Decision-making under uncertainty – a quantum circuit approach

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Decision-making under uncertainty – a quantum circuit approach

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ABSTRACT

Decision's biggest challenge is uncertainty. We propose a quantum expected value theory for decision-making under uncertainty. Decision-making under uncertainty is the unification of people's subjective beliefs and the objective world. Quantum density operator as value operator which we call it quantum circuit is proposed to simulate people's subjective beliefs. Quantum circuit guides people to choose corresponding actions based on their subjective beliefs through objective world. The quantum circuit is a decision tree constructed from quantum gates and logic operations. The genetic programming can be used to optimize and auto-generate quantum circuits. Our proposed synapse's quantum circuit hypothesis states that the combination of neurotransmitter and receptor functioned as quantum circuit open or close ion channels probabilistically which determines the excitation or inhibition of synapse.

Introduction

Blasé Pascal and Pierre de Fermat proposed the expected value theory, the expected utility theory was proposed by Daniel Bernoulli[1]. John Von Neumann and Oskar Morgenstern axiomatized the objective expected utility theory[2] and Leonard Savage expanded it into the subjective expected utility theory[3]. Decision-making can be viewed as a two-phase process: evaluation process and selection process. Classic decision theory[4] holds that rational-economic person knows the utility function as well as probability distribution through evaluation then maximizes utility by an optimal selection. Information completeness and selection consistence are required for expected utility theory, however in the real world information is rarely complete and consistency of selection cannot be guaranteed due to complexity. People sometimes show irrational behaviors, such as Allais Paradox[5] and Ellsberg Paradox[6]. In response to people's irrational behaviors, Daniel Kahneman and Amos Tversky proposed the prospect theory[7], Herbert Simon coined bounded rationality and satisficing theory[8]. Pierfrancesco La Mura proposed the quantum projective expected utility theory[9], and Jerome Busemeyer and Peter Bruza proposed the dynamic quantum model of decision[10].

At the neurobiological level, scientists are trying to apply quantum theory to reveal how the brain makes decisions. Roger Penrose and Stuart Hameroff collaborated to produce a theory known as orchestrated objective reduction[11, 12]; Henry Stapp[13] hypothesized that the information transmission at the synaptic boutons obeys the quantum mechanism; Quantum tunneling equation was used by Friedrich Beck and John Eccles[14] to describe exocytosis triggered by action potential; Mathew Fisher[15] hypothesized that phosphorus nuclear-spin may constitute qubits in the brain for information transmission; Andrei Khrennikov[16] et al. attempted to use the quantum state superposition representing action potentials to construct the quantum-like decision theory at the neuron level.

Classic decision theory is a "black box", scientists do not know what really happens inside the box. We believe that the results of decisions are the unity of subjective beliefs and objective facts, and the value of observed results is the bridge between those two different worlds. The decision's "black box" can be opened through the bridge of the value.

Results

Quantum expected value theory

Decision-making under uncertainty is a process of unifying people's subjective beliefs and objective world. Usually people subjectively choose an action a_i from $\{a_1, \dots a_m\}$ where nature's objective state is in q_j of $\{q_1, \dots q_n\}$ when decisions were made, and the result of the decision value matrix v_{ij} depends on both the state of the nature and choice of brain shown in table 1. $\{\omega_j, j = 0,1,\cdots n\}$ the objective frequency of occurrence of different natural states and $\{p_i, i = 0,1,\cdots m\}$ the subjective probability of different beliefs held by people. Objective probability ω_j and value v_{ij} are observable, while subjective probability p_i is generally not directly measurable which the major problem of studying decision is.

Action \ State		q_1	$\cdots$	q_j	$\cdots$	q_n
		ω_1		ω_j		ω_n
a_1	p_1	v_{11}	$\cdots$	v_{1j}	$\cdots$	v_{1n}
$\vdots$			$\ddots$			
a_i	p_i	$\vdots$		v_{ij}		$\vdots$
$\vdots$					$\ddots$	
a_m	p_m	v_{m1}	$\cdots$	v_{mj}	$\cdots$	v_{mn}

Table 1 State-action-value decision table

We can represent natural state in terms of the Hilbert state space[17, 18, 19] as follows:

$$|\psi\rangle = \sum_j c_j|q_j\rangle, \sum_j |c_j|^2 = 1 \tag{1}$$

Hilbert strategy space is used to represent a person's strategies which guide the action of the person:

$$|S\rangle = \sum_i \mu_i|a_i\rangle, \sum_i |\mu_i|^2 = 1 \tag{2}$$

Usually the information of decision under uncertainty is incomplete; the result of decision can be represented by a mixed state's density operator as value operator. Value operator is a sum of projection operators which projects a person's beliefs onto an action of choice based on nature states:

$$\widehat{V} = \sum_i p_i|a_i\rangle\langle a_i| = \sum_i p_i|A_i\rangle, \sum_i p_i = 1 \tag{3}$$

Quantum expected value can be represented as follows:

$$< \widehat{V} > = \langle\psi|\widehat{V}|\psi\rangle = \sum_k c_k^*\langle q_k| \sum_i p_i|a_i\rangle\langle a_i| \sum_j c_j|q_j\rangle \tag{4}$$

$|q_j\rangle, j = 1,\cdots,n$ is a set of quantum orthonormal basis, $\langle q_k|q_j\rangle = 0,\ \ \text{if } j \neq k$. So we have:

$$< \widehat{V} > = \sum_i p_i \sum_j |c_j|^2|\langle a_i|q_j\rangle|^2 = \sum_i p_i \sum_j \omega_j v_{ij} \tag{5}$$

Where $|A_i\rangle = |a_i\rangle\langle a_i|$ is Von Neumann's projection operator, $p_i = |\mu_i|^2$ is a person's subjective belief in choosing an action a_i; $\omega_j = |c_j|^2$ is the objective frequency at which natural state is in q_j; value matrix $v_{ij} = |\langle a_i|q_j\rangle|^2$ is the results of decision when a person choose an action a_i based on natural state q_j. Quantum expected value suggests that a subjective and objective unified result is obtained through people's beliefs which are based on natural states.

Quantum circuit is used to simulate people's subjective beliefs which guide their actions

Value operator $\widehat{V}$ can be constructed from the basic quantum gates and logic operations to form a quantum decision tree, which we call it quantum circuit shown in Figure 1. The decision process of a person can be simulated by the evolution of the quantum circuit according to the external environment. A final decision is equivalent to a "quantum measurement" performed on the strategy state by the brain which chooses an action a_i with probability p_i and a value observed.

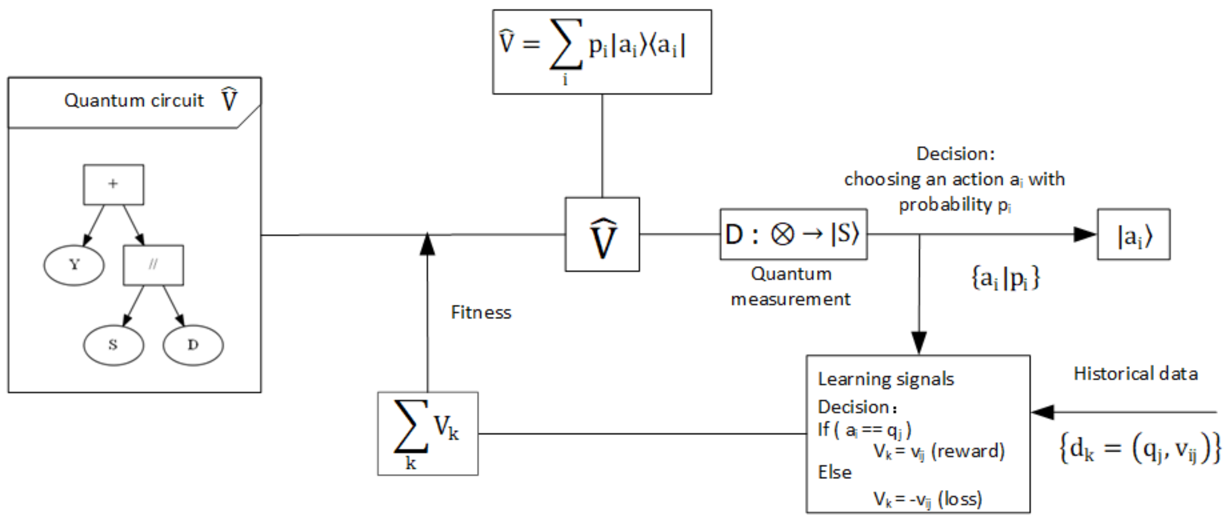

Figure 1 A person's decision process simulated by quantum circuit

How to search and optimize the quantum circuit to obtain a satisfied value? The genetic programming (GP) is the answer. On the basis of genetic algorithm (GA) proposed by John Holland[20, 21], John Koza[22, 23] extended GA into GP for optimization and auto-generation of programs. A tree structure is used for encoding by GP, which is particularly appropriate to solve hierarchical and structured complex problems. By using selection, crossover and mutation, GP continually evolve and auto-generate the optimized solution.

Quantum circuit is a quantum gate decision tree composed of different nodes and branches which connect nodes. There are two types of nodes, non-leaf nodes and leaf nodes. In a quantum decision tree, the non-leaf nodes are composed of the operation set F = {+, *, //}; the leaf nodes are composed of the data set T= {H, X, Y, Z, S, D, T, I}. The construction process of a quantum circuit is to randomly select a logical symbol from the operation set F= {+,*, //} as the root of quantum decision tree, and then grow corresponding branches according to the nature of the operation symbol and so on until a leaf node is reached. Value of measured results is used as fitness function in GP to optimize quantum circuit. The purpose of GP iterative evolution is to find a satisfactory quantum circuit through learning historical data shown in Figure 1. Quantum circuit is a mapping of objective historical data to people's subjective beliefs.

Algorithm: quantum circuit evolution

Input:

- Training data set $\{d_k = (q_j, v_{ij})\}$ which includes N samples, each sample consists of natural state and value.
- Setting
  1) Operation set $F = \{+,\ *,\ //\}$; "+" -> ADD; "*" -> MULTIPLY; "//" -> OR.
  2) Data set $T = \{H, X, Y, Z, S, D, T, I\}$, eight basic quantum gates[24, 25, 26]
     $$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\ X = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\ Y = \begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}\ Z = \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\ S = \begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}\ D = \begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\ T = \begin{bmatrix}1 & 0\\ 0 & e^{i\pi/4}\end{bmatrix}\ I = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
  3) Crossover probability = 70%; Mutation probability = 5%.

Initialization:

- Population: randomly create 100 quantum circuits.

Evolution:

- for each generation:
  a) Calculate fitness for each quantum circuit as follows:
     Risk aversion fitness $= (\sum_k V_k)/\max_{\text{loss}}\{V_k\}$, $\max_{\text{loss}}\{V_k\}$ is the biggest loss of the decisions.
  b) According to the quality of fitness:
     i. selection: selecting parent quantum circuits
     ii. crossover: generate a new offspring using the roulette algorithm based on crossover probability
     iii. mutation: randomly modify parent quantum circuit based on mutation probability
- until a satisfactory quantum circuit is evolved

Output:

- A quantum circuit of best fitness as follows:

$$\text{qCircuit} = \underset{\text{qGate} \in (F \cup T)}{\text{argmax}} \ (\text{fitness})$$

Example of quantum circuit of futures trading

We can represent future market states $|\psi\rangle$ in terms of the Hilbert state space, Hilbert strategy space $|S\rangle$ is used to represent trader's strategies of trading securities, quantum circuit $\hat{V}$ as density operator which projects a trader's beliefs onto an action of buying or selling a security according to market environment:

$$|\psi\rangle = c_1|q_1\rangle + c_2|q_2\rangle \tag{6}$$

$$|S\rangle = \mu_1|a_1\rangle + \mu_2|a_2\rangle \tag{7}$$

$$\hat{V} = p_1|a_1\rangle\langle a_1| + p_2|a_2\rangle\langle a_2| \tag{8}$$

Where $|q_1\rangle$ indicates a state in which the market is rising and $|q_2\rangle$ indicates a state in which the market is falling; $|a_1\rangle$ represents trader's action to buy and $|a_2\rangle$ represents trader's action to sell; p_1 represents the subjective probability which a trader choose to buy and p_2 represents the subjective probability which a trader choose to sell. If market is rising and a trader chooses to buy, the value V_k is positive and the trader gets an award, otherwise value V_k is negative and the trader losses money; if market is falling and a trader chooses to sell, the value V_k is positive and the trader gets an award, otherwise value V_k is negative and the trader losses money. GP can be used to search the locally optimal quantum circuit:

$$\text{qCircuit} = \underset{\text{qGate} \in (F \cup T)}{\text{argmax}} \left(\left(\sum_k V_k \right) \Big/ \max_{\text{loss}}\{V_k\} \right) \quad \max_{\text{loss}}\{V_k\} \text{ is the biggest loss of trading} \tag{9}$$

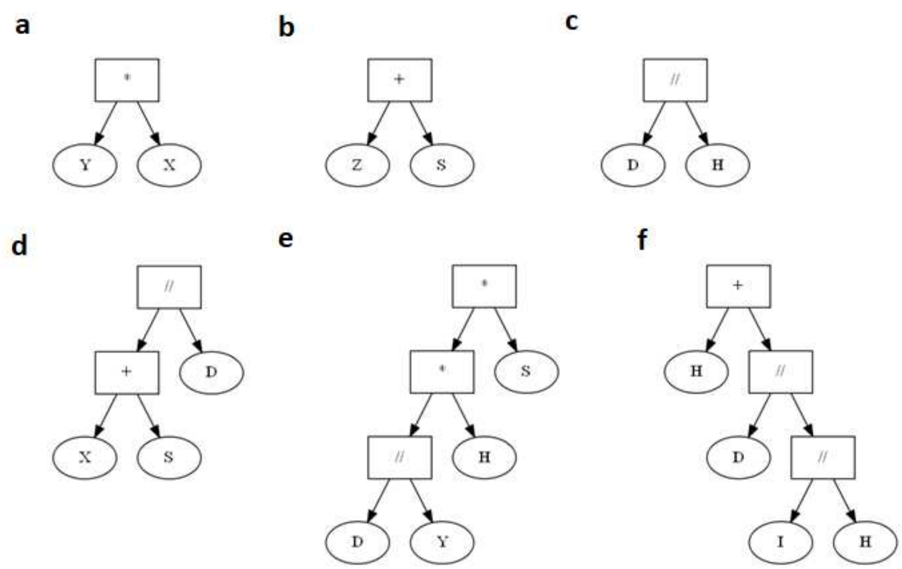

Figure 2 Trader's beliefs simulated by quantum circuits (**a**) $\text{qCircuit} = (Y * X)$ (**b**) $\text{qCircuit} = (Z + S)$ (**c**) $\text{qCircuit} = (D // H)$ (**d**) $\text{qCircuit} = \big((X + S) // D\big)$ (**e**) $\text{qCircuit} = \Big(\big((D // Y) * H\big) * S\Big)$ (**f**) $\text{qCircuit} = \Big(H + \big(D // (I // H)\big)\Big)$

As shown in Figure 2, there are a few strategies with different subjective beliefs that trader took on trading securities.

- $\text{qCircuit} = (Y * X) \ \rightarrow \ \hat{V} = 0.5|a_1\rangle\langle a_1| + 0.5|a_2\rangle\langle a_2|$ 50% belief to buy, 50% belief to sell
- $\text{qCircuit} = (Z + S) \ \rightarrow \ \hat{V} = 0.67|a_1\rangle\langle a_1| + 0.33|a_2\rangle\langle a_2|$ 67% belief to buy, 33% belief to sell
- $\text{qCircuit} = (D // H)$

1) $D \ \rightarrow \ \hat{V} = 0.5|a_1\rangle\langle a_1| + 0.5|a_2\rangle\langle a_2|$ 50% belief to buy, 50% belief to sell

2) $H \rightarrow \hat{V} = 0.5|a_1\rangle\langle a_1| + 0.5|a_2\rangle\langle a_2|$ 50% belief to buy, 50% belief to sell

- qCircuit = $\big((X + S) \,//\, D\big)$

1) $(X + S) \rightarrow \hat{V} = 0.69|a_1\rangle\langle a_1| + 0.31|a_2\rangle\langle a_2|$ 69% belief to buy, 31% belief to sell

2) $D \rightarrow \hat{V} = 0.5|a_1\rangle\langle a_1| + 0.5|a_2\rangle\langle a_2|$ 50% belief to buy, 50% belief to sell

- qCircuit = $\big(((D \,//\, Y) * H) * S\big)$

1) $\big((D * H) * S\big) \rightarrow \hat{V} = 0.55|a_1\rangle\langle a_1| + 0.45|a_2\rangle\langle a_2|$ 55% belief to buy, 45% belief to sell

2) $\big((Y * H) * S\big) \rightarrow \hat{V} = 0.45|a_1\rangle\langle a_1| + 0.55|a_2\rangle\langle a_2|$ 45% belief to buy, 55% belief to sell

- qCircuit = $\big(H + \big(D \,//\, (I \,//\, H)\big)\big)$

1) $(H + D) \rightarrow \hat{V} = 0.5|a_1\rangle\langle a_1| + 0.5|a_2\rangle\langle a_2|$ 50% belief to buy, 50% belief to sell

2) $(H + H) \rightarrow \hat{V} = 0.5|a_1\rangle\langle a_1| + 0.5|a_2\rangle\langle a_2|$ 50% belief to buy, 50% belief to sell

3) $(H + I) \rightarrow \hat{V} = |a_1\rangle\langle a_1|$ 100% belief to buy

Synapse's quantum circuit hypothesis

As shown in Figure 3, neuron transmits information through synapse with the help of neurotransmitters stored in vesicles at the end of axons.[27, 28] Action potentials propagating along axons release neurotransmitters, and receptors on the postsynaptic membrane respond specifically to certain neurotransmitters to form ion channels with switching function to excite or inhibit of postsynaptic neurons. When the ion channel is opened, the incoming ions will reduce the synaptic potential difference inside and outside the cell membrane, which is called depolarization, resulting in the excitation; the outgoing ions will increase the potential difference, which is called hyperpolarization, resulting in the inhibition of neurons. While the ion channel is opened or closed, the synaptic potential changes continuously to form excitation or inhibitory synaptic inputs into the neuron body for integration, when integrated synaptic potential exceeds the threshold, the all-or-nothing action potential will be triggered. Information transmissions between neurons is to repeatedly convert digital signals into analog signals, and then restore stimulated signals into digitals, as John Von Neumann suggested[29].

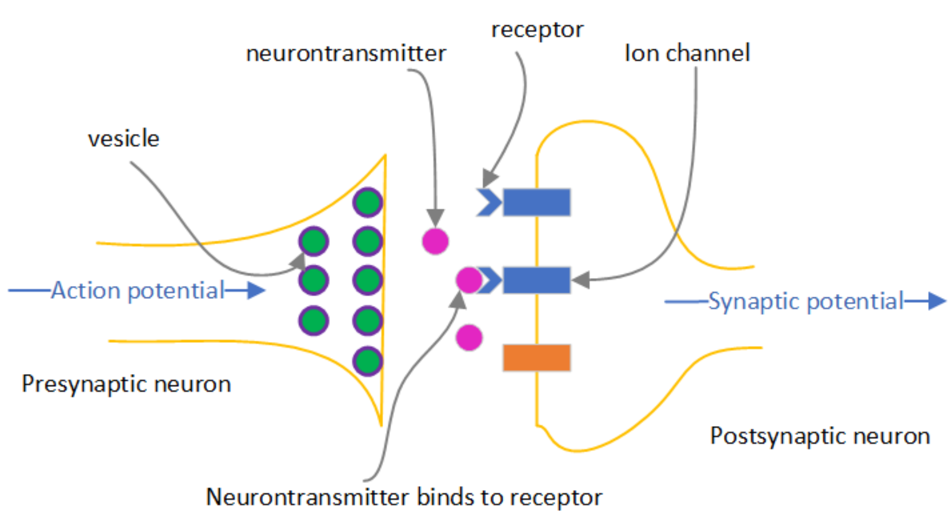

Figure 3 Synapse structure and functions

We can analogy that a single ion channel as a "trader", an ion channel has two "actions" to choose: open or close, equivalent to trader's buy or sell actions. Synapse can be compared with market; the nature states of synapse are excitation and inhibition, equivalent to market's rising or falling. Synaptic potential can be analogized to market's price and the change of synaptic potential can be compared to value of profit or loss. We can represent synapse states $|\psi\rangle$ in terms of the Hilbert state space, Hilbert strategy space $|\mathrm{S}\rangle$ is used to represent ion channel's "strategies" which decide if open or close, Neurotransmitter combined with receptor functioned as quantum circuit can be represented as density operator $\widehat{\mathrm{V}}$ which projects an ion channel's "beliefs" onto an open or close action:

$$|\psi\rangle = c_1|q_1\rangle + c_2|q_2\rangle \tag{10}$$

$$|S\rangle = \mu_1|a_1\rangle + \mu_2|a_2\rangle \tag{11}$$

$$\widehat{V} = p_1|a_1\rangle\langle a_1| + p_2|a_2\rangle\langle a_2| \tag{12}$$

Where $|q_1\rangle$ indicates a state in which the synapse is exciting and $|q_2\rangle$ indicates synapse is inhibiting; $|a_1\rangle$ refers to open ion channel and $|a_2\rangle$ refers to close ion channel; p_1 represents the "subjective" probability which an ion channel opened and p_2 represents the probability ion channel closed. The binding of neurotransmitter and receptor as a "quantum measurement" completes a decision-making by opening or closing an ion channel and the synaptic potential changed as value observed. It is the natural selection that has evolved the corresponding neurotransmitter and their receptors as the fittest "quantum circuit" to guide the neuron's decision-making. If synapse is exciting and an ion channel chooses to open, the value V_k is positive and the ion channel gets an award, otherwise value V_k is negative and the ion channel gets a punishment; if synapse is inhibiting and an ion channel chooses to close, the value V_k is positive and the ion channel gets an award, otherwise value V_k is negative and the ion channel gets a punishment. The "quantum circuit" formed by neurotransmitter and receptor evolves as described in equation (9).

Discussion

Human beings record a large amount of data through the observation of the world. It is through the study of the recorded data that human beings gradually understand the objective world and build a subjective simplified "world model" in our brain. We make decisions by considering both the world's objectivity and the subjectivity of our beliefs. Observed value is a bridge between objective world and our subjective consciousness.

Natural state describes objective world, we assume that an uncertain natural state can be represented by superposition of all possible states. Strategy state describes subjective world, we also assume that undecided decision state can be represented by superposition of all possible actions. The information we can get regarding decision is observed value, and value can be represented by a quantum density operator as value operator. Value operator represents our subjective beliefs of taking actions. The process of decision is equivalent to evolution of the mixed state's density operator based on nature states, and the final decision is to perform a "quantum measurement" on the strategy state which chooses an action probabilistically and get a value.

We proposed a quantum circuit decision theory based on quantum expected value. Instead of value function used in reinforcement learning[30], value operator is used to simulate brain's beliefs under uncertainty. Classical decision theory asks the bit question, 0 or 1? There are only two possible answers - 0 or 1. Quantum decision theory asks the qubit question, 0 and 1? Now, the answer could have infinite possibilities. Quantum circuit decision theory has inherent uncertainty due to superposition of quantum states and an observable result is obtained probabilistically from the "collapse" of the state. Scientists tend to think that the brain's decision is computing rather than reasoning, quantum circuit computes the probability of taking an action due to incomplete information, and the result usually cannot be given by a definite cause, but can only be obtained probabilistically.

Descartes' dualism holds that the body is separate from the soul, and he further postulated that matter and consciousness are exchanged at the pineal body of the brain. Until now the mind-body problem[31, 32,33,34,35] is still a mystery. We believe that the information of the objective world is encoded in action potential, and probabilistically transformed into subjective signals at synapse. Our proposed synapse's quantum circuit hypothesis states that the combination of neurotransmitter and receptor functioned as quantum circuit "subjectively" controls the opening and closing of ion channel which to decide the firing rate of neurons. The firing pattern of the neural network expresses the subjective decision of the brain, that is, consciousness converted from the matter.

In the future, we plan to implement quantum circuit evolution algorithm, and use the k-data of securities traded at the Shanghai Futures Exchange for quantum circuit learning and optimization.

Method

The value operator of a trader is a 2x2 matrix, and the matrix needs to be diagonalized first and then normalized to get probability $p_1$and p_2 as follows:

$$\hat{V} = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} \xrightarrow{\text{diagonalization}} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \xrightarrow{\text{normalization}} \begin{bmatrix} p_1 & 0 \\ 0 & p_2 \end{bmatrix} = p_1|a_1\rangle\langle a_1| + p_2|a_2\rangle\langle a_2| \tag{13}$$

Where $|a_1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, |a_2\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix};\ |a_1\rangle\langle a_1| = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, |a_1\rangle\langle a_2| = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$. If the matrix has two different eigenvalues, the matrix can be diagonalizlied by solving eigenvalues:

$$\begin{vmatrix} v_{11} - \lambda & v_{12} \\ v_{21} & v_{22} - \lambda \end{vmatrix} = 0 \ \rightarrow A\lambda^2 + B\lambda + C = 0 \tag{14}$$

$$\lambda_1 = 0.5 \times \left(-B + \sqrt{B^2 - 4C}\right) \text{ and } \lambda_2 = 0.5 \times \left(-B - \sqrt{B^2 - 4C}\right) \tag{15}$$

Where $B = -(v_{11} + v_{22}), C = (v_{11}v_{22} - v_{12}v_{21})$. The normalized p_1 and p_2 are given by:

$$p_1 = |\lambda_1|^2/(|\lambda_1|^2 + |\lambda_2|^2) \text{ and } p_2 = |\lambda_2|^2/(|\lambda_1|^2 + |\lambda_2|^2) \tag{16}$$

If quantum circuit is $\hat{H} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, we have two different eigenvalues $\lambda_1 = 1$ and $\lambda_2 = -1$, the normalized probability $p_1 = 0.5$ and $p_2 = 0.5$..

$$\hat{H} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \xrightarrow{\text{diagonalization}} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \xrightarrow{\text{normalization}} \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} = 0.5|a_1\rangle\langle a_1| + 0.5|a_2\rangle\langle a_2|. \tag{17}$$

Data availability

No datasets were generated or analyzed during the current study.

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Author contributions statement

All authors conducted the research and contributed to the development of the model. HX contributed as an expert in quantum theory and non-linear science. LX contributed to the research from the aspects of quantum computing, decision theory and neuroscience. The idea to use quantum circuit to simulate people's beliefs was proposed by LX. LX wrote this manuscript. All authors reviewed the manuscript.

Additional information

Competing interests: The authors declare no competing interests.