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## Research Article

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# The Effect of Far-field Cylindrical Scatterers on Radio Direction Finding and Beamforming Estimated by Cylindrical Array of Dipoles

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**Abstract:** Direction finding using a cylindrical array of dipoles has been implemented in presence of large cylindrical scatterers. The effect of the large cylindrical scatterers is considered by cylindrical harmonic expansion of the scattered field. These fields are computed analytically. The direction finding process is verified by a comparison with phase difference direction finding method implemented in HFSS simulator. The error induced in the direction finding mechanism by the scattered fields is estimated through Monte-Carlo simulations. Interfering signals are introduced and a beamforming process is implemented to evaluate the corresponding signal to interference ratios. The method alleviates the use of high computational cost simulations for scatterer effect estimation. The method could be used in wireless communication and smart antenna applications.

**Keywords:** Scattering, Far-field, Direction Finding, MUSIC Algorithm, Beamforming, Mutual coupling, 3D dipole array, Monte-Carlo simulation.

## 1. Introduction

The smart antennas are widely established in different wireless systems to enhance the spectrum efficiency by increasing the signal to noise ratio. The interest in smart antennas for airplane and aerospace applications such as localization, tracking, navigation and surveillance systems is growing increasingly [1]. For this purpose, the radio direction finding and the beamforming methods were studied to improve the capabilities of antenna arrays in different airplane and airport services [2], [3], [4]. In a more realistic situation such as airports, the effects of the scatterer in the presence of the mutual coupling between the antenna elements are not negligible and it should be considered in the corresponding models [5]. A maximum likelihood DOA (Direction of Arrival) estimator was established in the presence of a local scatterer without considering the mutual coupling effects in [6]. The studies in [7] and [8] proposed an iterative algorithms based on the self-calibration methods for direction finding to remove the effects of the mutual coupling and the 3D scatterer. These iterative methods had a limited application where the time delay/computation time is important. As the electrical size or the number of scatterers are increased, the efficiency of the algorithms in [7] and [8] are decreased. The issue of the far-field large scatterer has not been addressed in [7] or [8]. In [8] the authors established a calibration method to mitigate the multipath signal components in presence of scatterer for direction finding and localization. The method in [8] was a time consuming method due to its numerical electromagnetic tools, especially when the number of scatterers were increased. In addition, the authors in [9] simulated the problem to localize its near-field multi-paths without any far-field scattering components. The effects of large far-field scatterer in DOA estimation for antenna arrays has not been studied as well.

The effects of the coherent local scattering in adaptive beamforming were considered as a mismatch in construction of the IPN (Interference-Plus-Noise) covariance matrix in [10]. The proposed IPN covariance matrix in the algorithm of [10] consisted of the two parts. The first part was obtained by Capon spectrum estimator integrated over a region separated from the direction of the desired signal and the second part was

acquired by removing the desired signal component from the sample covariance matrix [10]. The approach to combine the two parts to obtain a good performance in different situations is also an open topic for the researchers. In [11] the authors conducted a cost-effective low rank algorithm for designing a robust adaptive beamforming and the presence of local scatterer was modelled by steering vector mismatches.

The effect of a cylindrical reflector/scatterer at the proximity of three dimensional cylindrical array was formulated for direction finding and beamforming algorithms in the presence of mutual coupling effects [12].

In this paper the effect of large far-field scatterers at the proximity of the antenna array in presence of mutual coupling is presented. In this model the scattered field is analytically formulated in terms of the cylindrical harmonics and then the total field is computed for direction finding and beamforming. In an earlier study, the scatterer problem was approximated as a cylinder and the cylindrical harmonic expansion was used for modelling the scattered signal [7]. As mentioned above the algorithm in [7] was iterative and did not guarantee the convergence. It also required a large number of antennas and examined only for near-field scatterers.

In this paper a number of large cylindrical scatterers are considered and using an array with a limited number of elements the problem of direction finding and beamforming is addressed using the standard MUSIC algorithm in presence of mutual coupling. Finally, the direction finding error and the signal to interference ratio is estimated.

In the following section we present the algorithms of direction finding and beamforming in presence of large cylindrical scatterers and we compare the direction finding results with the HFSS simulations. For this purpose, we have implemented a direction finding process using two orthogonal dipole arrays within the HFSS simulator. In section 3 the RMSE (Root Mean Square Error) values of the direction finding and the SIR (Signal to Interference ratio) values of the beamforming in the presence of the scatterers and mutual coupling effects are computed for a 3D geometry. Section 4 draws the conclusion.

## 2. Methods to Implement the Direction Finding and the Beamforming Algorithms in Presence of Large Cylindrical Scatterers

In this section, we first consider the model for scattering of a plane wave by a number of parallel large circular cylinders. Each cylinder is defined by its radius and axis coordinate  $(\rho'_i, \varphi'_i)$  with respect to the global cylindrical system  $(\rho, \varphi)$  as shown in Fig.1. Let consider a number of  $M$  cylindrical scatterers [13] and the receiving array is considered at the origin of the coordinates.

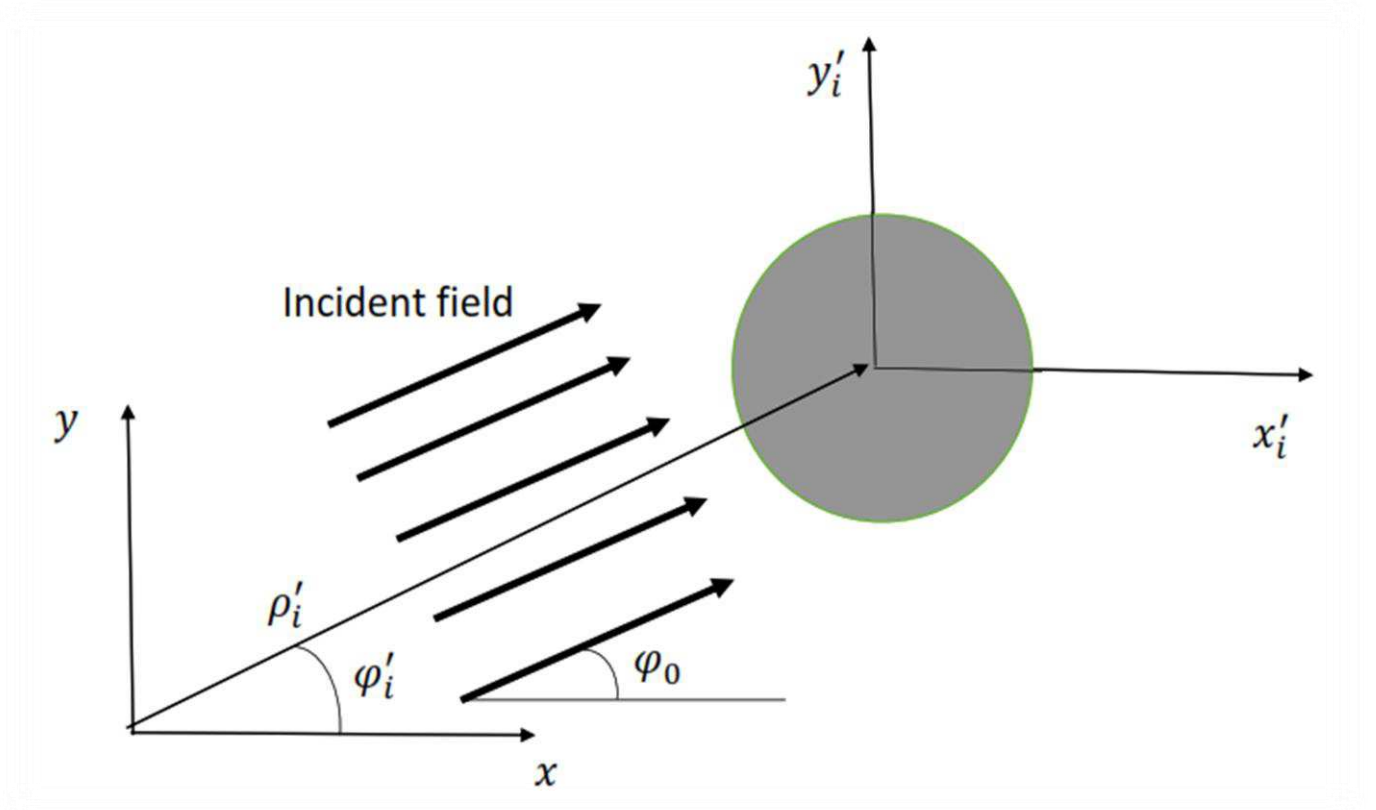


Fig.1. The cross section of circular cylinder “i”

Considering the TMz incident field, the incident electric field of a plane wave on the cylinder “i” is expressed as an expansion of Bessel functions in the  $(\rho_i, \varphi_i)$  cylindrical coordinate system as [13]:

$$E_z^{inc}(\rho_i, \varphi_i) = E_0 e^{jk_0 \rho_i' \cos(\varphi_i' - \varphi_0)} e^{jk_0 \rho_i \cos(\varphi_i - \varphi_0)} = E_0 e^{jk_0 \rho_i' \cos(\varphi_i' - \varphi_0)} \sum_{n=-\infty}^{n=+\infty} j^{-n} J_n(k_0 \rho_i) e^{jn(\varphi_i - \varphi_0)} \quad (1)$$

Where  $k_0$  is the free space wave number and  $\varphi_0$  is the angle of incidence of the plane wave. The expression of the scattered field for perfectly conducting cylinder “i” is given by [13], [16].

$$E_{z,i}^s(\rho_i, \varphi_i) = E_0 e^{jk_0 \rho_i' \cos(\varphi_i' - \varphi_0)} \times \sum_{n=-\infty}^{n=+\infty} \frac{(-j)^{-n} J_n(k_0 a)}{H_n^2(k_0 a)} H_n^2(k_0 \rho_i) e^{jn(\varphi_i - \varphi_0)} \quad (2)$$

Where  $a$  is the radius of a large cylindrical scatterer in the far-field of the antenna array and it is considered to be small relative to the wavelength. After computing the scattered field by each of the cylinders one can replace the local coordinate  $(\rho_i, \varphi_i)$  by global coordinate  $(\rho, \varphi)$  using the following equations,

$$\begin{aligned} x &= \rho \cos(\varphi) - \rho_i' \cos(\varphi_i') \\ y &= \rho \sin(\varphi) - \rho_i' \sin(\varphi_i') \\ \rho_i &= \sqrt{x^2 + y^2} \\ \varphi_i &= \tan^{-1} \frac{y}{x} \end{aligned} \quad (3)$$

Considering  $M$  scatterers, at any point the total electric field is

$$E_z(\rho, \varphi) = E_z^{inc}(\rho, \varphi) + \sum_{i=1}^M E_{z,i}^s(\rho, \varphi) \quad (4)$$

Now for considering the effect of scatterers in direction finding and beamforming processes we should use the equation (4) for the total incident field on the antenna array.

For direction finding purpose a cylindrical array of 6 half-wave dipole antennas is considered in presence of a few large cylindrical scatterers in the far-field region. The distance between the antenna elements is  $\frac{\lambda}{2}$  and thus the radius of the cylindrical array ( $rc$ ) is  $\frac{\lambda}{2}$  according to (5).

$$r_c = \frac{\lambda/2}{2\sin(\pi/6)} \quad (5)$$

As we desire to have the scatterers in the far-field of the array and the scatterers to be adequately large, here the distance between the cylindrical scatterer and the centre of the cylindrical array is considered to be  $10\lambda$  and the height of the cylindrical scatterer is considered to be  $10\lambda$ . The geometry of the cylindrical array including one cylindrical scatterer is shown in Fig.2.

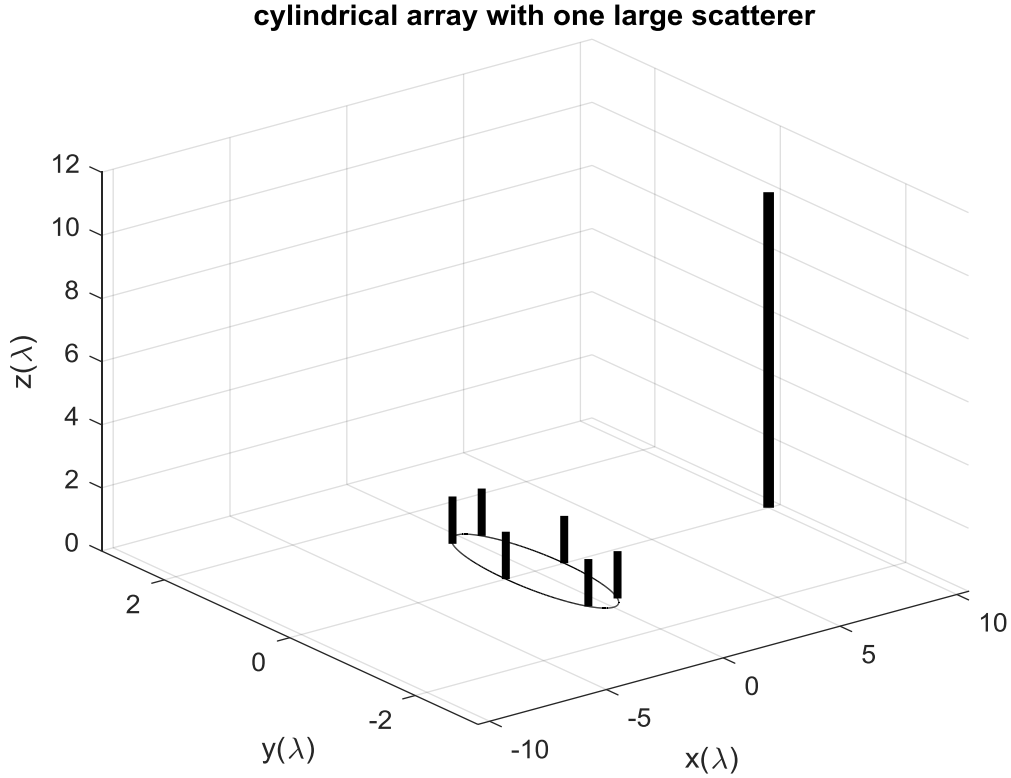


Fig.2. The geometry of the cylindrical array with a single cylindrical scatterer in this work

The search function of the modified MUSIC method (to perform the direction finding algorithm) as a function of the array steering vector  $\mathbf{a}(\theta, \varphi)$  can be expressed as (6) to include the mutual coupling effect [14].

$$\mathbf{P}_{MUSIC}(\theta, \varphi) = \frac{1}{|\mathbf{a}^H(\theta, \varphi)(\mathbf{Z}_0^{-1})^H \mathbf{E}_n \mathbf{E}_n^H \mathbf{Z}_0^{-1} \mathbf{a}(\theta, \varphi)|} \quad (6)$$

Here the direction finding algorithm is performed in presence of the incident field and the scattered field. In order to validate our model including the scatterer effect we compare our results with the HFSS simulations. For this purpose, we use the geometry of Fig.2 for direction finding with modified MUSIC algorithm. As an example the azimuth and elevation angles of the incident signal are considered to be  $\varphi = 140^\circ$  in horizontal plane ( $\theta = 90^\circ$ ). The number of snapshots is 100 and the value of SNR is 10 dB in simulation. The modified MUSIC Pseudospectrum as a function of azimuth angle for the defined geometry is shown in Fig.3.

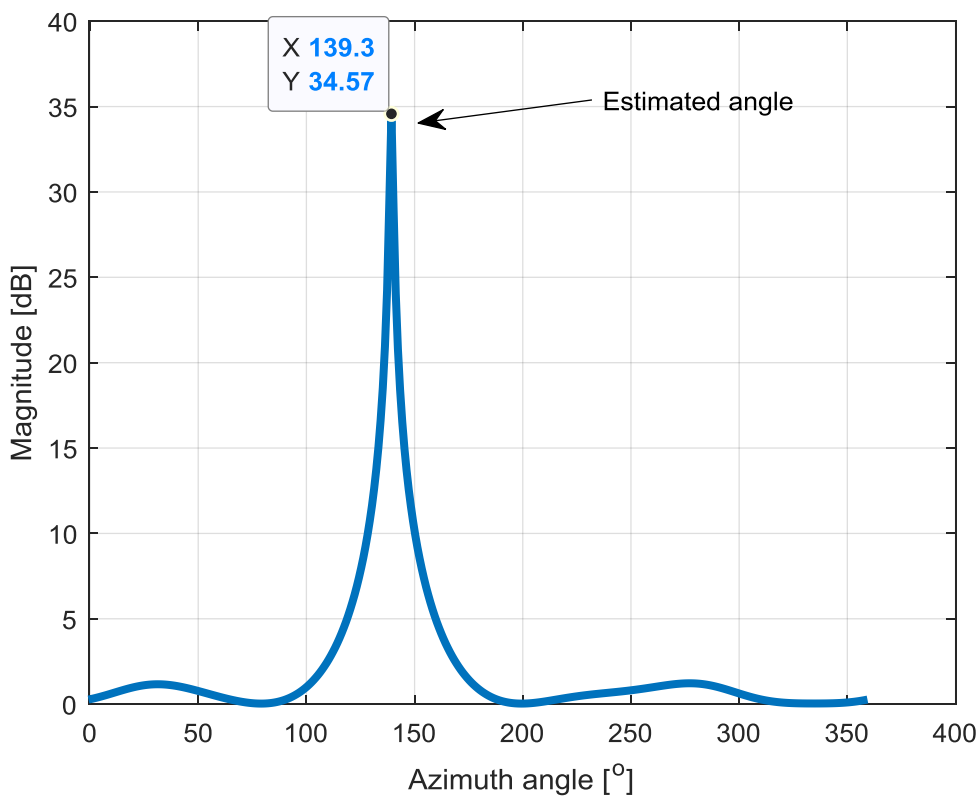


Fig.3. The magnitude of pseudospectrum function of MUSIC DOA algorithm as a function of azimuth angle with a single cylindrical scatterer

As shown in Fig.3 the proposed model estimates the azimuth direction of arrival angle properly in the presence of the far-field cylindrical scatterer. To verify this model, we implement two orthogonal array of dipoles and we estimate the angle of arrival by HFSS simulation, through phase comparison DF (Direction



Finding) as described in the following. For this purpose, we consider the geometry of the array with four half-wave dipole antennas in presence of the large far-field scatterer as in Fig.4. Here the centre frequency in HFSS simulations is equal to 1 GHz and the element's distances in each pair (for avoiding phase ambiguity) is considered to be  $0.35\lambda$ . The height and the position of the far-field scatterer are the same as before.

**Two orthogonal dipole arrays used for HFSS simulations with one scatterer**

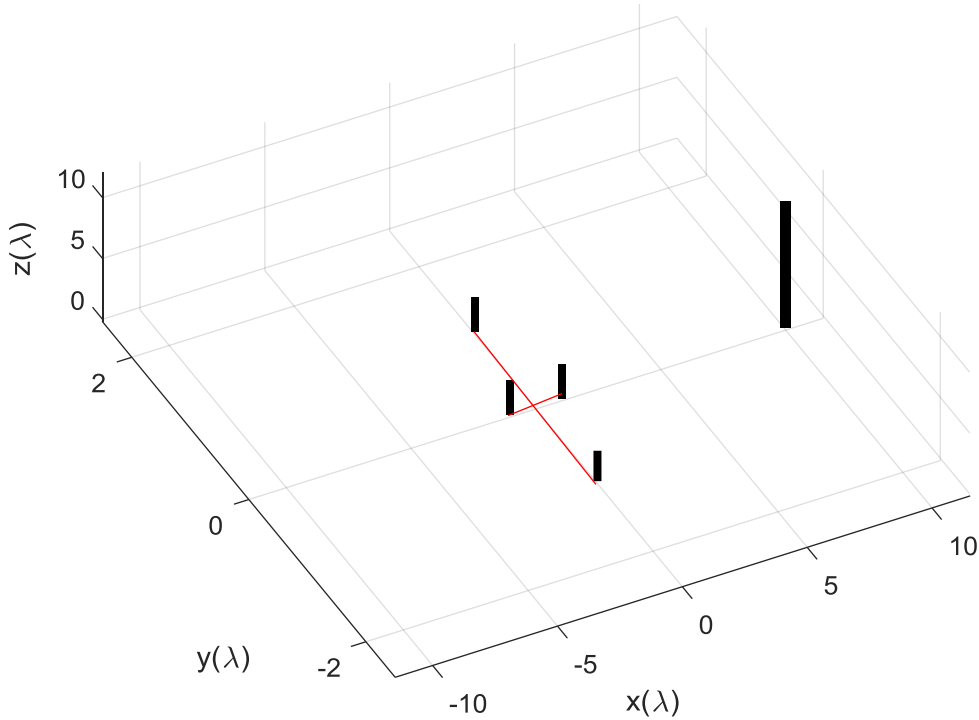


Fig.4. The geometry of the array with a single cylindrical scatterer and two orthogonal dipole arrays used for HFSS simulations

To estimate the direction of arrival (by phase comparison DF method) using the geometry of Fig.4 we first calculate the phase difference of the received signal for each dipole pair in the horizontal plane ( $\theta = 90^\circ$ ). For this array the phase difference between the received signal of the elements  $\Delta\varphi_{13}$  and  $\Delta\varphi_{24}$  are given by (7) and (8),

$$\Delta\varphi_{13} = \varphi_1 - \varphi_3 = kd_s \cos(\varphi_0) \quad (7)$$

$$\Delta\varphi_{24} = \varphi_2 - \varphi_4 = kd_s \sin(\varphi_0) \quad (8)$$

Where  $k$  is the wave-number of the incident signal,  $d_s$  is the dipoles baseline and the  $\varphi_0$  is the estimated azimuth angle. Using HFSS simulation results in equations (7) and (8), the azimuth angle of arrival is estimated about  $\varphi_0 = 137.12^\circ$  where the incident angle was at  $\varphi = 140^\circ$ .

To certify more our verifications we consider a number of incident angles  $\varphi_0 \in (0, \pi)$  for direction finding with proposed method compared to the phase difference method (implemented by HFSS). The estimated angle in each case was computed through both methods and the results of the estimated angles are depicted in Fig.5.

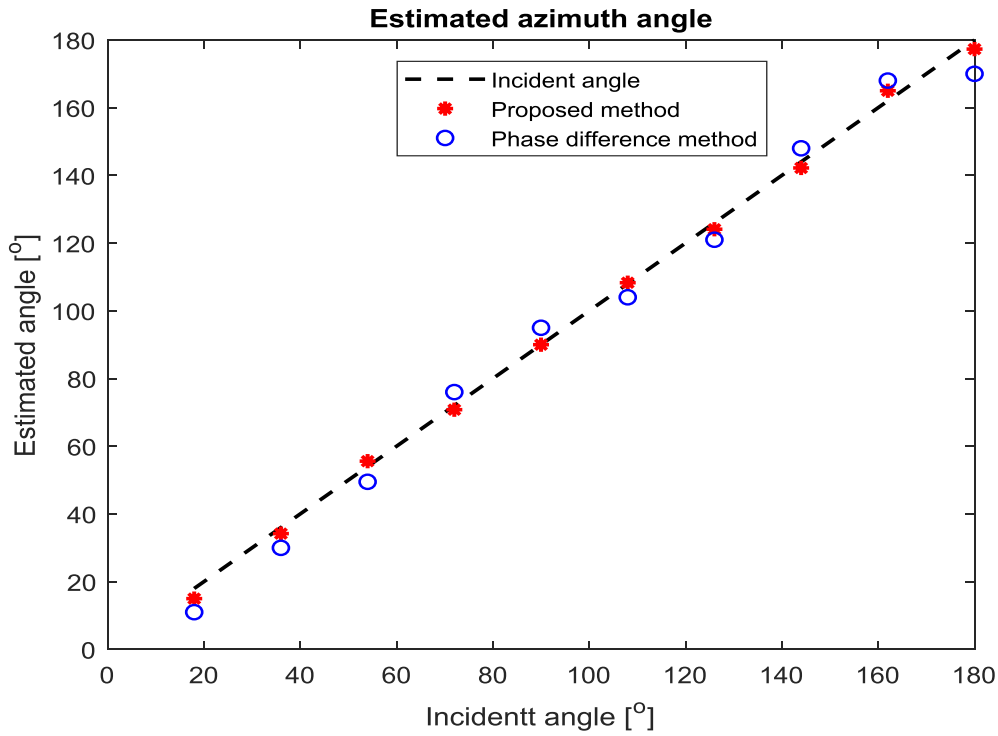


Fig.5. Estimated angle of arrivals versus the true incident angle with proposed method compared to the phase difference method (implemented by HFSS) with a single scatterer

The simulations show that the proposed method follows the HFSS simulations quite adequately. Now we consider a scenario with 3 far-field cylindrical scatterers as in Fig.6 and compare the angle of arrivals estimated by the proposed method with those of the phase difference method for a number of incident angles. The estimated angles are depicted in Fig.7.

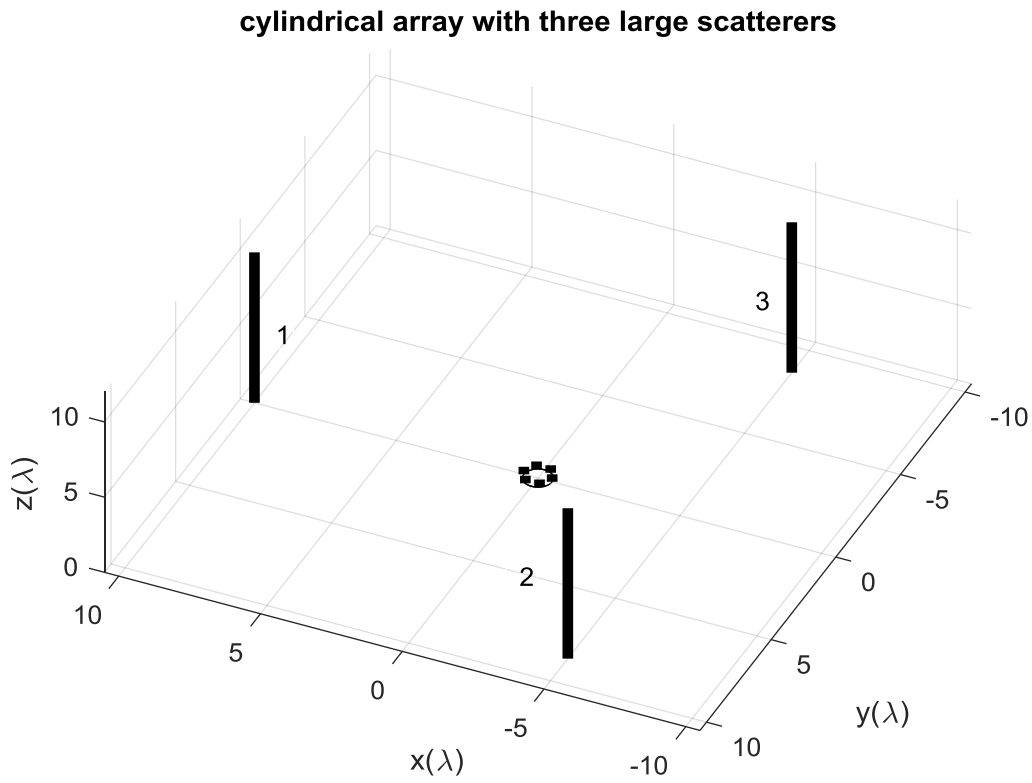


Fig.6. The geometry of cylindrical array with three cylindrical scatterers

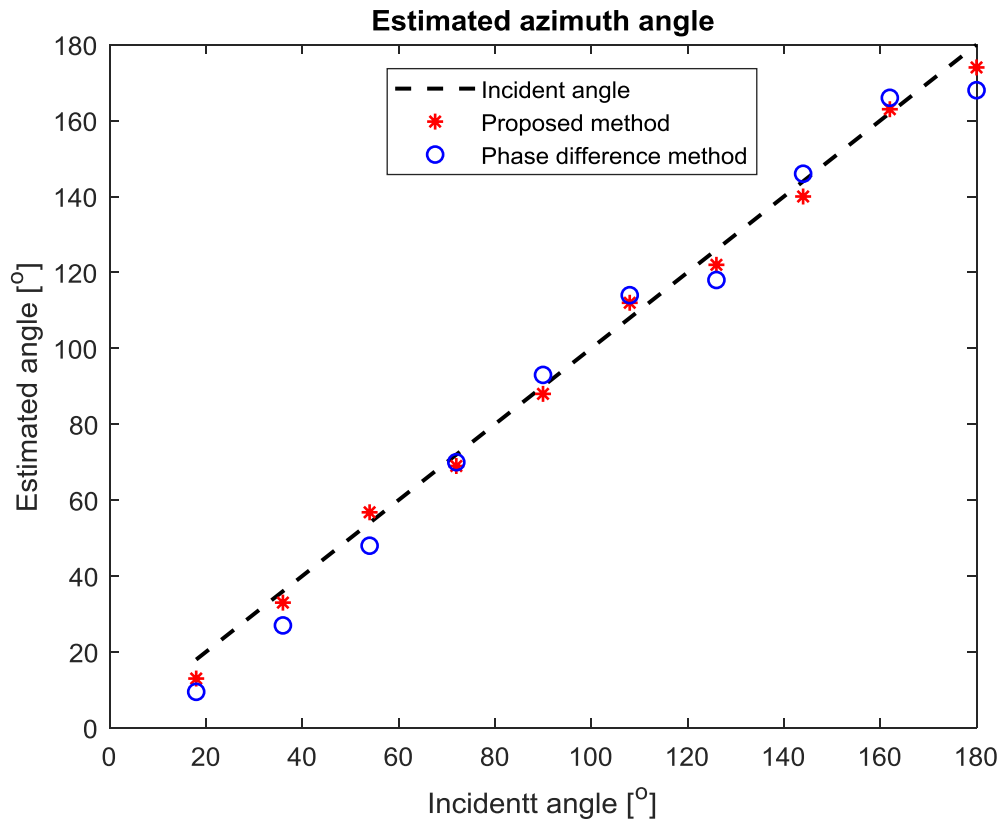


Fig.7. Estimated angle of arrivals versus the true incident angle with proposed method compared to the phase difference method (implemented by HFSS) with three far-field scatterers

As a scenario including an interfering signal, we consider the geometry of Fig.6 for direction finding with the modified MUSIC algorithm including the effects of far-field scatterers for azimuth and elevation angles of the signal and the interference as listed in Table 1.

Table 1 The sample azimuth and elevation angles of the signal and the interference for direction finding

Angle	Azimuth	Elevation
Signal	140	65
Interference	50	15

For these simulations the number of snapshot is considered as 100 and the value of the SNR is considered to be 10 dB. The scattered field by three cylindrical scatterers are combined in (4) to achieve the total electric field impinging on the array and the two-dimensional MUSIC Pseudospectrum as a function of azimuth and elevation angles is shown in Fig.8.

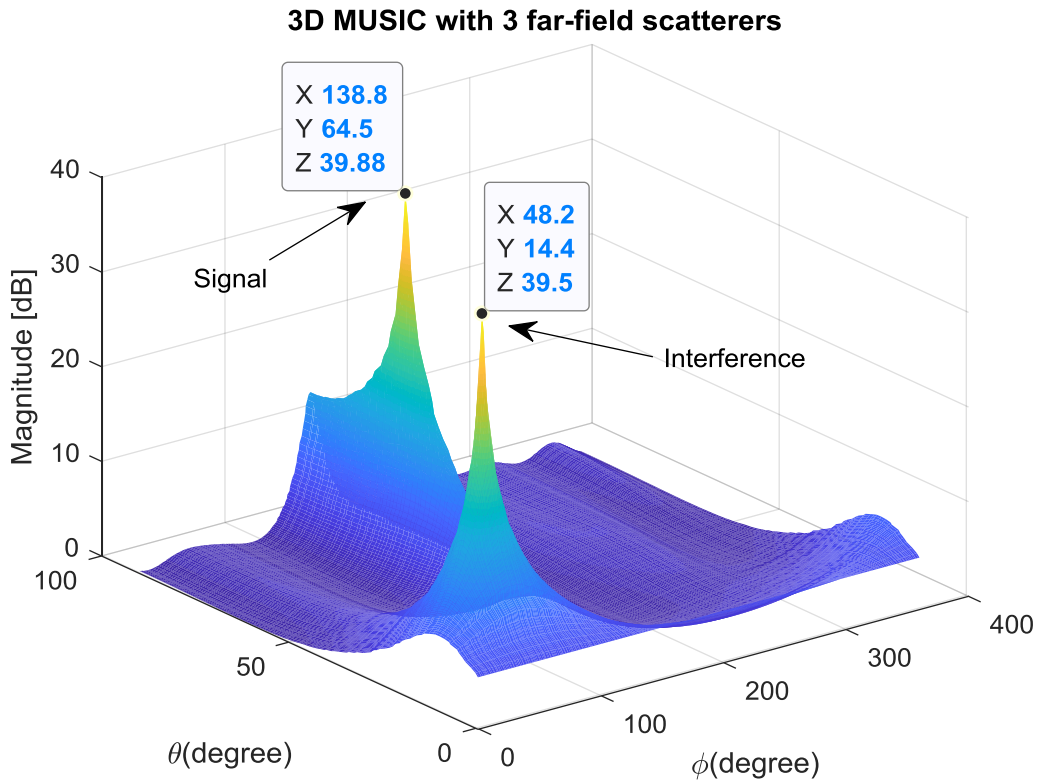


Fig.8. MUSIC Pseudospectrum function in presence of three far-field scatterers for one incident signal and one interference

As shown in Fig.8, the proposed method can determine the angle of arrival for the signal and the interference in presence of the far-field scatterers and mutual coupling effects with minor errors. In this research the sub-optimal beamforming method of [15] was modified for considering the effects of scatterers by changing the form of the received signals as described by equation (4). To evaluate the efficiency of the

proposed method in beamforming process, we consider the geometry of dipoles as in Fig.6 and the scenario of angle of arrivals as in Table.1. After the calculation of the optimum weights, the normalized radiation pattern as a function of the azimuth and elevation angles is depicted in Fig.9. The radiation pattern in dB scale is shown in Fig.10 for better understanding of the null depths (the direction of interference).

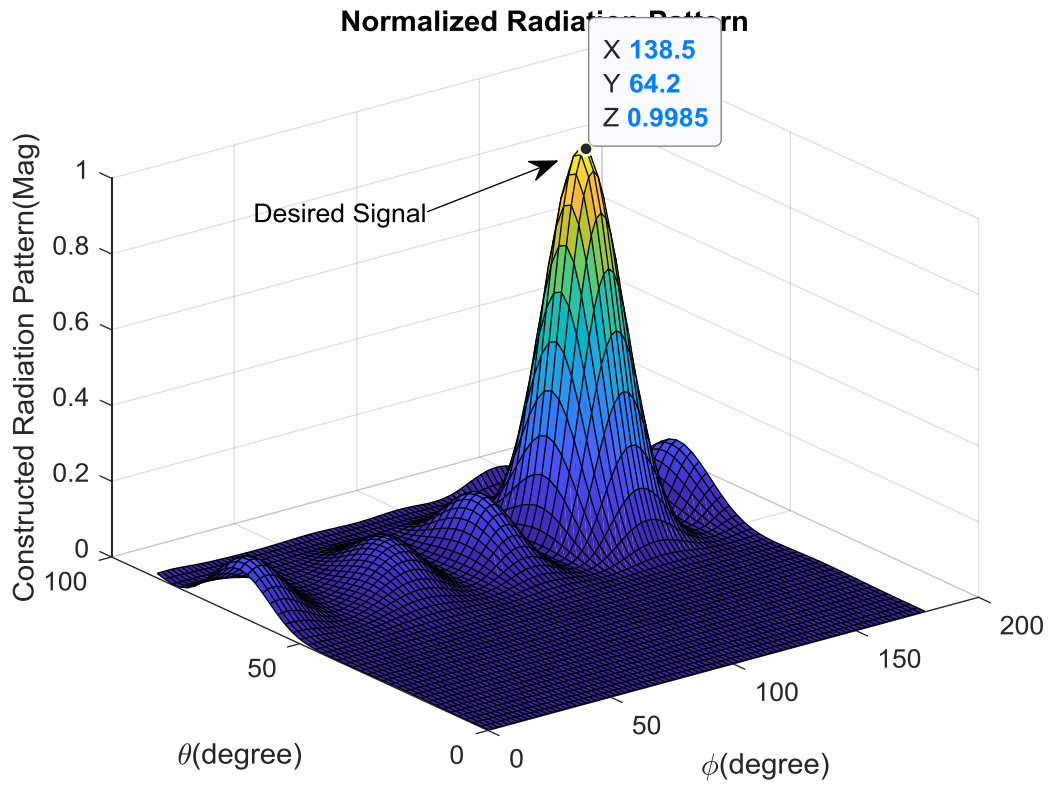


Fig.9. Normalized radiation pattern of beamforming in presence of three far-field scatterers (linear scale)

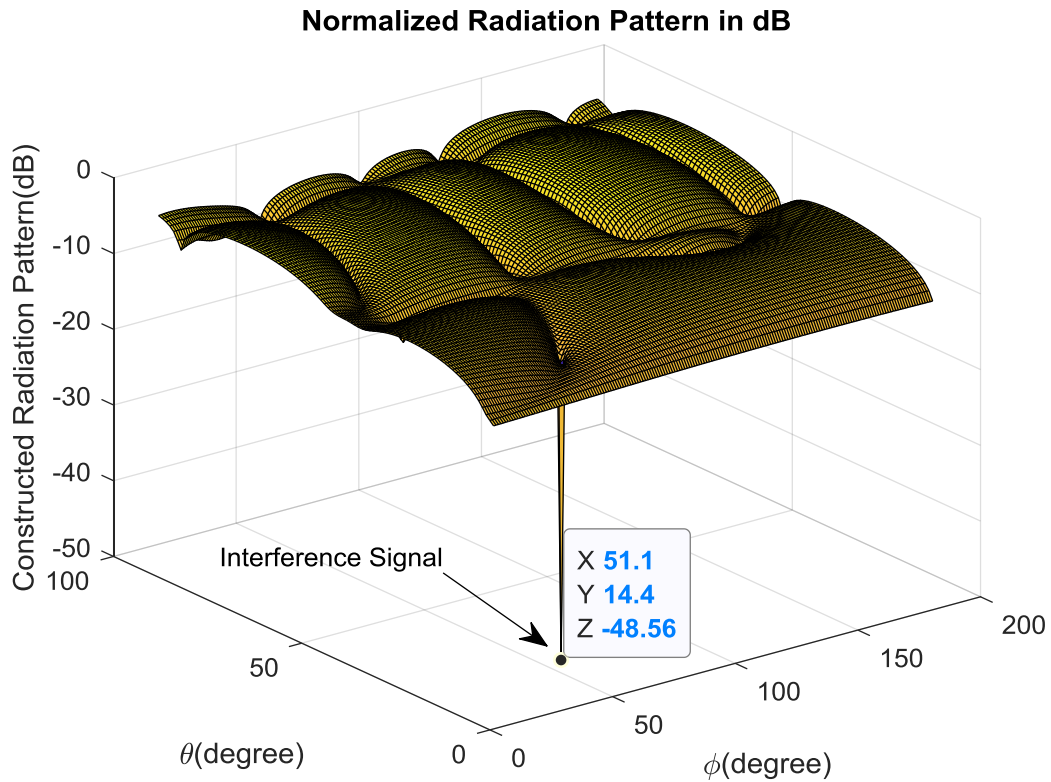


Fig.10. Normalized radiation pattern of beamforming in presence of three far-field scatterers in dB scale to demonstrate the null depth at the direction of the interference

As it is evident in Fig.9 and Fig.10 the beamforming process has quite satisfactorily optimized the direction of the main beam and the main null.

Instead of a particular scenario, to examine the efficiency of the proposed method in direction finding and beamforming, we have performed a number of Monte-Carlo simulations for different direction of arrivals. The comparison of these Monte-Carlo simulations with and without scatterers are conducted in section 3.

### 3. Results and Discussion: Direction Finding RMSE and Received SIRs Estimations

In this section we compare the two different 3D geometries with and without the scatterers to examine the effects of the scatterers on the RMSE values of proposed direction finding algorithm. The RMSE of the arrival angles is computed as follows [14].

$$RMSE = \frac{1}{2TN} \sum_{n=1}^T \sum_{i=1}^N (\widehat{\theta}_i^n - \theta_i^n)^2 + (\widehat{\varphi}_i^n - \varphi_i^n)^2 \quad (9)$$

Where,  $(\varphi_i^n, \theta_i^n)$  are the random angles of incidence signal and interferences and the  $(\widehat{\varphi}_i^n, \widehat{\theta}_i^n)$  are the corresponding estimated angle of arrivals. In (9) T and N are the number of random scenarios and the number of sources. The Monte-Carlo simulations are conducted for 3D array geometry of cylindrical dipoles as in Fig.11 with and without scatterers. We use a cylindrical array including 6 elements in each plane separated by  $\frac{\lambda}{2}$  distance. The distance between consecutive planes is  $\lambda$ . The distance between the cylindrical scatterer and the axis of the cylindrical array is considered to be order of  $10\lambda$  and the height of the each cylindrical scatterer is quite large (several  $10\lambda$ 's).

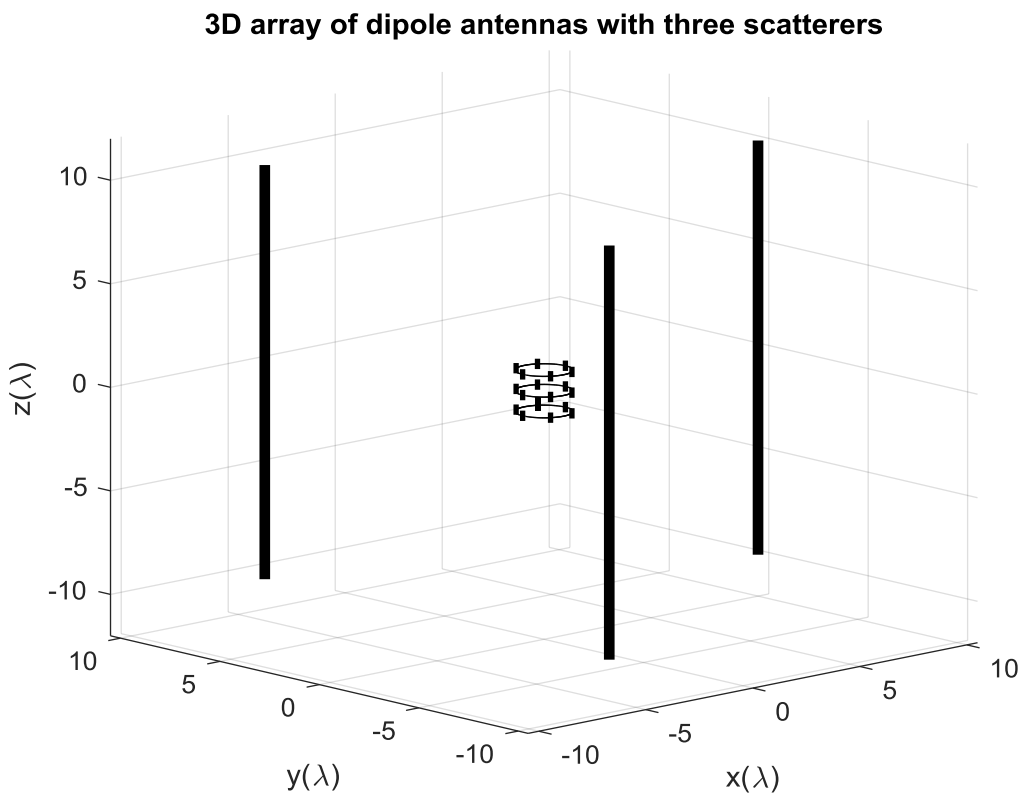


Fig.11. Three dimensional cylindrical array of half-wave dipole antennas in presence of three cylindrical large scatterers



To sketch the RMSE values versus the number of snapshots we consider  $N=5$  uncorrelated signal sources with 10000 random scenarios for azimuth and elevation angles with the incidence angles  $\theta_i^n \in [-\frac{\pi}{2}, \frac{\pi}{2}]$  and  $\varphi_i^n \in [0, 2\pi]$ . The signal to noise ratio is fixed at  $\text{SNR}=7$  dB and the simulations are conducted for the number of snapshots between 20 and 200. These simulations are conducted for different distances  $d = 10\lambda, 15\lambda, 20\lambda, 40\lambda$  between the cylindrical scatterers and the axis of the cylindrical array. The results are shown in Fig.12.

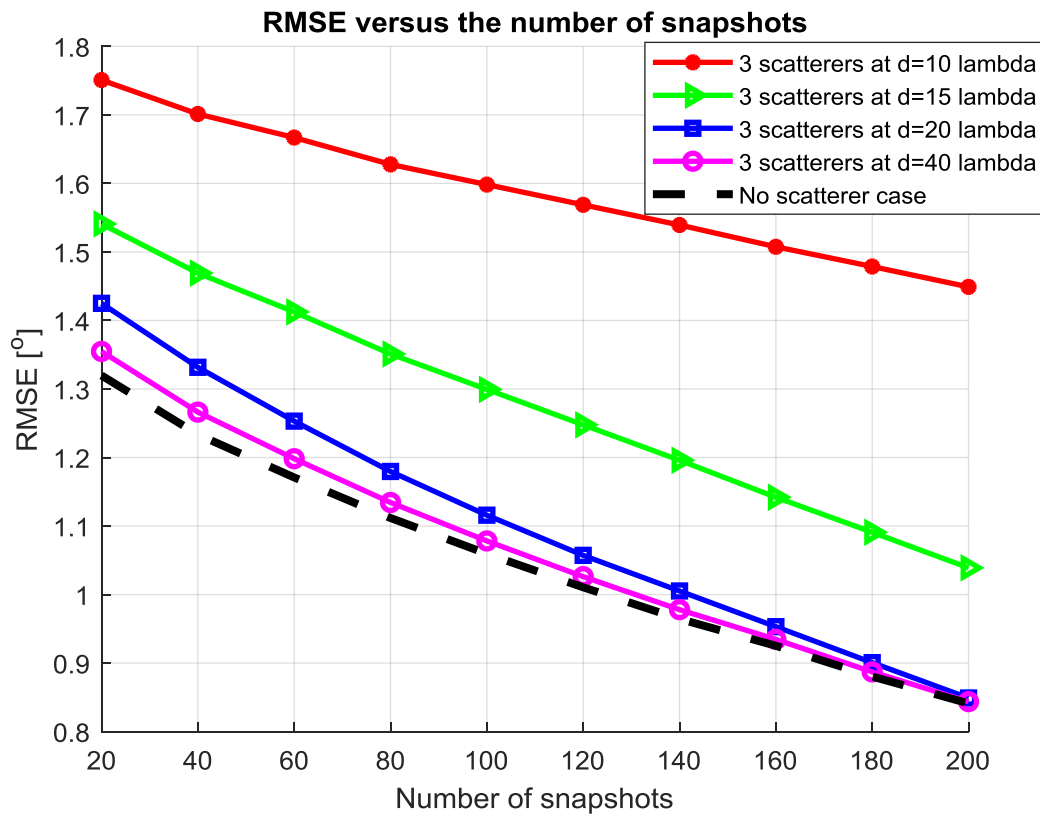


Fig.12. RMSE values versus the number of snapshots using the geometry of Fig.11 in presence of scatterers at  $d = 10\lambda, 15\lambda, 20\lambda, 40\lambda$  distances compared to no scatterer case, at fixed  $\text{SNR}=7$  dB.

The comparison of RMSE values with and without the scatterers as a function of the SNR are sketched in Fig.13 at fixed number of snapshots ( $L=100$ ) for 10000 random scenarios for azimuth and elevation angles.

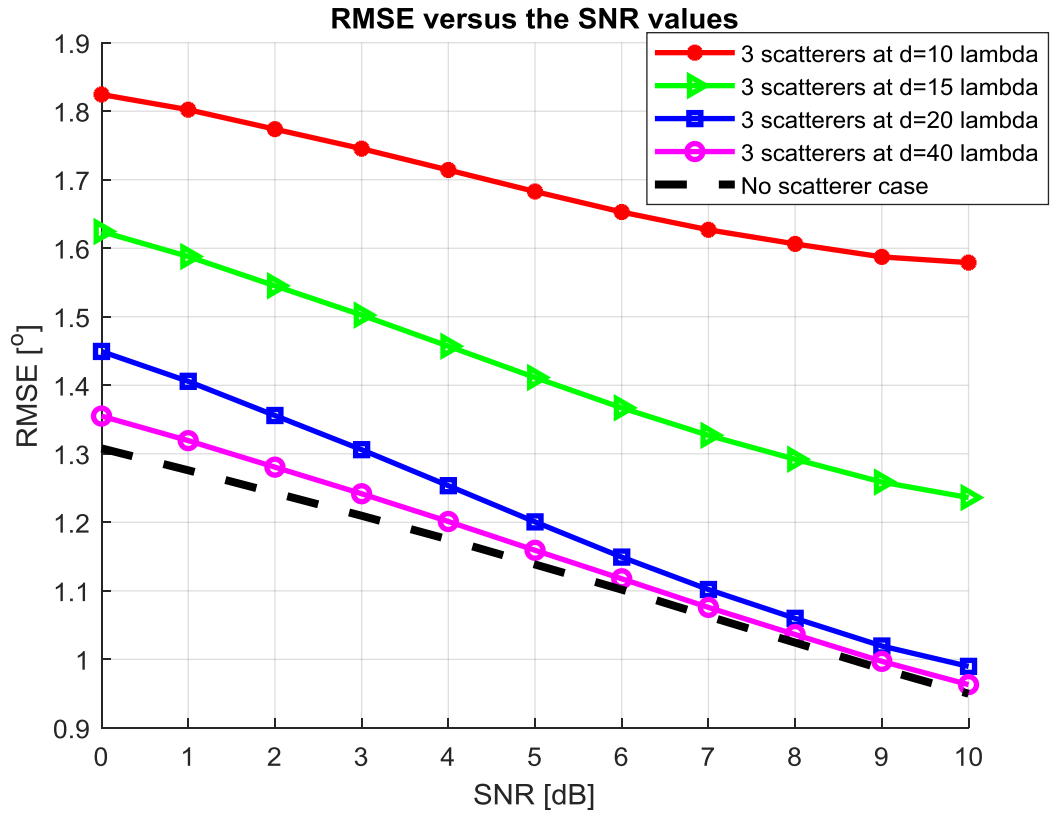


Fig.13. RMSE values versus the SNR using the geometry of Fig.11 in presence of far-field scatterers at  $d = 10\lambda, 15\lambda, 20\lambda, 40\lambda$  distances compared to no scatterer case, at fixed number of snapshots( $L=100$ )

As it is shown in Fig.12 the rms error is decreased with increasing the number of snapshots while it is increased by reducing the distance of the scatterers. Furthermore, in Fig.13 it is shown that the rms error is decreased with increasing the SNR and it is increased by reducing the scatterers distance.

In this section we use a modified sub-optimal beamforming method [15] to evaluate the efficiency of the proposed method with three far-field scatterers. The beamforming process is conducted using estimated angle of arrivals and the mean values of SIR versus the number of interferences are shown in Fig.14.

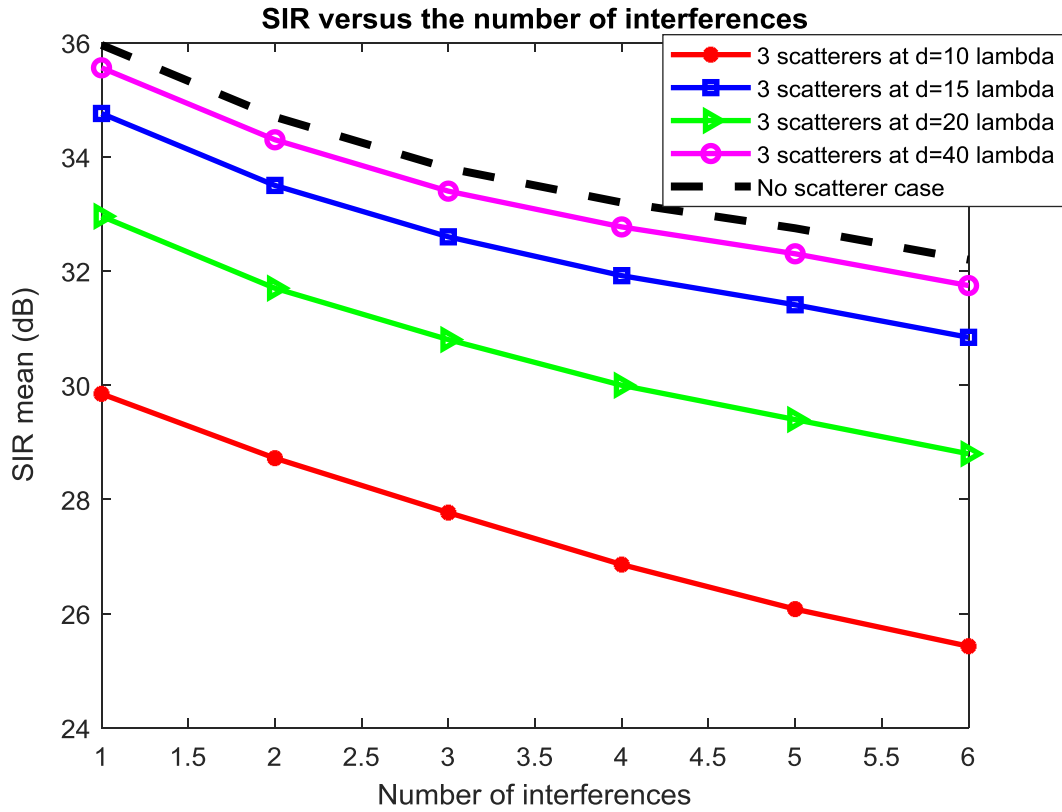


Fig.14. Mean values of the SIR versus the number of interferences in presence of the scatterers at  $d = 10\lambda, 15\lambda, 20\lambda, 40\lambda$  compared to no scatterer case

As it is seen in Fig.14 with three scatterers at a distance of  $10\lambda$  about 6 dB decrease in SIR is observed compared to no scatterer case while increasing the distances of the scatterers to  $20\lambda$  about 1.5 dB decrease in the SIR is observed compared to the no scatterer case. In all cases the SIR is monotonically decreased with the number of interferences.

#### 4. Conclusion

In this work the effect of a limited number of large far-field cylindrical scatterers on the process of direction finding and beamforming by an array of cylindrical dipoles was studied. For verification purpose we have compared the direction finding results with those of phase comparison method implemented by HFSS. The comparison results were satisfactory. Through our analytical calculations we have considered

the forward scattered fields by the cylindrical scatterers to estimate the angle of arrivals of the signal and the interferences to perform the beamforming process consequently.

We performed Monte-Carlo simulations to estimate the RMSE values in direction finding and the SIR in beamforming. Here the computations showed the increase in RMSE values in direction finding and the consequent decrease in SIR evaluations.

Given the fact that we have used an analytical cylindrical harmonic expansion for the scattered fields the problem of direction finding in presence of far-field scatterers does not need high computational burden (compared to the HFSS or other full wave numerical electromagnetic simulator). By this method one can increase the number of the scatterers as desired without increasing the computational load. As analytical formulation is used for the scattered fields, it is possible to consider more complex scatterers using several number of conducting cylinders.

Our study showed that the effect of large scatterers can become insignificant for large distances of order of several  $10\lambda$ 's.

The cylindrical scatterer method could be used in wireless communications, for smart antennas and eventually for the field test of direction finders at the airports or large field test areas.

### **Abbreviations**

SIR: Signal to Interference Ratio, MUSIC: Multiple Signal Classification, HFSS: High Frequency Simulation Software, rms: root mean square, RMSE: Root Mean Square Error, DOA: Direction of Arrival, DF: Direction Finding, SNR: Signal to Noise Ration, 3D: 3 Dimensional, IPN: Interference-Plus-Noise.

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### **Competing Interest**

The authors declare that they have no competing interests.

### **Authors' contributions**

Sarah Poormohammad carried out the implementation of the algorithms in MATLAB and HFSS, participated in the alignment and drafted the manuscript. Forouhar Farzaneh participated in the design of the study and helped to draft the manuscript. Ali banai participated in the sequence alignment.

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## 5. Figure Title and Legend

Figure 1 title: The cross section of circular cylinder “i”

Figure 1 legend: The cross section of circular cylinder “i”

Figure 2 title: Cylindrical array with one large scatterer

Figure 2 legend: The geometry of the cylindrical array with a single cylindrical scatterer in this work

Figure 3 title: The magnitude of pseudospectrum function of MUSIC DOA algorithm

Figure 3 legend: The magnitude of pseudospectrum function of MUSIC DOA algorithm as a function of azimuth angle with a single cylindrical scatterer

Figure 4 title: Two orthogonal dipole arrays used for HFSS simulations with one scatterer

Figure 4 legend: The geometry of the array with a single cylindrical scatterer and two orthogonal dipole arrays used for HFSS simulations

Figure 5 title: Estimated azimuth angle

Figure 5 legend: Estimated angle of arrivals versus the true incident angle with proposed method compared to the phase difference method (implemented by HFSS) with a single scatterer

Figure 6 title: Cylindrical array with three large scatterers

**Figure 6 legend: *The geometry of cylindrical array with three cylindrical scatterers***

Figure 7 title: Estimated azimuth angle

Figure 7 legend: Estimated angle of arrivals versus the true incident angle with proposed method compared to the phase difference method (implemented by HFSS) with three far-field scatterers

Figure 8 title: 3D MUSIC with 3 far-field scatterers

Figure 8 legend: MUSIC Pseudospectrum function in presence of three far-field scatterers for one incident signal and one interference

Figure 9 title: Normalized radiation pattern

Figure 9 legend: Normalized radiation pattern of beamforming in presence of three far-field scatterers (linear scale)

Figure 10 title: Normalized radiation pattern in dB

Figure 10 legend: Normalized radiation pattern of beamforming in presence of three far-field scatterers in dB scale to demonstrate the null depth at the direction of the interference

Figure 11 title: 3D array of dipole antennas with three scatterers

Figure 11 legend: Three dimensional cylindrical array of half-wave dipole antennas in presence of three cylindrical large scatterers

Figure 12 title: RMSE versus the number of snapshots

Figure 12 legend: RMSE values versus the number of snapshots using the geometry of Fig.11 in presence of scatterers at  $d = 10\lambda, 15\lambda, 20\lambda, 40\lambda$  distances compared to no scatterer case, at fixed SNR=7 dB.

Figure 13 title: RMSE versus the SNR values



Figure 13 legend: RMSE values versus the SNR using the geometry of Fig.11 in presence of far-field scatterers at  $d = 10\lambda, 15\lambda, 20\lambda, 40\lambda$  distances compared to no scatterer case, at fixed number of snapshots( $L=100$ )

Figure 14 title: SIR versus the number of interferences

Figure 14 legend: Mean values of the SIR versus the number of interferences in presence of the scatterers at  $d = 10\lambda, 15\lambda, 20\lambda, 40\lambda$  compared to no scatterer case