Possible existence of dark matter admixed pulsar in the Disk region of the Milky Way galaxy

Nilofar Rahman
Aliah University, Kolkata
Masum Murshid
Aliah University, Kolkata
Sajahan Molla
New Alipore College, Kolkata
Mehedi Kalam (✉ kalam@associates.iucaa.in)
Aliah University, Kolkata

Research Article

Keywords: Compact star, Dark matter, Mass function, Radius Compactness Red-shift

Posted Date: April 20th, 2022

DOI: https://doi.org/10.21203/rs.3.rs-1566806/v1

Possible existence of dark matter admixed pulsar in the Disk region of the Milky Way galaxy

Nilofar Rahman[1] • Masum Murshid[1] •
Sajahan Molla[2] • Mehedi Kalam[1]

Abstract In our previous study (Eur Phys J Plus 135:362, 2020 & Eur Phys J Plus 135:637, 2020), we have discussed about the possible existence of the dark matter admixed pulsars, located in dwarf as well as in massive spiral galaxies(based on Singular Isothermal Sphere dark matter density profile) and in Milky Way galaxy(based on Universal Rotational Curve dark matter density profile). In this article, we use the Navarro-Frenk-White(NFW) dark matter density profile to get analogous results for the pulsars in the Disk region of the Milky Way galaxy. These findings may be treated as a useful complements to the previous findings. We conclude from our findings that there is a notable possibility of the presence of dark matter admixed pulsars in all the region of the galaxies.

Keywords Compact star Dark matter Mass function Radius Compactness Red-shift

1 Introduction

The unique properties of compact objects are getting more attention for the last few decades. White dwarfs, strange stars, neutron stars are falling into the category of compact objects. These stars become stable as the outward degeneracy pressure from the Fermi gas balances the inward gravitational force. The Fermi gas in white dwarf and neutron star are consists mostly of electrons and neutrons respectively. The strange stars are consist of strange quark matter. The gravitational force is the main cause for the neutron star to be bound. On the other hand, a strange star is more bound than a neutron star due to the strong interaction along with gravitational. The source of enormous energy required for the formation of a strange star actually comes from the super luminous supernovae Leahy (2008). A strange star and a neutron star can be differentiated based on their vanishing surface energy density Haensel et al. (1986); Alcock et al. (1986); Farhi & Jaffe (1984); Postnikov et al. (2010); Dey et al. (1998). According to Lattimer and Prakash Lattimer & Prakash (2007) the radius and mass of a neutron star having a particular Equation of State (EoS) depend on its central density.The solutions of Tolman-Oppenheimer-Volkoff (TOV) equations can lead us to the theoretical estimation of the mass and radii of spherically symmetric compact stars. It can also be measure from pulsar timing, surface explosions, thermal emission from cooling stars and gravity wave emissions. It is a bit challenging for us to fix the EoS due to the complex structure of the starÖzel (2006); Özel et al. (2009); Özel & Psaitis (2009); Özel et al. (2010); GÖver et ai. (2010a,b). The radius of the compact star is not completely known to us, whereas the mass can be estimated due to it's presence in binaries Heap & Corcoran (1992); Lattimer & Prakash (2005); Stickland et al. (1997); Orosz & Kuulkers (1999); Kerkwijk et al. (1995).Due to some observational constrains, we need to study theoretically on the stellar structure of the newly discovered stellar objects Rahaman et al. (2012a,b); Kalam et al. (2012, 2013a,b, 2014a,b, 2016, 2017); Jafry et al. (2017); Hossein et al. (2012); Lobo (2006); Bronnikov & Fabris (2006); Maurya et al. (2016); Dayanandan et al. (2016); Maharaja et

---

Nilofar Rahman
rahmannilofar@gmail.com
Masum Murshid
masum.murshid@wbscte.ac.in
Sajahan Molla
sajahan.phy@gmail.com
Mehedi Kalam
kalam@associates.iucaa.in

[1]Department of Physics, Aliah University, IIA/27, New Town, Kolkata 700160, India

[2]Department of Physics, New Alipore College, L Block, New Alipore, Kolkata 700053, India

al. (2014); Ngubelanga et al. (2015); Paul et al. (2015); Pant et al. (2014); Bhar et al. (2017); Kalam (2018); Molla (2019); Islam (2019); Hendi et al. (2016, 2017); Panah (2017).

In 1933, the concept of dark matter was introduced by Zwicky when he was studying the dynamic properties of the Coma galaxy cluster Zwicky (1933, 2009). Rubbin and Ford Rubin & Ford (1970) has come to the same conclusion about the dark matter through the optical studies of galaxies (e.g. M31).Some discussion on this topic are available in the literatureOlive (2003); Munoz (2004). Though the nature and origin of dark matter is not clear till now, some scientist has given new concepts on dark matter explaining it's properties Taoso et al. (2008); Lopes & Silk (2010); Kouvaris & Tinyakov (2010); Turck-Chièze (2012); Lopes & Silk (2014); Lopes et al. (2014); Brito et al. (2015, 2016); Martins et al. (2017). Although normal matters have no such direct interaction with the dark matter,stellar objects have some remarkable gravitational effect due to dark matter Li et al. (2012); Wang et al. (2012); Panotopoulos & Lopes (2017a). The gravitational effect of fermionic dark matter influences the physical properties of the strange stars Narain et al. (2006); Leung et al. (2012); Mukhopadhyay & Bielich (2016); Panotopoulos & Lopes (2017b); Leung et al. (2011). Spergel and Steinhardt has also introduced the concept of self-interaction of dark matterSpergel & Steinhardt (2000).

Some astrophysicist Wang et al. (2012); Leung et al. (2012); Panotopoulos & Lopes (2017b); Leung et al. (2011); (2017); Sandin & Ciarcelluti (2009); Mukhopadhyay et al. (2017); Rezaei (2017, 2018); Takisa et al. (2020) have worked on dark matter neutron star.Inspired with their work, we have investigated Molla et al. (2020) about the existence of dark matter in pulsars by using Singular Isothermal Sphere (SIS) density profile. We have assumed the pulsars to be made of ordinary matter admixed with dark matter having density distribution as

$$\rho_d(r) = \frac{K}{2\pi G r^2}$$

where K is the velocity dispersion. Dark matter contribution actually comes from the rotational curve fitting of the SPARC sample of galaxiesLelli et al. (2016). We have shown the possibility of existence of dark matter with ordinary matter in the pulsars namely PSR J1748-2021B in NGC 6440B, PSR J1911-5958A in NGC 6752, PSR B1802-07 in NGC 6539 and PSR J1750-37A in NGC 6441 galaxies.

We have also investigated about the existence of dark matter admixed pulsars in Milky Way galaxy by using the Universal Rotation Curve(URC) dark matter density profile Rahman et al. (2020).We have used the URC dark matter density profile as

$$\rho_d(r) = \frac{\rho_0 r_0^3}{(r + r_0)(r^2 + r_0^2)}$$

where r_0 is the core radius and ρ_0 is the effective core density. For Milky Way galaxy(in a particular case), $r_0 = 9.11kpc$ and $\rho_0 = 5 \times 10^{-24} \left(\frac{r_0}{8.6kpc}\right)^{-1} gm/c.c.$ Maccio (2012); Castignani et al. (2012). The pulsars which we have studied in the Milky Way are PSR J0740+6620, PSR J1012+5307, PSR J0751+1807 and PSR J1614-2230.

In the present article we have considered the Navarro-Frenk-White(NFW) density profile, as it is a well accepted (dark matter) model particularly in the disk regions of the galaxy. In this article, we will study the pulsars namely PSR J0045-7319, PSR J0537-6910 which are located in the disk region of the Milky way galaxy. In NFW density profile, the dark matter density can be represented as Navarro et al. (1996)

$$\rho_d(r) = \frac{\rho_s}{\frac{r}{r_s}(1 + \frac{r}{r_s})^2} \tag{1}$$

where r_s is the scale radius and ρ_s is the effective density. Particularly for the Milky way galaxy $r_s = 20kpc$ and $\rho_s = 0.26GeV/c.c.$ Razeira & Mesquita (2011). The actual aim of this article is to investigate the possible existence of dark matter admixed pulsars namely, PSR J0045-7319, PSR J0537-6910 which are located in the disk region of the Milky way galaxy.

2 Interior Spacetime

The interior spacetime associate with the spherically symmetric pulsar Takisa et al. (2019); Matondo et al. (2018); (2015); Maurya et al. (2019); Dayanandana (2017); Maurya (2017); Gedela et al. (2018); Singh (2017); Rahaman et al. (2017); Jasim et al. (2016); Takisa & Maharaj (2016); Maurya et al. (2016); Singh et al. (2017a) has been described as

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2d\theta^2 + r^2\sin\theta^2 d\phi^2 \tag{2}$$

In the interior of the isotropic pulsar, we consider energy-momentum tensor as

$$T^{\mu}_{\nu} = diag(-\rho, p, p, p) \tag{3}$$

where ρ is the energy density and p is the isotropic pressure. The Einstein's field equations corresponding to the line element (2) in geometric unit ($G = c = 1$) can be written as

$$8\pi\rho = e^{-\lambda}\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2} \tag{4}$$

$$8\pi p = e^{-\lambda}\left(\frac{\nu'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2} \tag{5}$$

$$8\pi p = \frac{e^{-\lambda}}{2}\left(\frac{(\nu')^2 - \lambda'\nu'}{2} + \frac{\nu' - \lambda'}{r} + \nu''\right) \tag{6}$$

These equations approve the fact that the topology of spacetime is affected by the matter distribution.

According to Heintzmann metric Heintzmann (1969)

$$e^{\nu} = A^2(1 + ar^2)^3 \tag{7}$$

$$e^{-\lambda} = 1 - \frac{3ar^2}{2}\left[\frac{1 + C(1 + 4ar^2)^{-\frac{1}{2}}}{1 + ar^2}\right] \tag{8}$$

where A (dimensionless) , C (dimensionless) and a ($length^{-2}$) are constants.

If we consider the pulsars are made of ordinary matter mixed with dark matter, then the effective density and pressure can be expressed as

$$\rho_{eff} = \rho + \rho_d$$

$$p_{eff} = p - p_d$$

The pressure appear due to dark matter as $p_d = m\rho_d$, where m is a constantBarranco (2013).

In the presence of dark matter (1), the Einstein's field equations takes the form

$$\rho = \frac{1}{8\pi}\left[e^{-\lambda}\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2}\right] - \frac{\rho_s}{\frac{r}{r_s}(1+\frac{r}{r_s})^2} \tag{9}$$

$$p = \frac{1}{8\pi}\left[e^{-\lambda}\left(\frac{\nu'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2}\right] + \frac{m\rho_s}{\frac{r}{r_s}(1+\frac{r}{r_s})^2} \tag{10}$$

3 Study of Physical Properties

3.1 Energy density and Pressure

From the plot of energy density and pressure (see the Fig.1 and 2), we see that at the center, energy density and pressure are maximum and both decreases monotonically towards the boundary. Therefore, the energy density and pressure are well behaved in the interior of stellar structure. Here, we have taken the constants $a = 0.00138889km^{-2}$, $C = 1.34164$, $m = 0.025$, $r_s = 20kpc$ and $\rho_s = 0.26GeV/c.c.$ in such a way that all other required conditions must obeyed including the pressure, which goes to zero at the boundary.

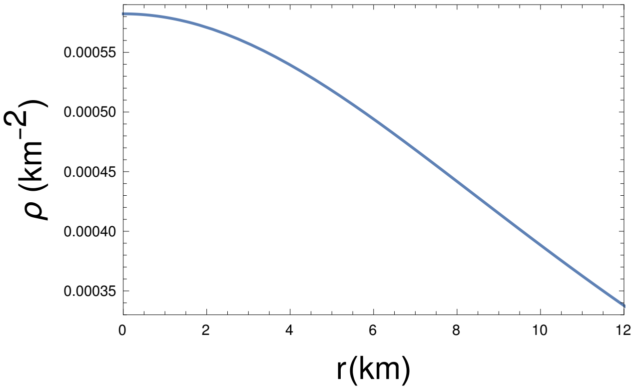

Fig. 1 Energy density (ρ) variation with radial distance (r) for $a = 0.00138889km^{-2}$, $C = 1.34164$, $m = 0.025$, $r_s = 20kpc$ and $\rho_s = 0.26GeV/c.c.$

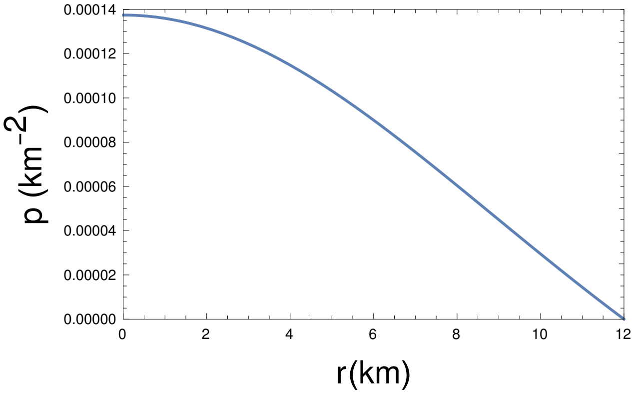

Fig. 2 Pressure (p) variation with radial distance (r) for $a = 0.00138889km^{-2}$, $C = 1.34164$, $m = 0.025$, $r_s = 20kpc$ and $\rho_s = 0.26GeV/c.c.$

3.2 Energy conditions

Fig. 3 indicate that all the energy conditions namely null energy condition(NEC), weak energy condition (WEC), strong energy condition (SEC) and dominant energy condition (DEC) are satisfied at the interior of the pulsar.

(i) NEC: $\rho \geq 0$
(ii) WEC: $\rho + p \geq 0$, $p \geq 0$
(iii) SEC: $\rho + p \geq 0$, $\rho + 3p \geq 0$
(iv) DEC: $\rho > |p|$

3.3 Matching conditions

For a stellar body, at the boundary ($r = R$), the interior metric must matched with the exterior Schwarzschild metric

$$ds^2 = -(1 - \frac{2M}{r})dt^2 + \frac{dr^2}{(1 - \frac{2M}{r})} + r^2(d\theta^2 + \sin\theta^2 d\phi^2)$$

PSR	Distanc (kpc)	Observed MassHo (2015)	Radius from model (km)	Compactness from model	RedShift from model
J 0045-7319	57	$1.58^{+0.34}_{-0.34}$	$10.8436^{+0.88}_{-0.98}$	$0.215152^{+0.027}_{-0.029}$	$0.324885^{+0.067}_{-0.064}$

Table 1 Evaluated parameters for pulsar J 0045-7319.

PSR	Distance (kpc)	Equation of State	Observed MassHo (2015)	Radius from model (km)	Compactness from model	RedShift from model
J 0537-6910	52.122	BSk 20	$1.83^{+0.04}_{-0.04}$	$11.5007^{+0.10}_{-0.10}$	$0.234957^{+0.003}_{-0.003}$	$0.373495^{+0.0079}_{-0.0079}$
J 0537-6910	52.122	BSk 21	$2.11^{+0.04}_{-0.05}$	$12.1875^{+0.094}_{-0.12}$	$0.25564^{+0.003}_{-0.003}$	$0.430441^{+0.0083}_{-0.01}$
J 0537-6910	52.122	APR	$2.05^{+0.04}_{-0.03}$	$12.0441^{+0.096}_{-0.072}$	$0.251328^{+0.003}_{-0.002}$	$0.417985^{+0.0083}_{-0.006}$

Table 2 Evaluated parameters for pulsar PSR J 0537-6910.

(11)

Assuming the continuity of metric function g_{tt}, g_{rr} and $\frac{dg_{tt}}{dr}$ at the boundary $(r = R)$, we get

$$e^{\nu(R)} = 1 - \frac{2M}{R} \tag{12}$$

$$e^{-\lambda(R)} = 1 - \frac{2M}{R} \tag{13}$$

$$\nu' e^{\nu(R)} = \frac{2M}{R^2} \tag{14}$$

This yields the gravitational mass of the pulsar as

$$M = \frac{3aR^3}{4}\left[\frac{1 + C(1 + 4aR^2)^{-\frac{1}{2}}}{1 + aR^2}\right] \tag{15}$$

Solving eqn.(12-14) and considering $p_{|r=R} = 0$, we get

$$A = \frac{\sqrt{R - 2M}}{3\sqrt{3}}\sqrt{\frac{\left(7M(R + r_s)^2 - R\left(4\pi m\rho_s R r_s^3 + 3R^2 + 6Rr_s + 3r_s^2\right)\right)^3}{R(2M - R)^3(R + r_s)^6}} \tag{16}$$

$$a = -\frac{-4\pi m\rho_s R^2 r_s^3 + MR^2 + 2MRr_s + Mr_s^2}{R^2\left(-4\pi m\rho_s R^2 r_s^3 + 7MR^2 + 14MRr_s + 7Mr_s^2 - 3R^3 - 6R^2 r_s - 3Rr_s^2\right)} \tag{17}$$

$$C = \frac{\left(4\pi m\rho_s R^3 r_s^3 - 8M^2(R + r_s)^2 + 3MR(R + r_s)^2\right)}{R\left(M(R + r_s)^2 - 4\pi m\rho_s R^2 r_s^3\right)}\sqrt{\frac{3R\left(-4\pi m\rho_s R r_s^3 + R^2 + 2Rr_s + r_s^2\right) - 3M(R + r_s)^2}{R\left(4\pi m\rho_s R r_s^3 + 3R^2 + 6Rr_s + 3r_s^2\right) - 7M(R + r_s)^2}} \tag{18}$$

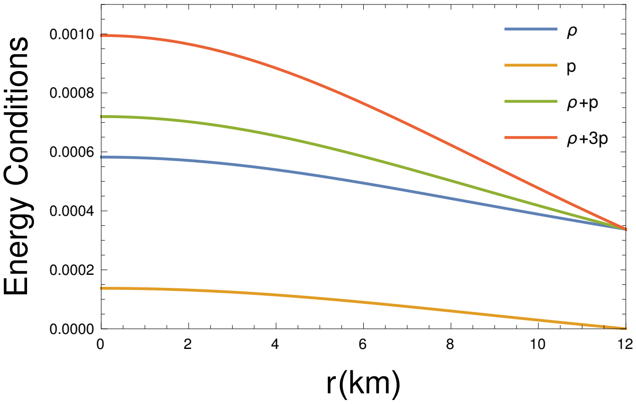

Fig. 3 Energy Condition variation with radial distance at the pulsar interior for $a = 0.00138889km^{-2}$, $C = 1.34164$, $m = 0.025$, $r_s = 20kpc$ and $\rho_s = 0.26GeV/c.c.$

3.4 Mass-Radius relation and Surface red-shift

Here, we have calculated the radial dependence gravitational mass function M(r) as

$$M(r) = 4\pi\int_0^r \rho_{eff} \ \ \tilde{r}^2 d\tilde{r} = \frac{3ar^3\left[1 + C(1 + 4ar^2)^{-\frac{1}{2}}\right]}{4\left(1 + ar^2\right)} \tag{19}$$

The mass function $M(r)$ has been plotted in Fig. 4. Therefore, the compactness of the pulsar can be written as

$$u(r) = \frac{M(r)}{r} = \frac{3ar^2\left[1 + C\left(1 + 4ar^2\right)^{-\frac{1}{2}}\right]}{4\left(1 + ar^2\right)} \tag{20}$$

Dark Matter Profile	Radius from model (km)	Compactness from model	RedShift from model
NFW	$10.8436^{+0.88}_{-0.98}$	$0.215152^{+0.027}_{-0.029}$	$0.324885^{+0.067}_{-0.064}$
URC	$9.45519^{+0.81}_{-0.90}$	$0.246746^{+0.029}_{-0.033}$	$0.405098^{+0.089}_{-0.083}$
SIS	$10.0366^{+0.84}_{-0.93}$	$0.232452^{+0.028}_{-0.031}$	$0.367048^{+0.078}_{-0.074}$

Table 3 Comparison between the parameters evaluated for different dark matter profiles (pulsar PSR J 0045-7319).

The surface red-shift corresponding to the compactness can be written as

$$Z_s = \frac{1}{\sqrt{1-2u}} - 1 = \frac{1}{\sqrt{1 - \frac{3ar^2\left(\frac{C}{\sqrt{4ar^2+1}}+1\right)}{2(ar^2+1)}}} - 1 \quad (21)$$

From Fig.5, we see that the compactness $u(r) = \frac{M(r)}{r}$ is an increasing function of radial parameter. In our model, the maximum value of $u(r)$ is 0.25 which satisfies Buchdahl limit $[\frac{M(r)}{r} < \frac{4}{9}]$ Buchdahl (1959). The gravitational red-shift at the surface comes out as $Z_s = 0.414214$ as shown in Fig.6, which is lesser than the allowed maximum limit $(Z_s \leq 0.85)$ Haensel (2000).

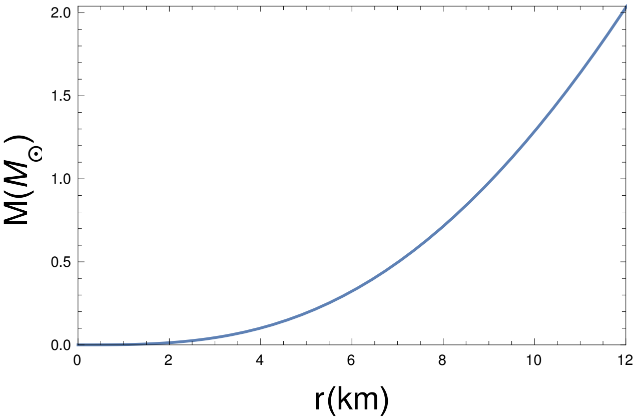

Fig. 4 Radial dependence Mass function, $M(r)$ for $a = 0.00138889km^{-2}$, $C = 1.34164$, $m = 0.025$, $r_s = 20kpc$ and $\rho_s = 0.26GeV/c.c.$

3.5 TOV Equation

For isotropic stellar body, the generalized TOV equation is written as

$$\frac{dp}{dr} + \frac{1}{2}\nu'(\rho + p) = 0 \quad (22)$$

The stable equilibrium condition has been found[see Fig.7] under gravitational force, F_g and hydrostatic force, F_h of the stellar body by the following equation

$$F_h + F_g = 0 \quad (23)$$

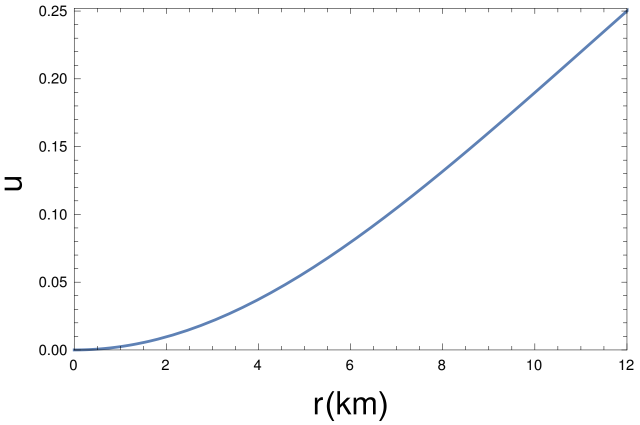

Fig. 5 Radial dependence Compactness $(u(r))$ for $a = 0.00138889km^{-2}$, $C = 1.34164$, $m = 0.025$, $r_s = 20kpc$ and $\rho_s = 0.26GeV/c.c.$

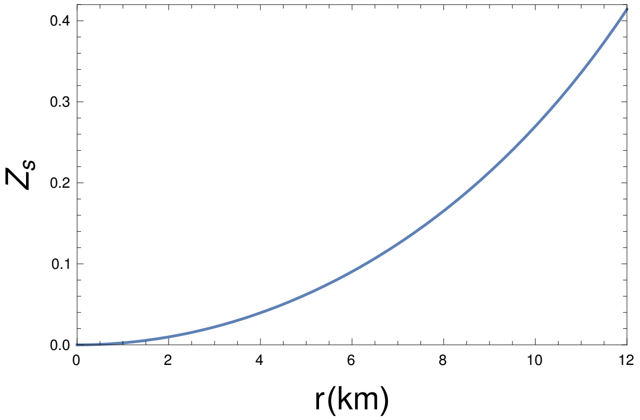

Fig. 6 Radial dependence Red-shift (Z_s) for $a = 0.00138889km^{-2}$, $C = 1.34164$, $m = 0.025$, $r_s = 20kpc$ and $\rho_s = 0.26GeV/c.c.$

where

$$F_g = -\tfrac{1}{2}\nu'(\rho + p) \quad (24)$$

$$F_h = -\tfrac{dp}{dr} \quad (25)$$

3.6 Speed of Sound and Adiabatic Index

Here, we have seen that our dark matter admixed pulsar model has satisfied (see the Fig. 8) the speed of sound,

$0 \leq v^2 = (\frac{dp}{d\rho}) \leq 1$ condition Herrera (1992); Abreu et al. (2007).

The infinitesimal radial adiabatic perturbation has to be checked if one needs to tune the model further. This concept was introduced by Chandrasekhar Chandrasekhar (1964). Later Bardeen et al., Knusten, Harko and Mak Bardeen et al. (1966); Knutsen (1988); Mak & Harko (2013) has used this stability condition for several astrophysical cases. For the radial stability, the adiabatic index for stellar body should be $\gamma = \frac{\rho+p}{p}\frac{dp}{d\rho} > \frac{4}{3}$. We see from Fig.9 that $\gamma > \frac{4}{3}$ through out the stellar interior. Therefore, we can say that our dark matter admixed pulsar model is also stable under radial perturbation.

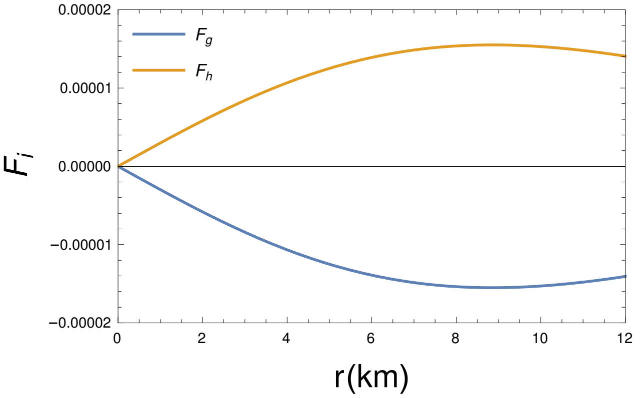

Fig. 7 Radial dependence of Gravitational force(F_g) and Hydrostatic force(F_h) for $a = 0.00138889 km^{-2}$, $C = 1.34164$, $m = 0.025$, $r_s = 20kpc$ and $\rho_s = 0.26 GeV/c.c.$

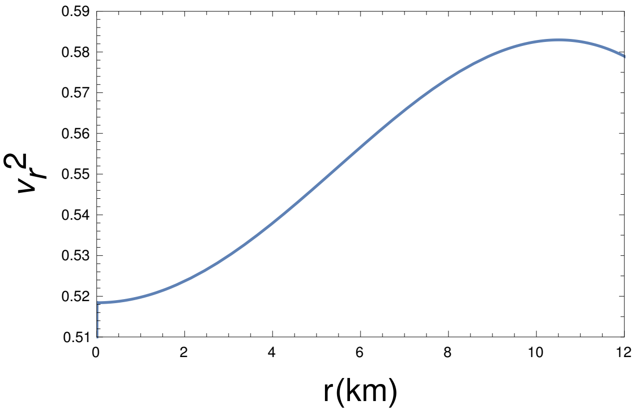

Fig. 8 Radial dependence of Sound Velocity (V_r^2) for $a = 0.00138889 km^{-2}$, $C = 1.34164$, $m = 0.025$, $r_s = 20kpc$ and $\rho_s = 0.26 GeV/c.c.$

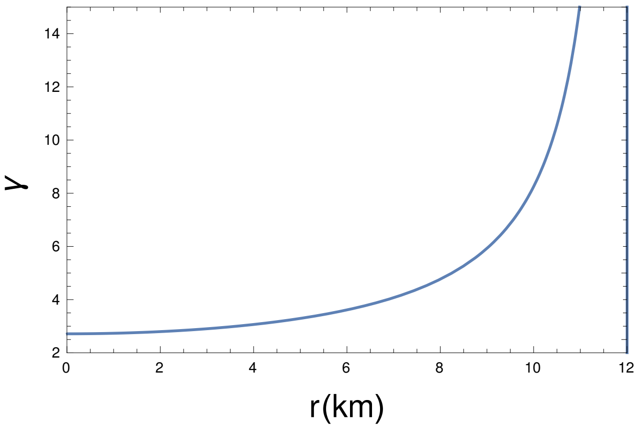

Fig. 9 Radial dependence of Adiabatic Index (γ) for $a = 0.00138889 km^{-2}$, $C = 1.34164$, $m = 0.025$, $r_s = 20kpc$ and $\rho_s = 0.26 GeV/c.c.$

4 Discussion and concluding remarks

In our previous study Molla et al. (2020); Rahman et al. (2020),we have considered a two-fluid pulsar model, assuming it to be made of ordinary matter admixed with dark matter.We have investigated the dark matter based on -

(i)the Singular Isothermal Sphere (SIS) profile for pulsars in the galactic halo region of different galaxiesMolla et al. (2020).

(ii) the Universal Rotational Curve (URC) profile for pulsars in the galactic halo region of Milky Way galaxyRahman et al. (2020).

Here we have investigated the dark matter based on the Navarro-Frenk-White(NFW) Navarro et al. (1996) dark matter density profile(as it is quite acceptable dark matter model in the disk region of the galaxy) for pulsars located in disk region of the Milky Way galaxy.

In this article, as earlier, we have also considered the two-fluid dark matter admixed pulsar model and the interior spacetime of the pulsar is described by Heintzmann IIa metric. Being inspired with the work done by some astrophysicist Wang et al. (2012); Leung et al. (2012); Panotopoulos & Lopes (2017b); Leung et al. (2011); (2017); Sandin & Ciarcelluti (2009); Mukhopadhyay et al. (2017); Rezaei (2017, 2018); Takisa et al. (2020), we have investigated about the existence of dark matter admixed pulsar in the disk region of the Milky Way galaxy. We see that the density and pressure at the interior of the pulsar are well behaved (Fig. 1,2).Pressure and density are both maximum at the centre and monotonically decreases towards the boundary. Here, we assume the value of the constants ($a = 0.00138889 km^{-2}$, $C = 1.34164$,$m = 0.025$,$r_s =$

Equation of State	Dark Matter Profile	Radius from model (km)	Compactness from model	RedShift from model
BSk 20	NFW	$11.5007^{+0.10}_{-0.10}$	$0.234957^{+0.003}_{-0.003}$	$0.373495^{+0.0079}_{-0.0079}$
BSk 20	URC	$10.0604^{+0.09}_{-0.09}$	$0.268595^{+0.003}_{-0.003}$	$0.469936^{+0.0107}_{-0.0106}$
BSk 20	SIS	$10.6601^{+0.096}_{-0.097}$	$0.253485^{+0.003}_{-0.003}$	$0.424174^{0.009}_{-0.009}$
BSk 21	NFW	$12.1875^{+0.094}_{-0.12}$	$0.25564^{+0.003}_{-0.003}$	$0.430441^{+0.0083}_{-0.01}$
BSk 21	URC	$10.698^{+0.088}_{-0.11}$	$0.291234^{0.003}_{-0.004}$	$0.547587^{+0.012}_{-0.014}$
BSk 21	SIS	$11.3142^{+0.090}_{-0.11}$	$0.275374^{+0.003}_{-0.004}$	$0.491953^{+0.0101}_{-0.0124}$
APR	NFW	$12.0441^{+0.096}_{-0.072}$	$0.251328^{+0.003}_{-0.002}$	$0.417985^{+0.0083}_{-0.006}$
APR	URC	$10.5645^{+0.089}_{-0.067}$	$0.286529^{+0.003}_{-0.002}$	$0.530439^{+0.0114}_{-0.008}$
APR	SIS	$11.1774^{+0.091}_{-0.069}$	$0.270817^{+0.003}_{-0.002}$	$0.477046^{+0.0099}_{-0.007}$

Table 4 Comparison between the parameters evaluated for different dark matter profiles (pulsar PSR J 0537-6910).

Dark Matter Profile	a (in km^{-2})	C	m	K	r_s (in kpc)	ρ_s (in $\frac{GeV}{c.c.}$)	r_0 (in kpc)	ρ_0 (in $\frac{gm}{c.c.}$)
NFW	0.00138889	1.34164	0.025	$*$	20	0.26	$*$	$*$
URC	0.003	0.8	0.1	$*$	$*$	$*$	9.11	5×10^{-24}
SIS	0.002	1.14	0.01	10^{-7}	$*$	$*$	$*$	$*$

Table 5 The values of the metric parameters and dark matter parameters used in different star modellings.

$20kpc$ and $\rho_s = 0.26GeV/c.c.$) in such a way that all physical required conditions must satisfy. Kindly note that the value of r_s and ρ_s taken here are applicable for Milky Way galaxy and we have checked our model with the pulsars namely PSR J0045-7319 and PSR J0537-6910 located in the disk region of Milky Way galaxy.

Our pulsar model also satisfies all the energy conditions, generalized TOV equation. From the mass function (eqn. 19), all desired interior features of a pulsar can be evaluated which satisfies Buchdahl mass-radius relation ($\frac{2M}{R} < \frac{8}{9}$) (Figs. 4, 5). Fig.6 shows that the value of surface gravitational red-shift, $Z_s = 0.414214$, which is much less than the maximum allowed value ($Z_s \leq 0.85$)Haensel (2000). From our mass function graphs Fig.4-5,eqn.(19)-(21) and Fig. 6, we have obtained the radii, compactness and surface red-shift of pulsars namely PSR J0045-7319 and PSR J0537-6910 located in the disk region of the Milky Way galaxy. The detail evaluated chart is shown in Table 1,2.

Now, in comparison, if we use URC dark matter density profile ($\rho_d = \frac{\rho_0 r_0^3}{(r+r_0)(r^2+r_0^2)}$)[by taking metric constants $a = 0.003km^{-2}$, $C = 0.8$, $m = 0.1$ and core radius,$r_0 = 9.11$kpc, effective core density,$\rho_0 = 5 \times 10^{-24} \left(\frac{r_0}{8.6kpc}\right)^{-1} gm/c.c.$Rahman et al. (2020)] or SIS dark matter density profile ($\rho_d = \frac{K}{2\pi G r^2}$)[by taking velocity dispersion, $K = 10^{-07}$ (as for spiral galaxies $K \sim 10^{-07}$ Molla et al. (2020))and metric constants $a = 0.002km^{-2}$, $C = 1.14$ and $m = 0.01$] instead of NFW density profile and applied it to the same pulsars, located in Milky Way galaxy, we have got the comparison chart shown in Table 3,4 and Table 5.

If we compare Table 3 and Table 4, we see that pulsars radii are minimum in URC profile, whereas it is maximum in NFW profile (due to change in mass function graph) and as a result of that compactness, surface red-shift are highest in URC profile and lowest in NFW profile (since compactness and red-shift are directly depend on the radius of the star). Moreover, it is to be mentioned that for using URC profile or NFW profile, core radius/scale radius and effective core density/effective density of Milky Way galaxy are known to us. But for other galaxies, these essential parameters are not known. Therefore, we cannot investigate the pulsars located in other galaxies by using URC/NFW profile. However, in SIS model, we will be able to study the pulsars located in different galaxies since K (velocity dispersion) value of dwarf galaxies and spiral galaxies are available to us (can be calculated from the fitting of the rotation curves of the SPARC sample of galaxiesLelli et al. (2016)).Although, Navarro-Frenk-White(NFW) density profile is a well accepted dark matter model particularly for the disk regions of the galaxyNavarro et al. (1996).

Therefore, our result confirms the possible existence of dark matter admixed pulsars (by using NFW density profile), located in the disk region of the Milky Way galaxy.Earlier we have got the similar results for the pulsars based on URC (Universal Rotation Curve) density profile, located in the Milky Way galaxy Rahman et al. (2020)and SIS (Singular Isothermal Sphere) density profile, located in dwarf and massive spiral galaxies Molla et al. (2020). Hence, we may conclude with a big hope of possible existence of dark matter admixed pulsars present in all the region of the galaxies.

5 Funding

NR is thankful to UGC MANF (MANF-2018-19-WES-96213) for providing financial support. MM is thankful to CSIR (Grand No.-09/1157(0007)/ 2019-EMR-I) for providing financial support. MK and SM is grateful to the Inter-University Centre for Astronomy and Astrophysics (IUCAA), Pune, India for providing Associateship programme.

6 Author Contribution

Nilofar Rahman: Writing-Original draft preparation, Software, Validation, prepared Graphs. Masum Murshid: Software, Validation, Formal Analysis, prepared Table- 4 and 5. Sajahan Molla: Software, Validation, Formal Analysis, prepared Table-1, 2 and 3. Mehedi Kalam: Conceptualization, Supervision, Writing- Reviewing and Editing, Visualization, Investigation.

7 Data Availability Statement

This manuscript has no associated data or the data will not be deposited. We have not used any data in this paper. All the figures in this paper are generated analytically and numerically using Mathematica.

References

S. Molla, B. Ghosh, M. Kalam, Eur.Phys. J. Plus **135**, 362 (2020)

N. Rahman, S. Molla, M. Kalam, Eur.Phys. J. Plus **135**, 637 (2020)

D. Leahy, R. Ouyed, Mon. Not. R. Astron. Soc. **387**, 1193 (2008)

P. Haensel, Z.L. Zdunik, R. Schaeffer, Astron. Astrophys. **160**,121,(1986)

C. Alcock, E. Farhi, A. Olinto, Astrophys. J. **310**, 261 (1986)

E. Farhi, R.L. Jaffe, Phys. Rev. D **30**, 2379 (1984)

S. Postnikov, M. Prakash, J.M. Lattimer, Phys. Rev. D **82**, 024016 (2010)

M. Dey, I. Bombaci, J. Dey, S. Ray, B.C. Samanta, Phys. Lett. B **438**, 123 (1998)

J.M. Lattimer, M. Prakash, Phys. Rep. **442**, 109 (2007)

F. *Ö*zel, Nature **441**, 1115 (2006)

F. *Ö*zel, T. G*Ö*ver, D. Psaitis, Astrophys. J **693**, 1775 (2009)

F. *Ö*zel, D. Psaitis, Phys. Rev. D **80**, 103003 (2009)

F. *Ö*zel, G. Baym, T. G*Ö*ver, Phys. Rev. D **82**, 101301 (2010)

T. G*Ö*ver, F. *Ö*zel, A. Cabrera-Lavers, Astrophys. J **712**, 964 (2010a)

T. G*Ö*ver, P. Wroblewski, L. Camarota, F. *Ö*zel, Astrophys. J **719**, 1807 (2010b)

S.R. Heap, M.F. Corcoran, Astrophys. J. **387**, 340 (1992)

J.M. Lattimer, M. Prakash, Phys. Rev. Lett. **94**, 111101 (2005)

D. Stickland, C. Lloyd, A. Radzuin-Woodham, MNRAS **286**, L21 (1997)

J.A. Orosz, E. Kuulkers, MNRAS **305**, 132 (1999)

J.H. Van Kerkwijk, J. van Paradijis, E.J. Zuiderwijk, A & A **303**, 497 (1995)

F. Rahaman et al., Gen. Relativ. Gravit. **44**, 107 (2012a)

F. Rahaman et al., Eur. Phys. J. C **72**, 2071 (2012b)

M. Kalam et al., Eur. Phys. J. C **72**, 2248 (2012)

M. Kalam et al., Int. J. Theor. Phys. **52**, 3319 (2013a)

M. Kalam et al., Eur. Phys. J. C **73**, 2409 (2013b)

M. Kalam et al., Eur. Phys. J. C **74**, 2971 (2014a)

M. Kalam et al., Astrophys. Space Sci. **349**, 865 (2014b)

M. Kalam et al., Mod. Phys. Lett. A **31**(40), 1650219 (2016)

M. Kalam et al., Mod. Phys. Lett. A **32**(4), 1750012 (2017)

M.A.K. Jafry et al., Astrophys. Space Sci. **362**(10), 188 (2017)

Sk M Hossein et al., Int. J. Mod. Phys. D **21**, 1250088 (2012)

F. Lobo, Class. Quantum. Grav. **23**, 1525 (2006)

K. Bronnikov, J.C. Fabris, Phys. Rev. Lett. **96**, 251101 (2006)

S. K. Maurya, Y.K. Gupta, Baiju Dayanandan, Saibal Ray, Eur. Phys. J. C **76**, 266 (2016)

B. Dayanandan, S.K. Maurya, Y.K. Gupta, T.T. Smitha, Astrophys Space Sci. **361**, 160 (2016)

S.D. Maharaja, J.M. Sunzub, S. Ray, Eur. Phys. J. Plus **129**, 3 (2014)

S. Ngubelanga, S.D. Maharaj, S. Ray, Astrophys. Space Sci. **357**, 74 (2015)

B.C. Paul, P.K. Chattopadhyay, S. Karmakar, Astrophys. Space Sci. **356**, 327 (2015)

N. Pant, N. Pradhan, M.H. Murad, Int. J. Theor. Phys. **53**, 11 (2014)

P. Bhar, K. N. Singh, N. Sarkar, F. Rahaman, Eur. Phys. J. C **77**, 9 (2017)

M. Kalam, *et al.*, Res.Astron.Astrophys. **18**, No.3, 025 (2018)

S. Molla, *et al.*, Res.Astron.Astrophys. **19**, No.2, 026 (2019)

R. Islam, *et al.*, Astrophys.Space Sci. **364**,No.7, 112 (2019)

S.H.Hendi, *et al.*, JCAP **09**, 013 (2016)

S.H.Hendi, *et al.*, JCAP **07**, 004 (2017)

B. Eslam Panah, *et al.*, Astrophys.J. **848**,No. 1, 24 (2017)

F. Zwicky, Helv. Phys. Acta **6**, 110 (1933)

F. Zwicky, Gen. Relativ. Gravit. **41**, 207 (2009)

V.C. Rubin, W.K. Ford Jr., Astrophys. J. **159**, 379 (1970)

K.A. Olive, arXiv:astro-ph/0301505

C. Munoz, Int. J. Mod. Phys. A **19**, 3093 (2004)

M. Taoso, G. Bertone, A. Masiero, J. Cosmol. Astropart. Phys. **03**, 022 (2008)

I. Lopes, J. Silk, Astrophys. J. **722**, L95 (2010)

C. Kouvaris, P. Tinyakov, Phys. Rev. D **82**, 063531 (2010)

S. Turck-Chièze, I. Lopes, Res. Astron. Astrophys. **12**, 1107 (2012)

I. Lopes, J. Silk, Astrophys. J. **786**, 25 (2014)

I. Lopes, K. Kadota, J. Silk, Astrophys. J. Lett. **780**, 2 (2014)

R. Brito, V. Cardoso, H. Okawa, Phys. Rev. Lett. **115**, 111301 (2015)

R. Brito, V. Cardoso, C.F.B. Macedo, H. Okawa, C. Palenzuela, Phys. Rev. D **93**, 044045 (2016)

A. Martins, I. Lopes, J. Casanellas, Phys. Rev. D **95**, 023507 (2017)

A. Li, F. Huang, R.X. Xu, Astropart. Phys. **37**, 70 (2012)

X. Li, F. Wang, K.S. Cheng, J. Cosmol. Astropart. Phys. **10**, 031 (2012)

G. Panotopoulos, I. Lopes, Phys. Rev. D **96**, 023002 (2017a)

G. Narain, J. Schaffner-Bielich, I.N. Mishustin, Phys. Rev. D **74**, 063003 (2006)

S.C. Leung, M.C. Chu, L.M. Lin, Phys. Rev. D **85**, 103528 (2012)

P. Mukhopadhyay, J. Schaffner-Bielich, Phys. Rev. D **93**, 083009 (2016)

G. Panotopoulos, I. Lopes, Phys. Rev. D **96**, 083004 (2017b)

S.C. Leung, M.C. Chu, L.M. Lin, Phys. Rev. D **84**, 107301 (2011)

D.N. Spergel, P.J. Steinhardt, Phys. Rev. Lett. **84**, 3760 (2000)

A.V. Maccio et al., Astrophys. J. Lett. **744**, L9 (2012)

G.Castignani,N. Frusciante, D. Vernieri, P. Salucci, Nat. Sci **4**, 265 (2012)

J.F. Navarro, C.S. Frenk and S.D.M. White, Astrophys. J. **462** ,563 (1996)

M. Razeira and A. Mesquita, Int. J. Mod. Phys. E **20**,109-116 (2011)

P.M. Takisa, S.D. Maharaj, L.L. Leeuw, Eur. Phys. J. C **79**, 8 (2019)

D.K. Matondo, S.D. Maharaj, S. Ray, Eur. Phys. J. C **78**, 437 (2018)

S.K. Maurya, Y.K. Gupta, S. Ray, S.R. Chowdhury, Eur. Phys. J. C **75**, 389 (2015)

S.K. Maurya, A. Banerjee, M.K. Jasim, J. Kumar, A.K. Prasad, A. Pradhan, Phys. Rev. D **99**, 044029 (2019)

B. Dayanandana, S.K. Maurya, T.T. Smitha, Eur. Phys. J. A **53**, 141 (2017)

S.K. Maurya, Eur. Phys. J. A **53**, 89 (2017)

S. Gedela, R.K. Bisht, N. Pant, Eur. Phys. J. A **54**, 207 (2018)

K.N. Singh, N. Pant, M. Govender, Chin. Phys. C **41**(1), 015103 (2017)

F. Rahaman, S.D. Maharaj, I.H. Sardar, K. Chakraborty, Mod. Phys. Lett. A **32**(8), 1750053 (2017)

M.K. Jasim, S.K. Maurya, Y.K. Gupta, B. Dayanandan, Astrophys. Space Sci. **361**, 352 (2016)

P.M. Takisa, S.D. Maharaj, Astrophys. Space Sci. **361**, 262 (2016)

S.K. Maurya, M.K. Jasim, Y.K. Gupta, T.T. Smitha, Astrophys. Space Sci. **361**, 163 (2016)

K.N. Singh, N. Pradhan, N. Pant, Pramana J. Phys. **89**, 23 (2017a)

H. Heintzmann, Z. Phys.**228**, 489 (1969)

J. Barranco et al., arXiv:1301.6785V1 [astro-ph.CO]

H.A. Buchdahl, Phys. Rev. **116**, 1027 (1959)

P. Haensel, J.P. Lasopa, J.L. Zdunik, Nucl. Phys. Proc. Suppl. **80**, 1110 (2000)

L. Herrera, Phys. Lett. A **165**, 206 (1992)

H. Abreu, H. Hernandez, L.A. Nunez, Class. Quantum. Gravit. **24**, 4631 (2007)

S. Chandrasekhar, Astrophys. J. **140**, 417 (1964)

J.M. Bardeen, K.S. Thorne, D.W. Meltzer, Astrophys. J. **145**, 505 (1966)

H. Knutsen, MNRAS **232**, 163 (1988)

M.K. Mak, T. Harko, Eur. Phys. J. C **73**, 2585 (2013)

James M. Lattimer, Ann.Rev.Nucl.Part.Sci. **62** (2012) 485-515 e-Print: 1305.3510 [nucl-th]

W. C. G. Ho, C. M. Espinoza, D. Antonopoulou, N. Andersson, Pinning down the superfluid and measuring masses using pulsar glitches. Sci. Adv. 1, e1500578 (2015).

G. Panotopoulos, I. Lopes, Phys. Rev. D **96**, 083013 (2017)

F. Sandin, P. Ciarcelluti, Astropart. Phys. **32**, 278 (2009)

S. Mukhopadhyay, D. Atta, K. Imam, D.N. Basu, C. Samanta, Eur. Phys. J. C **77**, 440 (2017)

Z. Rezaei, Astrophys. J. **835**, 33 (2017), arXiv:1612.02804 [astro-ph.HE]

Z. Rezaei, Int. J. Mod. Phys. D **27**, 1950002 (2018), arXiv:1807.01781 [astro-ph.HE]

P.M. Takisa, L.L. Leeuw, S.D. Maharaj, Astrophys. Space Sci. **365**, 164 (2020)

F. Lelli, S.S. McGaugh and J.M. Schombert, Astrophys. J. **152**, 157 (2016)

This manuscript was prepared with the AAS LATEX macros v5.2.