

Transport of topological edge states in 2D Zigzag edge Tungsten Ditelluride Nanoribbon

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Abstract

In this paper, we have investigated the transport of topological edge states in 2D Zigzag edge Tungsten Ditelluride Nanoribbon (ZTDNR). We have found that zigzag edge nanoribbon (NR) of Tungsten Ditelluride develops topological edge states in the presence of intrinsic spin orbit interaction (SOC). We have used three band tight binding model for the electrons of d_{z^2} , d_{xy} , and $d_{x^2-y^2}$ orbitals with SOC for calculating band structure of NR and Non Equilibrium Greens Function (NEGF) formalism for transport in the NR. We have investigated transport in a pristine device, transport in the presence of a finite potential barrier, transport with constriction within the device and transport with edge imperfections.

Keywords:

Topological Insulator, Spin orbit coupling, Tungsten ditelluride, Non-equilibrium greens function, Nanoribbon.

1. Introduction

Two-dimensional materials([1]-[4]) are attractive for their use in next-generation nanoelectronic and optoelectronic devices because, compared to one-dimensional materials, it is relatively easy to fabricate complex structures from them. Recently a new class of two-dimensional (2D) materials—the monolayer transition metal dichalcogenides (TMD) ([5]-[7]) has received great attention for their significant potential as the channel material in emerging nanoelectronic devices. 2D TMDs exhibit many rich properties such as excellent mobility [8] at room temperature, the unsaturated magneto resistance, gas sensor ([9]-[10]),

FET([11]-[12]), memoristor [13] and super conductivity [14] making them promising candidates for electronic and optoelectronic applications.

Some TMDs have been theoretically and experimentally proven as topological insulator. In 2D topological insulators (TI), the nontrivial edge states support quantum spin Hall (QSH) [15] effect. TI has a pair of edge states of opposite spins where the electrons with opposite spins will propagate along opposite direction. This property enables the topological edge states immune to scattering from non-magnetic impurities([16]-[19]). Since the early proposal of QSH effect in graphene, spin-orbit coupling (SOC) has played a key role to ma-

27 nipulate the properties of a material. Materials with
 28 strong SOC drastically differ from other materials and
 29 exhibit some novel phenomena([20]-[21]). So, a num-
 30 ber of 2D systems are being examined. A few of them
 31 are confirmed experimentally to have large SOC and
 32 direct band gap. Among them Tungsten Ditelluride
 33 (WTe_2) exhibits properties like having direct bandgap
 34 in bulk monolayer, breaking of inversion symmetry due
 35 to the trigonal prismatic geometry and the largest SOC
 36 strength among TMDs([22]-[23]). These properties ren-
 37 der it as a rich and interesting platform to study QSH
 38 effect([24]-[25]).

39 In this study, we have presented the transport prop-
 40 erties of 2D Zigzag nanoribbon of WTe_2 confirming
 41 it as a topological insulator. We have used three band
 42 tight binding model for simulating the monolayer WTe_2
 43 nanoribbon as has been used by other researchers([26]).
 44 The parameters of this TB model are obtained by fitting
 45 the DFT analysis and used for low energy description of
 46 single layered 2D materials([26]). So, This TB model is
 47 sufficient to capture the band-edge properties including
 48 the energy dispersion. For describing the transport, in
 49 this work the NEGF [27] formalism has been used. The
 50 transport properties of WTe_2 nanoribbon are studied for
 51 different scenarios.

52 2. Theory

53 TMDs are referred to generically as MX_2 , where M
 54 represents the transition metal and X the chalcogen. A
 55 monolayer can be seen as a lattice of M and X atoms.
 56 Each M (Tungsten, W) atom is connected with neighbor-

57 ing X (Ditelluride, Te) atoms. These materials have two
 58 different kinds of unit cells depending on the geometry
 59 of the coordination of M atom with the X atoms. One of
 60 them has a trigonal prismatic structure where every M
 61 atom is connected with six X atoms and it is called $2H$
 62 (hexagonal symmetry) or $3R$ (rhombohedral symmetry).
 63 The other one is octahedral coordination. Its structure
 64 will be $1T$ (tetragonal symmetry)[28]. In this study,
 65 we have considered trigonal prismatic structure($2H$ or
 66 hexagonal symmetry), with sites M and X . Figure 1
 67 shows a top-down view of the monolayer crystal struc-
 68 ture.

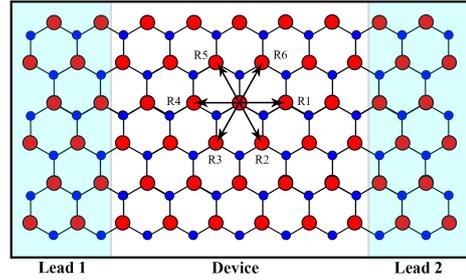


Figure 1: Lattice structure of TMD

69 For modeling the Hamiltonian of nanoribbon of
 70 TMDs , We have considered a three-band tight-binding
 71 (TB) model including spin. The conduction and
 72 valence-band edges are predominantly contributed by
 73 the d_{z^2} , d_{xy} and $d_{x^2-y^2}$ orbitals of M atoms. So the TB
 74 model is constructed using these three orbitals based on
 75 the symmetries of the monolayers. Due to the heavy
 76 transition-metal M atom, TMDs spin orbital coupling
 77 can be large. The large SOC of monolayer MX_2 is a
 78 fascinating feature. For our study, SOC effects are stud-
 79 ied under the approximation of on-site spin-orbit inter-

80 actions of M atoms. In this paper, we have demonstrated
81 different features of edge states for WTe_2 nano ribbon
82 by altering the device geometry. We have considered
83 constriction, edge imperfection and barrier. In simula-
84 tion, constriction has been implemented by removing a
85 chunk of atoms at one edge of the NR. Edge imperfec-
86 tions can happen during fabrication process in the NR.
87 We introduced it by removing random atoms from the
88 edge. For demonstrating barrier, we increased the en-
89 ergy of some particular atoms. These transport charac-
90 teristics of the nanoribbon have been simulated using
91 NEGF formalism. The NEGF formalism enabled us to
92 describe the quantum transport of electron for nanoscale
93 devices.

94 3. Mathematical model

We have used NEGF formalism in this work. We can calculate electron density and transmittance of the electrons inside a device by using this formalism([29]-[32]). The Green's function, G^R is the impulse response of a device. We can calculate the green's function of the device from the device Hamiltonian, H and the self energies of the interaction i.e. Σ_{l1} (left lead) and Σ_{l2} (right lead). Now the green's function, G^R can be calculated using the following equation([29]-[32]):

$$[EI - H - \Sigma_{l1} - \Sigma_{l2}]G^R = I \quad (1)$$

In the above eq.(1), E is the energy of electrons that are introduced into the device through the lead and I denotes identity matrix. The energy E is around the Fermi

energy level of the device. To get Hamiltonian of the device, we have used tight binding model. We can express Hamiltonian through the following equation([33])
:

$$H_o = \sum_i \sum_j a_{i\alpha}^\dagger E_{\alpha\beta}(r_{ij}) a_{j\beta}; r_{ij} = r_i - r_j \quad (2)$$

In eq.2, α and β are indices for orbitals. $E_{\alpha\beta}(r_{ij})$ is the next nearest neighbor M-M 3×3 Hamiltonian matrix. We assume $r_{ij}=R_l$ and $l=1,2,\dots,6$. R_l indicates the next nearest lattice vectors as has been shown in fig.1. The building blocks of H_0 are 3×3 matrices [33]. When we introduce spin-orbit coupling, the building blocks of H become 6×6 matrices. For spin, we use six bases and they are $|d_{z^2}, \uparrow\rangle, |d_{xy}, \uparrow\rangle, |d_{x^2-y^2}, \uparrow\rangle, |d_{z^2}, \downarrow\rangle, |d_{xy}, \downarrow\rangle, |d_{x^2-y^2}, \downarrow\rangle$. There are three up spin bands and three down spin bands [30]. Now we can get the SOC contribution to the Hamiltonian as([33]):

$$H' = \lambda L.S = \frac{\lambda}{2} \begin{bmatrix} L_z & 0 \\ 0 & -L_z \end{bmatrix} \quad (3)$$

where,

$$L_z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 2i \\ 0 & -2i & 0 \end{bmatrix} \quad (4)$$

L_z is a matrix of z component of the orbital angular momentum in bases of d_{z^2}, d_{xy} , and $d_{x^2-y^2}$ and λ characterizes the strength of the SOC. Now we can get the full

TB Hamiltonian with SOC as follows([33]):

$$H = I_2 \otimes H_o + H' = \begin{bmatrix} H_o + \frac{\lambda}{2}L_z & 0 \\ 0 & H_o - \frac{\lambda}{2}L_z \end{bmatrix} \quad (5)$$

95 In Eq.5, I_2 is the 2×2 identity matrix and H_o is Hamil-
96 tonian of the device without SOC as has been shown in
97 eq.(2).

98 The electron correlation function, G^n and the hole
99 correlation function, G^p are computed by the following
100 equations ([29]-[32],[34]):

$$G^n = G^R \Sigma^{in} G^A \quad (6)$$

$$G^p = G^R \Sigma^{out} G^A \quad (7)$$

Here $G^A = [G^R]^\dagger$. G^A represents the advanced green
function. Σ_{in} and Σ_{out} are the scattering functions. They
describe the rate at which electrons are scattered in and
out of the device. They can be calculated by the follow-
ing equations([29]-[31]):

$$\Sigma^{in} = \Sigma_{l1}^{in} + \Sigma_{l2}^{in} \quad (8)$$

$$\Sigma^{out} = \Sigma_{l1}^{out} + \Sigma_{l2}^{out} \quad (9)$$

$$\Sigma_{l1}^{in} = f_1 \Gamma_{l1} \quad (10)$$

$$\Sigma_{l1}^{out} = (1 - f_1) \Gamma_{l1} \quad (11)$$

$$\Sigma_{l2}^{in} = f_2 \Gamma_{l2} \quad (12)$$

$$\Sigma_{l2}^{out} = (1 - f_2) \Gamma_{l2} \quad (13)$$

$$\Gamma_{l1} = i[\Sigma_{l1} - \Sigma_{l1}^\dagger] \quad (14)$$

$$\Gamma_{l2} = i[\Sigma_{l2} - \Sigma_{l2}^\dagger] \quad (15)$$

In the above equations, f_1 and f_2 denote probability
of finding electrons in the left and right lead respec-

tively. The transmittance can also be calculated from
the following equation ([32]) :

$$T = \text{trace}(\Gamma_{l1} G^R \Gamma_{l2} G^A) \quad (16)$$

4. Result and discussion

4.1. Electronic Band Structure

The electronic band structure of a material allows us
to envision the allowed states of electrons in the mate-
rial. We have presented the electric band structure us-
ing three band tight binding model. This model only
considers the next nearest neighbour hopping of M(W)
atoms. We have shown electronic band structure of
ZTDNR in Fig.2. This is for 16-zigzag-ribbon i.e. there
are 16 number of atoms in one column (unit cell of NR).

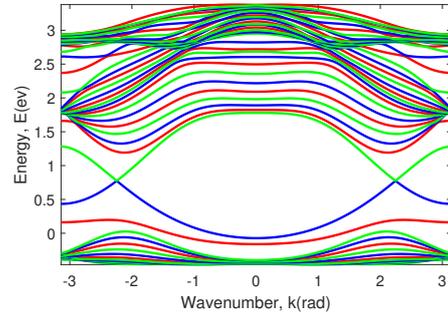


Figure 2: Band Structure of ZTDNR without SOC for one spin

110 There is an establishment of Dirac cone in fig.2. The
111 blue and green bands in the middle of fig.2 are for d_{z^2}
112 and $d_{x^2-y^2}$ electrons. They describe the energy disper-
113 sion relationship of edge states at the two edges of the
114 NR. In fig.3, we have demonstrated the electronic band
115 structure of the NR with SOC effect. From fig.3, we
116 observe that SOC splits the dirac cone and creates a di-
117 rect band gap among the d_{z^2} and $d_{x^2-y^2}$ bands. There is
118

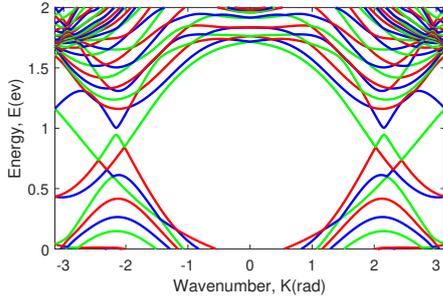


Figure 3: Band Structure of ZTDNR with SOC for both spins

119 a linear energy region of the $d_{x^2-y^2}$ band represented by
 120 the green curve between the direct bandgap. The $d_{x^2-y^2}$
 band represents the topological edge state in ZTDNR.

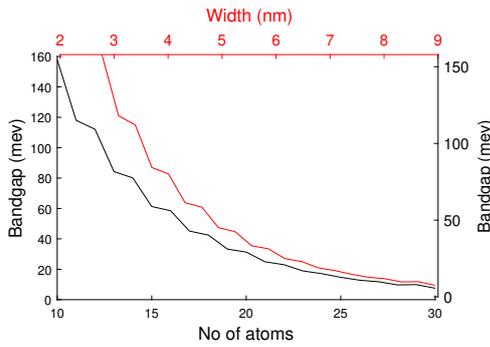


Figure 4: Bandgap (TI region) as a function of width (red curve) and no of atoms (black curve) for ZTDNR with SOC

121
 122 One of the exciting properties of ZTDNR is to control
 123 its direct bandgap. We observe a dependency of
 124 the direct bandgap of zigzag NR with the width. From
 125 fig.4, it can be said that the bandgap shrinks with the in-
 126 crease of the width of zigzag NR. Modern technologies
 127 of fabrication have reached a level where 2D material
 128 NR with a particular atomic width can be grown ([35]).
 129 Here, we consider sixteen atoms along the width of the
 130 NR. It is about 4.624nm wide and has a band gap of
 131 approximately 50mev.

132 Such opening of gap in the metallic part of band

133 structure and concomitant appearance of topological
 134 Dirac cone like bands has not been seen in any other
 135 material in our knowledge. It can be mentioned that by
 136 applying suitable gate voltage it is easier for 2D ma-
 137 terials to shift the band structure without changing the
 138 Fermi level ([36]). So it is possible to use these topo-
 logical states for transport of electrons.

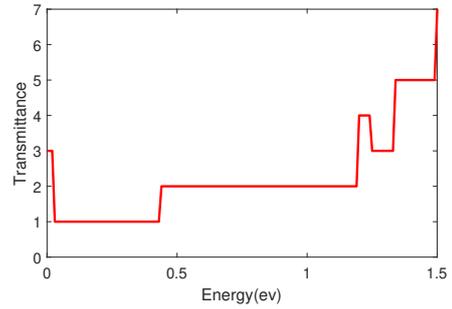


Figure 5: Transmittance vs Energy of ZTDNR without SOC

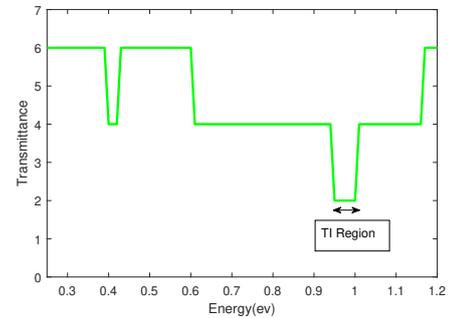


Figure 6: Transmittance vs Energy of ZTDNR with SOC

139 We have shown two plots of transmittance vs energy
 140 without and with SOC in fig.5 and fig.6 respectively.
 141 These plots are for 16-ZTDNR. We can observe the in-
 142 crease of transmittance after introducing SOC as shown
 143 in fig.6 because we consider up spin and down spin elec-
 144 trons for SOC. But transmittance at an energy range
 145 from 0.95eV to 1eV is 2 for two plots. We find topologi-
 146 cal edge properties at this energy range after introducing

148 SOC in 16-ZTDNR. So, we are interested to investigate
 149 the transport properties at this energy range.

150 4.2. Transport in a pristine device

151 In this section, we have investigated the transport
 152 property of a pristine device for ZTDNR with SOC.
 153 We have found six electron density profiles for ZTDNR.
 154 They are three electron density profiles of up spin elec-
 155 trons and three electron density profiles of down spin
 156 electrons. In fig.7, we have presented the electron den-
 157 sity profile of one of three up spin electrons. In fig.8 we
 158 have also shown the electron density profile of one of
 three down spin electrons.

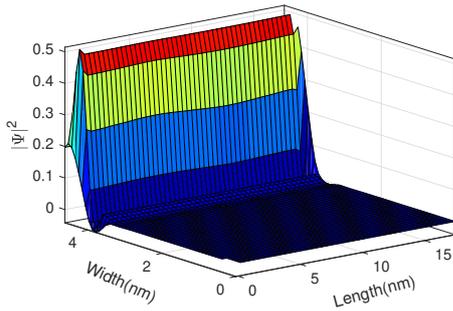


Figure 7: Electron density profile for ZTDNR up spin

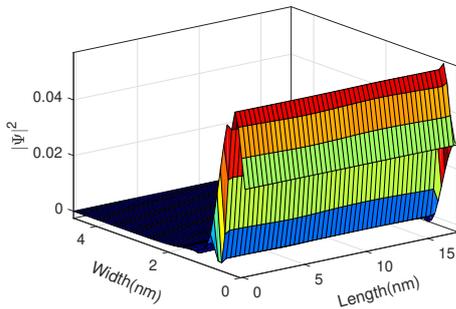


Figure 8: Electron density profile for ZTDNR down spin

159
 160 Three electron density profiles for each spin electron
 161 have electron distribution at same edge of the device.

162 So, we have demonstrated only one electron density
 163 profile for each spin electrons. From the figures, we ob-
 164 serve finite electron distribution only along one of the
 165 edges of the device for a particular spin. For the elec-
 166 tron density profiles in fig.7 and fig.8, electrons enter
 167 the device through the left lead and leave through the
 168 right lead. We have chosen the energy to be 0.98eV. If
 169 we change the direction of electrons passing through the
 170 device in such a way that electrons are entering the de-
 171 vice from the right lead and leaving from left lead, elec-
 172 tron density profile will also change from one edge to
 173 another. No back scattering occurs for these edge states
 174 and these edge states are behaving as quantum spin hall
 175 states.

176 4.3. Transport in the presence of barrier inside the de- 177 vice

178 In this section, we have investigated the phenomenon
 179 of Klein tunneling in the presence of barrier inside the
 180 device. Klein tunneling represents a phenomenon in
 181 which the electrons can pass through a high potential
 182 barrier and transmittance of the electrons is not depen-
 183 dent on the height and width of barrier. Hence transmit-
 184 tance always remains same. Here, we have considered
 185 an electron energy 0.975ev which is in the TI energy
 186 range for 16-ZTDNR and have put barrier inside the de-
 187 vice. We have demonstrated two electron density pro-
 188 files for up spin electron and down spin electron in fig.9
 189 and in fig.10 respectively.

190 From the above figures, we observe that electrons can
 191 pass through the device and travel with full transmit-
 192 tance ($T=2$). We have also increased the height and

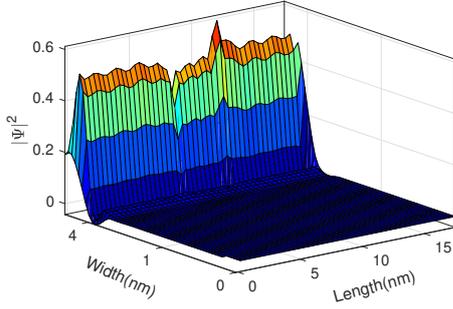


Figure 9: Electron density profile for ZTDNR up spin in the present of barrier inside the device

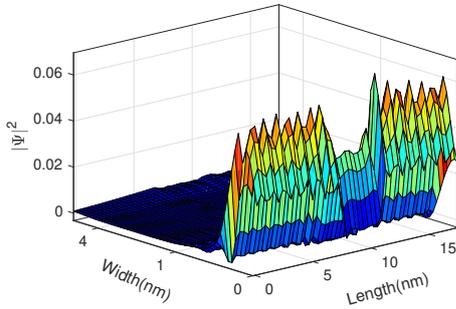


Figure 10: Electron density profile for ZTDNR down spin in the present of barrier inside the device

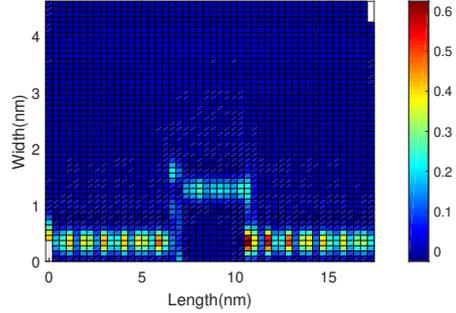


Figure 11: Electron density profile for ZTDNR with constriction

206 In fig.11, We observe that transportation is not af-
 207 fected by the existence of constriction inside the device
 208 and the electrons can pass following the edges of con-
 209 striction and conduct with full transmittance ($T=2$). The
 210 electrons are not back scattered. This is coming from
 211 transport of topological edge states.

193 width of the barrier. But transmittance of the electrons
 194 is always 2 and does not depend on the height and width
 195 of the barrier. Therefore this phenomenon is Klein tun-
 196 neling and this is coming from linear dispersion rela-
 197 tionship of topological states.

198 4.4. Transport in the presence of constriction inside the 199 device

200 We have introduced constriction in the device by re-
 201 moving a chunk of atoms from the channel. This is done
 202 to observe how the electron wave nature behaves in the
 203 presence of an extreme disturbance. We have shown an
 204 electron density profile in fig.11 with the presence of
 205 constriction.

212 4.5. Transport in the presence of edge imperfections in 213 one edge of the device

214 Edge imperfections can affect the transmission of the
 215 electrons through the device. First we demonstrate a
 plot of transmittance vs energy without SOC in fig.12.

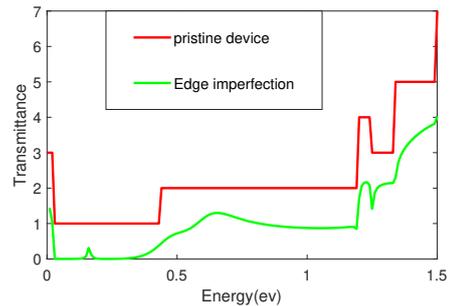


Figure 12: Transmittance vs energy without SOC for ZTDNR

216 From Fig.12, We observe that transmittance of the elec-
 217 trons is being affected by edge imperfections. Trans-
 218 mittance with edge imperfections is smaller than that of
 219

220 the pristine device for the whole energy range. Then
 221 in fig.13, we have presented the plot of transmittance
 vs energy with SOC. Here the transmittance with edge

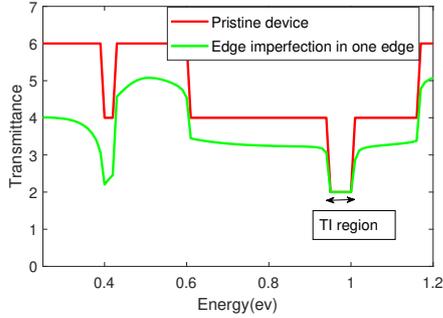


Figure 13: Transmittance vs energy with SOC for ZTDNR

222 imperfections in one edge is still smaller than transmit-
 223 tance of the pristine device for most of the energy range.
 224 In our device, electrons travel from left lead to right
 225 lead. During its travel, it faces obstacle at the pres-
 226 ence of edge imperfections and these obstacles cause
 227 reflection of electrons. Therefore, the electrons cannot
 228 conduct with full transmittance and transmittance of the
 229 electrons decreases. We get a small energy range from
 230 0.95eV to 1eV for which transmittance remain same as
 231 pristine device without being affected by edge imper-
 232 fections. This is the TI energy range for 16-ZTDNR.
 233
 234

235 We have also presented two electron density profiles
 236 in ZTDNR for up spin and down spin with edge imper-
 237 fections for this energy range in fig.14 and in fig.15 re-
 238 spectively. The electron density profiles in fig.14 and
 239 fig.14 show us that electrons can transport along the
 240 edge avoiding the imperfections with full transmittance
 241 ($T=1$) at the edge of the NR. The total transmittance of
 242 both spin electrons is 2.

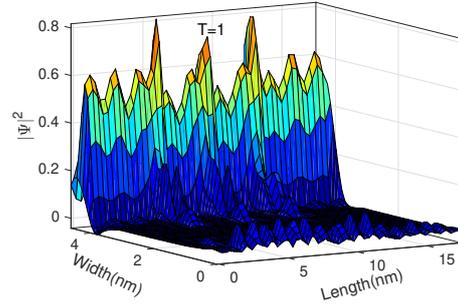


Figure 14: Electron density profile for ZTDNR up spin with edge imperfections in one edge ($T=1$)

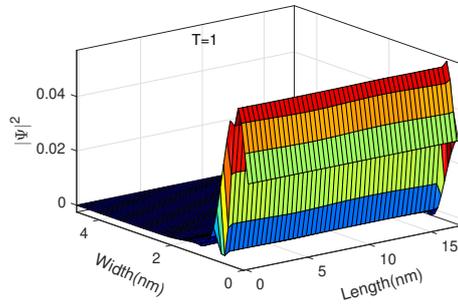


Figure 15: Electron density profile for ZTDNR down spin with edge imperfections in one edge ($T=1$)

4.6. Transport in the presence of edge imperfections in both edges of the device

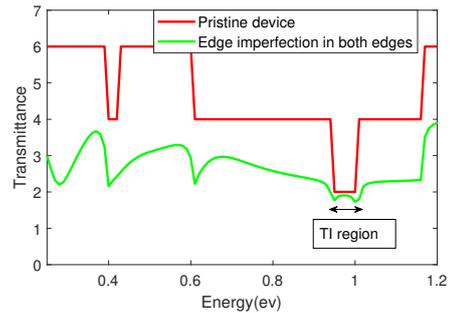


Figure 16: Transmittance vs energy with SOC for ZTDNR with imperfections in both edges

245 In this section, we have investigated the transport
 246 property of ZTDNR with edge imperfections in both
 247 edges of the device. We have demonstrated a plot of
 248 transmittance vs energy with SOC in fig.16.

249 In fig.16, we have observed that transmittance for TI
 250 energy region is being affected and is getting decreased
 251 by edge imperfections. But in fig.13, we observe that
 252 transmittance of TI energy region is not affected by edge
 253 imperfections in one edge of the device. We have also
 254 demonstrated the electron density profile of ZTDNR for
 255 up spin and down spin with edge imperfections in both
 256 edges of the device for TI energy region in fig.17 and in
 fig.18 respectively.

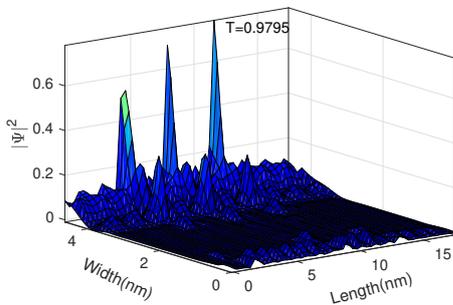


Figure 17: Electron density profile for ZTDNR up spin with edge imperfections in both edges (T=1.959)

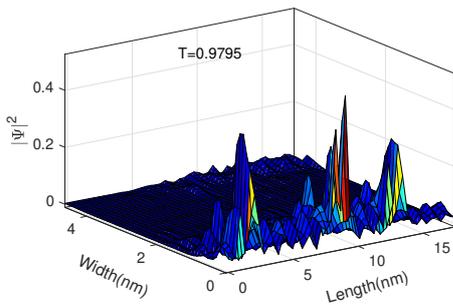


Figure 18: Electron density profile for ZTDNR down spin with edge imperfections in both edges (T=1.959)

257
 258 In the above figures, we have observed the local-
 259 ized anderson states ([37]) around imperfections. Then
 260 we observe that electrons can transport along the edge
 261 avoiding the imperfections and the total transmittance
 262 of both spin electrons is 1.959.

263 5. Conclusion

264 We have found that zigzag edge nanoribbon of Tung-
 265 sten Ditelluride with intrinsic SOC assumes topologi-
 266 cal insulator properties. The topological insulator states
 267 are quantum spin hall states. They show the character-
 268 istic edge states. It is interesting that the zigzag edge
 269 nanoribbon of Tungsten ditelluride shows edge states
 270 and klein tunneling even without intrinsic SOC, though
 271 these edge states do not show topological properties.
 272 We have shown computational results for transport in
 273 ZTDNR in the presence of constriction, potential barrier
 274 and edge imperfections. These results confirm the TI
 275 properties of ZTDNR. Interestingly when we put imper-
 276 fections in one edge only, the NR shows perfect trans-
 277 mittance but when we put imperfections in both edges,
 278 the transmittance decreases. At present, we do not have
 279 any explanation for this behavior. But we think there is
 280 more intricacies involved in the transport of electrons in
 281 ZTDNR that need to be investigated.

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