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# Stochastic Model Predictive Control Strategy with Short-term Forecast Optimal SOC for a Plug-in Hybrid Electric Vehicle

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# Stochastic Model Predictive Control Strategy with Short-term Forecast Optimal SOC for a Plug-in Hybrid Electric Vehicle

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**Abstract:** Both the stochastic traffic information and State of Charge (SOC) are great impact on the performance of the plug-in parallel hybrid electric vehicle (PHEV). The application of velocity prediction offers the forthcoming trip information directly to estimate the driving power and arrange the SOC. To address this issue, a Stochastic Model Predictive Control (SMPC) based energy management strategy **(**EMS) considering short-term forecast optimal SOC is proposed. Firstly, a multiple linear regression of engine and battery is developed for SMPC oriented application, respectively. Then, the velocity prediction model is developed based Markov Chain and the reference SOC optimized by Dynamic Programming (DP) using the forthcoming information. On this basis, the SMPC based EMS with the short-term optimal SOC is constituted. Finally, the various prediction horizon and driver styles are discussed to validate the proposed strategy. The normal MPC and the DP are used as benchmark strategies for comparison to evaluate the evolution of short-term optimal SOC and performance of SMPC based EMS. The test results indicate that the SMPC with the short-term optimal SOC made it possible to promote the EMS with capable of significantly improving the fuel economy of the plug-in PHEV.

**Keywords:** Plug-in hybrid electric vehicle; Energy management strategy; Stochastic Model Predictive Control; Velocity prediction; Multiple linear regression

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## 1 Introduction

The Plug-in Hybrid Electric Vehicles (HEVs) have attracted wide attention due to their advantage of lower fuel consumption and emission [1]. The most important issue for PHEVs is the development of an adaptive Energy Management strategy (EMS) to satisfy the power demand while improving fuel economy [2,3].

Existing optimal EMS can be mainly classified into offline and online optimizations [4,5]. The objective of offline optimization is to optimize the cost function with the given driving cycles [6]. The Dynamic Programming (DP) algorithm is employed to optimize the power distribution and configuration of hybrid energy storage system (HESS) [7,8], and the life cycle cost can be significantly reduced. A hybrid algorithm based on the global searching quickly by genetic algorithm (GA) and higher parallel processing capacity by enhanced ant colony algorithm is proposed to optimize the control parameters [9]. The particle swarm optimization (PSO) algorithm is used to optimize the battery parameters [10] and control parameters based on driving pattern recognition [11]. However, the real-time performance of these offline optimizations generally can't be satisfied caused by the high dependence on specific driving conditions and huge calculation boundary. And it should be noticed that DP generally employed as benchmark strategies to exam the performance of other strategies.

Compared with offline optimization, the online optimization makes it possible to realize the real-time performance of strategy. The equivalent consumption minimum strategy (ECMS) can find approximately global optimal solution without considering the future driving cycle, but the biggest challenge is how to determine the optimal value of equivalent factor (EF) [12]. Here the reference SOC is an indispensable method, a hierarchical predictive EMS is conducted by combining long-term SOC planning and short-term condition prediction [13]. The linear and nonlinear SOC trajectory plan method with considering trip distance and real-time traffic information is designed in [14] and [15]. The result shows that the predicted driving conditions and reference SOC trajectory can significantly improve the fuel economy. However, the driving behaviors have a significant influence on fuel economy, which should be taken into consideration. In Ref. [16], the learning vector quantization is established to identify the driving pattern and update the optimal EF. The Reinforcement Learning (RL) algorithm is a machine learning method which is employed to updating the strategy with the change of driving conditions by combining the power request Transition Probability Matrix (TPM). [17,18]. However, the real implemented of RL is limited by the high dimensional and continuous state variables and action variables.

The Model Predictive Control (MPC) as a multi-input and multi-output control algorithm can be applied in EMS successfully, such as nonlinear MPC and Stochastic MPC (SMPC) [19-21]. It should be noticed that the key points of MPC are as follows: velocity predictive model and reference standard. The accuracy of prediction model seriously affects the performance of MPC. Several kinds of frequently used forecasting models are follows: Markov prediction models [22,23], neural network models [24], the results show that these models have a good performance on prediction. However, most of them can't consider the driver behaviors, which have a significant influence on the dynamic parameters, such as the velocity and request power of vehicle.

The reference trajectory is the major factor limiting the practical application of MPC since it is difficult to preview the future driving conditions in the early time. With fast development of the vehicle to everything (V2X), which makes it possible to take the real-time traffic information into account

during the control process [25]. An appropriate reference SOC trajectory is an effective approach to improve the performance of MPC. The exiting research on SOC trajectory planning are mainly concentrate on finding the solution by global algorithm [26] while considering the traffic information [27]. In Ref. [28,29], DP is employed as the solution algorithm to find the SOC reference with considering the driving conditions. In Ref. [30], the optimal SOC trajectory is found by quadratic programming method. The results show that the MPC-based strategy can reach 92.83% of the fuel economy of the DP, and the calculation time can be reduced than the traditional MPC. However, the real time application is still difficult due to the huge computation boundary of global algorithm.

As discussed above, a superior EMS should take driver behavior and traffic information into account, which is the main motivation of this research.

This study proposed a stochastic model predictive control-based EMS with short-term prediction optimal SOC (SMPC-SOC for short) for a plug-in parallel HEV. The main contributions of this study are attributed to the following three aspects: (1) The Markov Chain (MC) considering the driver's style is proposed to improve the accuracy of velocity prediction model. (2) The approach to acquire the reference SOC optimized by the DP in the future short-term trip information with the help of the velocity prediction is developed, which provides a novel method to define the SOC constraint range for each control horizon of MPC and reduce the calculation time. (3) The SMPC-SOC strategy integrated with the velocity prediction and short-term preview optimal reference SOC is proposed.

The remainder of this paper is organized as follows. Section 2 describes the modeling of the powertrain and the main components. In section 3, the velocity prediction is implemented based on the MC model considering driver style. The reference SOC is optimized by the DP in the future short-term partial trip information. Finally, the proposed strategy based SMPC-SOC is presented. In section 4, the validation of velocity prediction with various prediction horizon and driver styles is presented. A normal MPC and DP are introduced as the benchmark strategies for a comparison to the proposed strategy, and the results of different strategies are discussed and analyzed. Section 5 summarizes the conclusions.

## 2 Development of Plug-in HEV Model

### 2.1 Vehicle Powertrain Model

The SMPC-SOC based strategy is proposed for the powertrain of a plug-in parallel HEV equipped with navigation system, as described in Figure 1. The plug-in PHEV consists of engine, battery pack, traction motor and torque coupling. There is a clutch between the engine and torque coupling, which is engaged or disengaged to switch the parallel mode to pure electric mode. The main specifications of the plug-in HEV are presented in Table 1.

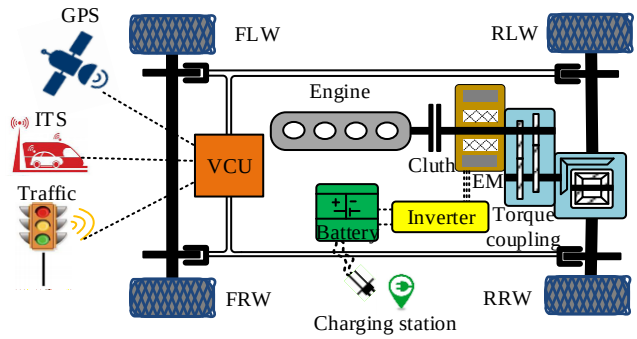


**Figure 1** Plug-in parallel hybrid electric vehicle system structure

**Table 1** Main components and parameters of PHEV

| Main components | Parameters |
|---|---|
| Engine | Max power 41 kW, Max torque 80 Nm |
| EM | Rated power 25 kW, Maximum torque 148 Nm |
| Battery | $LiFePO_4$ Battery Rated capacity 20 Ah |
| Vehicle Mass | 1800 kg |
| Aerodynamic Drag Coefficient | 0.25 |
| Windward Area | 1.9 $m^2$ |
| Final Drive Ratio | 11 |
| Wheel radius | 0.275 m |

A piecewise linear regression model is introduced to improve real-time implementing of MPC. The vehicle quasi-static longitudinal dynamics model is developed from the perspective of engine component. Defining the engine speed as the state variable, the expression as follows:

$$\begin{cases} \dot{\omega}_{eng} = \dot{\omega}_{mot} \\ I_t \cdot \dot{\omega}_{eng} = (T_{eng} + T_{mot} - T_r / i_0) \\ I_t = (I_{eng} + I_{mot} + I_w / i_0^2 + mr^2 / i_0^2) \\ T_r = \left\{ \frac{1}{2} \rho A C_d v^2 + mg[f \cdot \cos(\alpha) + \sin(\alpha)] \right\} \cdot r \end{cases} \tag{1}$$

where $I_t$ is the total moment of inertia, $i_0$ is the transmission ratio, $I_{eng}$ is the engine's moment of inertia 0.100 $kg \cdot m^2$, $I_{mot}$ is the motor's moment of inertia 0.0203 $kg \cdot m^2$, $I_w$ is the wheel's moment of inertia 3.1039 $kg \cdot m^2$, and $mr^2$ is the vehicle's linear acceleration mass converted to the wheel's moment of inertia 211.75 $kg \cdot m^2$, $\omega_{eng}$ is the engine speed, $T_{eng}$ is the output torque of the engine, $T_{mot}$ is the motor torque , and $T_r$ is the driving resistance torque of the vehicle, $\rho$ is the air density, $A$ is the windward area, $C_d$ is the coefficient of air resistance, $v$ is the vehicle speed, $f$ is the rolling resistance, $\alpha$ is the road slope ratio, $r$ is the wheel radius.

### 2.2 Multiple Linear Regression Model of Engine

Based on the brake specific fuel consumption (BSFC) of the engine, the multiple linear regression model of engine is developed for the MPC oriented application, as shown in Eq. (2). The specific realization of multiple linear regression model is referred to Ref. [31], Figure 2 shows the BSFC of engine acquired by multiple linear regression.

$$\begin{cases} \dot{m}_f = a_1 \cdot \omega_{eng} + b_1 \cdot T_{eng} + c_1 \\ a_1 = 0.0044,\ \ b_1 = 0.0273,\ \ c_1 = -1.3818 \\ R^2 = 0.886,\ \ F = 345.7,\ \ f = 0 \end{cases} \tag{2}$$

where $a_1$ , $b_1$, and $c_1$ are the coefficients of multiple linear regression equation. $R^2$ is the coefficient of determination, the value approaching 1 indicates that the fitting effect is perfect. The F-test value is greater than the threshold $f$ , which means the results of the multiple linear regression meet the requirements well.

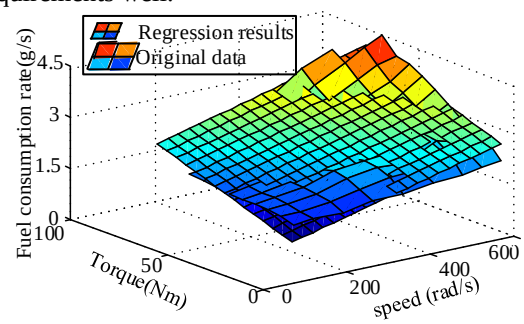


**Figure 2** Linear regression model of engine BSFC

### 2.3 Battery Multiple Linear Regression Model

The equivalent circuit model of battery and motor is constructed in Figure 3. The battery and motor evolving dynamics is expressed as follows:

$$\dot{SOC} = -\frac{U_{oc} - \sqrt{U_{oc}^{\ 2} - 4 \cdot P_{batt} \cdot R_{batt}}}{2C_{batt}R_{batt}} \tag{3}$$

$$P_{batt} = \begin{cases} \omega_{mot} \cdot T_{mot} \cdot \eta_{mot}\ , P_{batt} > 0 \\ \omega_{mot} \cdot T_{mot} / \eta_{mot}\ , P_{batt} < 0 \end{cases} \tag{4}$$

where, $U_{oc}$ is the open circuit voltage, $R_{batt}$ is the battery internal resistance, $C_{batt}$ is the maximum battery capacity, $P_{batt}$ is the battery discharge power, $\omega_{mot}$ is the motor speed, and $\eta_{mot}$ denotes the motor efficiency.

The energy consumption per unit time $dSOC$ is calculated by equation (3) and (4), as described in Figure 4. It can be observed that $dSOC$ is linearly dependent on motor torque under a certain motor speed, and this linear relationship varies with the value of SOC caused by the internal resistance, and open circuit voltage is varied with the battery SOC.

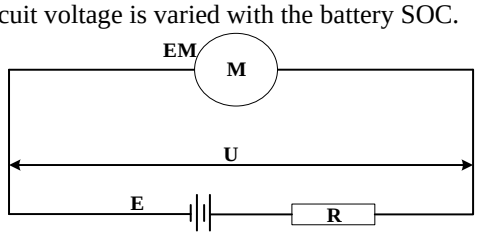


**Figure 3** The working principle of battery and motor

The resistance and open circuit voltage trajectory under different SOC are depicted in Figure 5. The battery SOC change range is generally between 0.3 and 0.9, the changes of resistance and open circuit voltage is negligible during this range. Consequently, the regression model of battery is obtained based on multiple linear regression as follows:

$$\begin{cases} \dot{SOC} = a_5 \cdot \omega_{mot} + b_5 \cdot T_{mot} + c_5 \\ a_5 = -6.7798 \cdot 10^{-7},\ \ b_5 = -3.8258 \cdot 10^{-6},\ \ c_5 = 4.4927 \cdot 10^{-5} \\ R^2 = 0.988,\ \ F_{test} = 200,\ \ f = 0 \end{cases} \tag{5}$$

where, $\omega_{mot}$ is the motor speed, $T_{mot}$ is the motor torque.

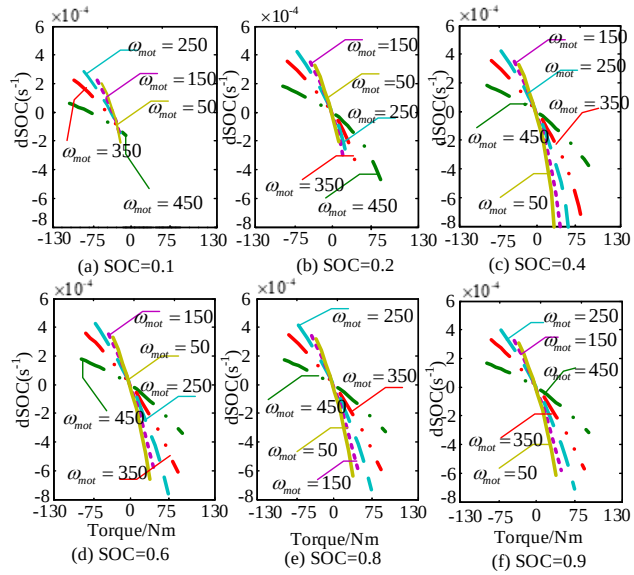


**Figure 4** Relationship between $dSOC$ and motor torque under different SOC.

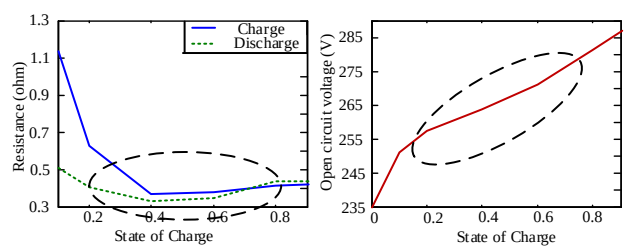


**Figure 5** Battery internal resistance and open circuit voltage and SOC

## 3 Stochastic Model Predictive Control Strategy

A SMPC-based EMS is proposed according to the framework of the MPC. In the upper layer, the future speed in a sampling time is predicted by the MC model with considering the traffic information, and DP is employed to seek the optimal solution of reference SOC. In the lower layer, the hierarchical is developed based on the predicted velocity and SOC reference.

### 3.1 Markov Chain-Based Velocity Prediction Model

During the practical driving process, the traffic conditions and the driver styles are the important factors affecting the current vehicle driving statements. The typical urban high-speed driving cycle and urban low-speed driving cycle are selected as the original driving cycles for study, the driver style is divided into three styles including: aggressive, moderate, gentle. Inspired by the stretching method which applies to driving condition diagram(v-t) proposed by reference [32], driver style coefficient with driving condition is as follows:

$$\begin{cases} v_{cycle}^{\sigma}(t) = v_{cycle}(t)\cdot\sigma \\ t_{cycle}^{\sigma} = t_{cycle}/\sigma \\ a_{cycle}^{\sigma}(t) = a_{cycle}(t)\cdot\sigma^2 \\ a_{\min}(t) < a_{cycle}^{\sigma}(t) < a_{\max}(t) \\ v_{\min}(t) < v_{cycle}^{\sigma}(t) < v_{\max}(t) \\ \sigma > 0 \end{cases} \tag{6}$$

where $\sigma$ is the driver style coefficient, $a_{cycle}(t)$ is the original acceleration, $t_{cycle}$ is the time range of the original driving cycle, $a_{cycle}^{\sigma}(t)$ and $t_{cycle}^{\sigma}$ is the acceleration and time range when the driver's style coefficient $\sigma$, $a_{\min}(t)$ and $a_{\max}(t)$ is the current minimum and maximum acceleration. The driver style is determined by the change rate value of the acceleration. Consequently, the selected two typical driving cycles have changed with the driving styles, as shown in Figure 6.

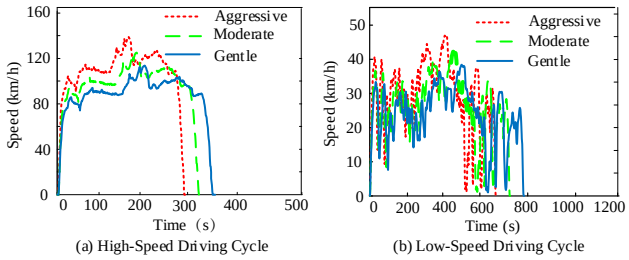


**Figure 6** Driving cycle of different driving styles

The Transition Probability Matrix (TPM) under three driver styles are described in Figure 7.

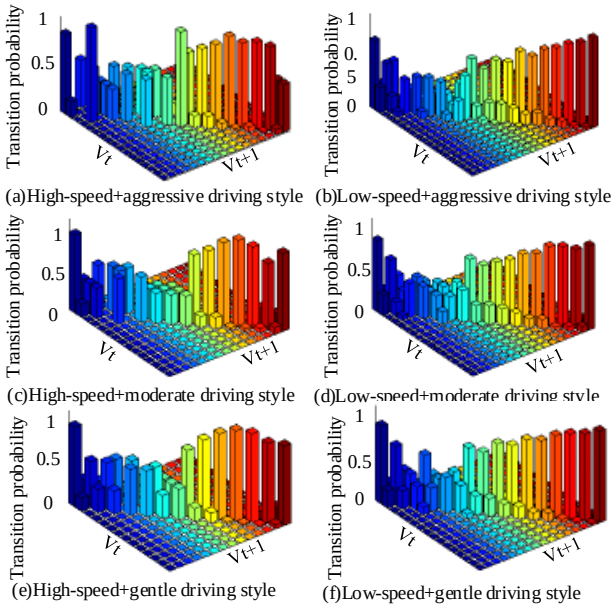


**Figure 7** TPM of different driving styles in high and low speed

The TPM of the next state is determined by the current state, the future velocity can be obtained by MC [33] as follows:

$$\begin{cases} v \in \{v^1, v^2, \cdots, v^M\} \\ P_{i,j} = P\{v(t+1) = v^j \mid v(t) = v^i\} \\ P_{i,j} = \dfrac{m_{i,j}}{m_i} \end{cases} \tag{7}$$

where, the speed is divided into a finite number of M states, $P_{i,j}$ is the transition probability from current state $v^i$ to next state $v^j$ at time t+1, $m_i$ is the number of times that state transfers from $v^i$ to $v^j$.

### 3.2 Reference SOC Optimized by DP

To further improve the performance of SMPC, the reference SOC trajectory with considering future traffic information provided by velocity prediction model is proposed, and the DP is employed to search the solution of SOC reference, the schematic diagram is shown in Figure 8.

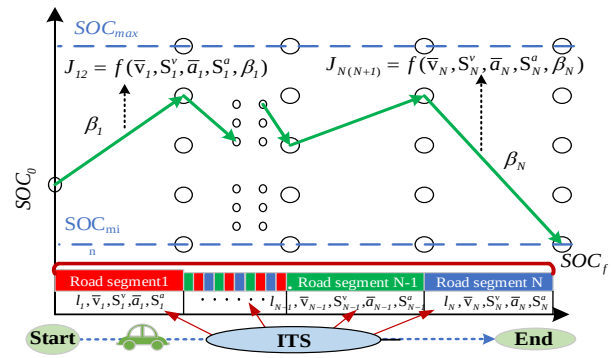


**Figure 8** Reference SOC Planning Based on Driving Information

*(1) Traveling information*

The entire trip is divided into N segments (the N would be less than 10 to ensure the real-time application of DP). the length of each segment $\{l_1, l_2, \ldots l_{N-1}, l_N\}$, the average speed of each segment is $\{\bar{v}_1, \bar{v}_2 \cdots \bar{v}_{N-1}, \bar{v}_N\}/\sigma$, the variance of vehicle speed is $\{S_1^v, S_2^v, \ldots S_{N-1}^v, S_N^v\}/\sigma^2$, average acceleration of each segment is $\{\bar{a}_1, \bar{a}_2 \cdots \bar{a}_{N-1}, \bar{a}_N\}/\sigma^2$, variance of acceleration for each segment is $\{S_1^a, S_2^a, \ldots S_{N-1}^a, S_N^a\}/\sigma^4$. The calculation results of the above parameters under the two driving cycles are listed in Table 2.

**Table 2** Speed Information of three driver styles at high and low speed conditions based on its information

| Driving cycle / Driver style | High-speed(10.1Km) | Low-speed(11.7Km) |
|---|---|---|
| aggressive | $\bar{v}$=30.76,$S^v$=49.46<br>$\bar{a}$=0,$S^a$=0.5775 | $\bar{v}$=14.68,$S^v$=26.10<br>$\bar{a}$=0,$S^a$=0.3960 |
| normal | $\bar{v}$=27.65,$S^v$=41.41<br>$\bar{a}$=0,$S^a$=0.2528 | $\bar{v}$=13.22,$S^v$=21.12<br>$\bar{a}$=0,$S^a$=0.2489 |
| gentle | $\bar{v}$=25.15,$S^v$=33.57<br>$\bar{a}$=0,$S^a$=0.2504 | $\bar{v}$=12.02,$S^v$=17.44<br>$\bar{a}$=0,$S^a$=0.1791 |

*(2) State transition*

The battery SOC is defined as the state variable, energy consumption rate $\beta$ as the penalty coefficient. The SOC of next route segment $N$ is determined by the $\beta$, and the travel time of segment $N$ with considering driver style can be expressed as:

$$\begin{cases} \mathrm{SOC}(N) = \mathrm{SOC}(N-1) - t_N \beta_N \\ t_N = l_N / (\sigma \bar{v}_N) \end{cases} \tag{8}$$

where $t_N$ is the time span of segment $N$.

*(3) Sub-cost function*

The velocity and acceleration of each segment can be regarded as normally distributed and independent of each other, the distribution of velocity and acceleration with considering driver's styles as follows:

$$v \sim N(\frac{\bar{v}}{\sigma}, \frac{S^v}{\sigma^2}),\ a \sim N(\frac{\bar{a}}{\sigma^2}, \frac{S^a}{\sigma^4}) \tag{9}$$

where, $\bar{v}$ is the average speed, $S^v$ is the variance of velocity, $\bar{a}$ is the average acceleration, $S^a$ is the acceleration variance.

The distribution of engine speed can be obtained by conversion of the velocity distribution:

$$\omega_{eng} \sim N(\frac{C \cdot \bar{v}}{\sigma}, \frac{C^2 \cdot S^v}{\sigma^2}),\ C = i \cdot r \tag{10}$$

The distribution of torque request can be further calculated according to the dynamical equation:

$$\begin{cases} T_{req} \sim N(m\frac{\bar{a}}{\sigma} + D, \frac{m^2 \cdot S^a}{\sigma^2}) \\ D \approx \frac{1}{2}\rho_{air} A C_d \bar{v}^2 + fmg\cos(\alpha) + mg\sin(\alpha) \end{cases} \tag{11}$$

The average output power of motor at power consumption per unit time $\beta$ can be calculated with the battery linear regression equation (5):

$$\bar{T}_{mot} = (\beta - c_5 - a_5\bar{\omega}_{mot}) / b_5 \tag{12}$$

Combining the equation (11) and (12), the engine output torque can be expressed as:

$$\begin{cases} T_{eng} = T_{req} - \bar{T}_{mot} \\ T_{eng} \sim N(m\frac{\bar{a}}{\sigma} + D', \frac{m^2 \cdot S^a}{\sigma^2}), D' = D - \bar{T}_{mot} \end{cases} \tag{13}$$

According to the assumption that the distribution of velocity and acceleration are normal and independent of each other, the engine rotation speed and torque can be also regarded as normally distributed and independent of each other at a certain case $\beta$. Therefore, the fuel consumption and emission at $\beta$ in segment $N$ are estimated as follows:

$$\begin{cases} m_{fuel}^{N,\beta} = t_N \cdot \iint_M f_{\omega_{eng}}^{N,\beta} \cdot f_{T_{eng}}^{N,\beta} \cdot fuel(\omega_{eng}, T_{eng}) d\omega dT \\ M = \{(\omega_{eng}, T_{eng}) \mid \omega_{eng}^{\min} \le \omega_{eng} \le \omega_{eng}^{\max}, T_{eng}^{\min} \le T_{eng} \le T_{eng}^{\max}\} \end{cases} \tag{14}$$

where, $m_{fuel}^{N,\beta}$ is the fuel consumption of the engine in segment $N$ when the energy consumption per unit time is $\beta$; $fuel(\omega_{eng}, T_{eng})$ is the engine fuel consumption rate at engine rotation speed $\omega_{eng}$ and torque $T_{eng}$; $f_{\omega_{eng}}^{N,\beta}$ and $f_{T_{eng}}^{N,\beta}$ are the probability density functions of engine rotation speed and torque with $\beta$ in segment $N$; $t_N$ is the travel time for the segment $N$. Thus, the sub-cost function of dynamic programming (DP) is written as:

$$J_{N(N+1)}^{\beta} = m_{fuel}^{N,\beta} \tag{15}$$

*(3) Cost Function and Dynamic Programming Solution*

Based on the sub-cost function in equation (15), the cost function of DP can be written as:

$$J_{DP} = \sum_{p=1}^{N} J_{p(p+1)} \tag{16}$$

DP is applied to search the solution of control sequence $[\beta_1, \beta_2 \cdots \beta_{N-1}\beta_N]$ which minimize the value of $J_{DP}$:

$$[\beta_1, \beta_2 \cdots \beta_{N-1}\beta_N] = \arg\min \sum_{i=1}^{N} J_{i(i+1)}^{\beta} \tag{17}$$

The remaining battery SOC of each segment can be calculated by the control sequence $[\beta_1, \beta_2 \cdots \beta_{N-1}\beta_N]$, then the reference SOC trajectory can be obtained by interpolation.

### 3.3 Stochastic Model Predictive Control Strategy

The predicted velocity based on MC and reference SOC trajectory based on traffic information are taken as the SMPC reference, and the hierarchical SMPC-SOC strategy is established as shown in Figure 9. as follows: First, the initial battery SOC and driver style can be obtained by the method mentioned above, the reference SOC with considering traffic information, such as, average speed, acceleration and length of each segment is calculated. Considering the driver style and current speed, the future vehicle speed is predicted based on MC with the current driving conditions provided by the internet information. The above three parts are incorporated to establish the hierarchical SMPC-SOC strategy.

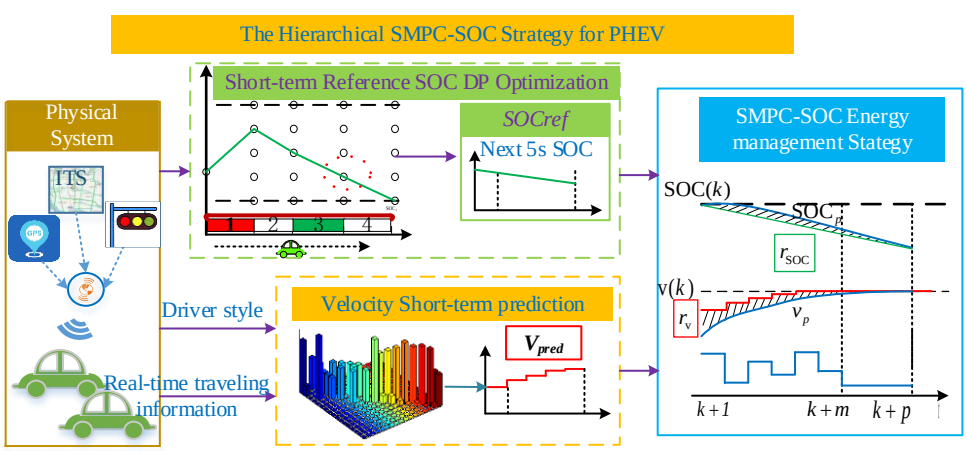


**Figure 9** The Process of the proposed SMPC-SOC strategy

The discrete state equation is established based on equations (1), (2) and (4), as follows:

$$\begin{cases} x(k)=[\omega_{eng}(k),m_f(k),\mathrm{SOC}(k)]^T \\ u(k)=[T_{eng}(k),\ T_{mot}]^T \\ x(k+1)=\underbrace{\begin{bmatrix}1&0&0\\a_1&1&0\\a_2&0&1\end{bmatrix}}_{A_s}x(k)+\underbrace{\begin{bmatrix}1/I_{total}&1/I_{total}\\b_1&0\\b_2&0\end{bmatrix}}_{B_u}u(k)+\underbrace{\begin{bmatrix}-T_r/(i\cdot I_{total})\\c_1\\c_2\end{bmatrix}}_{B_d} \\ y(k)=\begin{bmatrix}v(k)\\m_f(k)\\\mathrm{SOC}(k)\end{bmatrix}=\underbrace{\begin{bmatrix}r/i&0&0\\0&1&0\\0&0&1\end{bmatrix}}_{C_y}x(k) \end{cases} \tag{18}$$

where, $x$ is the state variable, $u$ is the control variable, $A_s$ is the state matrix, $B_u$ is the control matrix, $B_d$ is the input interference matrix, $C_y$ is the output matrix, and $v(k)$ is vehicle speed. According to the equation (18), the influence of the control $u$ on the objective function is neglected, and the optimization process of the model predictive control is established as follows:

$$\begin{cases} J(Z_k)=\sum_{i=0}^{N_p-1}\sum_{j=1}^{N_y}\{[(y_j(k+i+1|k)-r_j(k+i+1)]\}^2+\rho_\varepsilon\varepsilon^2 \\ Z_k=\{u(k|k),u(k+1|k)\cdots u(k+N_c-2|k), \\ \qquad u(k+N_c-1|k)\cdots u(k+N_p-1|k)\} \\ Z_k^*=\arg\min J(Z_k^*) \end{cases} \tag{19}$$

where, $N_p$ is the predicted horizon of 6 seconds, $N_y$ is the output number of 3; $y_j(k+i=1|k)$ is the predicted output of time $k+i+1$ at time $k$, $r_j(k+i=1|k)$ is the output reference of time $k+i+1$ predicted at time $k$; $\rho_\varepsilon$ and $\varepsilon$ are relaxation terms, $N_\varepsilon$ is the control horizon of 6 seconds, $Z_k$ is the control sequence $t$ at time $k$; $Z_k^*$ is the optimal control sequence, and $u_j(k+N_c+1|k)$ is the control acting on time $k+N_c+1$. The constraints of predictive control as follows:

$$\begin{cases} \omega_{eng}^{\min}(k)<\omega_{eng}(k)<\omega_{eng}^{\max}(k) \\ m_f(k)>0 \\ \mathrm{SOC}_{\min}(k)<\mathrm{SOC}(k)<\mathrm{SOC}_{\max}(k) \\ T_{eng}^{\min}(k)<T_{eng}(k)<T_{eng}^{\max}(k) \\ T_{mot}^{\min}(k)<T_{mot}(k)<T_{mot}^{\max}(k) \end{cases} \tag{20}$$

## 4 Validation and Discussion

To verify the effectiveness of the SMPC-SOC proposed in this research, validation was carried out under a comprehensive driving cycle, which consisting of typical urban high-speed driving cycle and urban low-speed driving cycle, defined as LHL for abbreviation. The effectiveness of the reference SOC estimation method based on internet information in the SMPC-SOC strategy is verified by using LHL driving cycle. By comparing simulation results of MPC and SMPC-SOC, the superiority of SMPC-SOC strategy is verified. Furthermore, DP is implemented as the benchmark strategy to verify the approximate global optimal fuel economy performance of the proposed SMPC-SOC strategy.

### 4.1 The results of various prediction horizons and driver styles

In this section, the Root Mean Square Error (RMSE) is implemented to evaluate the velocity predictive accuracy of the proposed MC driver model with short-term prediction. RMSE indicates the prediction accuracy by calculating the standard deviation of the error between the forecast value and the real value. In other words, RMSE is suitable to compare the preview with the real velocity by quantitative analysis. The RMSE of equation is defined as:

$$RMSE=\sqrt{\frac{\sum_{i=1}^{n}\left(v_{p,i}-v_{r,i}\right)^2}{n}} \tag{21}$$

where $v_{p,i}$ is predicted speed, $v_{r,i}$ is real speed and $n$ is the number of speed points.

Figure 10 demonstrates the results of velocity prediction. To clearly illustrate the performance of the proposed method, the validation of diverse driver styles and the preview horizons are tested respectively. The Figure 11 describes the relationship between the RMSE and prediction horizons for diverse driver styles in low-speed and high-speed. The results demonstrate an increased prediction error as the preview horizon grows and the prediction error is generally smaller at high-speed cycle than that of at low-speed cycle. Taking the prediction horizon of 6s as a case study, the stochastic driver model is used to forecast the speed in 6s prediction horizon.

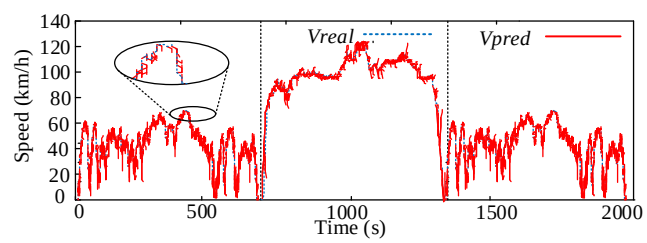


**Figure 10** The velocity prediction model based on MC

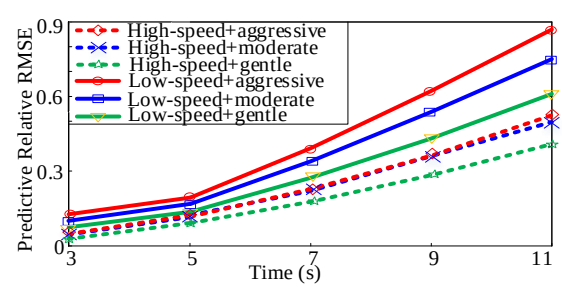


**Figure 11** The relationship between RMSE and prediction horizon

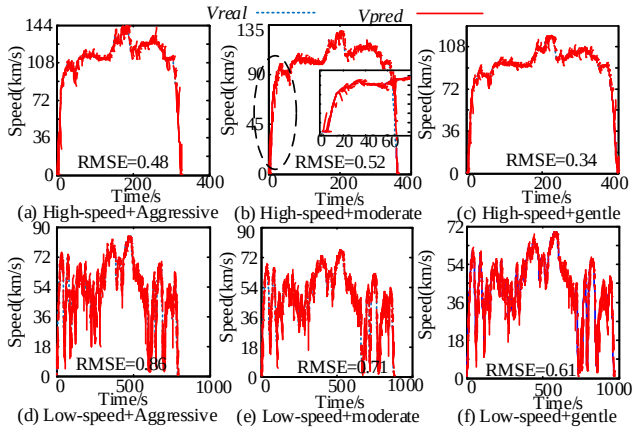


**Figure 12** The 6s preview horizon for three styles in low-speed and high-speed

The results are shown in Figure 12, the RMSE value of 6s prediction horizon reach maximum value of 0.86 in the case of aggressive style at low-speed driving cycle and minimum value of 0.34 in the case of gentle style at high-speed driving cycle. Furthermore, the moderate style is selected to verify the different prediction horizons at 3s, 6s and 9s, the results as shown in Figure 13. The RMSE values indicate that the prediction results become less accurate as the horizon increases. The results demonstrate that the MC has a great ability to preview the short-term future velocity with stochastic driver model. Therefore, this research applies the MC as the stochastic prediction approach combined with MPC to develop EMS for a plug-in PHEV. In the next section, the 6s prediction horizon will be applied to examine the performance of SMPC-SOC.

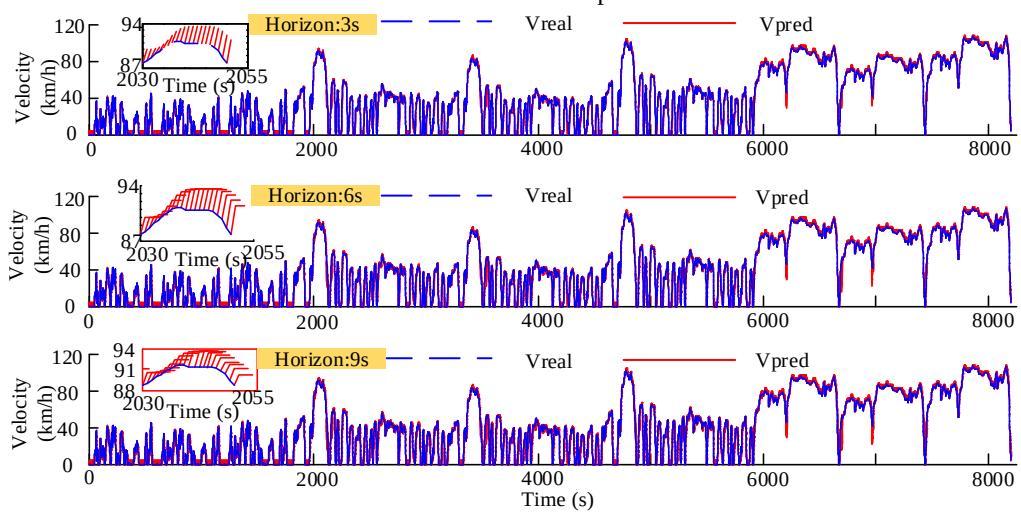


**Figure 13** Comparison between real and predicted speed under different prediction horizon

## 4.2 Validation of the Proposed Strategy with Reference SOC

In order to verify the effectiveness of the proposed reference SOC optimal method based on short-term preview by applying ITS, DP globally optimal method and normal MPC are implemented as the benchmark approach for comparison to evaluate the evolution of short-term optimal SOC. The LHL are applied to the numerical validation of SOC tendency, as described in Figure 14. In this research, four cases initial SOC of $SOC_0 = \{0.4, 0.6, 0.7, 0.8\}$ are selected as example to verify the reference SOC, and the optimal SOC trajectory generated by using short-term prediction horizon method is compared with the reference SOC trajectory obtained by equation (8).

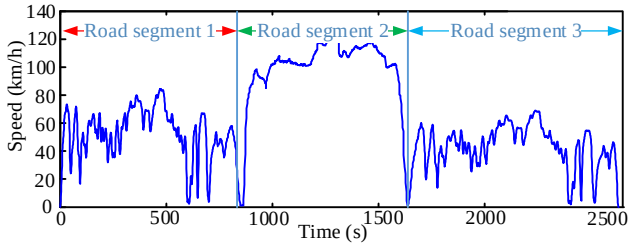


**Figure 14** Comprehensive driving cycle

The validation results are shown in Figure 15. It can be observed that the initial SOC in the high value case ($SOC_0 = \{0.7, 0.8\}$), the short-term prediction optimal SOC trajectory can almost follow the tendency of DP, but that of quite different from the normal MPC. These results of SOC profiles demonstrate that the high consumption of electric power appears in the case of frequent acceleration and deceleration over low-speed driving cycle, while the electrical energy tends to consume decrease significantly in the high-speed driving cycle. In the case of low $SOC_0$ ($SOC_0$=0.4), the short-term prediction optimal SOC trajectory can also follow well the tendency of DP, which means that the vehicle consumes more electricity at low speed and charges at high speed. The SOC of normal MPC differs significantly from the proposed SOC trajectory, especially when the $SOC_0$ is low and battery charging is carried out during the travel. Under different $SOC_0$, it is well known that the performance of SOC solved by dynamic programming is better than the performance of SOC in normal MPC[29]. The results demonstrate the trend of the short-term prediction optimal SOC is almost consistent with SOC of the globally optimal DP in all the different initial

SOC case. Hence, the proposed SOC operates as the approximate global optimal method, which will lead to the preferable fuel economy.

The 6s prediction horizon is selected for SMPC based EMS. The optimal SOC with Short-Term prediction is simplified to evaluate the fuel economy performance. Both the DP and the normal MPC are used as the benchmark strategies for comparisons to verify the effectiveness of the proposed strategy. The results of torque distribution between engine and motor are presented in Fig.16 respectively. The results of the engine operating points of the proposed strategy are locating more in the high efficiency area than that of a normal MPC are demonstrated in Figure 17. The results of fuel consumption will provide the evidences to further support the above evaluation.

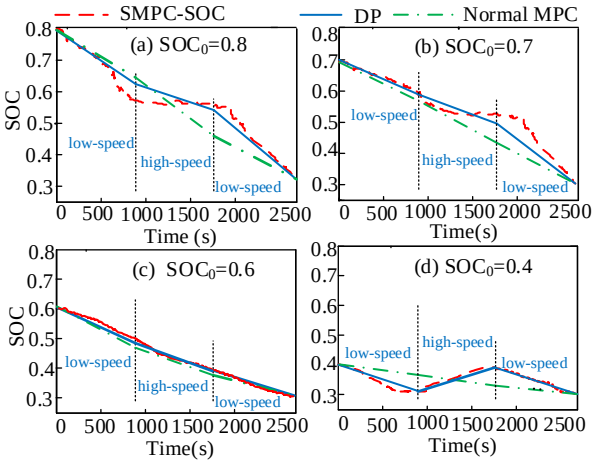


**Figure 15** Reference SOC and optimal SOC trajectory under different $SOC_0$

It is observed that the varying tendency of the SOC trajectories in the proposed strategy is different from that in normal MPC and is almost following the trend of the DP, as shown in Figure 18(a). The SOC profiles reflect that the torque distributions of the proposed strategy between engine and motor is more reasonable than that of a normal MPC, as shown in Figure 18(a). The results in Figure 18(b) presents the fuel consumption for the three control strategies. It can be observed that the fuel consumption of the proposed strategy is lower than that of the normal MPC and is approximate to that of DP. The fuel economy performance indicates the superiority of the proposed strategy is close to the globally optimization. The preferable fuel economy is contributed by the SMPC-based Markov Chain, which leads to the short-term prediction for obtaining the reference optimal SOC. The SMPC-SOC makes it possible to acquire the SOC evolution and results in the fuel consumption close to the globally optimization. As shown in Figure 18(b), the fuel consumption of a normal MPC is 2.63L, the proposed strategy is 2.3L and that of the DP is 2.11L. Compared with a normal MPC, the fuel consumption of our strategy is improved by about 12.5%.

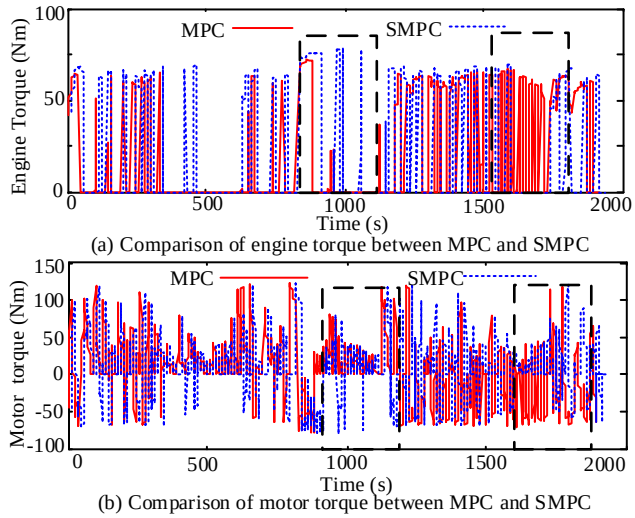


**Figure 16** The results of engine and motor torque for MPC and SMPC

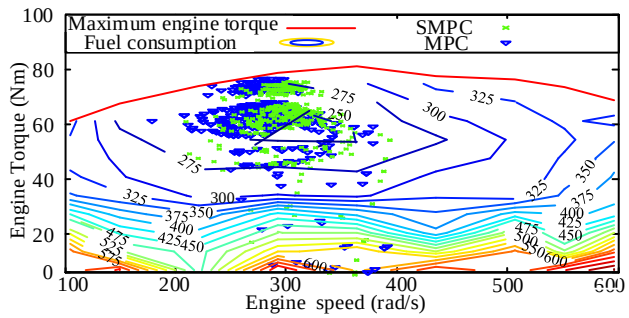


**Figure 17** Comparison of engine operating point between MPC and SMPC

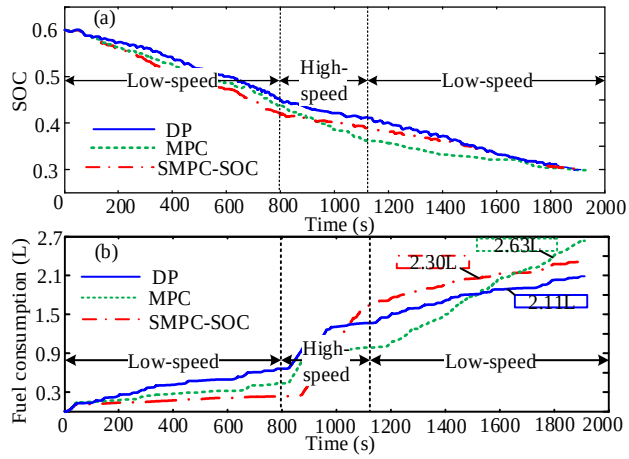


**Figure 18** The fuel economy results of SMPC, MPC and DP

Another comprehensive driving cycle constituted by a high-speed driving cycle and a low-speed driving is implemented to validate the adaptation of SMPC-SOC. The comprehensive driving cycle defined as HLH (High-speed, Low-speed, High-speed). And the driving cycle considering the driver style, defined as HLH-Agg for aggressive driving style, HLH-Mod for moderate driving style. The various initial SOC at the value of 0.6 to 0.9 by interval 0.1, implemented to validation of the proposed strategy, the results as shown in Figure 19. The fuel consumption of

HLH-Agg is higher than that of the HLH-Mod, and the SMPC-SOC performs well in fuel consumption at high SOC. The results demonstrate that the proposed SMPC-SOC with the capable to regulate the SOC. It is evident to observe that the different initial SOC values reach the same final SOC. This validation indicates that the proposed SMPC-SOC exhibits an advanced performance on regulation of SOC in both the high-speed and low-speed driving cycle considering the driver style.

Two low-speed and two high-speed driving cycles are implemented to evaluate the fuel economy performance of the proposed strategy. The two typical low-speed driving cycles are Urban Dynamometer Driving Schedule (UDDS) and Chinese Transition Bus Cycle (CTBC), the two typical high-speed driving cycles are Highway Fuel Economy Test (HWFET), US06 (This cycle is one of three included in the US EPA's Supplemental Federal Test Procedure). By comparing the validation results of a normal MPC and the DP, the adaption of the proposed strategy is further validated in other driving cycle. In this validation, the low initial SOC at the value of 0.4 and0.5 applied to the low-speed driving cycle, the high initial SOC at the value of 0.6 and 0.8 applied to the high-speed driving cycle.

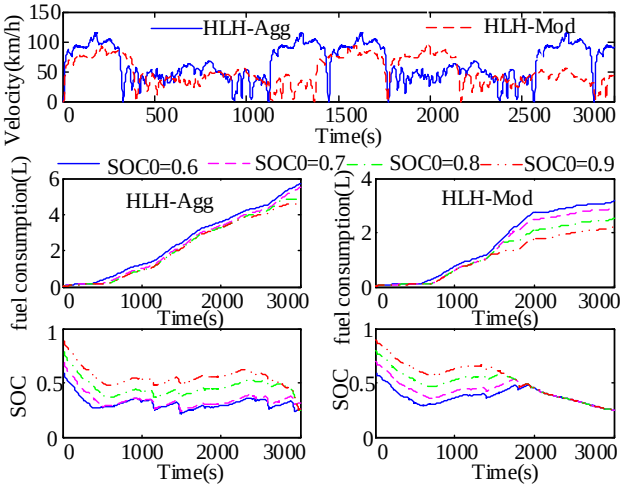


**Figure 19** The results of various initial SOC validation at 6s prediction

The validation results as show in Figure 20 and Table 3. The results demonstrate that the proposed strategy perform almost the same as the DP, and perform excellent than the normal MPC. Throughout the 3000s driving process of the four driving cycles, taking a low-speed driving cycle, UUDS for instance. In UUDS cycle at the initial SOC of value 0.4, for the proposed strategy, while the fuel economy performance is 6.287 L/100km, compared to normal MPC fuel economy performance of 7.129 L/100km and that of DP 6.056 L/100km. Specifically, the fuel economy of the proposed strategy compared with normal MPC improving by up about 11.8%, meanwhile, is closed to that of the globally optimal DP, by only decreasing 3.8%. Finally, various prediction horizons are applied to validate the short-term SOC prediction, as shown in Figure 21. The results indicate that the SOC at 6s prediction horizon exhibits the almost the same as that of DP. After the 6s, as the prediction horizon increases, the SOC trend performs far away from that of DP.

**Table 3** The fuel economy performance for various driving cycles

| cycles | Strategy | $SOC_0$ | $SOC_t$ | FE Performance /(L/100km) | FE promotion |
|---|---|---|---|---|---|
| UUDS | SMPC | 0.4 | 0.204 | 6.287 | [-] |
| | MPC | 0.4 | 0.257 | 7.129 | 11.8% |
| | DP | 0.4 | 0.199 | 6.056 | -3.8% |
| CTBC | SMPC | 0.5 | 0.211 | 3.706 | [-] |
| | MPC | 0.5 | 0.266 | 4.329 | 14.3% |
| | DP | 0.5 | 0.208 | 3.58 | -3.52% |
| HWFET | SMPC | 0.6 | 0.199 | 3.088 | [-] |
| | MPC | 0.6 | 0.199 | 3.5 | 11.77% |
| | DP | 0.6 | 0.199 | 2.89 | -6.85% |
| US06 | SMPC | 0.8 | 0.213 | 5.825 | [-] |
| | MPC | 0.8 | 0.258 | 6.527 | 10.7% |
| | DP | 0.8 | 0.197 | 5.524 | -5.45% |

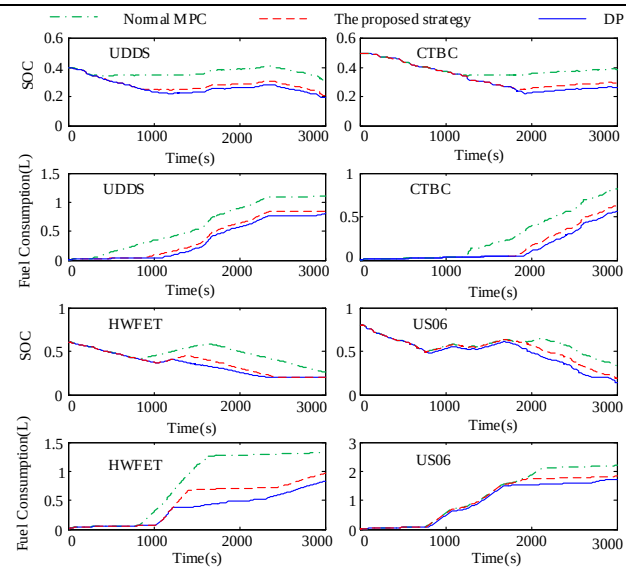


**Figure 20** The results for various driving cycle validation at 6s prediction

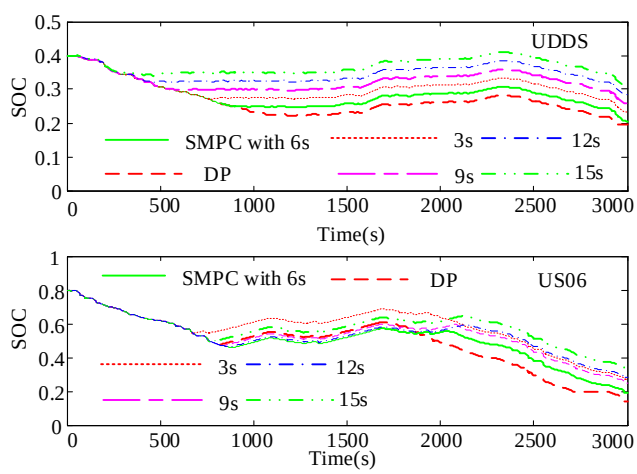


**Figure 21** The results of SOC for various prediction horizons

### 4.3 Hardware-in-loop test of SMPC-SOC strategy

A hardware-in-the-loop (HIL) experimental platform is constructed in Figure 22, which consists of both software and hardware components to further validate the fuel economy of the SMPC-SOC strategy.

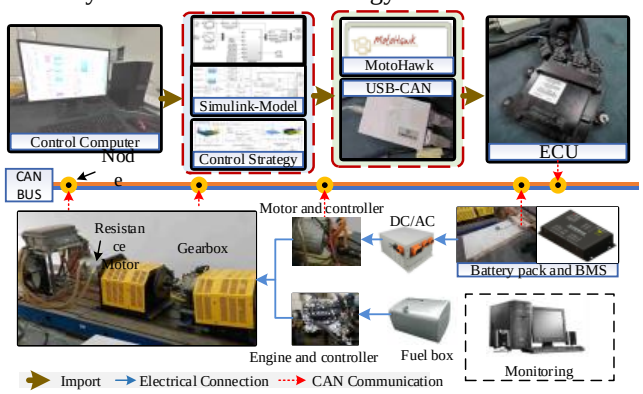


**Figure 22** The hardware-in-loop experiment bench.

DP is implemented as the benchmark strategy. From Figure 23, it can be shown that SOC trajectory of the HIL has basically the same trend as the simulated one in SMPC strategy, but the simulated SOC trajectory is closer to the DP SOC trajectory. Combining with Figure 24, it can be obtained that the fuel consumption of simulation is 2.30L and that of HIL is 2.37L. The simulated one has 3.1% lower fuel consumption than the HIL one, and 5.8% more compared with the DP one.

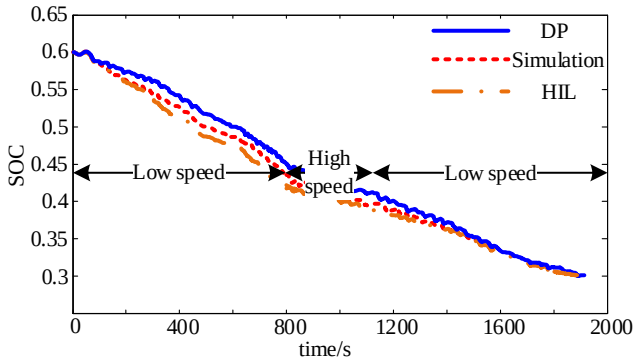


**Figure 23** Comparison of DP, Simulation and HIL in SOC trajectory when the initial SOC is 0.6.

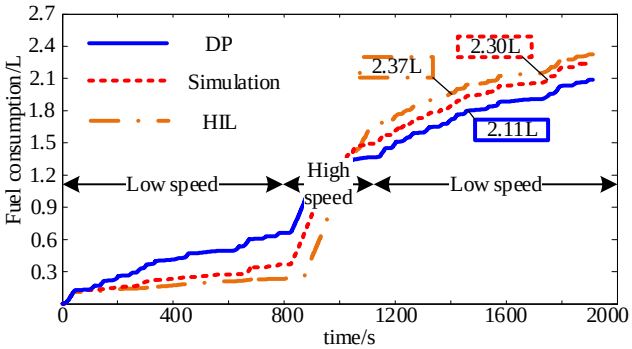


**Figure 24** Comparison of DP, Simulation and HIL in optimal fuel consumption when the initial SOC is 0.6.

## 5 Conclusion

This study focuses on improving fuel economy performance of a plug-in HEV. In order to address the influence stochastic traffic information and battery SOC, a SMPC with short-term prediction SOC based strategy is proposed. In this study, stochastic Markov driver model is applied to predict the future speed, and reference SOC is optimized by DP algorithm combined with internet information. Different prediction horizon and driving styles are applied to explore their relationships among fuel economy and prediction accuracy. A normal MPC and DP are applied as the benchmark strategies, for comparison purposes, to highlight that the proposed SMPC-SOC strategy is capable of significantly promoting the fuel economy performance of the PHEV, by means the short-term prediction optimal reference SOC. The validation results show that gentle driving styles and less prediction horizon make it possible for less fuel consumption. The proposed SMPC-SOC based on Markov stochastic driver model and the reference SOC obtained by internet information can improve fuel economy performance, which can make it closed to that of DP and perform well than a normal MPC.

Although this research has well validated the potential of the proposed SMPC-SOC, more endeavors will be made for the real-time predictive error feedback control of SOC and improving the accurate of large prediction horizon. A real vehicle validation could be also performed, to evaluate the presented strategy and demonstrate its real-time application potential, in the future studies.

## Declaration

### Availability of data and materials

Not applicable

### Competing interests

The authors declare no competing financial interests.

### Funding

This work was supported in part by the Natural Science Foundation of Fujian Province, China (Grant No. 2020J01449), National Natural Science Foundation of China (Grant No. 51505086). the Open Research Fund of Anhui Province Key Laboratory of Detection Technology and Energy Saving Devices, Anhui Polytechnic University, China (Grant No. JCKJ2021A04).

### Authors' contributions

The author' contributions are as follows: Lin X.Y. was in charge of the whole trial; Chen X.K. wrote the manuscript; Xu X.H. assisted with the part of software simulation and experimental analysis.

### Acknowledgements

Not applicable

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