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Abstract A Fabry-Perot (F-P) cavity with a low thermal noise limit 7.8×10^{-17} is investigated. The spacer aperture is composed of three holes, where a long and small hole in the middle, a shorter and larger hole on each side. The FS mirrors are placed in the larger holes of the spacer aperture, the FS rings are mounted on the end face of the spacer, and FS mirrors, aperture and FS rings are coaxial. It enables the cavity zero-crossing temperature easily tuning to above room temperature. In the meantime, with this design the machining error has little effect on the zero-crossing temperature of the cavity. In addition, it is convenient for manufacturing of long F-P cavity.

Keywords Fabry-Perot · zero-crossing temperature · Finite element analysis

1 Introduction

With the development of science and technology, the requirements for laser stability are more precise. Ultra-stable lasers are essential in optical atomic clocks [1], quantum information [2] and high-resolution laser spectroscopy [3, 4].A common method to obtain a highly stable laser is locking the laser to a highly stable Fabry-Perot (F-P) cavity by the Pound-Drever-Hall technique[5]. The dependence of the nominal resonant optical frequencies on the cavity length is $\Delta\nu/\nu = -\Delta L/L$; $\Delta\nu$ is the change of frequency and ΔL is the

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jitter of the cavity length. With this dependence, the stability of the laser frequency is translated into one of the F-P cavity length. The thermal noise limit is usually reduced by increasing the F-P cavity length, increasing the laser spot size, and using high mechanical Q materials[6]. Among these methods, materials have the greatest impact on the thermal noise limit. Conventional F-P cavity is made from ultra-low expansion glass (ULE). Its thermal noise limit is at the level of 10^{-15} 10^{-16}. In order to design a cavity with thermal noise limit of 10^{-17} magnitude, high Q fused silicon (FS) is selected as the substrate material of the cavity mirror.

When the substrate of the cavity mirror is FS, the thermal noise limit of cavity can be reduced to half of that when mirror is ULE. However, the thermal expansion coefficient (CTE) difference between FS and ULE is two orders of magnitude, which may cause stress due to temperature changes of the cavity, bending the mirror substrate and introducing greater effective CTE into the F-P cavity[7]. This problem can be mitigated by setting the operating temperature of the F-P cavity at ZCT of the effective CTE[8]. However, the ZCT of the composite cavity is about 10 °C lower than that of the ULE cavity spacer[7], which is in the range of 5-35 °C. In this way, the ZCT of the composite cavity is lower than room temperature. This means that in the experiment, the whole cavity needs to be cooled at the ZCT. But cooling the cavity is complex than heating it, and reducing the robustness of cavity system. These problems are disadvantage to miniaturized portable optical clock. Thus, the ZCT should be adjusted above the room temperature.

In order to raise the ZCT, a series of studies have been taken. Richard W. Fox et al. used the negative CTE of the ULE at room temperature, combined with the positive CTE of FS, to make the ZCT of the cavity higher than room temperature[9]. Thomas Legero et al. compensated for the ZCT by sticking the ULE ring on the FS mirror[7]. The ZCT of this F-P cavity was equivalent to that of the ULE spacer. Stephen Webster et al. put the FS cavity mirror into the optical hole of the cavity and fixed it on the ULE ring by optically contacting. At the same time, the ULE ring was optically glued on the ULE cavity, and the tuning range was achieved at about 30 K[10]. Similar to Stephen Webster's work, Zhang Jie et al. used the FS ring instead of the ULE ring to achieve the tuning range -10 °C to 23 °C[11]. These approaches have some limitations. The first method relies on the availability of ULE materials with negative CTE at room temperature, which require special selection and testing. This greatly increases the cost. The second method can only make the ZCT close to ULE spacer. It is not applicable when the cavity ZCT is lower than room temperature. In principle, the third method can be used to adjust the ZCT to about 30 °C. However, the cavity hole needed to be larger than the mirror diameter (25.4 mm), which is unsuitable for the long cavity machining. In this paper, the F-P cavity structure is redesigned with three stepped optical holes, which have three characteristics. (1) The cavity is easy machining and low-cost. (2) The tuning range of ZCT is -10 °C to 27 °C, which can adjust the cavity's ZCT, no matter what it is, above room temperature.

The cavity we designed is used to stabilize the clock laser in the aluminum ion optical clock. The aluminum ion clock laser at 267 nm is obtained by quadrupling the frequency 1068 nm laser. The laser frequency stability is achieved by locking the double frequency of the 534 nm laser on the F-P cavity. The stabilized 267 nm laser satisfies the aluminum ion clock experiment. In this paper, the second part introduces the structure of the F-P cavity, and the third part is ZCT compensation. Finally, the fourth part concludes this work.

2 Low-thermal noise F-P cavity

Thermal fluctuation is a fundamental phenomenon in the F-P cavity. At nonzero temperatures, the cavity has an inevitable mechanical thermal fluctuation which fundamentally limits the frequency stability, even if the CTE is zero. The mechanical losses ϕ of the spacer, mirror substrate, and reflective coating are the basic parameters for evaluating thermal noise of cavity. The thermal fluctuation spectrum of the F-P cavity is[6].

$$\sigma_{therm} = \sqrt{\frac{\ln 2 \cdot 8 \cdot K_B T}{\pi^{\frac{3}{2}} L^2} \cdot \frac{1-\sigma^2}{E\omega_0}(\phi_{sub} + \phi_{coat}\frac{2}{\sqrt{\pi}} \cdot \frac{1-2\sigma}{1-\sigma} \cdot \frac{d}{\omega_0})} \quad (1)$$

Here, K_B is the Boltzmann constant, T is the temperature, L is the cavity length, E is Young's modulus of the spacer, $\omega_0 = (\frac{L\lambda}{\pi}(\frac{R-L}{L})^{\frac{1}{2}})^{\frac{1}{2}}$ which is the beam radius, R is the curvature radius of concave mirror, σ is Poisson's ratio, d is the mirror coating thickness, and ϕ_{sub} and ϕ_{coat} are the mechanical losses of mirror substrate and mirror coating, respectively. Because the mechanical loss of FS (10^{-6}) is lower than that of ULE(1.6×10^{-5})[6], according to equation (1), it is better to choose FS as the cavity mirror substrate to obtain the cavity with low thermal noise.

From equations (1), we can see that the longer the cavity is, the smaller the thermal noise limit is. And the thermal noise limit is negatively correlated with the radius of curvature of the cavity mirror. The thermal noise limits of the cavity listed in Table1 with different materials and parameters. In order to obtain the stable laser of aluminum ion clock transition detection, the length and diameter of the cavity is selected as 300 mm and 150 mm. And the curvature radius of the concave mirror is 1 m. In addition, the substrate material of the cavity mirror is FS. The geometry of cavity is shown in Fig.1. With this design the thermal noise limit of the cavity is 7.8×10^{-17} when the ZCT is at 30 °C.

3 Zero-crossing temperature compensation

The length L of the F-P cavity varies with temperature due to thermal expansion:$\delta L/L = \alpha\delta T$. The thermal expansion coefficient of ULE ($\alpha_{ULE}T$))is a quadratic function of temperature T[7],

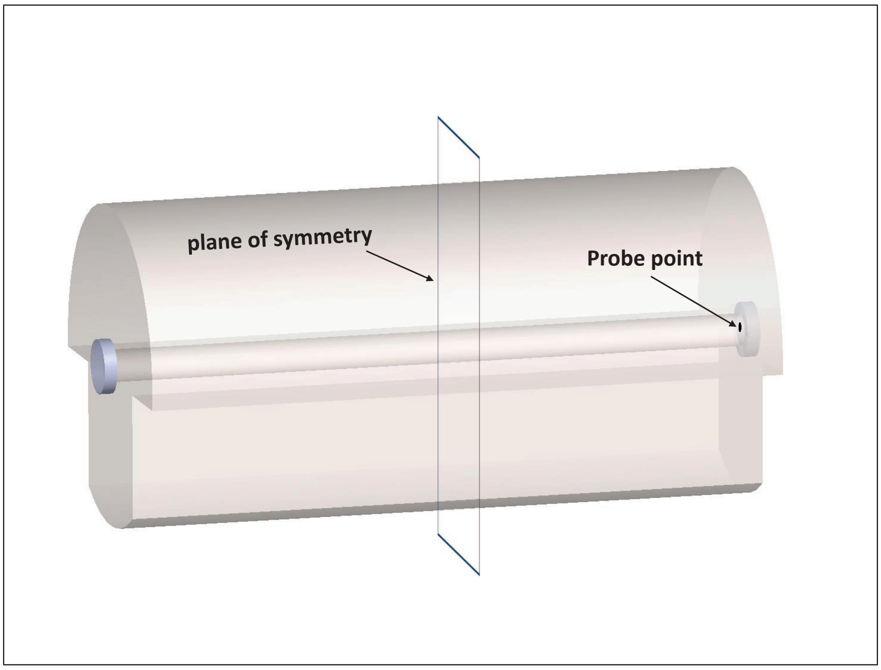

Fig. 1 Geometry of cavity.

$$\alpha_{ULE}(T) = a(T - T_0) + b(T - T_0)^2 \tag{2}$$

where a is $2.4 \times 10^{-9}/K^2$, b is $-10^{-11}/K^3$, T is cavity temperature, and T_0 is the zero-crossing temperature of the ULE. When $T = T_0$, the thermal expansion coefficient is 0, In this case, the temperature jitter does not affect the length of the cavity. In the experiment, the temperature of the cavity system is usually controlled at T_0 to reduce the influence of ambient temperature jitter.

The thermal expansion coefficient of FS is $\alpha_{FS} = 5 \times 10^{-7}/K^2$, which is two orders of magnitude larger than that of ULE. When the cavity spacer material is ULE, and the cavity mirror material is FS (composite cavity), the radial deformation of the cavity mirror caused by the temperature changes dT:

$$dR = (\alpha_{FS} - \alpha_{ULE})RdT \tag{3}$$

Because the spacer and the mirrors do not move relative, the radial pressure causes the deformation of the cavity axis

$$dB = \delta dR \tag{4}$$

δ is independent of temperature and relates to the geometry, size and material of the cavity. The dL of the composite cavity is given by the spacer's thermal expansion $L\alpha_{ULE}dT$ plus the induced axial displacement of the two mirrors 2dB. It can be described by an effective cavity CTE with

$$dL = L\alpha_{eff}dT \tag{5}$$

Let b=0, From equations (2)(3)(4)(5)we can obtain

$$\alpha_{eff} = (1 - 2\delta\frac{R}{L})a(T - T_0) + 2\delta\frac{R}{L}\alpha_{FS} \tag{6}$$

The equivalent zero expansion coefficient of the composite cavity $\alpha_{eff} = 0$ corresponds to the ZCT of T_0'. The offset of T_0' and T_0 is

$$\Delta T_0 = T_0' - T_0 = \frac{2\delta\frac{R}{L}\alpha_{FS}}{(-1 + 2\delta\frac{R}{L})a} \tag{7}$$

Since δ dependents on the geometry, size and material of the cavity, it is difficult to get the analytical solution through theoretical calculation. The values of δ under different conditions are obtained by using finite element analysis (FEA). In the simulation, because the cavity is axially symmetric (Fig.1), half of the cavity is selected as the research object. The displacement reference is the axially symmetry plane of the spacer (Fig. 2), and the temperature variation is set as 1 K. Moreover, the contact area of the FS mirror, ULE spacer and the FS ring are perfectly bonded such that there is no relative displacement. The cavity length variation dL is obtained by probing the axial displacement at the center of the cavity mirror's inner surface (Fig.1). The equivalent thermal expansion coefficient α_{eff} is calculated by the formula (6), and the δ is obtained by substituting α_{eff} into formula (7).

The deformation of the composite cavity is shown in Fig.2, and obviously the cavity mirror is bow. By calculating, we obtain $\delta = 0.0077$, $\alpha_{eff} = 2.8 \times 10^{-8}$ and $\Delta T_0 = -10.19K$. It means that the effective CTE of the composite cavity is relatively larger than that of ULE at room temperature, and the zero in the effective CTE still exists, but this is shifted to even lower temperature. The range of T_0 is 5 °Cto 35 °C, so the effective ZCT T_0' ranges from -5°C to 25°C, which close to or below room temperature. In this case, the cavity needs to be cooled. But cooling the cavity may cause additional problems, such as water condensation on the cavity shell. It is better to keep the temperature slightly higher than room temperature, such as 30 °C.

In order to tuning the zero-crossing temperature to 30 °C, ΔT_0 should be adjustable from -5 °C to 25 °C. This article is to find a cavity structure through finite element analysis. With this structure, the appropriate compensation ring size can be conveniently selected to achieve the ZCT of the cavity at about 30 °C. At the same time, the selected cavity structure is convenient for processing and manufacturing.

A solution to T_0' below room temperature is to optically contact a ULE ring behind each FS cavity mirror, which is shown in Fig.3 (a). Fig.3 (b),(c) and (d) is the results of the simulation. The temperature difference ΔT_0 changes with the external radius R, inner radius r, thickness h of the ring and diameter

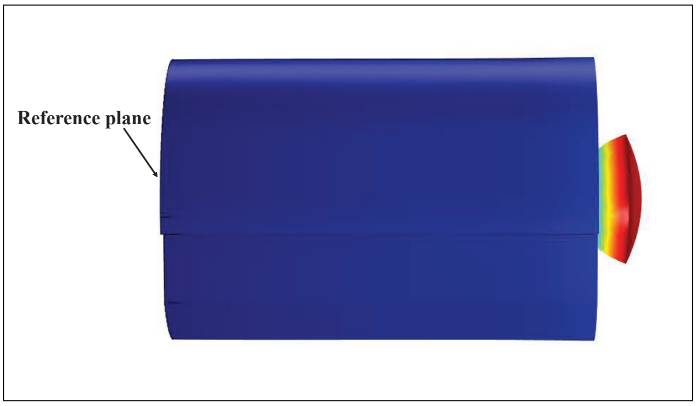

Fig. 2 FEM simulation of the elastic cavity deformation for 1 K above ZCT.

of the optical aperture bore. However, ΔT_0 at most is about 0 K no matter what the parameters are, which means that the ZCT is equivalent to the cavity spacer. The simulation result is in good agreement with reference [7]. If the ZCT of the ULE spacer is lower than room temperature, refrigeration is still required. Obviously, this scheme does not satisfy the requirements.

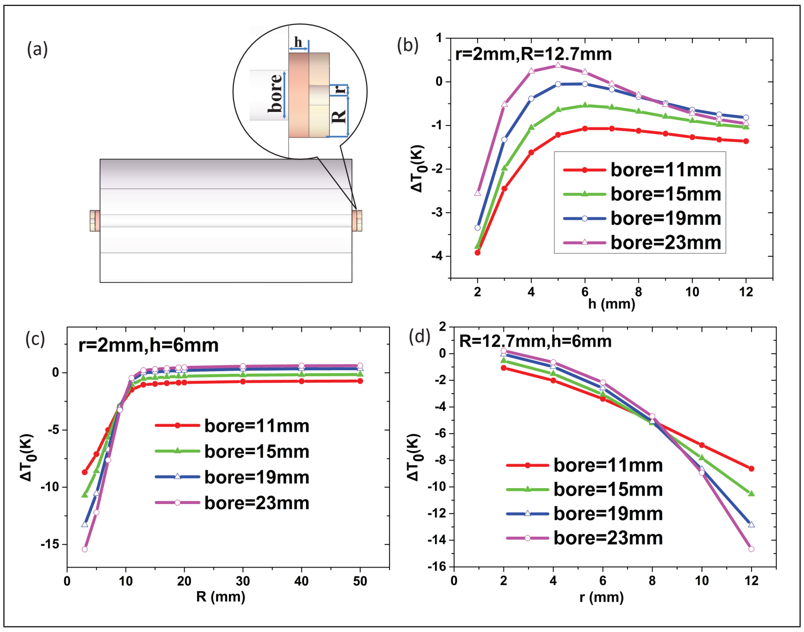

Fig. 3 (a)the structure of cavity. (b), (c) and (d) is the results of FEM simulations of the axial mirror displacement along a radial line on the mirror surface when an additional ULE ring was contacted behind the FS cavity mirrors. R: ring external radius; h: thickness of ring; r: ring inner radius, bore: diameter of the optical aperture.

We designed the aperture of spacer as stepped holes, and plug the mirrors into the lager holes. It is realized by mounting the FS mirror onto either the ULE or FS ring, and then fixing them on the spacer's end faces through the optical glue (Fig. 4). In this way, the thermal expansion direction of mirrors is opposite to that of spacer. Fig.5 are the simulation results when the cavity is with a ULE ring. Various ring thicknesses (h), external and inside radius (R, r), and large optical aperture's diameter and depth (bore, h1) of the spacer are used to simulate the ZCT. Of all these conditions, h1 and R have almost no effect on ΔT_0. ΔT_0 decreases with the decrease of bore, and it decays exponentially with h. r affects the variation range of ΔT_0. However, no matter what the sizes of ring are, ΔT_0 is always higher than 15 K. This configuration is only suitable for cavities with a ZCT below 15 °C.

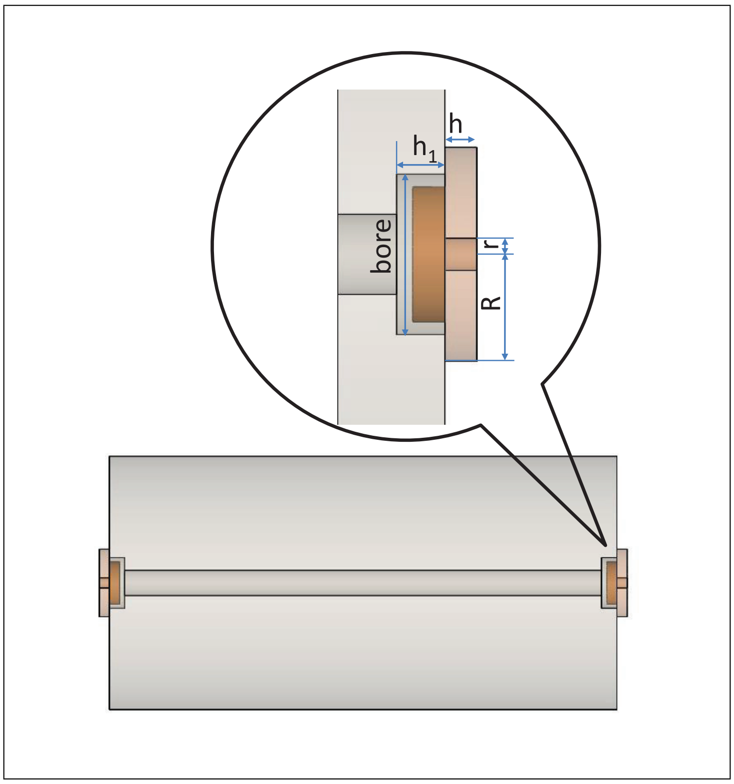

Fig. 4 The geometry of the cavity we designed.

We propose to replace ULE ring with FS ring to improve the second scheme. And the deformation of the ULE spacer can be inhibited with FS ring. This scheme is simulated, and the relevant results are shown in Fig.6. Firstly, the influence of h1 on ΔT_0 is calculated, and the result is shown in Fig. 6 (a). When h_1 changes from 7 mm to 140 mm, ΔT_0 only changes by about 1 K.

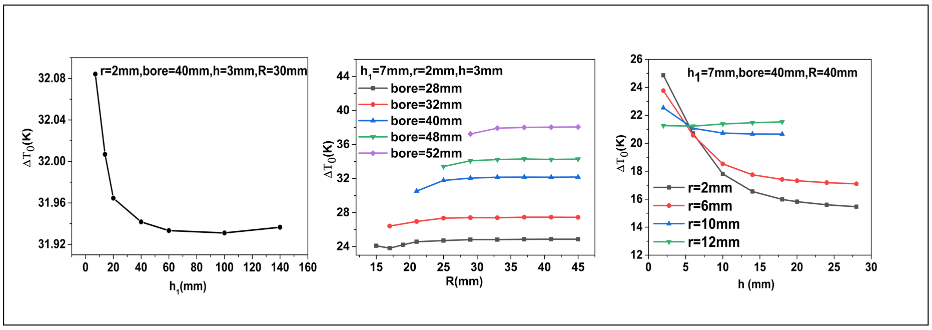

Fig. 5 FEM simulations of the axial mirror displacement along a radial line on the mirror surface when the rings are ULE. h_1: depth of large clear aperture; bore: diameter of the optical aperture.; R: ring external radius; h: thickness of ring; r: ring inner radius.

Thus, h_1 has little influence on ΔT_0. In order to facilitate processing, the h1 is selected as 7 mm. Secondly, We study the influence of both ring external radius R and diameter of large aperture bore on ΔT_0. ΔT_0 increases logarithmic with R, and bore affects the range and rise-rate of ΔT_0 (Fig.6(b)). The range and the rise-rate are increasing with bore. When bore is 28 mm and 40 mm, the variation range of ΔT_0 is 15 K and 30 K respectively. In order to satisfy the adjustment range and reduce the impact of ring sizes errors, we select bore as 40 mm. Thirdly, the influence of ring thickness h on ΔT_0 is studied, which is shown on Fig.6(c). h mainly affects the variation range of ΔT_0. For example, -5 K to 27 K when h=2 mm and -15 K to 20 K when h=3 mm. In order to improve the practicability of the scheme, the ring size with a wide range of ΔT_0 should be selected as far as possible because there are errors in the ring sizes, and h is selected as 3 mm. In this case, the adjustment range is -14 K to 22 K, slightly lower than the request range. According to Fig.6 (d), it can be improved by increasing r, and the adjustment range is -10 K to 27 K when r=6 mm, which is meeting the requirements. To sum up, the depth and diameter of cavity large clear aperture are 7 mm and 40 mm. And the ring parameters are r=6 mm, h=3 mm. The ring outer radius is chosen according to the ULE spacer ZCT.

4 Conclusions

We design a composite cavity with the thermal noise limit is 7.8×10^{-17} and it's ZCT above room temperature. The optical aperture are composed of a series of stepped holes, and the cavity mirror is placed in the large optical aperture and fixed on the cavity through the FS ring. Selecting the appropriate sizes of the ring, the zero-crossing temperature of the effective ZCT of the cavity can be easily tuned to above room temperature with any ULE spacer materials, as long as the zero-crossing temperature of the selected ULE spacer is known. Currently, the CTE zero crossing temperature of the ULE spacer can be measured with an accuracy of $\pm 1°$ C at reasonable cost. This design

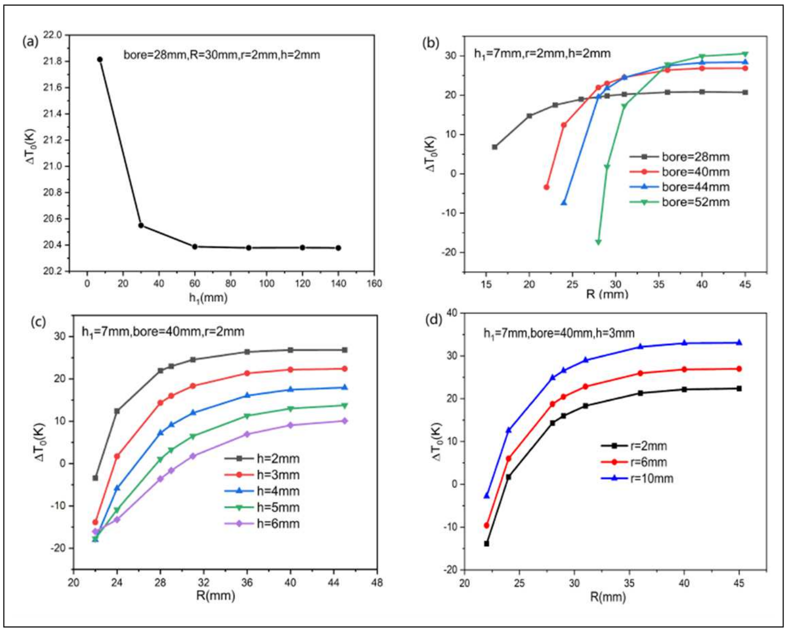

Fig. 6 FEM simulations of the axial mirror displacement along a radial line on the mirror surface when the rings are FS. h_1: depth of large clear aperture; bore: diameter of the optical aperture; R: ring external radius; h: thickness of ring; r: ring inner radius.

adopts the stepped holes structure, which is convenient for processing and reduces the cost. The desigined cavtiy is applicable for cavities with different lengths. In particular, long cavties have greater advantages using this desigin structure.

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References

1. A.W. Young, W.J. Eckner, W.R. Milner, D. Kedar, M.A. Norcia, E. Oelker, N. Schine, J. Ye, A.M. Kaufman, Nature **588**(7838), 408 (2020)
2. Y. Zeng, P. Xu, X. He, Y. Liu, M. Liu, J. Wang, D.J. Papoular, G.V. Shlyapnikov, M. Zhan, Physical Review Letters **119**(16) (2017)
3. R. Shaniv, N. Akerman, R. Ozeri, Physical Review Letters **116**(14) (2016)
4. R. Kaewuam, T.R. Tan, Z. Zhang, K.J. Arnold, M.S. Safronova, M.D. Barrett, Physical Review A **102**(4) (2020)
5. E.D. Black, American Journal of Physics **69**(1), 79 (2001)

6. k.Numata, K. A, J.Camp, Physical Review Letters **93**(250602), 3 (2004)
7. T. Legero, T. Kessler, U. Sterr, J.Opt.Soc.Am.B **27**(914) (2010)
8. W. Bian, Y. Huang, H. Guan, P. Liu, L. Ma, K. Gao, The Review of scientific instruments **87**(6), 063121 (2016)
9. R.W. Fox, Optics Express **17**(17), 15023 (2009)
10. S. Webster, P. Gill, Ieee, *Low-thermal-noise optical cavity* (Ieee, New York, 2010), pp. 470–473. IEEE International Frequency Control Symposium
11. J. Zhang, Y. Luo, B. Ouyang, K. Deng, Z. Lu, J. Luo, European Physical Journal D **67**(2) (2013)