrm{b}} + H'(u)] \\ 1 \end{bmatrix} \tag{8}$$

2.3 Mesh track on the gear flank

According to the theory of gearing [15], the necessary conditions of envelope existence may be given by equations as follows:

$$\mathbf{r_2}=\mathbf{r_1}+\mathbf{a}\mathrm{i} \tag{9}$$

$$\mathbf{n_1}+\mathbf{n_2}=0 \tag{10}$$

$$\mathbf{n} \cdot \mathbf{V^{(12)}} = 0 \tag{11}$$

where $\boldsymbol{r_1}$ is the coordinate of worm grinding wheel flank in coordinate system S_{p}, $\boldsymbol{r_2}$ is the coordinate of gear flank in coordinate system S_{p}, $\boldsymbol{n_1}$ is the normal vector of worm grinding wheel flank in coordinate system S_{p}, $\boldsymbol{n_2}$ is the normal vector of gear flank in coordinate system S_{p}. $\boldsymbol{n}$ is the normal vector of the contact point. $\boldsymbol{V^{(12)}}$ is the velocity vector at the contact point.

Eq.(7), Eq.(8), Eq.(9), Eq.(10) and (11) yield Eq.(12).

$$-n_{xp}\left[(1 - i_{21}cosQ)y_p + i_{21}z_p\sin Q\right] + n_{yp}\left[(1 - i_{21}\cos Q)x_p - ai_{21}\cos Q\right]$$

$$+n_{zp}i_{21}\left(x_p + a\right)\sin Q = 0 \tag{12}$$

By simplifying Eq. (12), the relationship among u, θ and Φ_1 can be obtained as follows:

$$\varepsilon = \arccos\left\{\frac{[r_b u+H(u)][r_b+H'(u)]i_{21}\sin Q}{p(1-i_{21}cos\Sigma)\sqrt{[H'(u)]^2+[r_b u+H(u)]^2}}\right\} + \arctan\frac{H'(u)}{[r_b u+H(u)]} \tag{13}$$

where $\varepsilon=u+\theta+\varphi_1$.

Eq.(12). and Eq.(13). yield Eq.(14).

$$\Theta = \frac{-n_{x0}(1-i_{21}\cos Q)y_0+n_{y0}(1-i_{21}\cos Q)x_0-n_{y0}ai_{21}\cos Q+n_{z0}i_{21}(x_0+a)\sin Q}{pn_{x0}i_{21}\sin Q} \tag{14}$$

Eq.(7), Eq.(13) and Eq.(14) yield mathematical model of mesh track on the gear flank

$$x_2 = r_{b2}\cos\varepsilon + (r_{b2}u + H)\sin\varepsilon$$

$$y_2 = r_{b2}\sin\varepsilon - (r_{b2}u + H)\cos\varepsilon \tag{15}$$

$$z_2 = p\Theta(u)$$

Because of the existence of the gear tooth profile modifications, the mesh track on the gear flank is not a straight line with slight slope alterations.

2.4 Gear flank twist model

Gear flank twist is the phenomenon that transverse profile of gear is twisted along axial direction. Gear flank twist is a principal error, influenced by profile modification. Fig.3 shows the gear flank twist. Where I and II refer to the gear end face. According to BS ISO 21771: 2007, gear flank twist can be calculated as follows:

$$T = \left|C_{\mathrm{H\alpha\,I}} - C_{\mathrm{H\alpha\,II}}\right| \tag{16}$$

where $C_{\mathrm{H\alpha I}}$ is the profile slope deviation on gear end plane I. $C_{\mathrm{H\alpha II}}$ is the profile slope deviation on gear end plane II.

When the grinding wheel's geometric center is in the middle of the breadth of the tooth, there is an axial distance S_{v} between the reference point and the midpoint of the breadth of the tooth. The reference point is the intersection of the meshing track and the reference circle.

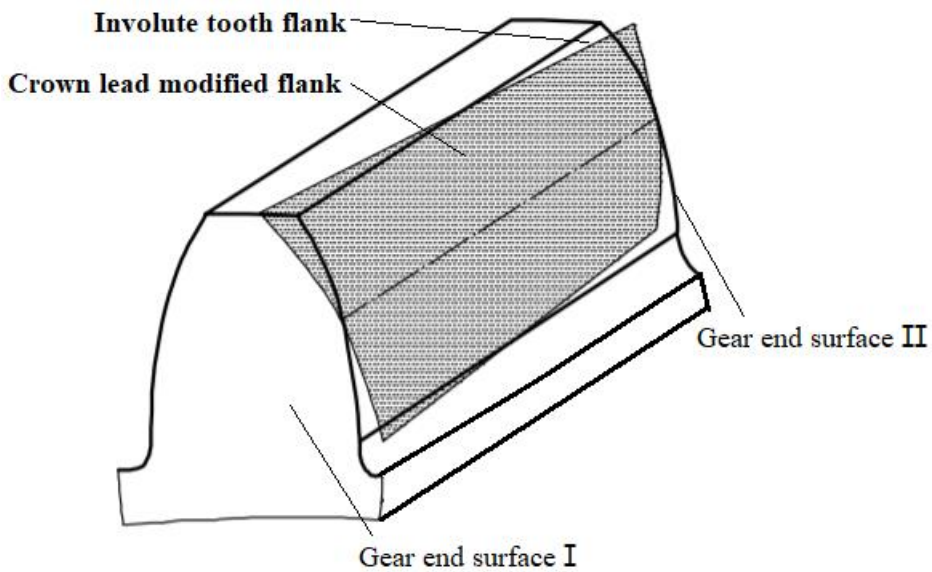


Fig.3 Gear flank twist

Assuming lead modification curve equation is as follows:

$$\mathrm{G}(z) = \frac{g_2}{l_4^2}(z + S_v)^2 - g_2 \tag{17}$$

where l_4 is half of the breadth of the tooth, g_2 is the maximal gear lead modification value.

Fig.4 shows the relationship between mesh track on the gear flank and gear flank twist. The figure shows the unfolded drawing of gear flank. The right figure shows the lead modification curve.

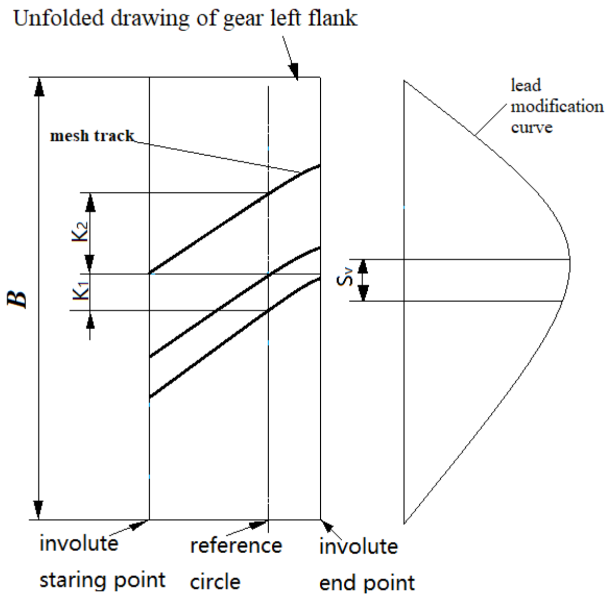


Fig.4 Tooth profile error

K_1 is the length of mesh track on the gear flank in the axial direction from involute starting point reference circle. K_2 is the length of mesh track on the gear flank in the axial direction from reference circle to involute end point. According to Yan [13], when the center of the grinding wheel is in the middle of the tooth width, there is a height difference S_V between the intersection of the mesh track and the reference circle and the midpoint of the tooth width. B is the facewidth.

According to Liu (2017), $C_{H\alpha I}$ and $C_{H\alpha II}$ can be calculated as follows:

$$C_{H\alpha\,I} = G\left(-\frac{B}{2}-K_1\right)-G(-\frac{B}{2}+K_2) \tag{18}$$

$$C_{H\alpha\,II} = G\left(\frac{B}{2}-K_1\right)-G(\frac{B}{2}+K_2) \tag{19}$$

where B is the facewidth.

3 .Influence of machine tool position error on tooth flank twist

The machine tool position errors have great influence on gear flank twist. So it is necessary to study the influence of machine tool position errors on gear flank twist.

The machine tool positon errors can be simplified into three position errors: relative position errors between worm grinding wheel and gear in Xp direction, Yp direction and Zp direction. The Xp, Yp and Zp directions can be seen in Fig.1.

The relative position error in the Xp direction is equivalent to the additional lead modification value, the influence of it on lead modification curve can be expressed as follows.

$$G(z)' = G(z) + \Delta G \tag{20}$$

$$\Delta G = \Delta x \cdot \sin\alpha \tag{21}$$

where α is the normal pressure angle of gear, Δx is the relative position error in Xp direction, ΔG is additional lead modification value. $G(z)$' is the lead modification curve equation considering the influence of the relative position error.

The relative position error in the Yp direction is equivalent to the flee cutter of the worm grinding wheel in axial direction, so the influence of relative position errors in the Yp direction can be ignored.

The relative position error in the Zp direction is equivalent to the translation of the tooth lead modification curve in the Zp direction, which causes the change of S_v . The influence of relative position error in the Zp direction on Sv can be expressed as follows.

$$Sv' = Sv \pm \Delta z \tag{22}$$

where Δz is the relative position error in the Z direction, Sv' is the Sv value considering the influence of the relative position error in the Z direction. When the tooth surface was analyzed, and the relative position error is in z direction, the Δz is a positive one.

4. Analyses of gear flank twist by orthogonal experiment

The orthogonal experiment is a statistical tool, adopted to investigate the influence of module (factor A), number of teeth (factor B), maximal profile modification value (factor C) and helix angle (factor D) on the gear flank twist, and to select the optimum parameters for gear.

4.1. Experimental conditions

The orthogonal experiments are based on an orthogonal array experimental design matrix, which is shown in Table 1.

Table 1. Levels and factors of gear flank surface

Level	Module (A)	Number of teeth (B)	Maximal profile modification value (C)	Helix angle (D)
1	4mm	17	4μm	14°

2	5mm	21	6μm	17˚
3	6mm	25	8μm	20˚

The gears were manufactured by worm wheel gear grinding machine. The gear flank twist was calculated by the method described above. The lead crowning of gear tooth flank was accomplished by varying the center distance between the wheel and work gear.

In order to cover all the levels in the present study, four levels of module, number of teeth, maximal profile modification value and helix angle were employed. Other necessary gear parameters are shown in the table 2.

Table 2. Basic parameters

Conditions	Values
Normal pressure angle	19˚
Facewidth	46mm
Tip diameter of worm grinding wheel	280mm
Maximal gear lead modification value	35μm
Wheel grinding worm's speed	63rpm

In total 9 experiments were designed by orthogonal method, which are shown in table 3. The Table 3 gives the various gear parameters for each experiment. The different units used here are: Module – mm, Maximal profile modification value -μm, Helix angle – ˚.

Table 3. Orthogonal experiment design

Experiment number	Module	Number of teeth	Maximal profile modification value	Helix angle
1	4	17	4	14
2	4	21	6	17
3	4	25	8	20
4	5	17	6	20
5	5	21	8	14
6	5	25	4	17
7	6	17	8	17
8	6	21	4	20
9	6	25	6	14

4.2. Parameters

There are two important parameters in a range analysis: K_{ji} and R_j. K_{ji} is defined as the sum of the evaluation indexes of all levels (i=1, 2, 3, 4) in each factor (j=A, B, C, D) and K_{ji} is used to determine the optimal level and the optimal combination of factors. The optimal level for each factor could be obtained when K_{ji} is the smallest. R_j is defined as the range between the maximum and minimum value of K_{ji} and is used for evaluating the importance of the factors, i.e. a larger R_j means a greater importance of the factor. For example, take the OA9 matrix. The calculation is shown below: For the factor of A:

$$K_{A1}=Y_1+Y_2+Y_3 \tag{30}$$

$$K_{A2}=Y_4+Y_5+Y_6 \tag{31}$$

$$K_{A3}=Y_7+Y_8+Y_9 \tag{32}$$

$$\overline{K_{A1}} = \frac{K_{A1}}{3} \tag{33}$$

$$\overline{K_{A2}} = \frac{K_{A2}}{3} \tag{34}$$

$$\overline{K_{A3}} = \frac{K_{A3}}{3} \tag{35}$$

$$R_A=\max(\overline{K_{Ai}})-\min(\overline{K_{Ai}}) \tag{36}$$

where K_{Ai} is the K value of the i level of the factor of A, $\overline{K_{ji}}$ is the mean value of K_{ji}; and Y_i is the value of the result of the No. i experiment. Other K values of the factors can be determined by the same calculation steps.

4.3. Results and discussions

According to the OA16 matrix, sixteen experiments were carried out and their product yield results were shown in Table 5.

Table 5. Results of Orthogonal experiments

Experiment number	Module	Number of teeth	Maximal profile modification value	Helix angle	Gear flank twist
1	4	17	4	14	4.7
2	4	21	6	17	6.7
3	4	25	8	20	9.2
4	5	17	6	20	8.9
5	5	21	8	14	6.8
6	5	25	4	17	9.6
7	6	17	8	17	8.8
8	6	21	4	20	12.4
9	6	25	6	14	9.3

$\overline{K_{1j}}$	6.87	7.47	8.90	6.93
$\overline{K_{2j}}$	8.43	8.63	8.30	8.37
$\overline{K_{3j}}$	5.08	9.37	8.27	10.17
R_i	3.35	1.90	0.63	3.24

As mentioned before, for each factor, the range value (R_j) indicates the significance of the factor's effect and a larger R_j means the factor has a bigger impact on the product yield. Therefore, compared with the range values of different factors (R_j), the factors' levels of significance are as follows: module (3.35) > helix angle (3.24) > number of teeth (1.90) > maximal profile modification value (0.63).

Table 6 a. Relationship between module and gear flank twist

Module	Gear flank twist
4	6.87
5	8.43
6	5.08

Table 6 b. Relationship between number of teeth and gear flank twist

Number of teeth	Gear flank twist
4	7.47
5	8.63
6	9.37

Table 6 c. Relationship between maximal profile modification value and gear flank twist

Maximal profile modification value	Gear flank twist
4	8.90
5	8.30
6	8.27

Table 6 d. Relationship between helix angle and gear flank twist

Helix angle	Gear flank twist

4	6.93
5	8.37
6	10.17

The mean values of each factor were shown in table. 6a-d. Based on the change in the mean value of each factor (table. 6a-d), it can be observed that the gear twist increased from 6.87 μm to 8.43 μm when the module increasing from 4 to 5. And the gear twist sharply decreased from 8.43 μm to 5.08 μm when the module increasing from 5 to 6. For number of teeth, the gear twist increased from 7.47 μm to 9.37 μm, with the module increasing from 17 to 25. Helix angle clearly influenced the gear flank twist. the gear twist slightly increased from 6.93 μm to 10.17 μm, with the helix angle increasing from 14° to 20°. And the gear twist slightly decreased from 8.90 μm to 8.27 μm, with the maximal profile modification value increasing from 4 μm to 8 μm. After the orthogonal experiments, the optimal level for each factor was determined as follows: module 6.0mm, number of teeth 17, maximal profile modification value 8 μm and helix angle 14°.

5. Verification of gear flank twist prediction model

The KX TWIN HS gear grinding machine was adopted as the gear processing machine tool in this paper, main movement axes of it are radial feed axis X-axis of grinding wheel, longitudinal feed axis Y-axis of tool past, axial feed axis Z-axis of tool past, rotation axis A-axis of grinding wheel, rotation axis B-axis of gear workpiece. The machining center is shown in Fig.5.

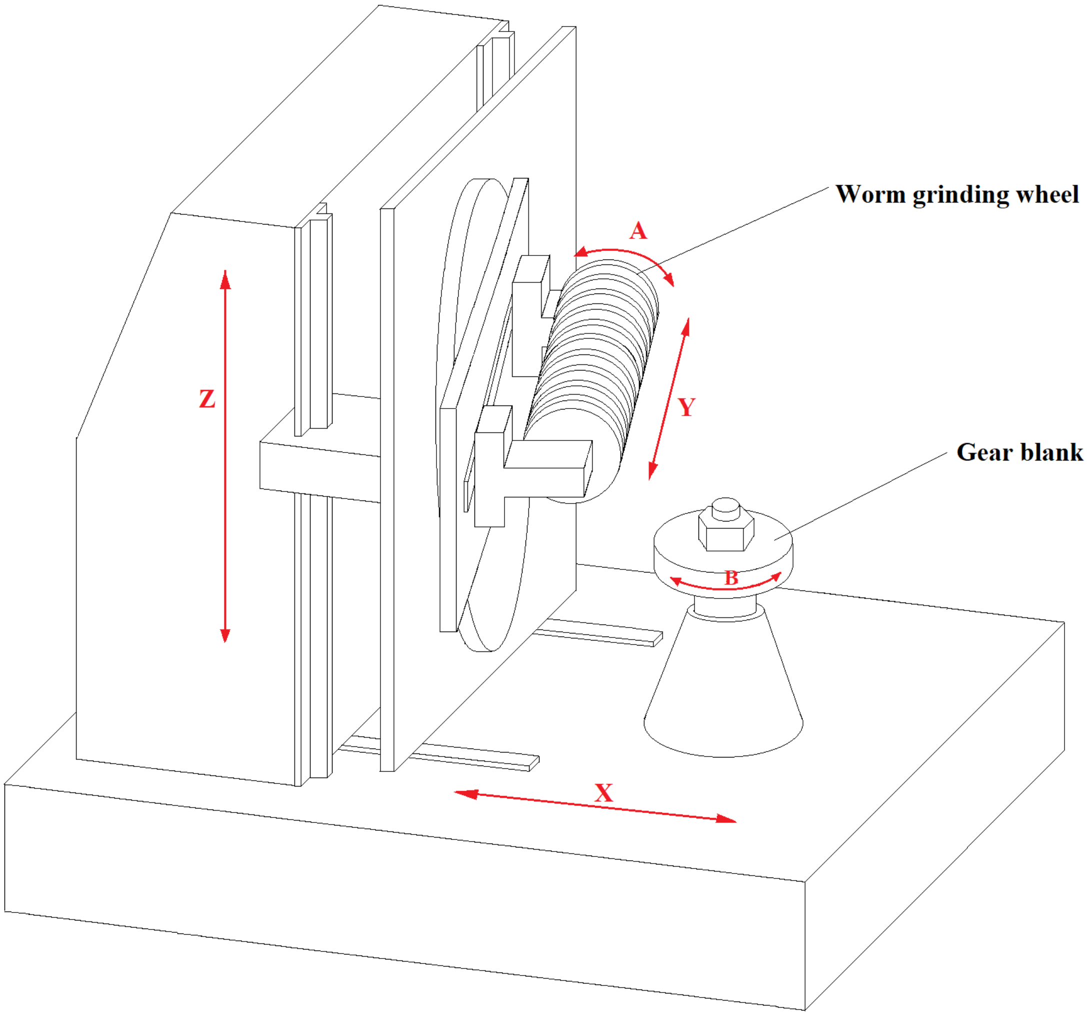


Fig. 5. Experimental setup

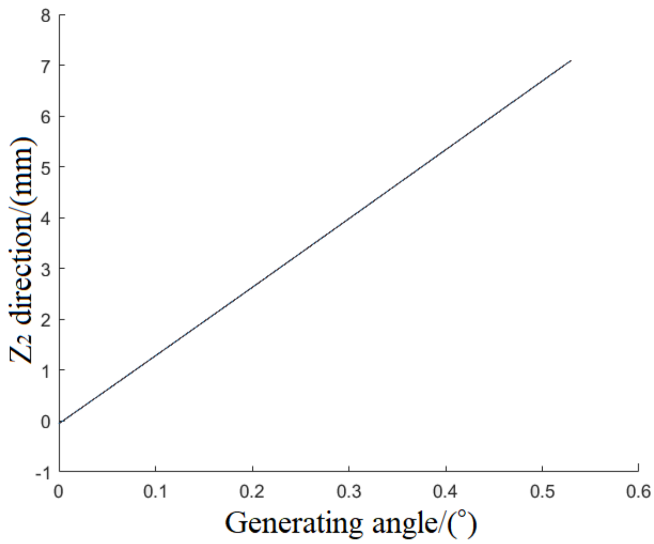
Two case-carburiged gears, named gear A and gear B, with profile and lead modification were manufactured. Gear A has one crown in axial direction. Gear B has two crown distributed symmetrically at both sides of gear in axial direction. The parameters of the gear A and gear B are listed in Tables 7.

Table 7 Parameters of gear A and gear B

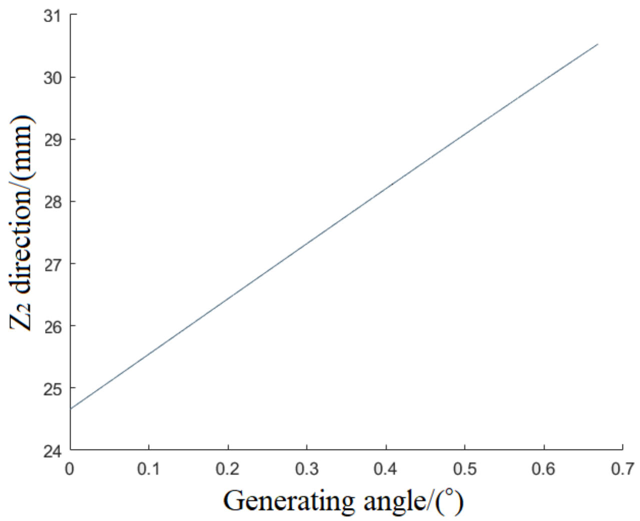
Conditions	Symbols	Gear A	Gear B
Module	m_n	4.25mm	4.4mm
Teeth number of gear workpiece	Z_2	21	17
Outside diameter of worm grinding wheel	D	280mm	98mm
Gear helix angle	β	17.81˚	14..85˚
Spindle speed	V	63m/s	63m/s
Normal pressure angle	α_n	19˚	21.5˚

Maximal modification value of gear tooth profile	h_2	6μm	6μm
Maximal helix modification value of gear	g_2	35μm	35μm
Tooth width	B	46mm	99mm

As shown in Fig.6, the mesh track on the two gear flank can be calculated by Matlab. X-axis and Y-axis representing generating angle of gear flank surface and mesh track in Z_2 direction. The values of S_v,K_1,K_2 can be obtained by analyzing the mesh track. For gear A, K_1=2.275mm，K_2=4.855mm，Sv=1.02mm. For gear B, K_1=3.61mm，K_2=2.26mm，Sv=0.89mm.



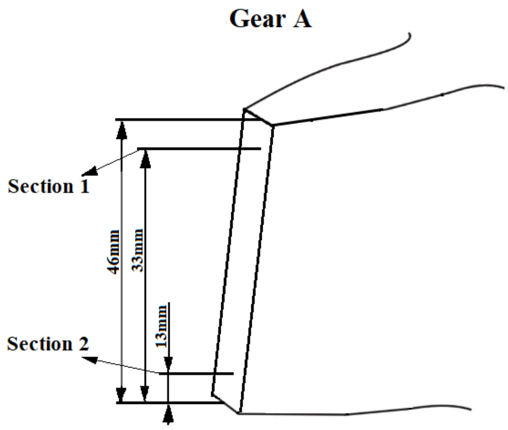
(a) Mesh track on gear A



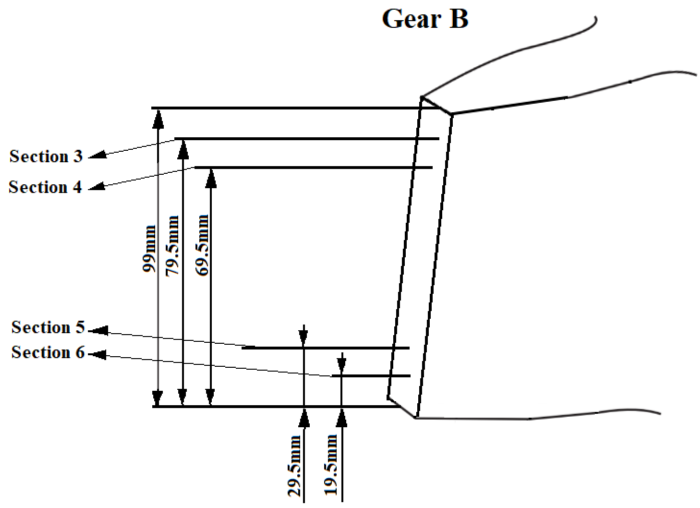
(b) Mesh track on gear B

Fig.6 Mesh track on the gear flank

As shown in Fig.7, to verify the mathematical model of gear flank twist, 6 sections were chosen to measure the profile slope deviations. Position 1 and position 2 are located on the flank face of gear A. Position 3-6 are located on the flank face of gear B.



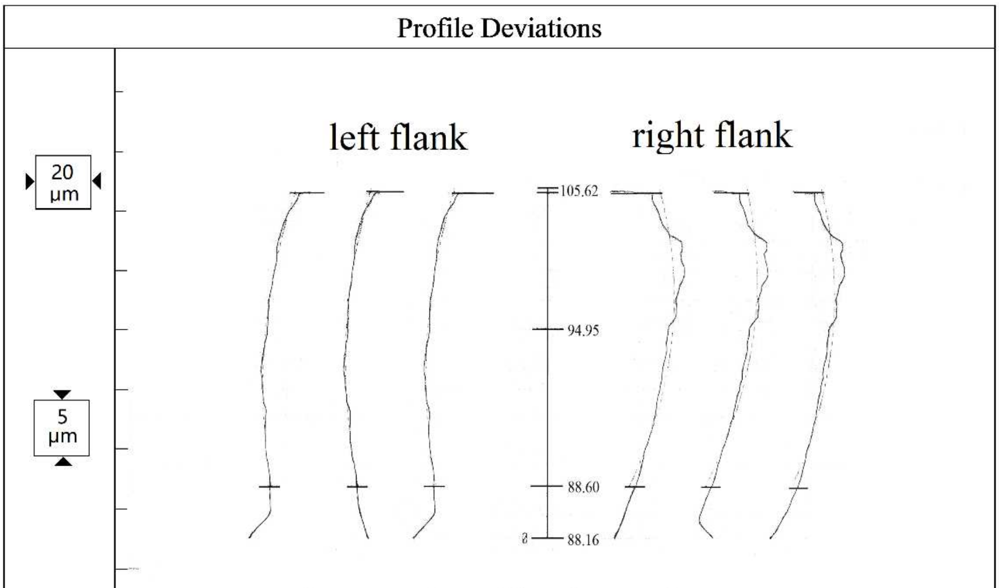
(a) Measured positions in gear A



(b) Measured positions in gear B

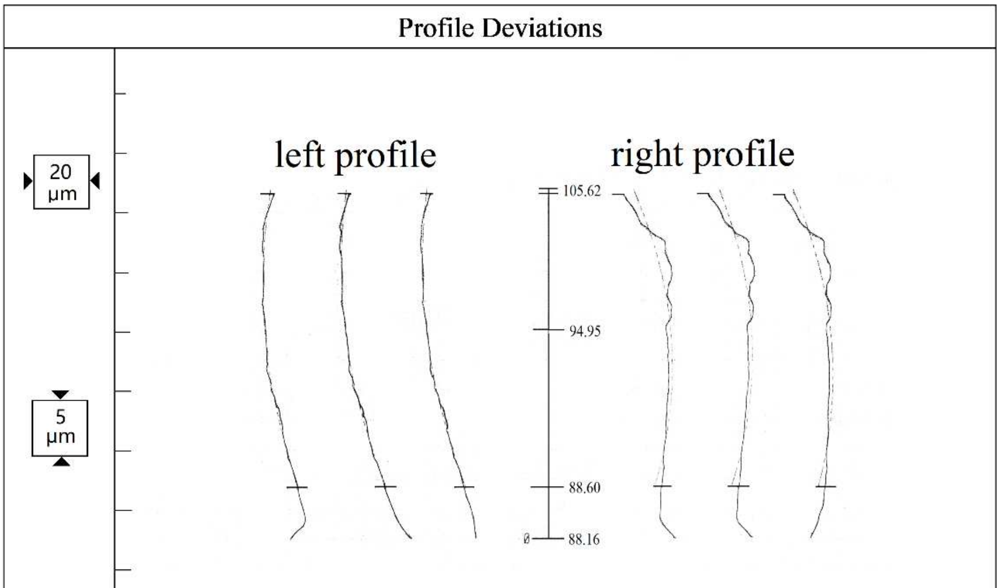
Fig.7 Measurement position of gear

The measurement data are shown in Fig. 8. The comparisons between experimental profile deviations and simulated profile slope deviations are shown in table 8. The data in table 8 is obtained by taking the average profile slope deviations of tooth 1, 8, 15 on the left flank. It can be found that experimental results and simulated results are nearly the same.



	15	8	1	Tooth	1	8	15
fHα	-9.0	-4.3	-6.7		12.1	16.6	12.3
Fα	11.3	7.0	10.7		13.2	16.5	13.3
ffα	3.0	3.5	4.7		8.5	8.5	8.4
Ca	7.2	7.2	7.0		10.6	10.4	10.2

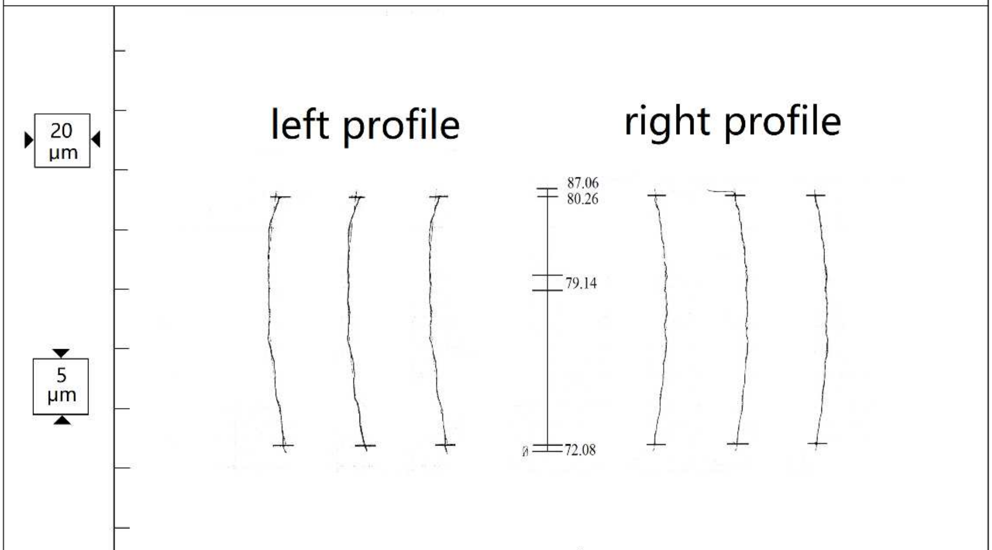
(a) Section 1



	15	8	1	Tooth	1	8	15
fHα	11.1	16.0	15.7		-7.5	-4.1	-9.5
Fα	10.3	14.5	14.0		14.8	12.6	16.4
ffα	2.8	2.5	4.5		9.7	10.8	9.3
Ca	7.4	7.7	7.3		10.6	10.4	10.2

(b) Section 2

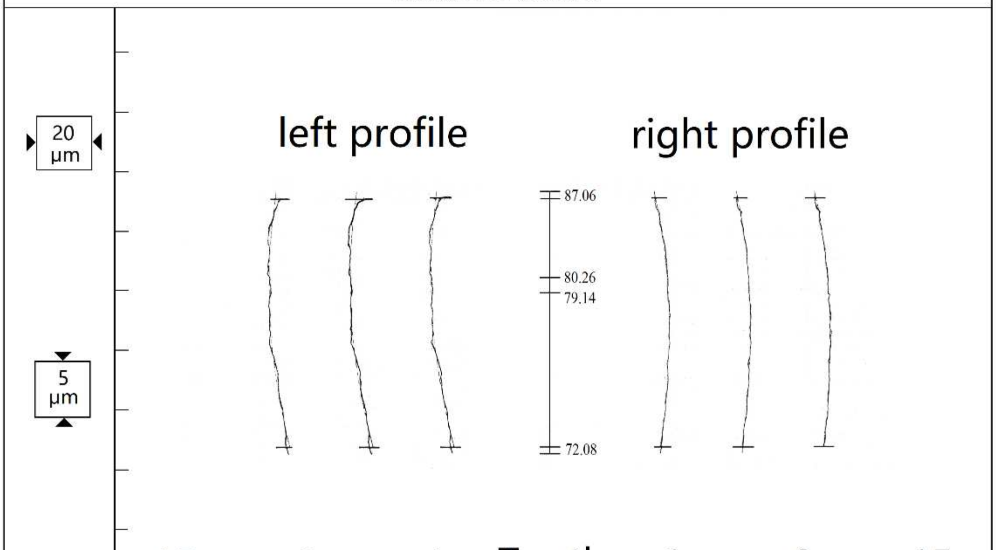
Profile Deviations



	15	8	1	Tooth	1	8	15
fHα	4.1	3.7	4.7		1.2	0.1	0.0
Fα	7.0	6.5	7.4		6.9	6.7	6.5
ffα	3.2	3.5	2.8		1.6	1.4	1.3
Ca	4.5	4.8	4.5		4.6	4.6	4.6

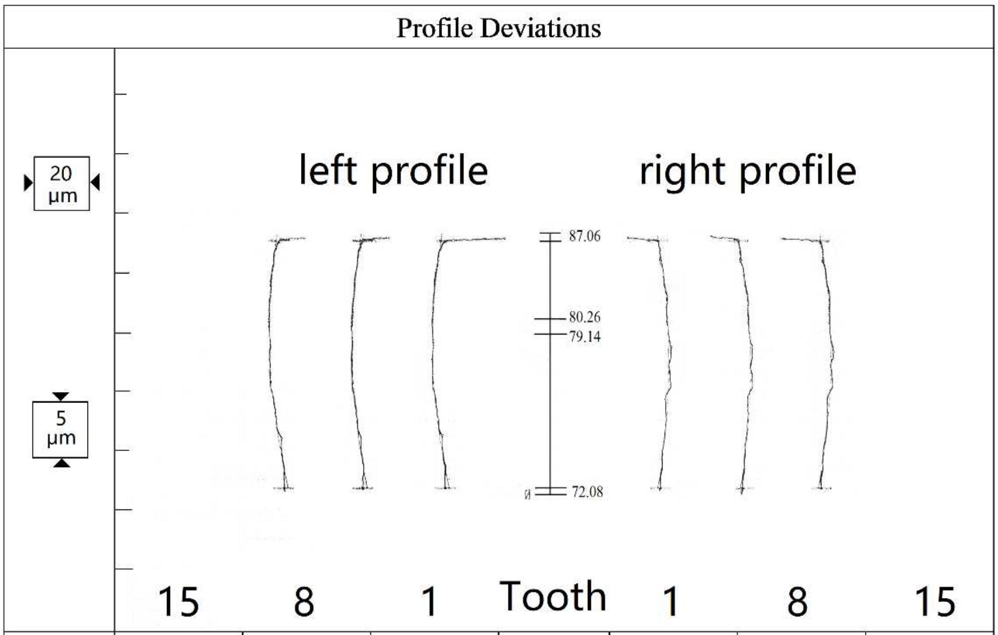
(c) Section 3

Profile Deviations



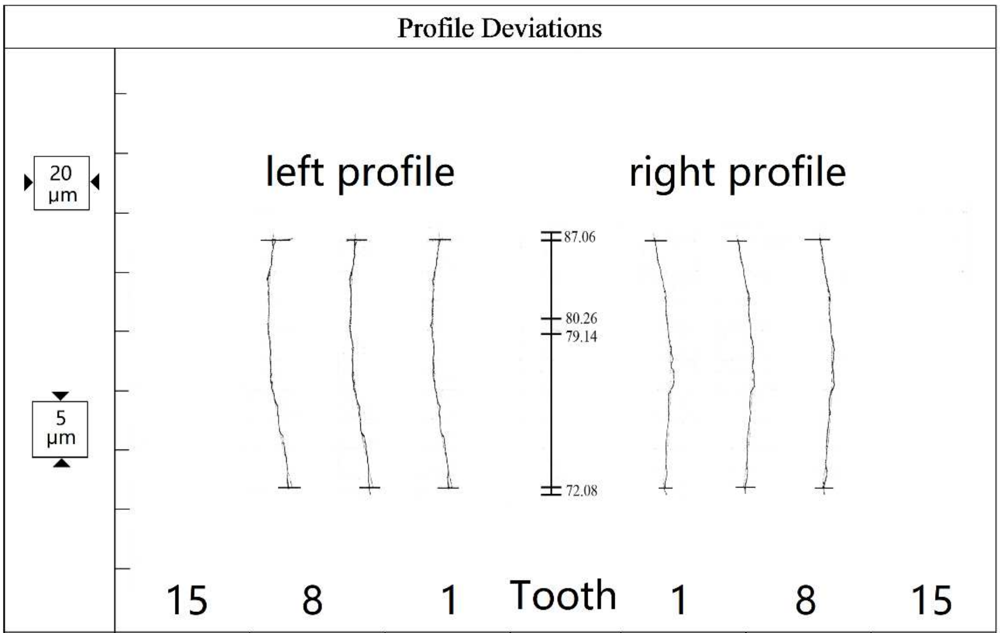
	15	8	1	Tooth	1	8	15
fHα	5.8	6.9	7.3		-1.6	-1.5	-3.0
Fα	8.1	8.9	9.2		6.9	7.0	7.9
ffα	3.6	8.7	3.7		1.4	1.3	1.3
Ca	4.8	5.0	4.7		4.5	4.4	4.4

(d) Section 4



	15	8	1		1	8	15
fHα	5.0	2.9	3.5		0.0	1.0	0.8
Fα	7.8	7.0	6.6		7.5	7.6	7.1
ffα	3.0	2.8	4.1		2.2	2.1	2.4
Ca	4.5	4.5	4.8		4.2	4.4	4.5

(e) Section 5



	15	8	1		1	8	15
fHα	7.7	7.3	5.6		-3.3	-2.1	-0.8
Fα	10.2	10.1	8.9		8.1	7.3	6.9
ffα	2.5	2.5	2.4		2.2	1.8	2.0
Ca	4.5	4.3	4.5		4.7	4.6	4.4

(f) Section 6

Fig.8 Measured data

Table 8 Comparisons between measured and calculated results

Section	Simulated profile deviations /μm	Experimental profile deviation /μm
1	6.30	6.67
2	10.10	14.27
3	5.70	6.87
4	3.80	3.80
5	5.70	6.67
6	3.80	4.17

By using Eq.16, the gear flank twist can be obtained. For gear A, the simulated gear flank twist is 3.8μm, the experimental gear flank twist is 7.6μm. For gear B, the simulated gear flank twist is 1.9μm, the experimental gear flank twist is 2.7μm. The errors are caused by the wear of worm grinding wheel and machine tool errors.

By measuring the profile deviations of chosen sections, compared them with the simulated results, the correctness of the model is proved.

6. Conclusions

A new prediction method for the gear flank twist for the generating grinding process is developed in this paper which is based on the mesh track calculation method. It takes into account the gear profile modifications, gear lead modifications and machine tool position error. The influence of four gear parameters on gear flank twist was studied using orthogonal experiments. The experiments showed that gear flank twist increased as gear's number of teeth and helix angle increased, and the gear flank twist decreased as gear's maximal profile modification value increased. When the gear's module increased, the gear flank twist increased first and then decreased when maximal profile modification value was more than 5mm. The sort order of parameters' levels of significance are as follows: module > helix angle > number of teeth > maximal profile modification value.

7.Declarations

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7.2 Competing Interests

The authors have no relevant financial or non-financial interests to disclose.

7.3 Availability of data and material

The authors have no financial or proprietary interests in any data and material discussed in this article.

7.4 Code availability

The authors have no financial or proprietary interests in any code discussed in this article.

7.5 Compliance with Ethical Standards

The authors have no competing interests to declare that are relevant to the content of this article.

All authors consent for the publication of this article.

7.6 Consent to participate

All authors consent to participate in the study.

7.7 Consent for publication

All authors consent for the publication of this article.

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