Arching Effect and Displacement on Theoretical Estimation for Lateral Force Acting on Retaining Wall

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Research Article

Keywords: Retaining wall, Displacement, Arching effect, Numerical model and analysis, Intermediate state

Posted Date: February 22nd, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-161970/v1>

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**Arching effect and displacement on theoretical estimation
for lateral force acting on retaining wall**

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Submitted on January 7, 2021

Abstract

A new approach is proposed to evaluate the non-limit active earth pressure in cohesive-frictional based on the horizontal slices method and limit equilibrium method. This approach takes into account the arching effect, displacement, average shear stress of the soil slice, rupture angle and tension cracks. The accuracy of the proposed method is demonstrated by comparing the experimental results and other theoretical methods. The comparison results show that the proposed approach is suitable for calculating the non-limit active earth pressure in cohesive-frictional soil and cohesionless soil. Additionally, the empirical formulations of the mobilized internal friction angle and soil-wall interface friction angle usually used to cohesionless soil are still applied to cohesive-frictional soil through comparison calculated results of other theoretical methods and finite element method. Some valid formulations of the rupture angle and tension cracks were derived considering the cohesion, wall height, and unit weight.

Keywords: Retaining wall; Displacement; Arching effect; Numerical model and analysis; Intermediate state

1 Introduction

An effective estimation of the lateral earth pressure is essential to the design of retaining walls for cohesive or cohesionless soil. The classical Coulomb (1776) theory or Rankine (1857) theory has been widely used as the simple model in practice. However, several laboratories and field results observed for retaining wall shown that the distribution of active earth pressure was nonlinear, and the lateral earth pressure behind the field wall was mostly in the intermediate state (Tsagareli 1965; Matsuo 1978; Bang 1985; Fang 1986; Zhou et al. 1990; Yue et al. 1992). Some scholars have proposed substantially advanced procedures for predicting the nonlinear distribution of the lateral

earth pressure in cohesionless soil (Handy 1985; Paik and Salgado 2003; Lu et al. 2012; Li et al. 2017; Chen et al. 2019). The research results showed that the soil arching led to the nonlinear distribution of active earth pressure (Fang, 1986).

In recent years, various research methods have been applied to calculate the displacement-dependent the lateral earth pressure and evaluate the working status of a retaining wall. These calculated methods of the lateral earth pressure considering the displacement mainly utilize a fitting method of measured data (Mei et al. 2009; Ni et al. 2018), a geotechnical parameter substitution method (Xu et al. 2013; Wang et al. 2015), and a linear model (Bang 1985; Golam et al. 2015), but the arching effect is hardly still considered in these methods.

In addition, there have been many research achievements such as seismic, unsaturated or reinforced active earth pressure in cohesive-frictional soil (Ahmadabadi and Ghanbari 2009; Kumar 2010; Vahedifard et al. 2015), but few studies on the lateral earth pressure of cohesive-frictional soil considering arching effect and displacement (Zhu et al. 2015; Rao et al. 2016). Moreover, many advanced methods were directly based on the assumption of a slip surface angle ($45^\circ + \phi/2$) or the simplified Coulomb rupture angle (Handy 1985; Paik and Salgado 2003; Venanzio 2010; Tu and Jia 2012), which ignored the influence of cohesion and soil-wall interface friction angle for the slip surface and tension cracks. In this paper, an analytical procedure based on static limit equilibrium methods and horizontal slices method is applied to evaluate the non-limit active earth pressure considering the arching effect, displacement, horizontal average shear stress of soil slice and tension cracks.

2 Mobilization of friction angle and cohesion

The lateral earth pressure with the wall movement is gradually variation from an at-rest state to an

active state (Bang 1985). The internal friction angle (φ_m), soil-wall interface friction angle (δ_m) and cohesion (c_m) with displacement are gradually mobilized from an initial value to a peak value (Chang 1997). Bang (1985) believed that there is a linear relationship between the mobilized internal friction and displacement, so he first derived the formulation of the non-limit active earth pressure in cohesionless soil. Based on Bang' study, Chang (1997) and Wang et al. (2015) constructed an empirical distribution mode of mobilized internal friction when the horizontal displacement (s) at the top of the wall exceeded the critical displacement (s_a). In practice, the relationship is not linear between the mobilized internal friction and displacement (Liu 2014; Chen et al. 2019). Furthermore, the critical displacement (s_a , s_c) of the critical internal friction angle and soil-wall interface friction angle are not consistent (Li et al. 2017).

An empirical model about the mobilized friction angle (φ_m , δ_m), the initial friction angle (φ_0 , δ_0) and the critical friction angle (φ , δ) can be obtained by Eq. (1) and Eq. (2) in cohesionless soil (Chang1997; Li et al. 2017).

$$\varphi_m = \begin{cases} \varphi_0 + \frac{4\arctan(s/s_a)}{\pi}(\varphi - \varphi_0) & (s \leq s_a) \\ \varphi & (s \geq s_a) \end{cases} \quad (1)$$

$$\delta_m = \begin{cases} \delta_0 + \frac{4\arctan(s/s_c)}{\pi}(\delta - \delta_0) & (s \leq s_c) \\ \delta & (s \geq s_c) \end{cases} \quad (2)$$

where s is the horizontal displacement of the wall at the arbitrary depth; s_a is the critical displacement condition corresponding to the maximum internal friction angle; and s_c is the critical displacement corresponding to the maximum soil-wall interface friction angle.

For normally consolidated soil, the internal friction angle (φ_0) under the initial state can be obtained by Eq. (3) when the influence of the initial soil-wall interface friction angle is not considered.

$$\varphi_0 = \arcsin\left(\frac{1 - k_0}{1 + k_0}\right) \quad (3)$$

where k_0 is the coefficient of the at-rest earth pressure, which can be estimated by the empirical formulation (for example cohesive soil $k_0 = 0.95 - \sin \varphi'$, cohesionless soil $k_0 = 1 - \sin \varphi'$ and overconsolidated soil $k_0 = 1 - \sin \varphi' (OCR)^{\sin \varphi}$) (Mayne and Kulhawy 1983; Chen et al. 2019).

Although most scholars have studied the relationship between the mechanical parameters (cohesion and friction angle) and displacement of a retaining wall under translational motion (T mode), few have studies cohesive-frictional soil in this way. Xu et al (2013) derived a formulation for calculating the mobilized internal friction angle of cohesive-frictional soil with displacement through the stress of the Mohr's circle, which is quite different from some advised approaches for cohesionless soil; but his calculation method for mobilized soil-wall interface friction angle is based on prior achievements for cohesionless soil.

For the relationship between the mobilized cohesion (c_m) and displacement, Xu et al. (2013) obtained the Eq. (5) by the geometric relationship in the stress of the Mohr's circle.

$$c_m = \frac{c \cdot \tan \varphi_m}{\tan \varphi} \quad (4)$$

For further illustration, the mobilizations of the friction angle in cohesive-frictional soil are presented based on the numerical results. The empirical model and a laboratory model test are used to verify the effectiveness of the numerical results. The finite element model is composed of a 10 m high cantilever retaining wall and silty sand (Matsuo 1978). The boundary conditions are: the thickness of the mesh was 1m to simulate the plane strain state; the total depth of the mesh and the width at the left side of retaining wall were two times the height of retaining wall. The top surface of the mesh is free, the bottom surfaces were fixed, and other sides can only rotate, as shown in Fig.1. The finite element mesh adopts a quadrilateral element. The element size is 0.5 m. The material

parameters are shown in Table 1. The calculated results of the finite element method are shown in Table 2.

Resultant forces of the lateral earth pressure from the at-rest to intermediate active state were recorded with the wall movement. It was observed that resultant forces of the lateral earth pressure decrease and remain at the same pace with the measured values in Fig 2. The shear force (T) and lateral resultant force (N) acting on retaining wall can be obtained by the finite element method. According to the shear force $T = N \tan \delta_m + (H - z) c_w$ in Fig.7 and the cohesion of soil-wall interface $c_w = c_m \tan \delta_m / \tan \varphi_m$ (Spangler and Handy 1984; Zhu et al 2014), the mobilized soil-wall interface friction angle (δ_m) based on Rankine's theory can be obtained by Eq. (5).

$$\delta_m = \arctan \frac{T}{N + cH \cot \varphi - \frac{2(c \cdot \cot \varphi)^2}{\gamma \cot \varphi_m \tan(\frac{\pi}{4} - \frac{\varphi_m}{2})}} \quad (5)$$

where T and N are the total vertical shear force and lateral resultant force in the finite element method, which acts on the back of retaining wall, respectively.

Fig. 3 shows that the mobilized soil-wall interface friction angle always keeps increasing with the displacement until the soil is at the limit state. The initial value of the soil-wall interface friction angle (δ_0) obtained by the finite element method is 3.28° , which is not consistent with half of the limited internal friction angle in Fang's (1986) experiment. Various errors in the model experiment may have caused certain deformation of the wall and soil, which make the experimental initial value of the soil-wall interface friction angle larger than the ideal initial value. However, for Matsuo's measured value, the soil-wall interface friction angle is too bigger than the theoretical value. After checking the Mohr's circle of stress for an arbitrary point near the retaining wall, this phenomenon is impossible (Paik and Salgado 2003). Moreover, according to the measured value and calculated

value by finite element method, the mobilized soil-wall interface friction angle is going to be smaller than the limit internal friction angle (φ), which is the same as what Fang (1986) and Li et al. (2017) observed in cohesionless soil. So the influence of the cohesion of the soil-wall interface on the soil-wall friction angle may be ignored in Matsuo's experiment, which causes the measured value of the soil-wall interface friction angle to be larger. By comparing the calculated values between the theoretical method and finite element method under different conditions, it can be seen that Eq. (2) is also applicable for cohesive-frictional soil. The initial soil-wall interface friction angle is advised to be 0° , and the critical displacement (s_c) is approximately $0.01H$.

Meanwhile, the resultant force of lateral earth pressure based on Rankine's theory can be expressed by Eq. (6).

$$N = \frac{1}{2} \left(H - \frac{2c_m}{\gamma\sqrt{K_a}} \right) (\gamma HK_a - 2c_m\sqrt{K_a}) \quad (6)$$

where H is the total height of the retaining wall, and γ is the unit weight of the soil.

By substituting Eq. (4) into Eq. (6), and solving a quadratic equation with one unknown, the mobilized internal friction angle (φ_m) can be obtained by Eq. (7).

$$\varphi_m = \arcsin \frac{\gamma H(\gamma H + 2c \cot \varphi) - 2\sqrt{\gamma N(2\gamma Hc \cdot \cot \varphi + 2c^2 \cot^2 \varphi + \gamma N)}}{(\gamma H + 2c \cot \varphi)^2 + 2\gamma N} \quad (7)$$

The variation of the mobilized internal friction angle with the wall movement is presented in Fig. 4. Comparing the calculating values of the three methods, it shows that Eq. (1) is also applied to cohesive-frictional soil. The initial internal friction angle can be appropriately calculated by Eq. (3). The critical displacement (s_a) is approximately $0.01H$, which is agreement with s_c . It shows that the internal friction angle and soil-wall interface friction angle will reach the limit state at the same displacement.

3 Theoretical considerations

The detailed derivation procedures for the non-limit active earth pressure in cohesive-frictional soil are described in the following section. To facilitate the earth pressure for cohesive-frictional soil considering the arching effect and displacement, the present study makes the following assumptions:

(1) The top of retaining wall is horizontal, and there is no load on it.

(2) The rigid plate, whose initial condition is upright, rotates around the heel of the wall.

(3) It is assumed that there is a polyline potential slip surface in cohesive-frictional soil, when retaining wall does not reach the limit equilibrium condition.

(4) The potential slip surface passes through the heel of the wall.

(5) The trajectory of the minor principal stress is an arc (Paik and Salgado2003).

3.1 Coefficient estimation of non-limit active earth pressure

Quinlan (1987) and Kingsley (1989) proved the shape of the soil arching is closer to the arc. According to Paik and Salgado's theory about soil arching, minor principal stress is always tangent to the arc, and major principal stress is always vertical to the arc tangent. As show in Fig. 5, the radius of the arc for the minor principal stress is r , and the centre of arc is the point O . The depth of tension cracks is z . The length of the trapezoidal layer element is L . The distance between the trapezoidal layer element and the top of the wall is y . The deflection angle between the major principal stress and the horizontal surface is θ_A at point A. For point B, the deflection angle between the major principal stress and the horizontal surface is θ_B . For the arbitrary point D , the deflection angle between the major principal stress and the horizontal surface is θ .

To simplify the following theoretical derivation, the stress state will be analyzed by the translational coordinate system in Fig. 6. The transition formulation of the old stress and the new

161 stress is as follows in Eq. (8).

$$162 \quad \begin{cases} \sigma' = \sigma + \sigma_0 \\ \tau' = \tau \end{cases} \quad (8)$$

163 where σ and τ are the normal stress and shear stress in the old coordinate system ($\tau O \sigma$), σ' and τ' is
164 the normal stress and shear stress in the new coordinate system ($\tau' O \sigma'$), respectively.

165 According to the Mohr's circle representation of stresses and stress analysis in Fig. 6, the normal
166 stress (σ_w) and shear stress (τ_w) acting on retaining wall at point A can be expressed by Eq.(9) (Tu et
167 al. 2012; Lou 2015).

$$168 \quad \begin{cases} \sigma'_w = \sigma'_1 \cos^2 \alpha + \sigma'_3 \sin^2 \alpha \\ \tau'_w = (\sigma'_1 - \sigma'_3) \sin \alpha \cos \alpha \end{cases} \quad (9)$$

169 where σ'_1 and σ'_3 are the major principal stress and minor principal stress in the new coordinate
170 system, respectively; α is the angle between the direction of the major principle stress and the
171 vertical direction of the retaining wall. According to the equation $\tau'_w = \sigma'_w \tan \delta_m$, α can be obtained
172 from the following relationship with the soil-wall interface friction angle.

$$173 \quad \alpha = \frac{1}{2} \arccos \frac{-(1 + K_a) \tan^2 \delta_m - \sqrt{(1 - K_a)^2 - 4 K_a \tan^2 \delta_m}}{(1 - K_a)(\tan^2 \delta_m + 1)} \quad (10)$$

174 Then the horizontal stress (σ_{Ah}) at point A can be obtained by Eq. (11).

$$175 \quad \sigma'_{Ah} = (\sigma'_1 \cos^2 \alpha + \sigma'_3 \sin^2 \alpha) \cos \varepsilon \quad (11)$$

176 where ε is the rotation angle of the wall.

177 Similarly, according to the stress analysis in Figure 6 (c), the normal stress (σ_s) and shear stress
178 (τ_s) acting on retaining wall at point B can be expressed by Eq. (12).

$$179 \quad \begin{cases} \sigma'_s = \sigma'_1 \sin^2(\frac{\pi}{4} - \frac{\varphi_m}{2}) + \sigma'_3 \cos^2(\frac{\pi}{4} - \frac{\varphi_m}{2}) \\ \tau'_s = (\sigma'_1 - \sigma'_3) \sin(\frac{\pi}{4} - \frac{\varphi_m}{2}) \cos(\frac{\pi}{4} - \frac{\varphi_m}{2}) \end{cases} \quad (12)$$

180 According to the Mohr's circle representation of the stresses in Figure 6(a), the horizontal normal

181 stress (σ_v') is related to the vertical normal stress (σ_h') by Eq. (13). Then the relationship can also be
 182 applied to an arbitrary point D in the AB trapezoidal layer element.

$$183 \quad \sigma'_1 - \sigma'_v = \sigma'_h - \sigma'_3 \quad (13)$$

184 Although the direction of the principal stress deflects after the arching effect appears, the
 185 horizontal stress of the arbitrary position is nearly equal in the AB trapezoidal layer element of the
 186 sliding wedge. This is different from the vertical stress (Chen et al, 2019).

187 Similarly, the horizontal normal stress, the vertical normal stress and the shear stress at an
 188 arbitrary point D of the trapezoidal layer element can be obtained from the following relationship:

$$189 \quad \begin{cases} \sigma'_{Dh} = \sigma'_1 \cos^2 \theta + \sigma'_3 \sin^2 \theta \\ \sigma'_{Dv} = \sigma'_1 \sin^2 \theta + \sigma'_3 \cos^2 \theta \\ \tau'_D = (\sigma'_1 - \sigma'_3) \sin \theta \cos \theta \end{cases} \quad (14)$$

190 In Fig. 5, the average vertical stress (σ_{av}') in the AB trapezoidal layer element can be derived from
 191 the total vertical stress divided by the width (L) (Rao et al. 2016).

$$192 \quad \sigma'_{av} = \frac{\int_L \sigma'_{Dv} dL}{L} = \frac{\int_{\theta_A}^{\theta_B} \sigma'_{Dv} r \sin \theta d\theta}{L} = \frac{\int_{\theta_A}^{\theta_B} (\sigma'_1 \sin^2 \theta + \sigma'_3 \cos^2 \theta) r \sin \theta d\theta}{L}$$

$$193 \quad = \sigma'_1 + \frac{\cos^3 \theta_A - \cos^3 \theta_B}{3(\cos \theta_A - \cos \theta_B)} (\sigma'_3 - \sigma'_1) \quad (15)$$

194 where the radius of the arc can be solved by Eq. (16), and the deflection angle of the stress (θ_A, θ_B)
 195 can be obtained by Eq. (17).

$$196 \quad r = \frac{L}{\cos \theta_A - \cos \theta_B} \quad (16)$$

$$197 \quad \begin{cases} \theta_A = \alpha + \varepsilon \\ \theta_B = \frac{\pi}{4} - \frac{\varphi_m}{2} + \beta \end{cases} \quad (17)$$

198 where β is the angle between the potential slip surface and horizontal surface.

199 The coefficient of the non-limit active earth pressure (K_{aw}) is defined as the ratio of the horizontal
 200 stress to the average vertical stress at a wall with soil-wall friction and the effects of the soil arching

201 (Handy, 1985; Zhu, 2014). Thus, K_{aw} can be derived from Eq. (18).

$$202 \quad K_{aw} = \frac{\sigma_{Ah}}{\sigma_{av}} = \frac{\sigma'_{Ah} - \sigma_0}{\sigma'_{av} - \sigma_0} = \frac{\sigma'_1 (\cos^2 \alpha + K_a \sin^2 \alpha) \cos \varepsilon - c_m \cot \varphi_m}{\sigma'_1 [1 + \frac{(K_a - 1)(\cos^3 \theta_A - \cos^3 \theta_B)}{3(\cos \theta_A - \cos \theta_B)}] - c_m \cot \varphi_m} \quad (18)$$

203 When a point in cohesionless soil reaches the limit state, the relationship between the major
204 principal stress and the minor principal stress is given as follows (Rao et al. 2016; Chen et al. 2019).

$$205 \quad K_a = \frac{\sigma'_3}{\sigma'_1} = \frac{1 - \sin \varphi_m}{1 + \sin \varphi_m} \quad (19)$$

206 The major principal stress (σ'_1) in cohesive-frictional soil can be obtained by Eq. (20).

$$207 \quad \sigma'_1 = \sigma_1 + \sigma_0 = \gamma y + c_m \cot \varphi_m \quad (20)$$

208 As shown in the above formulation, K_{aw} is related to the change with the depth, and it will cause
209 difficulties when solving the differential equation later. However, when the wall is in the initial
210 position, the effect of soil arching should not appear yet, and the coefficient of the non-limit active
211 earth pressure (K_{aw}) should keep pace with the coefficient of the at-rest earth pressure (K_0). In reality,
212 K_{aw} is inconsistent with the actual initial situation in Eq. (18), and it still continues to increase with
213 the increasing depth in cohesive-frictional soil. In addition, there are generally tension cracks in
214 cohesive-frictional soil on the top of retaining wall. When $y \geq z$ and $\varepsilon = 0$, the coefficient of the
215 non-limit active earth pressure (K_{aw}) is equal at the arbitrary depths after the displacement is
216 determined. If $y = H$, it can be substituted into Eq. (18), and it can be regarded the value as the initial
217 coefficient (K_0) of every point along the depth. Then, the coefficient of the average non-limit active
218 earth pressure (K_{awn}) of every point along the depth shows the same change as the wall movement.

$$219 \quad K_{awn} = \frac{\sigma_{Ah}}{\sigma_{av}} = \frac{\sigma'_{Ah} - \sigma_0}{\sigma'_{av} - \sigma_0} = \frac{(\cos^2 \alpha + K_a \sin^2 \alpha) \cos \varepsilon - \frac{c_m \cot \varphi_m}{\gamma H + c_m \cot \varphi_m}}{1 + \frac{(K_a - 1)(\cos^3 \theta_A - \cos^3 \theta_B)}{3(\cos \theta_A - \cos \theta_B)} - \frac{c_m \cot \varphi_m}{\gamma H + c_m \cot \varphi_m}} \quad (21)$$

220 3.2 Coefficient estimation of horizontal shear stress

221 The horizontal slices method has been widely used for determining active earth pressure at the
 222 intermediate state (Ahmadabadi et al. 2009; Chen et al. 2019), but the horizontal shear stress of the
 223 trapezoidal layer element is hardly considered. Actually, if the horizontal displacement of each point
 224 on the retaining wall is inconsistent, there must be a mutual dislocation among the soil slices. On the
 225 other hand, the horizontal shear stress of each point in the trapezoidal layer element is not equal
 226 because of the arching effect. Therefore, the horizontal average shear stress (τ_{am}) of the trapezoidal
 227 layer element can similarly be obtained from Eq. (22).

$$228 \quad \tau'_{am} = \frac{\int_L \tau'_D dL}{L} = \frac{\int_{\theta_A}^{\theta_B} \tau'_D r \sin \theta d\theta}{L} = \frac{(\sigma'_1 - \sigma'_3)(\sin^3 \theta_B - \sin^3 \theta_A)}{3(\cos \theta_A - \cos \theta_B)} \quad (22)$$

229 The coefficient of the average shear stress (K) can be defined as the ratio of the average shear
 230 stress (τ_{am}) of the trapezoidal layer element to the average vertical stress. Similarly, K is also related
 231 to the depth (y), but it can be replaced with the coefficient (K).

$$232 \quad K' = \frac{\tau'_{am}}{\sigma'_{av}} = \frac{(1 - K_a)(\sin^3 \theta_B - \sin^3 \theta_A)}{3(\cos \theta_A - \cos \theta_B) + (K_a - 1)(\cos^3 \theta_A - \cos^3 \theta_B)} \quad (23)$$

$$233 \quad K = \frac{\tau_{am}}{\sigma_{av}} = \frac{K' \sigma'_{av}}{\sigma_{av}} = K' + K' \frac{\sigma_0}{\sigma_{av}} \quad (24)$$

234 3.3 Tension cracks and rupture angle in cohesive-frictional soil

235 Because the depth of tension cracks at the top of retaining wall is not only related to the
 236 mechanical parameters of the soil, but it is also related to the mechanical parameters of the soil-wall
 237 interface. Moreover, tension cracks at the top of retaining wall are more likely to occur at contact
 238 position of the wall and soil in engineering practice. The cohesion (c_m) of the soil should be replaced
 239 by cohesion of soil-wall interface (c_w) in practical engineering application in Eq. (26). According to
 240 the boundary conditions: $y=z$, $P_{am}|_{y=z} = 0$ and $K_{aw} = 0$, substitute the conditions into Eq. (18), the depth
 241 (z) of tension cracks considering the arching effect can be calculated by Eq. (26).

$$z = \frac{c_w [1 - \cos \varepsilon (\cos^2 \alpha + K_a \sin^2 \alpha)]}{\gamma \tan \varphi_m \cos \varepsilon (\cos^2 \alpha + K_a \sin^2 \alpha)} \quad (25)$$

If the mechanical parameters of the soil were already known, the calculated values for tension cracks could be compared with different theoretical methods in Fig. 8. Because Rankine's theory considered the influence of the soil cohesion (c_m), Lou (2015) ignored the influence of the rotation angle (ε), while the proposed method considered the smaller cohesion of the soil-wall interface (c_w), the calculated values of the proposed method is smaller than other two approaches. Moreover, the depth of tensile cracks for Rankine and Lou's theory was about 1.9 m in the initial state, but it should be closer to 0 m in the practical initial situation. Therefore, the proposed method is more applicable for calculating the depth of tensile cracks under intermediate displacement.

It is well known that the rupture angle can be simply assumed to remain at $45^\circ + \varphi/2$ in Rankine's theory. Based on this assumption, some scholars researched the nonlinear distribution of active earth pressure (Paik and Salgado 2003; Tu and Jia 2012). Kuamr (2010) derived a formulation for the rupture angle considering tension cracks, cohesion and seismic load, but tension cracks are still obtained by the simple Rankine theory. Rao et al. (2016) also derived the calculated formulation for the rupture angle by the geometrical relationship with soil arching. To obtain a simple formulation for the angle of the potential slip surface (β) in cohesive-frictional soil, the proposed method still need to be assumed that the total weight of the slip zone (ABC) always remains constant from the initiation to failure.

In Fig.7, T_1 and T_2 are the shear forces in the BC and AB plane, N_1 and N_2 are both the normal forces across the contact surface BC and EB, respectively; Q and R are both the resultant forces acting on the sliding wedge.

According to the static equilibrium condition of the sliding wedge in Fig. 7, then

$$T_1 \cos \beta + N_2 \cos \varepsilon = N_1 \sin \beta + T_2 \sin \varepsilon \quad (26)$$

Based on the Mohr-Coulomb criterion, the shear forces (T_1, T_2) can be obtained by Eq. (27).

$$\begin{cases} T_1 = N_1 \tan \varphi_m + \frac{c_m(H-z)}{\sin \beta} \\ T_2 = N_2 \tan \delta_m \end{cases} \quad (27)$$

The normal force (N_1, N_2) along the contact surface can be expressed by Eq. (28).

$$\begin{cases} N_1 = \frac{G \cos \psi \cos(\varepsilon + \delta_m)}{\cos(\beta - \psi - \varepsilon - \delta_m)} \\ N_2 = Q \cos \delta_m \end{cases} \quad (28)$$

The total weight (G) of the soil wedge ($EBCD$) can be obtained by Eq. (29).

$$G = \frac{1}{2} \gamma H^2 (\cot \beta + \tan \varepsilon) - \frac{\gamma z^2}{2} \cot \beta \quad (29)$$

By Substituting Eq. (27), Eq. (28) and Eq. (29) into Eq. (26), resultant force of the lateral earth pressure (Q) can be derived from Eq. (30).

$$Q = \frac{1}{2} \lambda_1 \gamma H^2 (\cot \beta + \tan \varepsilon) - \frac{\lambda_1 \gamma z^2}{2} \cot \beta - \lambda_2 c \quad (30)$$

where λ_1 and λ_2 are the coefficients obtained by Eq. (31).

$$\begin{cases} \lambda_1 = \frac{\sin(\beta - \varphi_m) \cos \psi}{\cos(\beta - \psi - \varepsilon - \delta_m) \cos \varphi_m} \\ \lambda_2 = \frac{H - z}{\tan \beta \cos(\varepsilon + \delta_m)} \end{cases} \quad (31)$$

Because there is no cohesion in cohesionless soil, the angle between the reaction force (R) and normal BC plane is equal to the internal friction angle (φ_m). However, some scholars still ignored the influence of cohesion on the reaction force, when the soil is viscous (Xu et al 2013; Zhu et al. 2015). Actually, the relationship between the angle (ψ) and other factors is intricate in cohesive-frictional soil. Based on the equivalence principle of the shear strength (Zhao and Bai, 2004), the angle (ψ) of the reaction force R , internal soil friction (φ) and cohesion (c) can be established using the following relationship for Eq. (32).

$$N_1 \tan \psi = N_1 \tan \varphi_m + \frac{c_m(H-z)}{\sin \beta} \quad (32)$$

By Substituting Eq. (28) and Eq. (29) into Eq. (32), the following Eq. (33) can be obtained.

$$\tan \psi = \frac{\gamma(H+z) \cdot \tan \varphi_m + 2c_m[1 + \tan \beta \cdot \tan(\delta_m + \varepsilon)]}{\gamma(H+z) - 2c_m[\tan \beta - \tan(\delta_m + \varepsilon)]} \quad (33)$$

If the soil-wall interface is upright and smooth, the failure plane (β) is $45^\circ + \varphi/2$, and this condition can be substituted into Eq.(33); therefore, the angle (ψ) of the reaction force R will become a known parameter taking into account the influence of the cohesion by Eq. (34).

$$\tan \psi = \tan \varphi_m + \frac{2c_m[1 + \tan(\frac{\pi}{4} + \frac{\varphi_m}{2})]}{\gamma H} \quad (34)$$

According to Eq. (30), Q is a function about β . The depth (z) of tension cracks is also a function about β , but it is not related to the wall height. It is difficult to obtain a formulation about β by directly differentiating β from Eq. (30). However, if by substituting $H=y$ (depth) to Eq. (30), active earth pressure (p_a) can firstly be obtained through $p_a = \frac{\partial Q}{\partial y}$.

$$p_a = \frac{\partial Q}{\partial y} = \lambda_1 \gamma (\tan \varepsilon + \cot \beta) - \frac{c_m \cot \beta}{\cos(\varepsilon + \delta_m)} \quad (35)$$

Because the failure plane always passes through point B , the rupture angle (β) is related to the wall height (H) instead of the depth (y). Substitute Eq. (31) and $h=H$ into Eq. (35); then active earth pressure (p_a) is a function about $\cot \beta$.

Because $\cot \beta$ is monotonic functions about β , if only one value exists for $\cot \beta$ which can make p_a reach the extreme value, it is obvious that only an existed value for β can make p_a reach an extreme value. Since β satisfies the conditions: $0^\circ < \beta < 90^\circ$, $\cot \beta$ cannot be negative (Kumar 2010). When

$\frac{\partial p_a}{\partial(\cot \beta)} = 0$, only a limit value can be obtained for $\cot \beta$ by Eq. (36).

$$\cot \beta = -\tan(\psi + \varepsilon + \delta_m) + \sqrt{\frac{H\gamma[\tan(\psi + \varepsilon + \delta_m) - \tan \varepsilon][\tan(\psi + \varepsilon + \delta_m) \tan \varphi_m + 1]}{H\gamma \tan \varphi_m - c_m \cdot \tan \psi \tan(\delta_m + \varepsilon) + c_m}} \quad (36)$$

When $c=0$ and $\psi = \varphi_m$, Eq. (37) is the Coulomb solution of rupture angle by substituting them into Eq. (36) (Gong, 2014).

$$\cot\beta = -\tan(\varphi_m + \varepsilon + \delta_m) + \sqrt{[\tan(\varphi_m + \varepsilon + \delta_m) - \tan \varepsilon][\tan(\varphi_m + \varepsilon + \delta_m) + \cot \varphi_m]} \quad (37)$$

In Fig. 9, several theoretical methods for determining the rupture angle were compared. The results show that the potential rupture angle (β) continues to increase with displacement increase from a potential slip surface to the critical failure surface except for Rao's method, calculated values of the proposed method is consistent with Rankine solution and Zhu's method. When the shear strength of the soil will reach the limit state, the rupture angle for proposed method will gradually approach $45^\circ + \varphi/2$. Because Rao et al. (2015) derived a formulation of rupture angle based on the geometrical relationship with arching effect, but other methods obtained the formulation of rupture angle based on the limit equilibrium method, the calculated values of Rao's method were significantly larger than other methods. Moreover, the proposed method and Zhu's method considered the influence of the wall height, cohesion and unit weight; but other methods only considered the internal friction angle and the soil-wall friction angle.

3.4 Lateral earth pressure at the intermediate active state

To obtain active earth pressure at the intermediate state, a trapezoidal layer element is subjected to mechanical analysis, as shown in Fig. 10.

By establishing the static equilibrium in the horizontal direction, Eq. (38) can be obtained (Chen et al. 2019).

$$\tau_w dy \tan \varepsilon - \sigma_w dy + \sigma_s dy - \tau_s dy \cot \beta + \tau_{am} L - (\tau_{am} + d \tau_{am}) [L - dy(\tan \varepsilon + \cot \beta)] = 0 \quad (38)$$

where the length (L) of the trapezoidal layer element can be expressed by the formulation

$$L = (H - y)(\tan \varepsilon + \cot \beta).$$

According to the above assumptions, the stresses in the soil have the following relationships.

$$\tau_w = \sigma_w \tan \delta_m; \tau_s = \sigma_s \tan \varphi_m + c_m; \sigma_{Ah} = K_{avn} \sigma_{av} \quad (39)$$

From Eq. (9) and Eq. (11), the following equation can be obtained:

$$\sigma_w = \frac{K_{awn} \sigma_{av}}{\cos \varepsilon} + \sigma_0 \left(\frac{1}{\cos \varepsilon} - 1 \right) \quad (40)$$

Based on Eq. (24), the average shear stress of the trapezoidal layer element (τ_{am}) can be expressed as the following Eq. (41) by the average vertical stress (σ_{av}).

$$\tau_{am} = K \sigma_{av} \quad (41)$$

Eq. (39), Eq. (40) and Eq. (41) are substituted into Eq. (38). Considering that the second derivative and rotation angle (ε) are generally sufficiently small, their influence can be ignored to simplify the formulation. Therefore, Eq. (42) can be derived as follows:

$$\sigma_s = \frac{K' L d \sigma_{av}}{(1 - \tan \varphi_m \cot \beta) dy} + \frac{\sigma_{av} (K_{awn} - K' \cot \beta) - K' \cot \beta \sigma_0 + c_m \cot \beta}{1 - \tan \varphi_m \cot \beta} \quad (42)$$

For the trapezoidal layer element in Fig. 10, the static equilibrium in the vertical direction requires the use of Eq. (43):

$$\sigma_{av} L + dG - (\sigma_{av} + d \sigma_{av}) [L - dy(\tan \varepsilon + \cot \beta)] - \sigma_w dy \tan \varepsilon - \tau_w dy - \sigma_s dy \cot \beta - \tau_s dy = 0 \quad (43)$$

The above formulation is simplified; The following Eq. (44) can be obtained.

$$\frac{d \sigma_{av}}{dy} = \gamma + \frac{\sigma_{av}}{H - y} - \frac{\sigma_w (\tan \delta_m + \tan \varepsilon) + \sigma_s (\cot \beta + \tan \varphi_m) + c_m}{L} \quad (44)$$

Eq. (40) and Eq. (42) are substituted into Eq. (44), and Eq. (45) is derived as follows:

$$A \frac{d \sigma_{av}}{dy} = \gamma + B \frac{\sigma_{av}}{H - y} - \frac{C}{H - y} \quad (45)$$

where A , B and C are the constants, and they yield the following expressions:

$$\begin{cases} A = 1 + \frac{K'}{\tan(\beta - \varphi_m)} \\ B = 1 - \frac{K_{awn} (\tan \delta_m + \tan \varepsilon)}{\cos \varepsilon (\cot \beta + \tan \varepsilon)} - \frac{K_{awn} - K' \cot \beta}{\tan(\beta - \varphi_m) (\cot \beta + \tan \varepsilon)} \\ C = \frac{\sigma_0 (\tan \delta_m + \tan \varepsilon) (1 - \cos \varepsilon)}{\cos \varepsilon (\cot \beta + \tan \varepsilon)} + \frac{c_m - K' \sigma_0}{\tan(\beta - \varphi_m) (1 + \tan \beta \tan \varepsilon)} + \frac{c_m}{\cot \beta + \tan \varepsilon} \end{cases}$$

Because the boundary condition is the average vertical stress $\sigma_{av} = \gamma z$, when $y = z$. Substituting them

into equation (45), the average vertical stress can be expressed by Eq. (46).

$$\sigma_{av} = [\gamma z + \frac{\gamma(H-z)}{A+B} - \frac{C}{B}] (\frac{H-y}{H-z})^{\frac{-B}{A}} - \frac{\gamma(H-y)}{A+B} + \frac{C}{B} \quad (46)$$

When $0 \leq y < z$, the horizontal stress $\sigma_h=0$ because of tension cracks; when $z \leq y < H$, the horizontal stress $\sigma_h = K_{aw} \sigma_{av}$.

$$\sigma_h = \begin{cases} 0 & (0 \leq y < z) \\ K_{aw} \left\{ [\gamma z + \frac{\gamma(H-z)}{A+B} - \frac{C}{B}] (\frac{H-y}{H-z})^{\frac{-B}{A}} - \frac{\gamma(H-y)}{A+B} + \frac{C}{B} \right\} & (z \leq y < H) \end{cases} \quad (47)$$

In Fig.7, the angle between active earth pressure and the horizontal surface is $\delta_m + \varepsilon$ (Zhao and Bai 2004). Therefore, the non-limit active earth pressure (p_{am}) is derived from Eq. (48).

$$p_{am} = \begin{cases} 0 & (0 \leq y < z) \\ \frac{K_{aw}}{\cos(\delta_m + \varepsilon)} \left\{ [\gamma z + \frac{\gamma(H-z)}{A+B} - \frac{C}{B}] (\frac{H-y}{H-z})^{\frac{-B}{A}} - \frac{\gamma(H-y)}{A+B} + \frac{C}{B} \right\} & (z \leq y < H) \end{cases} \quad (48)$$

According to Eq. (48), the non-limit active earth pressure (p_{am}) with the displacement increases is possibly negative on the bottom of the wall as the displacement increases because of soil arching. Therefore, this part of the negative earth pressure should be ignored. Assuming that y_0 is a critical point of the lateral earth pressure from the bottom of the wall, the non-limit lateral resultant force (Q_m) is given as follows:

$$Q_m = \int_z^{H-y_0} p_{am} dy = \frac{K_{aw}}{\cos(\delta_m + \varepsilon)} \left\{ \left[\gamma z + \frac{\gamma(H-z)}{A+B} - \frac{C}{B} \right] \frac{A}{A-B} \times [(H-z) - y_0 (\frac{y_0}{H-z})^{\frac{-B}{A}}] + \frac{\gamma[y_0^2 - (H-z)^2]}{A+B} + \frac{C}{B} (H - y_0 - z) \right\} \quad (49)$$

4 Parametric studies

4.1 Cohesion

Fig.11 shows the distribution of active earth pressure along the depth of the wall with different values of cohesion. To obtain these data, the calculated parameters of the soil were based on the unit weight of the soil $\gamma=19.0\text{kN/m}^3$, internal friction angle $\varphi=30^\circ$, wall height $H=10\text{m}$, and limit soil-wall

365 interface angle $\delta=2\varphi/3$. Because active earth pressure at tension cracks is 0 kPa, the calculated value
366 of active earth pressure for the proposed method is not uniformly marked throughout this paper in
367 Fig.11.

368 According to Fig.11, active earth pressure gradually decreases with the increasing cohesion. The
369 distribution of active earth pressure still keeps linearly increasing at $0 \leq y \leq 0.6H$, it does not
370 significantly decrease until the obvious arching effect appears near the bottom of the wall. In
371 addition, because tension cracks are considered in the proposed method, which is different from the
372 Rankine theory, the changes of active earth pressure near the top is slight with the increasing
373 cohesion. With the increasing depth, the effect of the cohesion on the active pressure gradually
374 increases. From Fig.11, active earth pressure at the bottom appears a negative value with the
375 increasing cohesion, which shows that tension cracks appear when the cohesion $c \geq 20kPa$.
376 Because the peak point of active earth pressure keeps moving up with increasing cohesion, it shows
377 that the position of the vertical soil arching also continues moving up. The results show the cohesion
378 is an important factor for arching effect.

379 4.2 Friction angle

380 Fig.12 shows the distribution of active earth pressure along the depth of the wall with different
381 friction angles. To obtain these data, the calculated parameters of the soil are based on the unit
382 weight of the soil $\gamma=19.0 \text{ kN/m}^3$, cohesion $c=10 \text{ kPa}$, wall height $H=10\text{m}$, and limit soil-wall
383 interface angle $\delta=2\varphi/3$.

384 Similarly, active earth pressure gradually decreases with the increasing internal friction angle. The
385 upper active earth pressure basically increases linearly, and active earth pressure does not decrease
386 rapidly until the stress deflection obviously occurs near the bottom of the wall. Moreover, as the

387 internal friction angle increases, the height of tension cracks also increases gradually in Fig.12.

388 Fig.13 shows the variation of the average vertical stress with different depth and soil-wall
389 interface friction angle. The average vertical stress can be obtained from Eq. (48) as a function of the
390 depth and soil-wall interface friction angle. The calculated parameters of the soil were based on the
391 unit weight of the soil $\gamma=19.0 \text{ kN/m}^3$, cohesion $c=20 \text{ kPa}$, internal friction angle $\varphi=27^\circ$, initial
392 soil-wall interface friction angle $\delta_0=0^\circ$, limit soil-wall interface friction angle $\delta=18^\circ$ and wall height
393 $H=10\text{m}$.

394 According to Fig.13, when $\delta_m=0^\circ$, the average vertical stress linearly increases with increasing
395 depth until the depth $y=0.9H$. When $\delta_m \neq 0^\circ$, the average vertical stress obviously appears nonlinear
396 with increasing depth and soil-wall interface friction angle; it shows the significant reduction with
397 the increasing soil-wall interface friction angle, especially at the bottom. At the same time, it shows
398 that δ_m is also a crucial factor, due to causing a significant change for the position of the soil arching.

399 Fig.14 shows the coefficient variation of the average lateral active earth pressure with the different
400 soil-wall interface friction angle and displacement. K_{awn} can be obtained from Eq. (22) as a function
401 of the soil-wall interface friction angle and displacement. The calculated parameters of the soil were
402 based on the unit weight of the soil $\gamma=19.0 \text{ kN/m}^3$, cohesion $c=20 \text{ kPa}$, internal friction angle $\varphi=27^\circ$,
403 initial soil-wall interface friction angle $\delta_0=0^\circ$ and wall height $H=10\text{m}$. As shown in Fig.14, K_{awn} is
404 nonlinearly decreasing with the displacement; as the critical soil-wall interface friction angle (δ)
405 increases, the unit decrease in K_{awn} also decreases.

406 4.3 Horizontal shear stress between the slices

407 Fig.15 shows the coefficient variation of the average shear stress with different depth and
408 displacement. The coefficient of the average shear stress (K) can be obtained from Eq. (24), Eq. (48)

and Eq. (24) as a function of the depth and displacement. To obtain the data, the calculated parameters of the soil were based on the unit weight of the soil $\gamma=19.0 \text{ kN/m}^3$, cohesion $c=20 \text{ kPa}$, internal friction angle $\varphi=27^\circ$, initial soil-wall interface friction angle $\delta_0=0^\circ$, limit soil-wall interface friction angle $\delta=18^\circ$ and the wall height $H=10 \text{ m}$. The coefficient of the average shear stress at a certain depth continuously increases with increasing displacement, and it gradually tends to a constant value until the soil-wall mass arrives at the limit state. In addition, because of the increasing depth of tension cracks, the coefficient of the average shear stress increases significantly at the depth $y/H=0.1$ than other depth. From Fig.15, the coefficient (K) is always smaller than $\tan \varphi_m$. This result shows that horizontal shear failure will not occur under normal conditions.

Fig.16 shows the distribution of active earth pressure along the depth of the wall considering soil arching and horizontal shear stress or only soil arching. The parameters used for the soil are the same as those mentioned above.

According to Fig.16, active earth pressure considering soil arching with horizontal shear stress is smaller than that only considering soil arching. At the same time, this indicates that the influence of the average shear stress gradually weakens with the displacement, and the horizontal shear stress between the slices hinders the destruction of the soil. Therefore, the horizontal shear stress of the soil slice can be used as a safe reserve for retaining wall's stability under normal conditions.

5 Discussion and comparison of the results

To check the applicability of the proposed method in cohesive-frictional soil and cohesionless soil, the predictions from the derived equation were compared with experimental values and other theory methods.

First, in cohesive-frictional soil, Yue (1992) conducted 11 sets of centrifuge model tests by a

homemade hydraulic control device, which could control the displacement. The parameters of the soil were based on the unit weight of the soil $\gamma=18.6 \text{ kN/m}^3$, cohesion $c=38.2 \text{ kPa}$, internal friction angle $\varphi=22.7^\circ$, wall height $H=250\text{mm}$, and model rate of 80.18. In the test, the horizontal displacement of retaining wall arriving at the limit state is $s=2.7\text{mm}$. According to Fig.17, the measured values of lateral earth pressure are mainly distributed on both sides of the calculated values of the proposed method. The results showed that the calculated values of the proposed method were in good agreement with the measured values. The measured resultant force of active earth pressure was approximately 698.24 kN, the calculated value of the proposed method was 677.75 kN, and the calculated value of Rankine theory was 788.11 kN.

Secondly, in cohesionless soil, Tsagareli (1965) measured the distribution of active earth pressure under the translation mode. The parameters of the soil were based on the unit weight of the soil $\gamma=18.0 \text{ kN/m}^3$, cohesion $c=0 \text{ kPa}$, internal friction angle $\varphi=37^\circ$, wall height $H=4 \text{ m}$, and critical soil-wall interface angle $\delta=2\varphi/3$. From Fig.18, the calculated values of the proposed method were in good agreement with Rankine theory at $0 \leq y \leq 0.5H$. The significant differences appeared at the bottom due to the arching effect. The measured resultant force of active earth pressure was approximately 40.06 kN, the calculated value of the proposed method was 33.04kN, the calculated value of Rankine theory was 35.80 kN, and the calculated value of Handy's method was 29.58 kN.

Therefore, the calculated values of active earth pressure for the proposed method were basically consistent with the measured values and other theoretical methods.

6 Summary and Conclusions

The estimation of the lateral earth pressure acting on retaining wall is extremely important in practical engineering. The simple model for the classical Rankine theory and Coulomb theory has

453 been widely applied in practice. However, the lateral earth pressure behind the wall is mostly in the
454 intermediate state with the increasing displacement, and its distribution is nonlinear. Although some
455 had attempted to estimate active earth pressure considering arching effect, the displacement and the
456 effect of soil arching exist mutually, but most studies consider them separately and ignore the
457 influence of rotation angle and horizontal shear stress. In this paper, a new formulation for the
458 non-limit active earth pressure is derived for cohesive-frictional soil, which is based on the
459 horizontal slices and static equilibrium method. The relevant features of the proposed method are
460 summarized below:

461 (1) The influence of the displacement and the arching effect was considered in the proposed
462 method. Analytical expressions of the lateral non-limit active earth pressure were obtained for the
463 potential rupture angle, depth of tension cracks, rotation angle, and horizontal average shear stress.

464 (2) The variation of the mobilized internal friction angle and soil-wall interface friction angle for
465 cohesive-frictional soil with the displacement was performed by comparing the FEM and other
466 theoretical methods. It shows that the empirical formulations for the mobilized soil-wall interface
467 friction angle and internal friction angle are still applicable to cohesive-frictional soil.

468 (3) A formulation of the rupture angle was derived by considering the cohesion, soil-wall interface
469 friction angle, wall height and the unit weight. Depth of tension cracks was calculated considering
470 the influence of the arching effect and cohesion of the soil-wall interface. The formulation of the
471 rupture angle and tension cracks can better reflect the actual situation in cohesive-frictional soil by
472 compared with other methods.

473 (4) By comparing the experimental results and other theoretical methods, it was concluded that the
474 proposed analytical procedure could reliably calculate the non-limit active earth pressure, lateral
475 resultant force, rupture angle, depth of tension cracks, mobilized internal friction angle and soil-wall

476 interface friction in cohesive-frictional soil.

477 **Data Availability Statement**

478 All data generated or analyzed during this study are included within the article.

479 **Acknowledgments**

480 The study was supported by the National Natural Science Foundation of China (Grant 41272333),
481 National Program on Key Basic Research Project of China (Grant 2011CB013501). We express our
482 gratitude for this financial support.

483 [https://github.com/jjf524973903/Theoretical-analysis-for-lateral-earth-pressure-considering-arching-](https://github.com/jjf524973903/Theoretical-analysis-for-lateral-earth-pressure-considering-arching-effect-in-cohesive-soil)
484 [effect-in-cohesive-soil](https://github.com/jjf524973903/Theoretical-analysis-for-lateral-earth-pressure-considering-arching-effect-in-cohesive-soil)

485 **Conflict of interest**

486 The authors declare that they have no conflict of interest.

487 **Notation**

488 The following symbols are used in this paper:

489 H =total height of retaining wall

490 s =displacement of retaining wall

491 z =depth of tension cracks

492 b =width of tension cracks zone

493 y =arbitrarily depth of retaining wall

494 β =rupture angle

495 ε =rotational angle of retaining wall

496 γ =unit weight

497	c =cohesion
498	c_m = the mobilized cohesion
499	φ =internal friction angle
500	φ_0 =initial internal friction angle
501	φ_m =mobilized internal friction angle
502	Ψ =angle between slip surface and reaction force R
503	δ =critical soil-wall interface friction angle
504	δ_0 =initial soil-wall interface friction angle
505	δ_m =mobilized soil-wall interface friction angle
506	λ_1, λ_2 = coefficient
507	G = weight of sliding wedge (EBCD)
508	R = reaction force
509	Q = reaction resultant force of wall to the soil wedge
510	Q_m = non-limit lateral resultant force
511	T_1, T_2 = shear force in the BC and AB plane
512	N_1, N_2 = vertical force across the contact surface
513	P_a = active earth pressure
514	P_{am} = non-limit active earth pressure
515	s_a = critical displacement condition under which the soil will yield
516	s_c = critical displacement corresponding to the maximum soil-wall interface friction angle
517	k_0 = coefficient of the at-rest earth pressure
518	r = radius of the arc for the minor principal stress
519	L = length of the trapezoidal layer element

520 K_{aw} = coefficient of the lateral active earth pressure

521 K_{awn} = coefficient of the average non-limit active earth pressure

522 K = coefficient of the horizontal average shear stress

523 T = the shear force acting on retaining wall in the FEM method

524 N = resultant force of the lateral earth pressure acting on retaining wall in the FEM method

525 σ_1 = major principal stress

526 σ_3 = minor principal stress

527 σ_{av} = average vertical stress

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644 **Fig. 17** Comparison between predicted and experimental values obtained about lateral earth pressure
645 in cohesive-frictional soil

646 **Fig. 18** Comparison between predicted and experimental values obtained about active earth pressure
647 in cohesionless soil

648 **Table 1** Material parameters used in the finite element model

<i>Parameters</i>	<i>Silty sand</i>	<i>Concrete</i>	<i>Soil-wall interface</i>
<i>Unit weight (γ, kN/m³)</i>	19.40	25.00	/
<i>Cohesion(c, kPa)</i>	23.00	/	/
<i>Internal friction angle (ϕ, °)</i>	27.00	/	/
<i>Elastic modulus (E, MPa)</i>	0.75	30000	/
<i>Poisson ratio (ν)</i>	0.32	0.20	/
<i>Normal stiffness ratio coefficient</i>	/	/	1
<i>Tangential stiffness proportional coefficient</i>	/	/	0.1

649 **Table 2** Calculated results of the finite element model

<i>Calculation Steps</i>	<i>Max horizontal displacement s (mm)</i>	<i>Horizontal lateral resultant force N (kN)</i>	<i>Shear force T (kN)</i>	<i>Internal frictional angle ϕ_m (°)</i>	<i>Soil-wall interface frictional angle δ_m (°)</i>
<i>Step 1</i>	0	304.24	38.13	17.49	3.28
<i>Step 2</i>	4.1	294.43	41.88	17.90	3.67
<i>Step 3</i>	8.3	284.67	46.65	18.31	4.17
<i>Step 4</i>	12.5	274.94	52.22	18.73	4.76
<i>Step 5</i>	16.6	265.23	58.41	19.16	5.43
<i>Step 6</i>	21.2	255.55	65.06	19.59	6.18
<i>Step 7</i>	25.7	245.90	72.07	20.03	6.99
<i>Step 8</i>	30.2	236.28	79.33	20.48	7.86
<i>Step 9</i>	34.8	226.69	86.79	20.94	8.79
<i>Step 10</i>	39.5	217.11	94.41	21.41	9.78
<i>Step 11</i>	44.2	207.56	102.14	21.88	10.83
<i>Step 12</i>	49.0	198.03	109.97	22.37	11.93
<i>Step 13</i>	53.8	188.50	117.87	22.86	13.11
<i>Step 14</i>	58.7	178.99	125.83	23.37	14.34
<i>Step 15</i>	63.6	169.49	133.84	23.89	15.65
<i>Step 16</i>	68.5	160.00	141.88	24.43	17.04
<i>Step 17</i>	73.4	150.51	149.96	24.98	18.50
<i>Step 18</i>	78.4	141.03	158.06	25.55	20.06
<i>Step 19</i>	83.5	131.55	166.07	26.14	21.70
<i>Step 20</i>	90.4	121.92	171.16	26.75	23.10
<i>Step 21</i>	99.3	112.33	173.28	27.00	24.02

Fig. 1 Finite element analytical model for the lateral earth pressure

Fig. 2 Variation of resultant force of the lateral earth pressure with the wall movement

Fig. 3 Comparison of the mobilized soil-wall interface friction angles under different methods and conditions

657

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672

673 **Fig. 10** Forces acted on the AB trapezoidal layer element

674

675 **Fig. 11** Change of active earth pressure distribution with different cohesion

676

677 **Fig. 12** Change of active earth pressure distribution with different friction angles

Fig. 13 Variations of the average vertical stress with different depth and soil-wall interface friction angle

Fig. 14 Change in K_{awn} with different critical soil-wall interface friction angle and displacement

Fig. 15 Change in K with different depth and displacement

685

686 **Fig. 16** Change of the lateral earth pressure distribution considering arching effect and shear stress or
 687 only arching effect

Fig. 17 Comparison between predicted and experimental values obtained about lateral earth pressure in cohesive-frictional soil

Fig. 18 Comparison between predicted and experimental values obtained about active earth pressure in cohesionless soil

Figures

Figure 1

Finite element analytical model for the lateral earth pressure

Figure 2

Variation of resultant force of the lateral earth pressure with the wall movement

Figure 3

Comparison of the mobilized soil-wall interface friction angles under different methods and conditions

Figure 4

Comparison of the mobilized internal friction angles under different methods and conditions

Figure 5

Trajectory of the minor principal stress in the differential element of the sliding wedge

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Mohr's circle representation of the stresses on point A (a) and the stress analysis in the sliding wedge (b, c)

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