Teaching Behaviors in Dynamic Geometry Environment: An Investigation of Nine Chinese Mathematics Lessons in The Prospective of Instrumental Orchestration

Fangchun ZHU (fczhu@math.ecnu.edu.cn)
East China Normal University https://orcid.org/0000-0002-1365-9061
Xu Binyan
East China Normal University

Research Article

Keywords: Dynamic Geometry, Teaching practice with DGS, Instrumental orchestration, T-explain-the-screen, S-explain-the-screen

Posted Date: June 17th, 2022

DOI: https://doi.org/10.21203/rs.3.rs-1610616/v1

Abstract

This paper focuses on how Chinese mathematics teachers integrate dynamic geometry software in lessons. Instrumental Orchestration (IO) lens and its categorization are used to analyze 9 Chinese mathematics lessons. As a result, it shows that most orchestration types can be identified in this study with some different characteristics observed. And a more classic way is identified for Chinese teachers to use DGS which indicates DGS is assimilated into the current teaching process. Teachers more use dragging mode to show the initial and final state of the figures. We also re-organize IO types and divided explain-the-screen into T-explain-the-screen and S-explain-the-screen based on who can be the explainer. This indicates that these teachers try to change their teaching process from lecturing to facilitating student's learning.

Introduction

Information and communication technologies (ICT) become increasingly important in modern education. Teachers can use ICT as a tool to support students' learning; or a tool for teachers' personal productivity; or a medium for interacting and collaborating with colleagues (Ponte et al., 2002). In spite of its potential for teaching and learning, the integration of ICT into mathematics education lags behind the high expectations of researchers (Assude, 2007; Cayton et al., 2017; Drijvers et al., 2010). And how technology can be used effectively and the effects it brought into mathematics teaching and learning are important in mathematics education (Bretscher, 2014). Researchers think it is important for us to make empirical studies on the real use of technology in teaching and learning (Sinclair et al., 2016). However, in China, a few studies are about teacher's teaching practices with technology. So this research wants to fill in the gap. We take dynamic geometry software (DGS here and after) as an example to analyze how teacher integrates DGS into mathematics lessons and their opinions towards using DGS.

TEACHING PRACTICE WITH DYNAMIC GEOMETRY SOFTWARE

DGS is developed for visualization and for checking properties by dragging geometrical objects (Laborde & Laborde, 2008; Sinclair et al., 2016). Its main feature: dragging mode, a kind of direct manipulation towards the software, means the simultaneity between students' action and DGS feedback (Sinclair et al., 2016) which can solve the gap between experimental and theoretical mathematics (Leung, 2003). This mode is mainly used to: 1, check the correctness of the supposed (known) geometrical properties in the figure; 2, to look for new geometry properties through the perception of what remains invariant when dragging; and 3, to check whether the construction preserves its geometrical properties when dragging (Healy, 2000; Laborde, 2001). But the use of DGS by teachers is often limited to the first modality, students are expected to drag figures to confirm empirically the properties (Hölzl, 2001). Because teachers may find that students' activities with DGS may lead them to the production which is not their expectation (Olivero, 2002).

In China, Shang (2008) investigated how teachers integrated DGS in mathematics teaching. She observed 8 mathematics lessons and found DGS played mainly as a servant for teachers rather than a patterner. In these lessons, students manipulated DGS mainly for constructing different objects not for exploring mathematics knowledge. The author concluded that what teachers did is not integrating DGS in mathematics lessons but for replacing traditional teaching methods. This indicates that although researchers think DGS can change their teaching methods (Hu, 2005), promote the effects of mathematics teaching (Zhao et al., 2012), and let students explore mathematical knowledge by themselves (Fan, 2003), small amounts of teachers choose to let students manipulate DGS in the classrooms (Hu, 2005). And in these lessons, students have no chance to exchange their findings of special questions and even do not know whether what they have is right or not (Liu, 2009).

One of the reasons why teachers do not use DGS as researchers want is that teachers need to make a transformation between old and new didactical practices (Assude & Gelis, 2002; Lagrange et al., 2003). Another important reason is that teachers lack enough support for making educational use of DGS (de Castell et al., 2002; Norris et al., 2003). Teachers lack enough strategies to effectively use DGS or other technologies for learning mathematics (Niess et al., 2009). Like Kortenkamp et al. (2009) point out: "still, the adoption of DGS at school is often difficult. … Many teachers do not seem to know about the new possibilities……" (p. 1,150). Other factors, like the change of room location and physical layout, change of class organization and classroom procedures (Jenson & Rose, 2006), and the resources they need to use in the class such as mathematics tasks (Hegedus et al., 2017; Pierce & Stacey, 2013; Author, 2009) also affect teachers practice with DGS.

Thus, how to use DGS to create new opportunities for students' learning process (Angeli & Valanides, 2009) is a big problem for teachers. Some of the researchers began to think about how to help teachers to teach with DGS, such as building the repository which includes DGS to support their didactic process (Trgalová & Jahn, 2013). In order to help teachers to integrate technology into classrooms, Trouche (2004) introduces instrumental orchestration to describe how teachers orchestrate classrooms with technology. In this study, we want to make sense of teaching practice with DGS and answer the following questions: How teachers organize these nine mathematics lessons with DGS? And what characters can be identified in these mathematics lessons with DGS?

INSTRUMENTAL ORCHESTRATION

Instrumental orchestration comes from instrumental approach which acknowledge the complexity of using technologies in mathematics teaching and learning (Artigue, 2002). This approach focus on a psychological construction process called instrumental genesis during which an artifact comes into instrument. During this process the user develops a scheme to use the instrument for a specific task (Drijvers et al., 2010). In this scheme, users' technological knowledge about the artifact and specific knowledge about the domain like mathematics are intertwined. So that instrumental genesis is useful for us to analyze how to use technology in mathematics education.

There are many studies focus on the students' instrumental genesis in learning mathematics with technology. Researchers believe that students' instrumental genesis is guided by teachers. For example, teachers chose to use certain technological tools in order to guide students' learning process(Kendal & Stacey, 2002). So Trouche (2004) introduced instrumental orchestration to describe how the teacher can guide students' instruments genesis within the classroom.

Instrumental orchestration tries to answer questions about what technological artifacts mathematics educators should introduce to learners and what guidance they should provide so that learners can appropriate and use artifacts as instruments to mediate their activities with various artifacts (Drijvers et al., 2010; Trouche, 2004). it is defined as how teachers want to organize the class teaching with different kinds of artefacts available in learning environment in order to help students learn mathematics (Trouche, 2004). We can distinguish instrumental orchestration into three different elements: a didactic configuration, an exploitation mode and a didactical performance (Drijvers et al., 2010).

In this study, instrumental orchestration is used to describe how teachers organize classroom teaching with DGS. Instrumental orchestration tries to answer questions about what technological artifacts mathematics educators should introduce to learners and what guidance they should provide so that learners can appropriate and use artifacts as instruments to mediate their activities with various artifacts (Trouche, 2004). We can distinguish instrumental orchestration into three different dimensions:

A didactic configuration which means an arrangement of artefacts in the didactical environment, or in other words, a configuration of the teaching setting and the artefacts involved in it. Like in the musical metaphor of orchestration, the didactical configuration can be seen as choosing what musical instruments need to be included, and how to arrange them in space so that the different sounds result in polyphonic music, which in the mathematics classroom might come down to a sound and converging mathematical discourse.

An exploitation mode means the way teachers decide to exploit a didactical configuration for the benefit of his/her didactical intentions. This includes decisions on the way a task is introduced and worked through, on the possible roles to be played by the artefacts, and on the schemes and techniques to be developed and established by the students.

A didactical performance involves the ad hoc decisions taken while teaching on how to actually perform in the chosen didactic configuration and exploitation mode: what question to pose now, how to do justice to (or to set aside) any particular student input, how to deal with an unexpected aspect of the mathematical task or the technological tool, or other emerging goals.

According to Drijvers (2010, 2013), the following types can be used to describe different orchestration of classroom with technology in mathematics teaching: Technical-demo, Explain-the-screen, Guide-and-explain, Link-screen-board, Discuss-the-screen, Spot-and-show, Sherpa-at-work, Board-instruction (see Appendix). In this study, we would analyze Chinese mathematics lessons to check if all these types can be identified or if there are some new types or new characteristics can be found in Chinese mathematics lessons.

Method

Participants

5 Chinese secondary school mathematics teachers participate in this study because of their different teaching experiences with DGS. The following criteria were used to decide which teacher was chosen: first, the chosen teachers need to have different teaching experiences (both elder and young teachers); second, the teachers should know how to use DGS and want to use DGS in their teaching; third, all of the teachers teach in secondary schools and during this study they teach geometry.

The first two teachers are chosen (Teacher A and Teacher B) because of their sufficient teaching experiences and familiarity with DGS. Both of them come from one private school in Shanghai. Students in this school are at the top level in Shanghai. The school committee buys lots of software, like the Geometry Sketchpad, to support teachers develop their didactical methods. From their practices, we can find out some possible ways to integrate DGS into mathematics lessons. Another experienced teacher, teacher C, who has about 10 years of teaching experience, is also good at using DGS.

Young teachers (Teacher D and Teacher E), who have less teaching experience with DGS, are also important for us. They just began their teaching career three years before this study. By contrasting their practice with experienced teachers, we can know the different ideas of teaching mathematics with DGS and it can help teachers to reflect their teaching from a different point of view. Teacher D did not know too much about how to use DGS. But she is very interested in DGS and is willing to integrate it into her classes. Teacher E learned DGS in mathematics teaching in the university, but because he also just begins his career so he has not too much teaching experience with DGS.

Semi-structured Interview

All the teachers who attend this research would be interviewed before and after their mathematics lessons in order to analyze their opinions on using DGS. Before the mathematics lessons, a short interview with these teachers is made for collecting the documents including the list of tasks, lesson plans, and the DGS, and so on. After each lesson, all of the teachers also answer several questions about the teaching progress, the teaching objectives, the role of the DGS in the proposed tasks, and how they are organized and used (Sabra, 2011). And teachers also talked about their opinions on the constraints and affordances of DGS, their preparation work, the description of what they planned, the description of what actually happened, and the work done afterward (Bueno-Ravel & Gueudet, 2007).

Lesson observation

Altogether 9 lessons (Table 1) are recorded and analyzed based on instrumental orchestration. The following points are contained in the observing list: teacher's behavior in classroom teaching, interaction with students (Kynigos, 2015), the organization and configuration of the class, deviations from the forecast, and comments or possible remarks made by both teacher and students (Sabra, 2011). The objectives of lesson observation are observing the process of implementing the lesson plan and identifying students' and teacher's reactions which will eventually have effects on the use of DGS (Sabra, 2011).

Table 1
Basic information of the lessons

Content of lesson	Teacher	Grade
Moving point	Teacher A	Grade 8
Relation among the sides of triangle	Teacher A	Grade 7
Review test	Teacher B	Grade 9
Review test	Teacher B	Grade 9 (another class)
Review test	Teacher C	Grade 8
Review test	Teacher C	Grade 8
Congruent triangle 1	Teacher D	Grade 7
Congruent triangle 2	Teacher D	Grade 7
Isosceles triangle	Teacher E	Grade 7

Data and data analysis

According to the research questions, we collected the data includes task lists; interview audios and transcripts with teachers; lesson videos (9 lessons) which record all the interactions between teacher and students.

For interviews, first, all the interview audios were also transcribed in order to make further analysis. Second, the transcripts would be cut into short parts based on the interview questions. Third, each teacher's answer would be cut into sentences, and then each sentence would be abstracted into several keywords which can present the meaning of each sentence. Here is one example of a teacher's answer:

Which one is suitable? I like to use, if in preparation or in lecture, maybe PPT, if I have lectured for other teachers because at that time I just need to present. But in general lessons, I think I use [another software] more times. Because I can operate it during the lessons or I can let students operate it with an iPad.

First, we divided this transcription into the following sentences:

1 *Which one is suitable?*

2 *I like to use, if in preparation or in lecture, maybe PPT, if I have lectured for other teachers because at that time I just need to present.*

3 *But in general lessons, I think I use [one software] more times.*

4 *Because I can operate it during the lessons or I can let students operate it with an iPad.*

Then for each sentence, we found the first one is repeating what is asked during the interview, so it can be deleted. In the second one, the teacher mentions one software: PPT, which is used for educating other teachers. Then the third sentence, this teacher mentioned another software (the teacher forgot the name) which is held on an iPad for presenting the contents. This sentence is related to the software. The fourth one is describing how she uses the software during the lessons. So we need to focus on sentences 3 and 4 which are related to the research question in this study.

For lesson videos, first, we transcribed all the videos in order to analyze the interaction between teacher and students, the teacher's action with DGS, and the image on the screen according to the timeline. Second, these videos were cut into different episodes which are defined as the whole-class treatment of a single mathematics task with DGS. For example, if the lesson has three mathematics tasks, then three episodes would be identified. Then we would analyze the instrumental orchestration types which were used by these teachers. If each of these episodes contained different orchestration types, they would be cut into different sub-episodes, each of them with a unique type. After that, the frequency of each instrumental orchestration type would be counted and also their duration time which can make us have a general image of the organization of classes with DGS in these lessons. Finally, each episode would also be deep analyzed according to the three dimensions of instrumental orchestration: didactic configuration, exploitation mode, and didactical performance.

Results

This section first presents the general information of instrumental orchestration showed in the lessons. Then we show some characteristics of some main types of instrumental orchestration in these Chinese lessons.

General information of orchestration type

In table 2, we can see almost all of the 5 teachers spent most of the class time teaching with DGS (about 20 to 40 minutes each lesson[1]). *Explain-the-screen* (totally 39 times in these 9 mathematics lessons) or *guide-and-explain* (totally 24 times) are most identified. This indicated that it was the teacher who controlled DGS and students did not have too much time to use DGS.

Table 2
Orchestration type frequencies and duration time by teachers

Type of instrumental orchestration	Teacher				
	A	B	C	D	E
Technical-demo					
Explain-the-screen	12* (15min**)	8 (21min)	4 (13min)	12 (31min)	3 (5min)
Guide-and-explain	6 (16min)	16 (39min)			2 (7min)
Link-screen-board	1 (30sec)	5 (7min)	2 (13min)	1 (6min)	1 (3min)
Discuss-the-screen	2 (2min)	1 (1min)	1 (1min)		
Spot-and-show	2 (5min)				
Sherpa-at-work		3 (6min)		3 (12min)	
Board-instruction	2 (7min)	1 (3min)	2 (12min)	9 (33min)	4 (20min)
Note: * In each column, this number means how many times does each type appears in this teacher's lessons.					
** This number means how long does each type continues in the lessons.					

We also found some teachers organized whole-class discussions to let students take part in the learning activities (*discuss-the-screen*). Other types like *spot-and-show* (2 times in Teacher A's lesson)and *Sherpa-at-work* (totally 6 times in Teacher B and Teacher D's lessons) were used by several teachers. This phenomenon told us that these teachers began to notice students need to have more time to present their ideas or operate the software.

During the interview, these Chinese teachers, both young and experienced, consider DGS as a servant. They used DGS mainly for explaining mathematical contents. For example, Teacher B said: *Media I think it is just a supporter for teachers, but it will play an important role in the future. … But it still needs teachers' guidance and control in the lessons.* From the statement, Teacher B used supporter to describe the role of DGS in mathematics teaching and thought that the center of the class is still the teacher (Guerrero, 2010), she needs to think about how to control the use of this software to make their lessons more effective. Others like Teacher E and Teacher C also think DGS can help students to learn mathematics but they cannot rely on technologies to solve mathematics problems.

Characteristics of main orchestration types in this research

Based on the videos, most of the orchestration types mentioned by Drijvers can be identified in this research. To further investigate instrumental orchestration in these Chinese lessons and of its third element of didactical performance, in particular, some exemplary episodes of main orchestration types (most used by teachers) were analyzed more extensively.

From the videos, the didactic configuration is similar and not change in one lesson. each classroom contains a digital mathematics environment (computer, screen, internet, DGS, and so on), and also traditional resources like a board, chalk, textbooks, and so on. Some teachers (Teacher A and Teacher B) also use iPhones or iPad which are not contained in other classrooms. With this kind of configuration, all of the students can see the screen and follow what the teacher did clearly. Teachers prepare all of the resources or equipment they need before the class and never add or remove new ones during the teaching. Because the didactic configuration in one lesson almost never changes, so the following analysis of the teaching episodes in these lessons is mainly about exploitation mode and didactical performance.

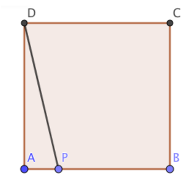
Explain-the-screen

This orchestration type is common in these Chinese lessons (Table 2). The exemplary episodes from Teacher A's lessons were presented following to analyze the interaction between teacher and DGS.

The exploitation mode includes his choice to operate DGS by himself to present tasks or mathematical contents. This mode lets students focus on the projection on the wall. Similar modes are also found in other teachers' classrooms when they need to explain the diagrams on the screen or make conclusions in the lessons.

The didactical performance starts with solving the following task:

Table 3: Task in explain-the-screen

	Task
	The task contains one moving point P which coincides with point A and moves from A to B to C to D. Students are required to find out the mathematical relation between the area of triangle APD and the length of the moving trace.

Teacher A first let one of the students explain how does point P move according to the task (Fig. 1). Teacher A let point P move from point A to B to C automatically by clicking the button. During that time, what he does is just listening what this student said without interrupting her explanation. And he does not comment on the answer to say if this student's answer is correct.

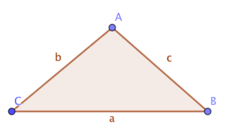
In Drijvers' work, *explain-the-screen* means it is the teacher who explains what happened on the screen, but we notice that in Teacher A's lesson, of course also in other experienced teachers' lessons, it is the student who makes an explanation about what happened on the screen. This indicates that students are not always in a passive position although they cannot manipulate DGS.

Guide-and-explain

This orchestration type shares a similar didactic configuration with *explain-the-screen* which includes a computer, DGS, and projecting facilities in this research. In this classroom setting, students can easily follow what teachers and other students said. The exploitation mode is also similar to *explain-the-screen*, which is a closed explanation based on what is on the screen. But the difference is in this type teachers always posed several questions for students to guide students' learning activity.

The following episode is an example of this orchestration type from Teacher A's first lesson. In this episode, Teacher A wants students to explain the relationships among the three sides of a triangle (Table 4).

Table 4: Task in guide-and-explain

	Task
	There is a triangle whose length of side AC and BC is determined (AC = 4, BC = 6). Point A could be moved while points B and C were stable. Try to find the maximum and minimum of the length of AB and what is the relation among the three sides

Teacher A starts his teaching with the following question: looking for the minimum of the length of AB and what is the relation among the three sides at that time. He let one of the students answer this question. At first, Teacher A did not operate the software, and when he noticed this student made some misunderstanding, he dragged the point to show why it is wrong. This student tried to find the minimum of side AB. He first gave a wrong answer: 2.2. Teacher A dragged point A to let the student notice the answer is not correct. During this episode, the interaction happened between two persons: the student chose to answer the questions and teacher A. Other students just listen to their talking and did not give any feedback. The explanation goes beyond techniques and involves mathematical content. This process gave students chances to present their own ideas.

The above two orchestration types are frequently used by these Chinese teachers. It reveals what these teachers think about the learning of mathematics. They believe students need to have enough time to present their voices which is important in learning mathematics. This analysis reveals one important characteristic of these teachers: they let students be more active by inviting students to explain their reasoning, and by giving their own explanations.

Spot-and-show

We choose this orchestration type to present here is because although it is seldom used in these lessons, it reveals a transition of Chinese mathematics teaching practice. In Drijvers's work, in this type, teachers find some interesting student work during lesson preparation and brought them to the classroom, and use it in a classroom discussion. But in this research, teachers find student work during the lessons.

A didactical configuration includes access to the students' work in the technological environment like in Teacher A's second lesson, he used a smartphone to photograph students' work and show it through the screen (Fig. 2). As exploitation modes, he chose this work when students solved the problems, and at the same time, he put them on the screen and made them a topic of the following discussion.

The didactical performance begins with having this student whose work is shown explain her answer, and Teacher A makes reactions or poses questions on the student's work to let her make deep thinking. In this episode, the topics came from the students' work based on the task situation. So these activities can be clarified into the type *spot-and-show* according to the characteristics.

Sherpa-at-work

This type is also not common in Chinese mathematics lessons. We chose one episode from Teacher B's lesson as an example to show what happened when she uses this type of instrumental orchestration.

In the *Sherpa-at-work*, a *Sherpa student* uses the technology to present his or her work or to carry out actions the teacher requests (Trouche, 2004). The didactical configuration in this research includes access to DGS, projecting facilities, and other tools which can access student work. And this so-called *Sherpa student* should be in control of using the technology with all the students able to follow his or her work. But in this study, we find a different picture: not the students but Teacher B played this *Sherpa student* role (Fig. 3). This is a special property that is different from Drijvers' work. As exploitation modes, Teacher B let one student tell her how to construct geometry objects according to the task with the help of DGS (Fig. 3) and she, herself carry out specific actions with the help of an iPad.

As a didactical performance, this student described how to construct this figure according to the feedback on the screen. The main role of the teacher in this episode is like a robot to make reactions according to the student's instruction with the software. According to the types of instrumental orchestration mentioned above, this episode can be put into *Sherpa-at-work*. This kind of instrumental orchestration happened three times in Teacher B's second lesson. Teacher B also posed sequences of questions to make this student explain her strategies.

These two orchestration types reveal that some of these teachers try to change their teaching process and let students operate DGS. But these teachers believe that students should use DGS after learning relative contents. Like what one teacher said: *After they (students) understand relative knowledge in the tasks, you can let them construct objects with DGS. For example, students need to know what is perpendicular bisector and some basic knowledge of the circle. And then they can use software to construct circles.* This may be one of the reasons why Teacher B just use Sherpa-at-work in her second lesson (the topic is a review test which means students have learned relative knowledge before this lesson).

Footnote

[1] In China, each lesson always continues 40 minutes in secondary school

Conclusion And Discussion

Presenting the initial and final state of the figure

With the help of DGS, geometry properties can be shown directly which makes it easy for students to understand. Students can see the whole changing process like watching films so we call it *film-dragging* (Olivero, 2002). While without the software, students need to draw several pictures to find how the geometry objects change. We call it *photo-dragging* (Olivero, 2002). It required students to have more skills to construct as accurate drafts as possible as they can. If they cannot do this, they will find more difficulties to solve the tasks. In this research, teachers use dragging mode more in a second way. They drag the object and show the results of why students are not correct when they find students have some troubles during the lessons. And the results from these lesson videos indicate teachers choose to show the initial and final state of the figures instead of showing the whole moving process, although the latter one is more important for student's learning.

It is because these teachers believe that "Doing mathematics means following rules laid down by the teacher; knowing mathematics means remembering and applying the correct rule when the teacher asks a question, and mathematical truth is determined when the answer is ratified by the teacher." (Lampert, 1990, p.31) So students' work is almost about memorizing what teachers presented without attention to why or when it makes sense to do so, even may not understand the purpose of them and such teacher-dominated actions may not help students employ their own methods to overcome difficulties (Assude, 2007; Erfjord, 2011).

Assimilate DGS into current teaching practices

Using DGS can make teachers think about new forms of practice and teachers need to adapt to these new tools rather than absorbing them (Olive & Makar, 2010). There are two ways for teachers to adapt new instruction practices: assimilation and adaptation, which come from the changes in their way of thinking (Niess, 2005). Researchers have found different strategies to integrate DGS into teaching (e.g. Gueudet & Trouche, 2011): First, in a more classic way, teachers introduce a new mathematics notion and build a figure with DGS and subsequently drag the points so that the students can make some conjectures and in the end let the students create proofs with paper-pencil. Second, they may construct a figure and let students discuss how to build the same figure with DGS, which can be called a black-box type.

From the data, these Chinese teachers always choose the first way to integrate DGS into the teaching process. One of the reasons is that there is only one computer in the classroom so that it is not easy for teachers to spend more time letting students control the software. This also indicated that these teachers are familiar with traditional teaching methods. And what they do is more like use DGS to fit their teaching process but not change their teaching with the help of DGS (Olive & Makar, 2010; Shang, 2008). In this study, they use DGS more to introduce mathematics notions or explain the solutions to the students. It proves that in these lessons, DGS is used as a supporter for mathematics teachers.

In fact, in China, students need to pass the exams without the help of DGS. So these teachers worried that students maybe can solve the problems with DGS, but without the software, they cannot solve them anymore. So in most of the lessons students cannot deal with mathematics tasks with the help of software and explore mathematics contents by themselves. On the other hand, these Chinese cases tend to view effective teaching as a teacher guide with a coherent structure. So that in these lessons, teachers preferred to make preparations for all of the things in the lessons and not change the physical configuration in their courses without leading any changes in learning-teaching routines(Palak & Walls, 2009). In this way, technological resources, like DGS, are assimilated into current practice (Olive & Makar, 2010).

This study puts attention to analyze different ways to integrate DGS into mathematics teaching. Many factors teachers need to consider when preparing to use DGS during the lessons, like the abilities of students, examinations, and so on. In this research, we do not make a deep analysis about how these factors influence teachers' practice in the classroom and how teachers deal with these factors with DGS. It may need further studies to find the relations between these factors and teacher's practice with DGS.

From lecturing to facilitating student's learning

From the lesson videos, It means that teachers always controlled DGS during the lessons (Drijvers et al., 2010), one of the reasons is that these lessons are often constrained by required content coverage, a rapid teaching pace, and large class size (Cai & Wang, 2010). And teachers considered DGS as a supporter tool for their teaching which makes our students lack enough time to operate DGS. So teachers preferred to use *explain-the-screen* and *guide-and-explain* to organize their lessons with DGS. It is not the case which researchers believe that interactivity in a mathematical digital environment means a reaction of the environment to the user's actions related to mathematics (Laborde, 2014).

But we noticed that although *explain-the-screen* always means more controlled by teachers, it does not mean students are always passive in these Chinese lessons. For example, teachers let students explaining what happened on the screen which indicates that in these lessons, teachers noticed that students is a partner in teaching and learning process who need chances to present their ideas. This is one of the characteristics of student-center learning environment (McDonough, 2012). So we think considered these two types as teacher-center type (Drijvers et al., 2010) is not suitable in Chinese cases. Instead, we choose to divided explain-the-screen into two cases: first, if the explainer is a teacher, then this means students cannot participate in the teaching activity, we call it *T-explain-the-screen* and consider it as a teacher-center type; second, when the explainer is student, which means students begin to attend the teaching process and play an important role, we call it as *S-explain-the-screen*. In this type, students can share thoughts with others and listening to others which is critical for developing mathematical proficiency (Bergem & Pepin, 2013). And because during this kind of teaching, students still do not operate DGS, we do not consider it as a student-center type directly, instead as a transition type between teacher-center and student center. Similarly, when teachers use *guide-and-explain*, it also indicates that students are facilitated by the teachers in learning mathematics through answering questions. We use the following figure (Fig. 4) to describe the relation among different types of instrumental orchestration.

Other student-center characteristics were also found in the study. Teacher D let students operate DGS during the lessons which make their students explore mathematical knowledge under the situation which teachers have prepared before the lessons (Alper Ardıç & İşleyen, 2017). Teacher B used group work, in her lessons, to make students share opinions, collaborate with each other and communicate (Arseven et al., 2016). With these methods, students become more active in learning mathematics. And in these teaching processes, DGS allows students to focus on deep conceptual understanding through decision makings like which tools they need to choose to solve the tasks in which technology can bridge the gap between concrete and abstract mathematics (Guerrero, 2010).

Experienced teachers always not only make lectures in lessons but also try to facilitate students by posing different questions. They have noticed that using technologies like DGS can support individualistic pedagogic practices. So they tried to make students reflect by themselves and to make them be more active in learning mathematics in order to facilitate students' learning process by using several student-centre learning methods like group work. In contrast, young teachers more like to control the whole teaching process by themselves. This is because they have fewer experiences to deal with the events which are not prepared by them before the lesson compared with experienced teachers. Also, they believe that technology needs to be used as a supporter for teaching and used after students mastering mathematical concepts and can solve problems by paper-pencil (Kastberg & Leatham, 2005). According to the findings of the study, although in these Chinese lessons, due to the limited number of technological tools and also insufficient time, students cannot manipulate DGS all the times, it does not mean student centered education is not possible. Teachers always think about how to modify their teaching process to apply student centered teaching, which indicated teachers' positive attitudes toward technology and instructional strategies which can help students' cooperative and project-based learning.

Declarations

On behalf of all authors, the corresponding author states that there is no conflict of interest.

Data Availability Statement:

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

References

1. Alper Ardıç, M., & İşleyen, T. (2017). High School Mathematics Teachers' Levels of Achieving Technology Integration and In-class Reflections: The Case of Mathematica. *Universal Journal of Educational Research*, 5(12B), 1–17. https://doi.org/10.13189/ujer.2017.051401
2. Angeli, C., & Valanides, N. (2009). Epistemological and methodological issues for the conceptualization, development, and assessment of ICT–TPCK: Advances in technological pedagogical content knowledge (TPCK). *Computers & Education*, 52(1), 154–168. https://doi.org/10.1016/j.compedu.2008.07.006
3. Arseven, Z., Şahin, Ş., & Kılıç, A. (2016). Teachers' Adoptation Level of Student Centered Education Approach.Journal of Education and Practice, 12
4. Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning*, 7(3), 245–274

5. Assude, T. (2007). Teacher's practices and degree of ICT integration. In *Proceedings of the fifth congress of the European Society for Research in Mathematics Education* (pp. 1339–1348). https://www.researchgate.net/profile/Teresa_Assude/publication/267237311_Teacher's_practices_and_degree_of_ICT_integration/links/54abf3d10cf25c4 practices-and-degree-of-ICT-integration.pdf
6. Assude, T., & Gelis, J. M. (2002). La dialectique ancien-nouveau dans l'intégration de Cabri-géomètre à l'école primaire. *Educational Studies in Mathematics*, 50(3), 259–287
7. Bergem, O. K., & Pepin, B. (2013). Developing Mathematical Proficiency and Democratic Agency through Participation – An Analysis of Teacher-Student Dialogues in a Norwegian 9th Grade Classroom. In B. Kaur, G. Anthony, M. Ohtani, & D. Clarke (Eds.), *Student Voice in Mathematics Classrooms around the World* (pp. 143–160). SensePublishers. https://doi.org/10.1007/978-94-6209-350-8_9
8. Bretscher, N. (2014). Exploring the quantitative and qualitative gap between expectation and implementation: A survey of English mathematics teachers' uses of ICT. In A. Clark-Wilson, O. Robutti, & N. Sinclair (Eds.), *The Mathematics Teacher in the Digital Era* (pp. 43–70). Dordrecht: Springer
9. Bueno-Ravel, L., & Gueudet, G. (2007). Online Resources in Mathematics: Teachers' Genesis of Use. In *Proceedings of the fifth congress of the European Society for Research in Mathematics Education* (pp. 1369–1378)
10. Cai, J., & Wang, T. (2010). Conceptions of effective mathematics teaching within a cultural context: Perspectives of teachers from China and the United States. *Journal of Mathematics Teacher Education*, 13(3), 265–287. https://doi.org/10.1007/s10857-009-9132-1
11. Cayton, C., Hollebrands, K., Okumuş, S., & Boehm, E. (2017). Pivotal teaching moments in technology-intensive secondary geometry classrooms. *Journal of Mathematics Teacher Education*, 20(1), 75–100. https://doi.org/10.1007/s10857-015-9314-y
12. de Castell, S., Bryson, M. K., & Jenson, J. (2002). Object Lessons: Towards an Educational Theory of Technology. *First Monday*, 7(1), 13
13. Drijvers, P., Doorman, M., Boon, P., Reed, H., & Gravemeijer, K. (2010). The teacher and the tool: Instrumental orchestrations in the technology-rich mathematics classroom. *Educational Studies in Mathematics*, 75(2), 213–234. https://doi.org/10.1007/s10649-010-9254-5
14. Drijvers, P., Tacoma, S., Besamusca, A., Doorman, M., & Boon, P. (2013). Digital resources inviting changes in mid-adopting teachers' practices and orchestrations. *ZDM*, 45(7), 987–1001. https://doi.org/10.1007/s11858-013-0535-1
15. Erfjord, I. (2011). Teachers' initial orchestration of students' dynamic geometry software use: Consequences for students' opportunities to learn mathematics. *Technology, Knowledge and Learning*, 16(1), 35–54
16. Fan, W. (2003). A Case Study of Using Geometric Sketchpad to Carry Out Exploratory Mathematics Learning. *China Educational Technology*, 4, 34–36. https://doi.org/10.3969/j.issn.1006-9860.2003.04.010
17. Guerrero, S. (2010). Technological Pedagogical Content Knowledge in the Mathematics Classroom. *Journal of Computing in Teacher Education*, 26(4), 132–139
18. Gueudet, G., & Trouche, L. (2011). Mathematics teacher education advanced methods: An example in dynamic geometry. *ZDM*, 43(3), 399–411. https://doi.org/10.1007/s11858-011-0313-x
19. Healy, L. (2000). Identifying and explaining geometrical relationship: Interactions with robust and soft Cabri constructions. In *Proceedings of the Conference of the International Group for the Psychology of Mathematics Education (PME)* (Vol. 1, pp. 1–103)
20. Hegedus, S., Laborde, C., Brady, C., Dalton, S., Siller, H. S., Tabach, M. … Moreno-Armella, L. (2017). Uses of Technology in Upper Secondary Mathematics Education. In S. Hegedus, C. Laborde, C. Brady, S. Dalton, H. S. Siller, M. Tabach, J. Trgalova, & L. Moreno-Armella (Eds.), *Uses of Technology in Upper Secondary Mathematics Education* (pp. 1–36). Springer International Publishing. https://doi.org/10.1007/978-3-319-42611-2_1
21. Hölzl, R. (2001). Using Dynamic Geometry Software to Add Contrast to Geometric Situations – A Case Study. *International Journal of Computers for Mathematical Learning*, 6, 63–86
22. Hu, W. (2005). *Research of Mathematical Inquiry Teaching Module Based on the Geometer's Sketchpad*. Guangxi Normal University
23. Jenson, J., & Rose, C. B. (2006). Finding space for technology: Pedagogical observations on the organization of computers in school environments. *Canadian Journal of Learning and Technology / La Revue Canadienne de l'apprentissage et de La Technologie*, *32*(1). https://doi.org/10.21432/T2NS3Z
24. Kastberg, S., & Leatham, K. (2005). Research On Graphing Calculators at the Secondary Level: Implications for Mathematics Teacher Education. *Contemporary Issues in Technology and Teacher Education*, 5(1), 25–37
25. Kendal, M., & Stacey, K. (2002). Teachers in transition: Moving towards CAS-supported classrooms. *ZDM – The International Journal on Mathematics Education*, 34(5), 196–203. https://doi.org/10.1007/BF02655822
26. Kortenkamp, U., Blessing, A., Dohrmann, C., Kreis, Y., Libbrecht, P., & Mercat, C. (2009). Interoperable Interactive Geometry for Europe—First Technological and Educational Results and Future Challenges of The Intergeo Project. *Proceeding of CERME 6-Sixth Conference of European Research in Mathematics Education*, 1150–1160
27. Kynigos, C. (2015). Designing Constructionist New Mediations for Creative Mathematical Thinking? *Constructivist Foundations*, 10(3), 305–313
28. Laborde, C. (2001). Integration of Technology in the Design of Geometry Tasks with Cabri-Geometry. *International Journal of Computers for Mathematical Learning*, 6(3), 283–317
29. Laborde, C. (2014). Interactivity and flexibility exemplified with Cabri. *Plenary Lecture given at the Asian Technology Conference in Mathematics*, 11
30. Laborde, C., & Laborde, J. M. (2008). The development of a dynamical geometry environment. In K. Heid, & G. Blume (Eds.), *Research on Technology and the Learning and Teaching of Mathematics, Vol. 2 Cases and Perspectives* (pp. 31–52). Information Age Publishing
31. Lagrange, J. B., Artigue, M., Laborde, C., & Trouche, L. (2003). Technology and Mathematics Education: A Multidimensional Study of the Evolution of Research and Innovation. In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.), *Second International Handbook of Mathematics Education* (pp. 237–269). Springer Netherlands. https://doi.org/10.1007/978-94-010-0273-8_9

32. Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal*, 27(1), 29–63
33. Leung, A. (2003). Dynamic Geometry and The Theory of Variation. In N. A. Pateman, B. J. Dougherty, & J. T.Zilliox (Eds.), *Proceedings of the 27th PME International Conference* (Vol. 3, pp. 197–204)
34. Liu, H. (2009). Integration of Information Technology and Mathematics Curriculum—Geometry Sketchpad—What Can Geometry Sketchpad Do for High School Mathematics Teaching. *The Monthly Journal of High School Mathematics*, 9, 36–38. https://doi.org/10.3969/j.issn.1004-1176.2009.09.017
35. McDonough, M. (2012). Applying learner-centered principles: From face to face instruction to a hybrid course learning format. *Journal of Learning in Higher Education*, 8(2), 31–39
36. Niess, M. L. (2005). Preparing teachers to teach science and mathematics with technology: Developing a technology pedagogical content knowledge. *Teaching and Teacher Education*, 21(5), 509–523. https://doi.org/10.1016/j.tate.2005.03.006
37. Niess, M. L., Ronau, R. N., Shafer, K. G., Driskell, S. O., Harper, S. R., Johnston, C. … Kersaint, G. (2009). Mathematics Teacher TPACK Standards and Development Model. *Contemporary Issues in Technology and Teacher Education*, 9(1), 4–24
38. Norris, C., Sullivan, T., Poirot, J., & Soloway, E. (2003). No Access, No Use, No Impact: Snapshot Surveys of Educational Technology In K-12. *Journal of Research on Technology in Education*, 36(1), 15–27
39. Olive, J., & Makar, K. (2010). Mathematical Knowledge and Practices Resulting from Access to Digital Technologies. In C. Hoyles, & J. B. Lagrange (Eds.), *Mathematics Education and Technology-Rethinking the Terrain: The 17th ICMI Study* (13 vol., pp. 133–177). Springer US
40. Olivero, F. (2002). *The proving process within a dynamic geometry environment* [PhD Thesis]. Graduate School of Education
41. Palak, D., & Walls, R. T. (2009). Teachers' Beliefs and Technology Practices: A Mixed-methods Approach. *Journal of Research on Technology in Education*, 41(4), 417–441
42. Pierce, R., & Stacey, K. (2013). Teaching with new technology: Four 'early majority' teachers. *Journal of Mathematics Teacher Education*, 16(5), 323–347. https://doi.org/10.1007/s10857-012-9227-y
43. Ponte, J. P. D., Oliveira, H., & Varandas, J. M. (2002). Development of Pre-Service Mathematics Teachers' Professional Knowledge and Identity in Working with Information and Communication Technology. *Journal of Mathematics Teacher Education*, 5, 93–115
44. Sabra, H. (2011). *Contribution à l'étude du travail documentaire des enseignants de mathématiques: Les incidents comme révélateurs des rapports entre documentations individuelle et communautaire* [Université Claude Bernard-Lyon I]. https://halshs.archives-ouvertes.fr/tel-00768508/
45. Shang, X. (2008). *The Integrationg of and Geometry Teaching in Middle School*. Southwest University. [Doctor]
46. Sinclair, N., Bussi, B., de Villiers, M. G., Jones, M., Kortenkamp, K., Leung, U., A., & Owens, K. (2016). Recent research on geometry education: An ICME-13 survey team report. *ZDM*, 48(5), 691–719. https://doi.org/10.1007/s11858-016-0796-6
47. Trgalová, J., & Jahn, A. P. (2013). Quality issue in the design and use of resources by mathematics teachers. *ZDM*, 45(7), 973–986. https://doi.org/10.1007/s11858-013-0525-3
48. Trgalova, J., Jahn, A. P., & Soury-Lavergne, S. (2009). Quality process for dynamic geometry resources: The Intergeo project. *Proceedings of CERME*, *6*. http://ife.ens-lyon.fr/publications/edition-electronique/cerme6/wg7-12-trgalovajahnssl.pdf
49. Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: Guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematical Learning*, 9(3), 281–307
50. Zhao, S., Du, W., & Lu, X. (2012). The Practice and Exploration of Geometer's Sketchpad in Junior Middle School Mathematics Teaching. *China Educational Technology*, 20(3), 104–107. https://doi.org/10.3969/j.issn.1006-9860.2012.03.020

Figures

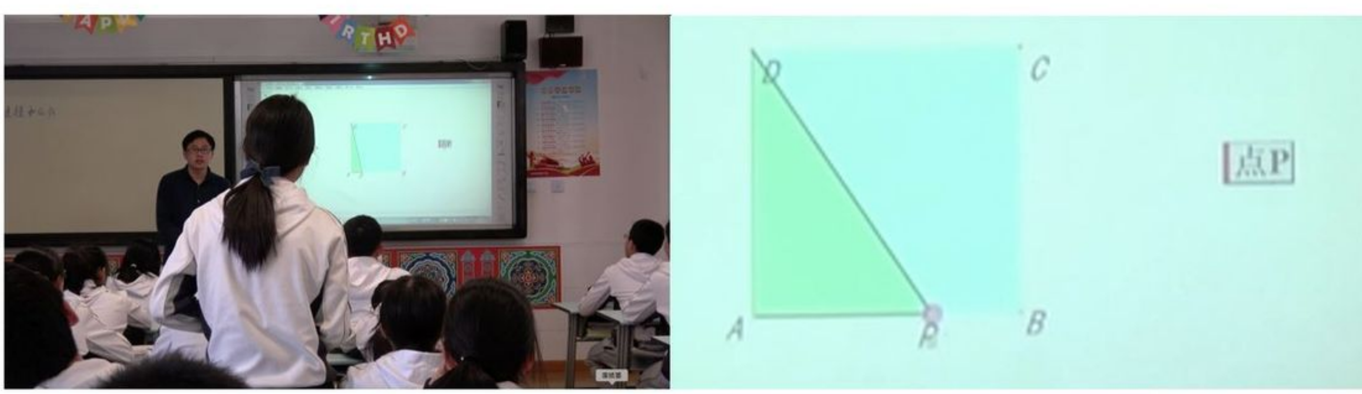


Figure 1

Explain-the-screen

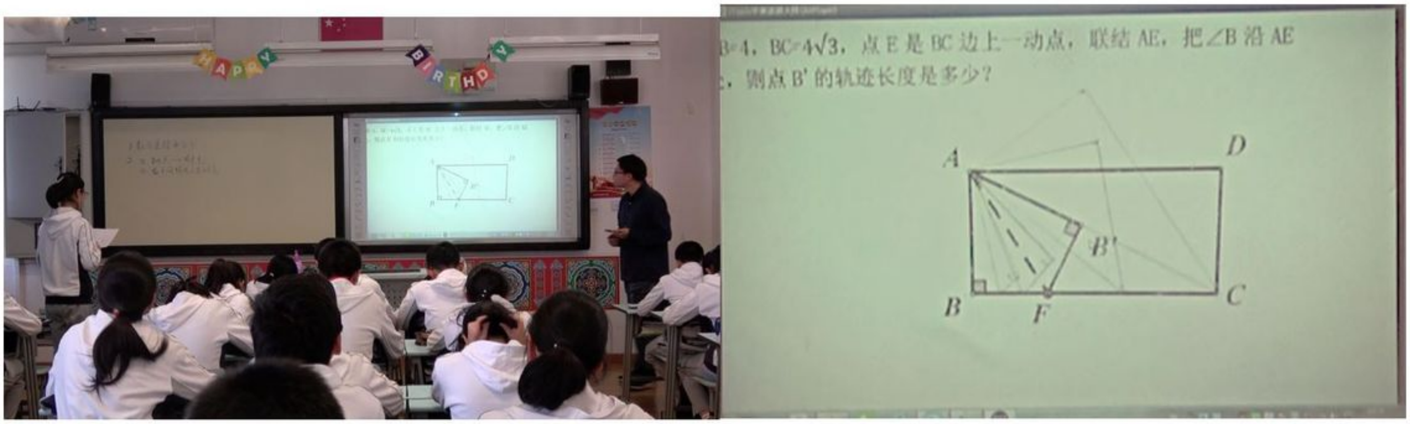


Figure 2

Spot-and-show

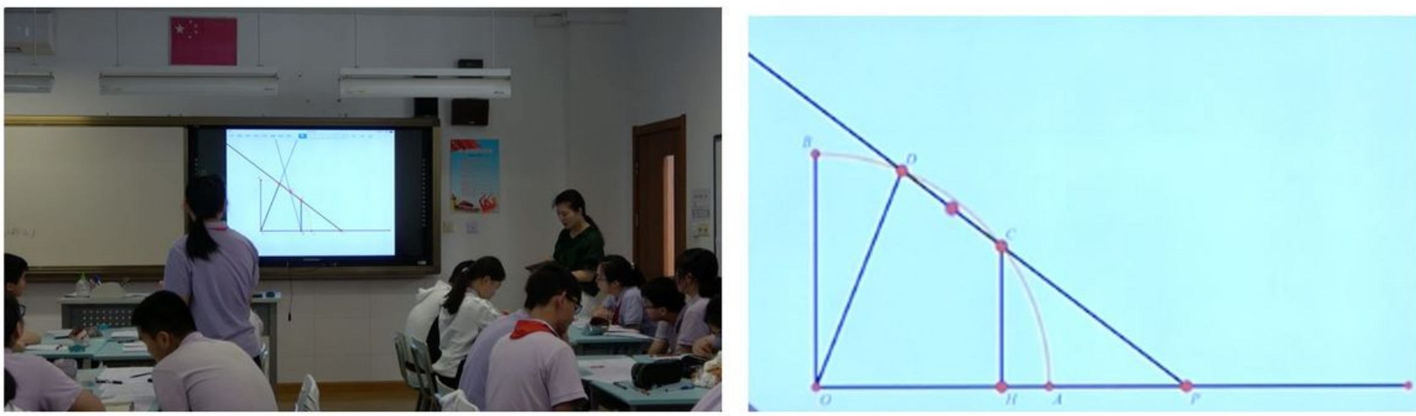


Figure 3

Sherpa-at-work

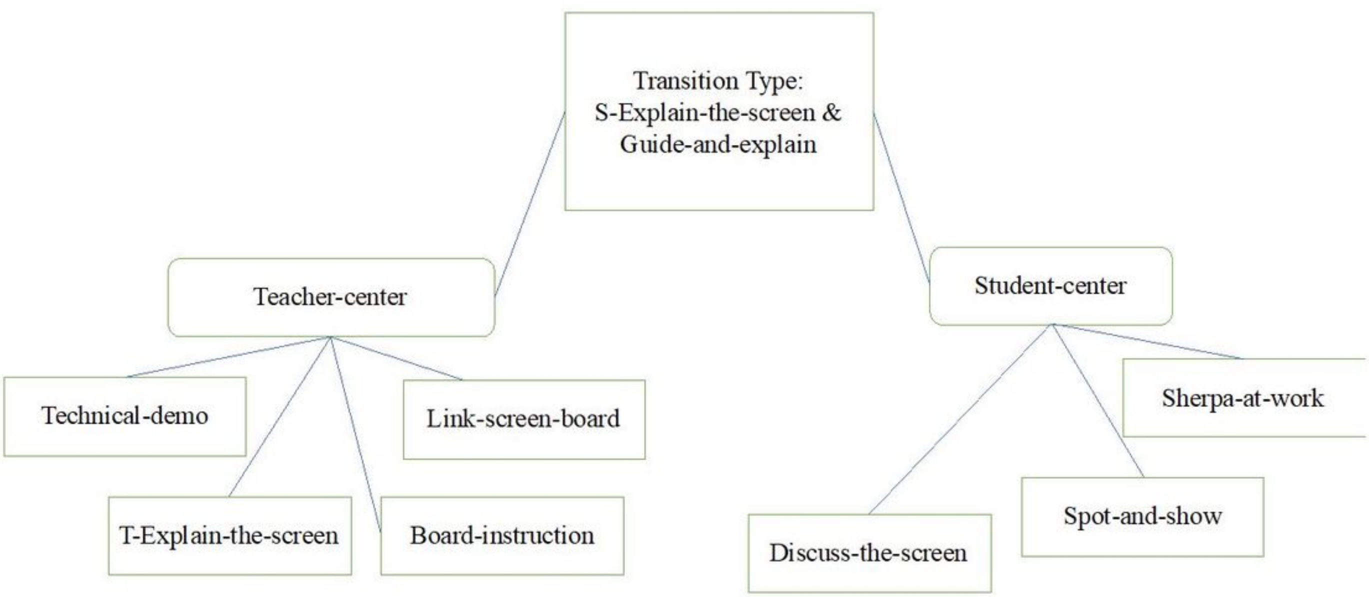


Figure 4

Teacher-center and student-center instrumental orchestration types

Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- Appendix.docx