

Nonlinear Dynamic Analysis of Shale Gas Engine Combustion Stability

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1 **Nonlinear dynamic analysis of shale gas engine combustion** 2 **stability**

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7 **Abstract:** The traditional analysis method of engine combustion cycle variation is a statistical
8 method based on a small amount of data. In essence, the obtained cycle variation is random
9 data. In order to reveal the dynamic nature of the cyclical changes during the combustion of a
10 shale gas engine, a nonlinear dynamics method was used to study the stability of the combustion
11 process. The motion law of the phase space trajectory is analyzed, the influence of the shale gas
12 composition on the trajectory distribution is analyzed, the return mapping point of the average
13 indicated pressure in the cylinder is discussed. The relationship between adjacent combustion
14 characteristic parameters is studied; the chaotic characteristics of the shale gas engine
15 combustion process are discussed. The results show that during the working process of the
16 shale gas engine, the in-cylinder pressure shows a similar quasi-periodic state in the entire phase
17 space, and the working process has a certain chaotic law; with the increase of the CH₄, N₂ and
18 CO₂ content in the shale gas, the combustion cycle variation increases, and the randomness of
19 the engine working process increases. The phase space trajectory shows a monotonously
20 increasing distribution of Poincaré mapping points on the ΣXY_+ section. With the increase of
21 the combustion cycle, the linear relationship of the scattered points gradually increases, and the
22 randomness of the combustion process increases. The return map points of the engine

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23 combustion characteristic parameters are distributed in a cluster. When the CH₄ content
24 increases, the distribution range of the average indicated pressure return map points increases.
25 With the increase of N₂ and CO₂ content, abnormal combustion phenomena such as partial
26 combustion or misfire occur during the engine combustion process, the uncertainty of the
27 combustion process increases, and the combustion stability decreases. With the increase of
28 engine speed, the correlation dimension and the maximum Lyapunov exponent increase, the
29 randomness of the combustion process increases, and the chaotic characteristics of the engine
30 working process are obvious; the time series of the cylinder pressure is more sensitive to the
31 content of inert gas. With the increase of N₂ and CO₂ content in the gas, the correlation
32 dimension and the maximum Lyapunov exponent increase significantly, the complexity of the
33 phase space trajectory increases, and the chaotic characteristics become more obvious.

34 **Keywords:** Shale gas engine, Nonlinear dynamic characteristics, Phase space structure,
35 Poincaré map, Return map

36

37 **1. Introduction**

38 The combustion speed of natural gas is slower than that of gasoline, which results in lower
39 thermal efficiency of the engine, larger combustion cycle changes, and poorer stability of the
40 engine's working process [1-4]. The composition of shale gas is similar to natural gas, the main
41 component is CH₄, the molecule contains only one C atom, and the H/C ratio is relatively high.
42 When applied to engines, it can improve the effective thermal efficiency of the engine and is a
43 potential engine alternative fuel [5-7]. The distribution position and accumulation process of
44 shale gas reservoirs are different, and the composition of shale gas is quite different. The content

45 of N_2 and CO_2 in shale gas in some areas is relatively high, leading to differences in the
46 propagation speed and flame structure of shale gas premixed flames, resulting in different
47 cyclical changes in the working process [8-9].

48 Changes in various components of gaseous fuel affect the calorific value of the fuel. The
49 combustion speed of different gas components is different, resulting in a difference in the
50 combustion process in the cylinder. By changing the composition of the combustible mixture,
51 the combustion process of the engine can be improved and the stability of the working process
52 can be improved [10-14]. Weaver and Santavicca et al. [15] analyzed the influence of N_2 content
53 on the formation process of the fire core and the pressure in the cylinder on an optical engine.
54 The research results show that with the increase of N_2 content, the maximum combustion
55 pressure in the cylinder and the peak pressure rise rate decrease, and the combustion speed
56 decreases, which leads to the increase of the combustion cycle variation coefficient of the
57 engine. Li et al. [16] mixed coal-bed methane with different components and carried out
58 experimental research on engine combustion and emission characteristics. The research results
59 showed that when the N_2 content in coal-bed methane increases, the maximum combustion
60 pressure and pressure rise rate decrease, the peak heat release rate decreases, the flame
61 development period is prolonged, the emission of HC and CO increases, and the emission of
62 NO_x decreases significantly. According to the research results, it can be found that the gas
63 composition has a certain influence on the combustion cycle of the engine. Due to the different
64 contents of N_2 and CO_2 in shale gas in different regions, it is necessary to study the influence
65 of gas composition on the combustion cycle of shale gas engines.

66 The traditional analysis method of engine combustion cycle variation is a statistical method

67 based on a small amount of data. In essence, the cycle variation obtained is random data. With
68 the development of nonlinear dynamics, many scholars have carried out related research work
69 on the nonlinear dynamics of combustion cycle changes. Daw et al. [17] used a zero-
70 dimensional simulation model to explore the dynamic characteristics between the cycle changes
71 before and after the ignition engine. The research results show that there are chaotic
72 characteristics between cyclic changes, which are different from typical chaotic models due to
73 the influence of combustion noise. Green et al. [18] used a linear Gaussian stochastic model to
74 study the dynamics of the combustion cycle of a spark-ignition engine. The research results
75 show that the combustion process in the cylinder is inconsistent with the linear Gaussian
76 random process, showing a certain nonlinear dynamic process. According to the research results
77 of scholars, it can be seen that the combustion process of spark-ignited engines has the
78 characteristics of complex nonlinear dynamics, and the method of nonlinear dynamics can be
79 used for research.

80 In order to characterize the nonlinear dynamics characteristics of the engine combustion
81 process, scholars have adopted different nonlinear dynamics research methods. Wangner et al.
82 [19] used non-linear dynamics research methods such as back mapping, entropy analysis, and
83 symbolic time analysis to explore the transient characteristics of the dynamic behavior of the
84 engine cycle. The dynamic characteristics of the change are gradually determined. When the
85 intensity of turbulence in the cylinder decreases, the combustion stability under lean burn
86 conditions decreases, and the noise in the dynamic characteristics of the cycle changes
87 gradually becomes obvious. The dynamic characteristics of the combustion cycle changes are
88 affected by the injection time. Yao et al. [20] analyzed the correlation between the characteristic

89 parameters of the combustion cycle and reconstructed the time series of the in-cylinder pressure.

90 The research results show that the average indicated pressure is linearly related to the maximum

91 combustion pressure. When the engine is running at a fast combustion rate and complete

92 combustion conditions, the maximum combustion pressure can fully characterize the

93 combustion cycle changes. When the combustion in the cylinder deteriorates, the maximum

94 combustion pressure cannot represent the combustion cycle changes. Yang et al. [21] studied

95 the influence of ethanol substitution rate on the dynamic characteristics of the combustion

96 system by using a nonlinear dynamics research method. The research results show that the

97 introduction of ethanol has a significant impact on the inter-cycle combustion changes of the

98 engine load and the system dynamics. Many scholars have not yet unified the opinions of

99 whether the combustion process of ignition engines has chaotic characteristics. Letellier et al.

100 [22] used the method of nonlinear dynamics to reconstruct the phase space of the time series of

101 the engine cylinder pressure, and performed the Poincaré mapping and the return mapping of

102 the phase space with an equivalent ratio of 0.8, and discussed the combustion dynamic

103 characteristics of cyclic changes. The research results show that there are random factors in the

104 combustion process of the engine, and the combustion process does not have chaotic

105 characteristics. Liu et al. [23] used the method of nonlinear dynamics to analyze the dynamic

106 nature of the cycle variation of different fuels in the engine. The research results show that the

107 phase space trajectory of the cylinder pressure has a certain chaotic characteristic, and the

108 structure of the initial stage of combustion is relatively tight. Poincaré map reduces the

109 complexity of the phase space structure; the engine combustion process has chaotic

110 characteristics. Wang et al. [24] studied the chaotic characteristics of the EFI natural gas engine

111 combustion system under different ignition advance angles. The research results show that in
112 the engine working process, the phase space attractor is in a limited range and has a complex
113 geometric structure that is twisted, crossed and folded. The correlation dimension of the
114 attractor is a fraction less than 2, and the maximum Lyapunov exponent is positive. The
115 combustion system shows obvious chaotic characteristics. Ding et al. [25] used the phase space
116 reconstruction method and the 0-1 test method to study the dynamic characteristics of the
117 premixed natural gas engine combustion process. The results show that by reconstructing the
118 phase space, the law of the combustion process can be clearly studied. The combustion process
119 of the premixed natural gas engine under different injection timing conditions is chaotic.

120 According to the research results of scholars, it can be seen that in the research methods of
121 nonlinear dynamics, phase space reconstruction, return mapping, entropy analysis, symbolic
122 time analysis and recursive mapping can study the combustion process of the engine. The
123 method of nonlinear dynamics is used to study engine cycle changes, ignoring the complex
124 physical and chemical models in the combustion process. Through the analysis of the time
125 series of in-cylinder pressure, the essential characteristics of the system in the process of in-
126 cylinder pressure change are proved, which provides a theoretical basis for the establishment
127 and improvement of the calculation model of cycle variation and the improvement of engine
128 combustion stability. At present, scholars focus on the research on the influence of engine
129 operating parameters on the dynamic characteristics of cycle changes. There is not much
130 research work carried out to explore the influence of gas composition on the dynamic
131 characteristics of combustion cycle changes using nonlinear dynamics. It is necessary to study
132 the non-linear dynamic characteristics of shale gas components on the engine combustion

133 process, to find out the nature of cyclic changes, and to provide a theoretical basis for improving
134 the combustion stability of shale gas engines.

135 In this paper, the phase space reconstruction of the cylinder pressure is carried out by the
136 delayed coordinate method, the motion law of the phase space trajectory is analyzed, the
137 trajectory is projected on the XY plane, and the influence of shale gas composition on the
138 trajectory distribution is studied; Σ_{XY+} is selected Poincaré section, analyzed the projection of
139 the phase space trajectory on the section; explored the return mapping point of the average
140 indicated pressure in the cylinder, studied the relationship between adjacent combustion
141 characteristic parameters; analyzed the correlation dimension and the largest Lyapunov
142 exponent, etc. The chaotic characteristic quantity of the combustion process, and the change
143 law of the chaotic characteristic of the shale gas engine combustion process is discussed.

144 **2. NONLINEAR DYNAMICS AND CHAOS THEORY**

145 2.1. Phase space reconstruction

146 The delay coordinate method in the phase space reconstruction theory is used to analyze the
147 time series of the engine cylinder pressure [26-27]. The first derivative and the second
148 derivative of the cylinder pressure time series have important physical meanings in the
149 combustion process. Using differential coordinates to reconstruct the phase space of the
150 cylinder pressure time series can better analyze the combustion dynamics. The reconstructed
151 phase space Ω_{XYZ} is shown in equation (1).

$$152 \quad \Omega_{XYZ} = \{(X, Y, Z) \in R^3\} \quad (1)$$

153 In order to facilitate the analysis of the changes in the phase space of different fuels, the

154 projection method is used to reduce the dimensions of the three-dimensional phase space
155 structure, as shown in equation (3).

$$156 \quad \Omega_{XY} = \{(X, Y) \in R^2\} \quad (2)$$

157 2.2. Nonlinear transformation

158 (1) Poincaré map

159 During the working process of the engine, the trajectories obtained through phase space
160 reconstruction are densely distributed, the structure is complex, and the dynamic details are not
161 clearly expressed. Direct observation is difficult. The Poincaré section method can effectively
162 analyze the law of motion in the cylinder. Poincaré cross-section refers to the two-dimensional
163 cross-section obtained by selecting a suitable plane in the three-dimensional phase space
164 structure [28-29]. The intersection of the phase space trajectory and the plane is the Poincaré
165 point. The reduction of the dimensionality simplifies the analysis of the trajectory to the analysis
166 of scattered points, which is simpler and more effective. The Σ_{XY+} Poincaré section is selected
167 to analyze the dynamics in the cylinder, as shown in equation (3).

$$168 \quad \Sigma_{XY+} = \{(X, Y) \in R^2 | Z=0, Y>0\} \quad (3)$$

169 (2) Return map

170 Return mapping is an important method to analyze the correlation between the front and rear
171 cycles of the engine [30]. The working process of the engine is a periodic movement, and the
172 average indicated pressure (IMEP) of two adjacent cycles can be used as the horizontal and
173 vertical coordinates to obtain a set of discrete return maps. This discards most of the in-cylinder
174 pressure information, but it is convenient and intuitive to study the dynamic characteristics of

175 combustion cycle changes. The return map of the average indicated pressure is shown in
176 equation (4) [31].

$$\begin{cases} X=IMEP_{(n)}; \\ Y=IMEP_{(n+1)}. \end{cases} \quad (4)$$

177

178 (3) Chaos analysis method

179 Chaos phenomenon refers to a special non-periodic motion that exists in a certain system,
180 which is similar to irregular, but has a sensitive dependence on initial conditions. It is a
181 manifestation of the inherent randomness of nonlinear dynamic systems [32-34]. According to
182 the phase space trajectory of the cylinder pressure in the combustion process of the engine, it
183 can be seen that the system moves around a limited range with "attraction", which is called an
184 attractor. The phase space trajectory is attracted by the attractor of the pressure in the cylinder.
185 The trajectory is close to the attractor, and the combustion process in the cylinder is stable on
186 the attractor. It is shown that the attractor represents the stable state of the system and is a phase
187 space region of the final destination of the dynamic system.

188 The correlation dimension and Lyapunov exponent are characteristic quantities that represent
189 the chaotic characteristics of the system [35]. The correlation dimension characterizes the
190 estimation of the complexity of the system, is sensitive to the behavior of the time course, and
191 can better reflect the dynamic characteristics of the system; The Lyapunov exponent can
192 quantitatively describe the separation speed of the chaotic motion trajectory, which is the basis
193 for judging the sensitivity of the chaotic motion to the initial conditions, and it describes the
194 instability of the motion trajectory.

195 3. Test equipment and plan

196 The test is aimed at a single-cylinder test gasoline engine. It adopts a simple structure and a
 197 low-cost modified intake port injection mode. The shale gas is fully mixed with air in the intake
 198 manifold and ignites. The cylinder diameter of the gasoline engine is 88mm, the calibrated
 199 speed is 3600r/min, and the calibrated power is 7.5kW. The specific technical parameters of the
 200 test engine are shown in **Table 1**.

201

Table 1 Test engine parameters

Parameter name	Parameter
Cylinder diameter (mm)	88
Piston stroke (mm)	64
Displacement (mL)	389
Compression ratio	8.1
Rated speed (r/min)	3600
Rated power (kW)	7.5
Maximum torsion speed (r/min)	2000~2600
Maximum torque (N•m)	26

202 The shale gas engine was operated under the maximum torque and calibration conditions,
 203 respectively, and the changes in the combustion process and the stability of the engine when
 204 the gas components such as CH₄, N₂ and CO₂ in shale gas changed were discussed.

205

Table 2 Each gas composition during the CH₄ content change test

Test plan	CH ₄	C ₂ H ₆	C ₃ H ₈

1	85%	10%	5%
2	90%	5%	5%
3	95%	5%	0%

206 According to the test requirements, the content of CH₄ in shale gas is 85%, 90%, and 95%,
 207 respectively. When the CH₄ content change test is carried out, the N₂ and CO₂ content are not
 208 considered. The test plan is shown in **Table 2**.

209 **Table 3** Each gas composition during the inert gas content change test

Test plan	CH ₄	C ₂ H ₆	C ₃ H ₈	N ₂	CO ₂
1	90%	5%	5%	0%	0%
2	85%	2.5%	2.5%	10%	0%
3	75%	2.5%	2.5%	20%	0%
4	85%	2.5%	2.5%	0%	10%
5	75%	2.5%	2.5%	0%	20%

210 When the inert gas content change test was carried out, the content of CH₄ in shale gas was
 211 90%, the content of N₂ and CO₂ were 0%, 10% and 20%, respectively. The test plan is shown
 212 in **Table 3**.



214

Fig. 1. DEWE800 combustion analyzer

215 During the test, data collection was mainly carried out on the cylinder pressure, and the
216 cylinder pressure curve for 120 continuous working cycles was collected. Cylinder pressure
217 data is measured by a Kistler6125B01 spark plug pressure sensor, and the cylinder pressure
218 signal is recorded by the DEWE800 combustion analyzer. As shown in **Fig. 1**, the crankshaft
219 angle generator interval is 1°CA.

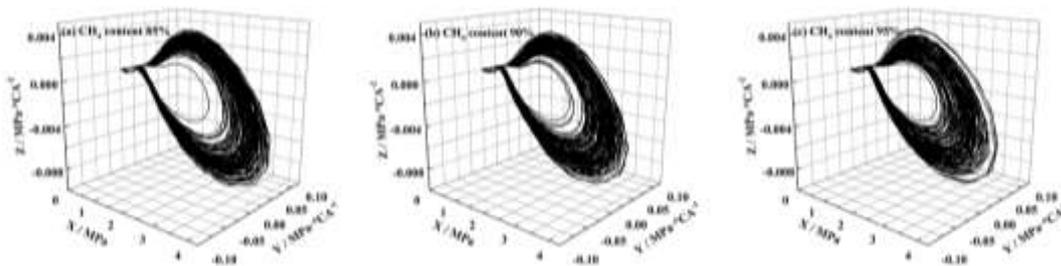
220 **4. Results and discussion**

221 4.1. Phase space analysis

222 (1) Phase space reconstruction

223 **Fig. 2** shows the reconstructed phase space trajectory of the in-cylinder combustion process
224 when the engine is running at the maximum torque and the CH₄ content in shale gas changes.
225 It can be seen from the figure that during the combustion process of a shale gas engine, each
226 cycle corresponds to a closed curve in the phase space, and each curve has a similar
227 development history, which is distributed in an irregular band in the phase space. The in-
228 cylinder state of the engine reflects a quasi-periodic state, and the dynamics shows a certain
229 chaotic law [36-37]; In-cylinder pressure is in the range of 0-2MPa, and the tip trajectory in the
230 phase space structure is highly coincident, indicating that the engine's intake and exhaust
231 process is stable. In the region of $Y > 0$, the trajectory first increases in the Z-axis direction and
232 then gradually decreases, which characterizes the mixture. In the compression stage before
233 combustion, the arc end of the curve is larger in the X-axis direction, which represents the

234 combustion stage of the engine. In the region of $Y < 0$, the trajectory gradually increases in the
235 Z-axis direction, which is characterized by the expansion of the mixture after combustion. The
236 phase space trajectory is relatively scattered in the arc area, indicating that combustion is the
237 main factor that causes the engine to work instability; with the increase of CH_4 content, the
238 trajectory of the arc end is gradually dispersed, indicating that the stability of the engine is
239 gradually decreasing.

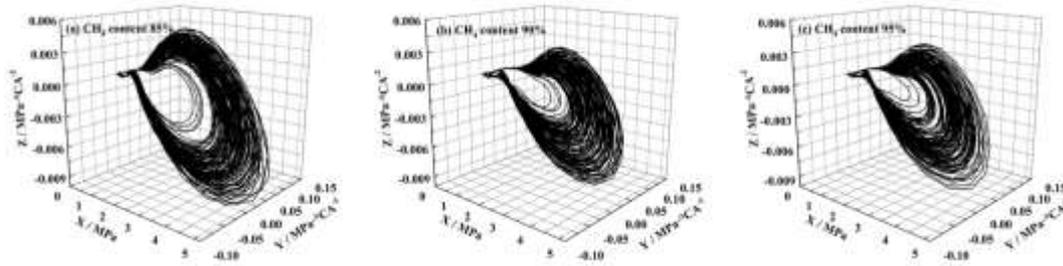


240

241 **Fig. 2.** The phase space of the combustion process in the cylinder under the maximum torque condition

242 **Fig. 3** shows the reconstructed phase space trajectory of the in-cylinder combustion process
243 when the engine is running under calibration conditions and the shale gas CH_4 content changes.
244 It can be seen from the figure that the phase space trajectory of the combustion in the cylinder
245 is closed and distributed periodically, showing a certain chaotic characteristic. As the CH_4
246 content in shale gas increases, the arc area gradually disperses; when the CH_4 content is
247 85% ,the reconstruction trajectory is distributed in a wide area in the phase space, the extreme
248 values in the X-axis and Y-axis directions are large, indicating that the shale gas burns quickly
249 in the cylinder, and the maximum explosion pressure and pressure rise rate are large; Compared
250 with the maximum torque condition, the phase space trajectory of the combustion process is
251 more scattered, indicating that the combustion stability of the engine decreases with the increase
252 of speed; when the CH_4 content is 95%, the value of individual trajectories in the X-axis

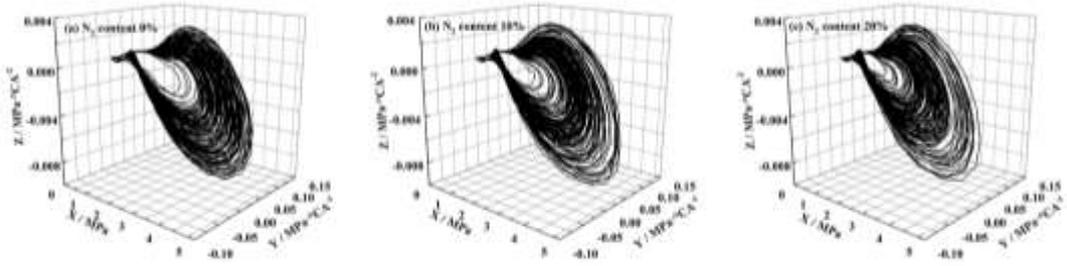
253 direction is lower, and the Y-axis The range of value change in the Z-axis and Z-axis directions
254 is narrow, which is a partial combustion cycle [38].



255

256 **Fig. 3.** The phase space of the combustion process in the cylinder under calibration conditions

257 **Fig. 4** shows the influence of the N₂ content in the shale gas on the phase space trajectory of
258 the combustion process in the cylinder when the engine is running under the calibrated
259 conditions. It can be seen from the figure that when the N₂ content is 0%, the phase space
260 trajectory distribution is relatively compact; when the N₂ content increases to 10%, the
261 distribution range of the phase space trajectory in the X-axis direction increases, and the
262 relatively dense area of the arc end trajectory is mainly distributed in the range of 3MPa, and
263 the number of partial combustion cycles increases; as the N₂ content increases to 20%, the phase
264 space trajectory is further dispersed, and the combustion stability decreases. The maximum
265 value in the X-axis direction is significantly reduced, indicating that the increase in N₂ content,
266 the decrease in the combustion speed of shale gas in the cylinder, the slower heat release process,
267 the decrease in the maximum explosion pressure, and the increase in the number of partial
268 combustion cycles. The maximum value of the arc area of individual working cycles in the X-
269 axis direction is less than 2MPa, and the Y-axis and Z-axis directions do not increase
270 significantly, which is a misfire cycle.



271

272 **Fig. 4.** The phase space of the combustion process in the cylinder when the N₂ content changes

273 **Fig. 5** shows the influence of the CO₂ content in shale gas on the phase space trajectory of

274 the combustion process in the cylinder when the engine is running under calibration conditions.

275 It can be seen from the figure that with the increase of CO₂ content, the phase space trajectory

276 gradually disperses. Compared with the influence of N₂ on the combustion phase space

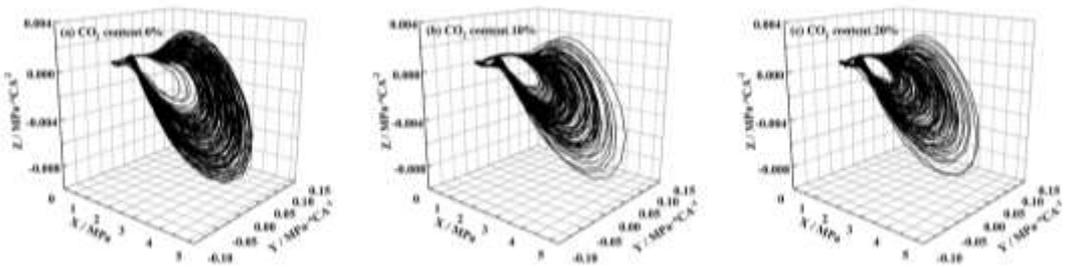
277 trajectory, under the same content condition, the phase space trajectory of CO₂ disperses and

278 the pressure rise rate decreases. The maximum value in the Y-axis direction decreases, and the

279 maximum explosion pressure in the cylinder decreases; during the intake and exhaust phases,

280 with the increase of CO₂ content, the distribution of the phase space trajectory does not change

281 much.



282

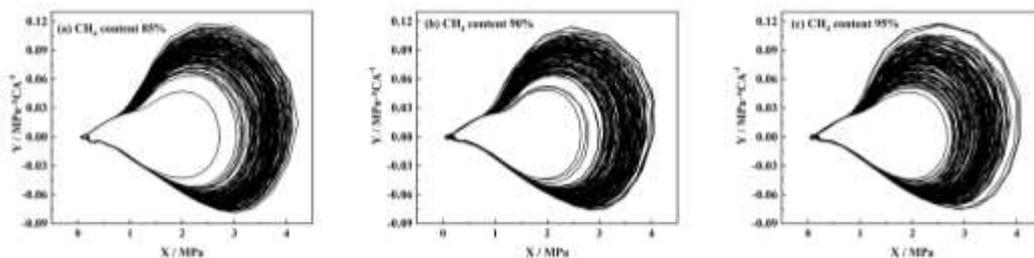
283 **Fig. 5.** The phase space of the combustion process in the cylinder when the CO₂ content changes

284 (2) XY plane projection

285 By reducing the phase space dimension of the combustion process by projecting on a plane,

286 the change rule of the phase space trajectory is discussed. **Fig. 6** shows the projection of the

287 phase space trajectory of the combustion process in the cylinder on the XY plane when the
 288 engine is operating at the maximum torque and the CH₄ content in the shale gas changes. It can
 289 be seen from the figure that the projection of the phase space trajectory on the XY plane is fan-
 290 shaped. The initial stage of the trajectory has a high degree of coincidence and a relatively tight
 291 structure. When combustion occurs, the trajectory rises rapidly and gradually diverges, and is
 292 relatively scattered at the maximum value of the Y axis. Obviously, the trajectory quickly
 293 overlaps after reaching the maximum value in the X-axis direction, and returns to the starting
 294 point to form a closed curve, indicating that the combustion process in the cylinder is chaotic;
 295 when the CH₄ content is 85%, the maximum value of the trajectory in the Y-axis direction is in
 296 the range of 0.07~0.11MPa/°CA, and the distribution area in the X-axis direction is 3.2~
 297 4.0MPa; with the increase of CH₄ content, the phase space trajectory gradually diverges, which
 298 is obvious at the maximum of the X-axis and Y-axis. When the CH₄ content is 95%, in the X-
 299 axis direction, the maximum value of the trajectory is distributed in the range of 2.8~3.8MPa,
 300 and the distribution range is increased by 25%. In the Y-axis direction, the maximum value is
 301 mainly distributed in the range of 0.05~0.12MPa/°CA .

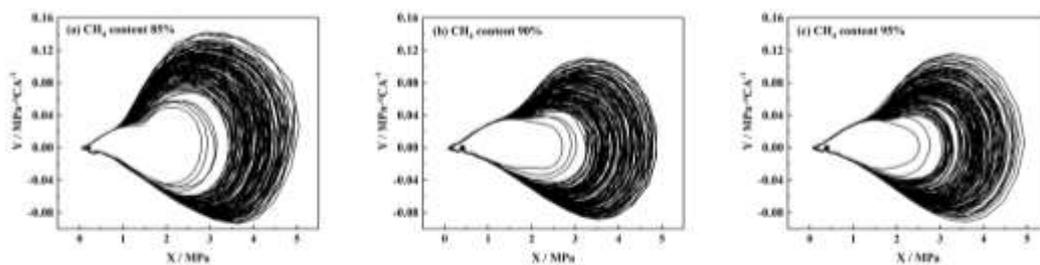


302

303 **Fig. 6.** Projection of phase space trajectory on XY plane for maximum torque condition

304 **Fig. 7** shows the projection of the phase space trajectory of the combustion process in the
 305 cylinder on the XY plane when the engine is running under the calibrated conditions and the

306 CH₄ content in the shale gas changes. It can be seen from the figure that as the engine speed
 307 increases, the flame propagation speed increases, the heat release rate increases, the turbulence
 308 intensity in the cylinder and the residual exhaust gas volume increase, resulting in an increase
 309 in the distribution range of the maximum explosion pressure and the pressure increase rate.
 310 Under calibration conditions, when the CH₄ content is 85%, in the X-axis and Y-axis directions,
 311 the maximum distribution is in the range of 3.5~5.0MPa and 0.06~0.14MPa/°CA; with the
 312 increase of CH₄ content, the value in the X-axis direction corresponding to the trajectory
 313 dispersion position increases, indicating that the flame retardation period is prolonged, and the
 314 decrease of the combustion speed causes the maximum value in the X-axis and Y-axis directions
 315 to decrease, the distribution range increases, and there is a partial cycle. The maximum
 316 explosion pressure of is relatively small, which is a partial combustion cycle. The maximum
 317 value of individual cycles in the X-axis direction is less than 2.5MPa, and the distribution range
 318 of the trajectory in the Y-axis direction is small, which is a misfire cycle [39].

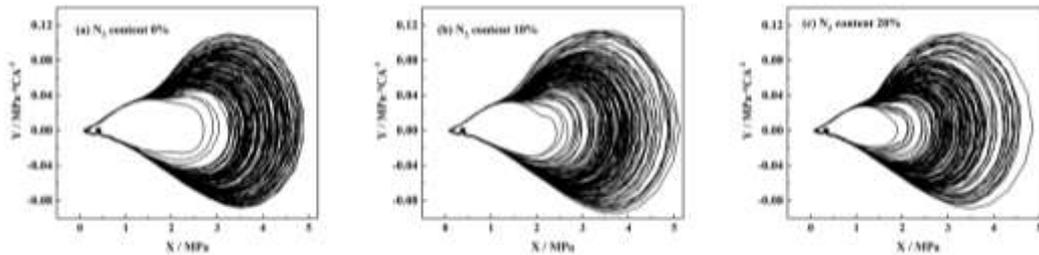


319

320 **Fig. 7.** The projection of the phase space trajectory on the XY plane of the calibration condition

321 **Fig. 8** shows the change rule of the projection of the phase space trajectory on the XY plane
 322 with the N₂ content when the engine is running under the calibrated condition. It can be seen
 323 from the figure that when the N₂ content is 0%, as the combustion process begins, the trajectory
 324 gradually diverges, indicating that the dynamic process in the cylinder is controlled by the

325 combustion phase; when the N_2 content increases to 10%, the trajectory diverges significantly
326 in the X-axis direction, and the maximum value is in the range of 2.5~5.1MPa, and the
327 maximum value in the Y-axis direction is in the range of 0.02~0.10MPa/°CA. The N_2 in the
328 gas suppresses the combustion process, the heat release process is prolonged, and there is a
329 partial combustion and misfire cycle; when the N_2 content increases to 20%, the phase space
330 trajectory further diverges, and the combustion speed decreases, resulting in a decrease in the
331 value of the trajectory in the dense area of the X-axis and Y-axis directions, an increase in
332 misfire cycles, and a significant decrease in engine working stability.

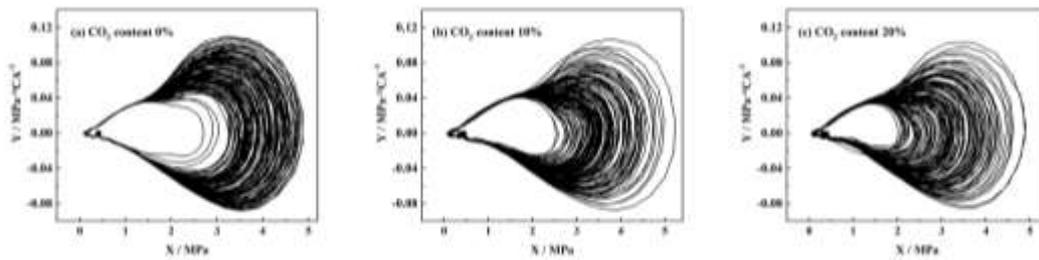


333

334 **Fig. 8.** Projection of phase space trajectory on XY plane when N_2 content changes

335 **Fig. 9** shows the variation of the projection of the phase space trajectory on the XY plane
336 with the CO_2 content when the engine is running under the calibrated conditions. It can be seen
337 from the figure that with the increase of CO_2 content, the degree of dispersion of the trace in
338 the later stage of combustion increases, and the distance from the obvious divergence area to
339 the combustion start point increases. The extension of the combustion duration leads to a
340 decrease in the stability of the engine and the reciprocating movement of the piston. The impact
341 on the dynamics process in the cylinder increases. The combustion cycle changes are mainly
342 affected by the flow field in the cylinder. The changes in the flow field characteristics during
343 the formation of the fire core have the greatest impact on the cycle changes; when the CO_2

344 content increases to 20%, the tangent slope of the trajectory dispersion position decreases, the
 345 combustion process is difficult, the flame development period is prolonged, and the number of
 346 partial combustion and misfire cycles increases significantly; the effect of CO₂ on the phase
 347 space trajectory dispersion is significantly stronger than N₂, the combustion stability of the
 348 engine is poor.



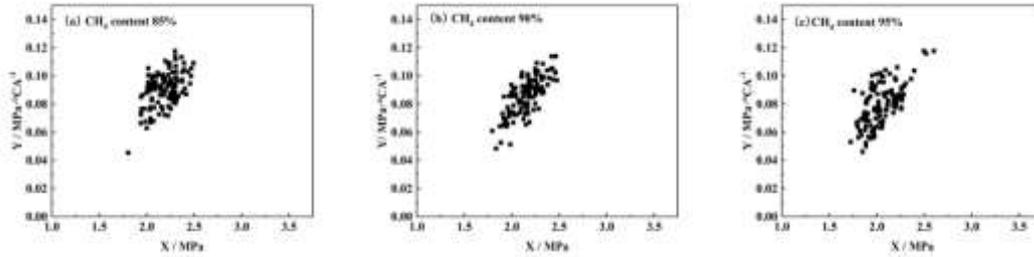
349

350 **Fig. 9.** Projection of phase space trajectory on XY plane when CO₂ content changes

351 4.2. Poincaré mapping

352 **Fig. 10** shows the Poincaré mapping of the phase space of the shale gas engine combustion
 353 process in ΣXY_+ when the engine is running at the maximum torque condition and the CH₄
 354 content changes. It can be seen from the figure that the Poincaré mapping points are distributed
 355 in an elliptical shape in a small area, indicating that the shale gas engine is operating at the
 356 maximum torque, and the phase space of the combustion process exhibits periodic chaotic
 357 motion in the Poincaré map of the cross-section ΣXY_+ [40]; when the CH₄ content is 85%, the
 358 distribution range in the X-axis direction is 1.9~2.7MPa, and the distribution range in the Y-
 359 axis direction is 0.06 ~0.12MPa/°CA⁻¹, when the content of CH₄ increases, the scatter points
 360 move to the lower left corner, the maximum burst pressure and pressure rise rate decrease, the
 361 combustion speed decreases, the distribution range is gradually slender, the periodicity weakens,
 362 and the combustion stability decreases. When the CH₄ content reaches 95%, the distribution

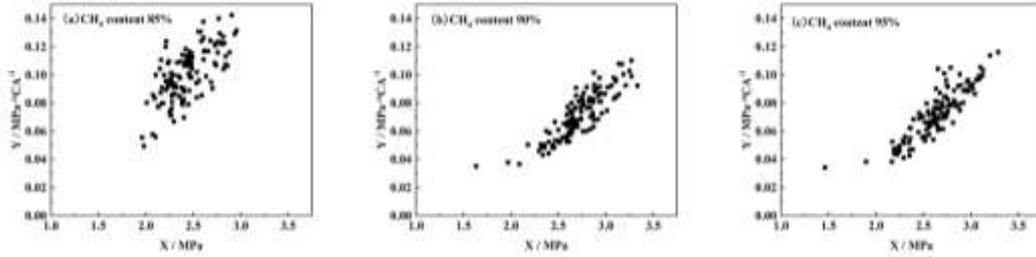
363 range of scattered points in the X-axis direction is 1.75~2.75MPa, and the distribution range
364 of the Y-axis direction is 0.04~0.12MPa/°CA⁻¹.



365

366 **Fig. 10.** Poincaré mapping of the phase space of the maximum torque condition on the section ΣXY_+

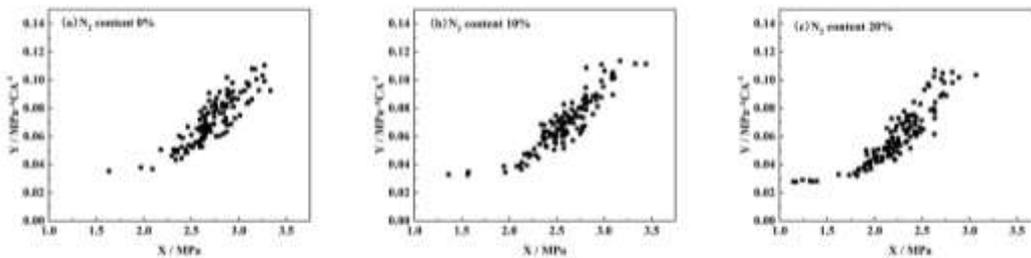
367 **Fig. 11** shows the Poincaré mapping of the phase space of the shale gas engine combustion
368 process in ΣXY_+ when the engine is running under the calibration conditions and the CH₄
369 content changes. It can be seen from the figure that when the CH₄ content is 85%, the
370 distribution range of the mapping points in the X-axis direction is 2.0~3.0MPa, and the
371 distribution range in the Y-axis direction is 0.04~0.14MPa/°CA⁻¹, as the rotation speed
372 increases, the distribution range of scattered points increases, the periodicity decreases, the
373 cyclic variation of combustion increases, the stability decreases, and the values in the X-axis
374 and Y-axis directions increase, indicating that the heat release rate increases, and the pressure
375 in the cylinder increases; when the CH₄ content is 90%, the mapping points are linearly
376 distributed, and the distribution law is monotonously increasing. The correlation between the
377 pressure in the cylinder and the pressure rise rate increases. At the lower left corner of the
378 distribution area, some scattered points rise upward; when the CH₄ content is 95%, the
379 periodicity of the mapping point distribution is weakened, indicating that the shale gas engine
380 is operating under the calibration conditions, and the Poincaré mapping of the combustion
381 process phase space on the section ΣXY_+ shows aperiodic chaotic motion.



382

383 **Fig. 11.** The Poincaré mapping of the phase space of the calibrated condition on the section ΣXY_+

384 **Fig. 12** shows the influence of N_2 content on the ΣXY_+ Poincaré mapping when the shale
 385 gas engine is running under calibration conditions. It can be seen from the figure that when
 386 shale gas contains N_2 , the distribution area of Poincaré mapping points is gradually slender,
 387 correlation coefficient between the pressure in the cylinder and the pressure rise rate increases,
 388 and the non-periodic distribution of the scattered points increases, the upward circulation in
 389 the lower left corner increases, and the distribution pattern of the scattered points gradually
 390 becomes arc-shaped, and the densely distributed areas are concentrated in the ranges of 2.2~
 391 3.0MPa and 0.03~0.10MPa/°CA⁻¹, in-cylinder pressure and pressure rise rate decrease; when
 392 the N_2 content is 20%, the combustion speed decreases, and the arc-shaped distribution law is
 393 more obvious, indicating that the non-periodical distribution of the mapping points increases,
 394 the combustion uncertainty increases, and the stability of the work decreases.

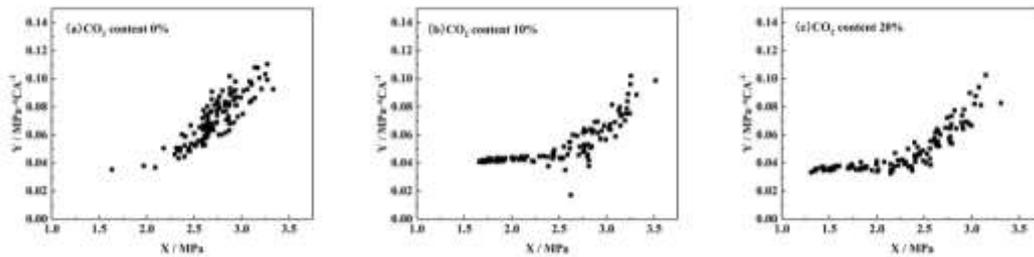


395

396 **Fig. 12.** Poincaré mapping of the phase space on the section ΣXY_+ when the N_2 content changes

397 **Fig. 13** shows the influence of CO_2 content on the ΣXY_+ Poincaré mapping when the shale

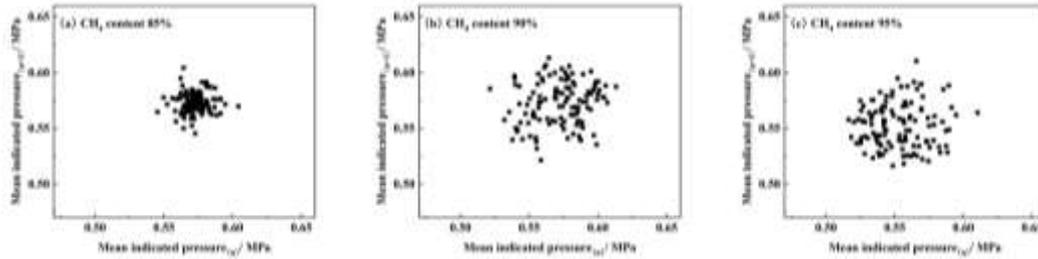
398 gas engine is running under calibration conditions. It can be seen from the figure that when
 399 shale gas contains CO₂, the distribution area of the mapping points increases, and the number
 400 of scattered points in the upper left corner increases significantly, and the arc-shaped
 401 distribution law is obvious, showing strong non-periodic chaotic characteristics. Compared
 402 with the same content of N₂, it can be seen that the densely distributed area of the mapping
 403 points moves to the lower part of the graph, indicating that CO₂ has a stronger inhibitory effect
 404 on the combustion rate than N₂, and the cylinder pressure and pressure rise rate drop
 405 significantly; when shale gas contains 20% CO₂, the distribution range of scattered points is
 406 about 1.3~3.2MPa and 0.03~0.10MPa/°CA⁻¹, the non-periodical combustion cycle is further
 407 enhanced, and the working stability is weakened.



408
 409 **Fig. 13.** Poincaré mapping of the phase space on the section ΣXY_+ when the CO₂ content changes

410 4.3. return mapping

411 The average indicated pressure comprehensively characterizes the pressure fluctuations of
 412 the entire cycle, and is closely related to the output power of the engine, and is an important
 413 indicator for studying engine cycle fluctuations [41-43]. **Fig. 14** shows the return map of the
 414 average indicated pressure when the engine is running at maximum torque and the content of
 415 CH₄ changes.



416

417 **Fig. 14.** The return map of the average indicated pressure time series under the maximum torque

418

condition

419

It can be seen from the figure that when the CH₄ content is 85%, the return mapping points

420

are distributed in a circular area within the range of 0.55 to 0.58 MPa, which indicates that the

421

average indicated pressure is strongly connected during the operation of the engine, and the

422

combustion cycle changes are determined. The combustion process is stable; when the content

423

increases to 90%, within the circular range of 0.53~0.61MPa, the mapping points are densely

424

distributed, and the average indicating pressure of most cycles shows a strong connection. The

425

working process is determined, and some mapping points are randomly distributed outside the

426

circular area, showing the randomness of the front and back cycles in the combustion cycle

427

change process, and the working process of the engine becomes uncertain [44]; when the CH₄

428

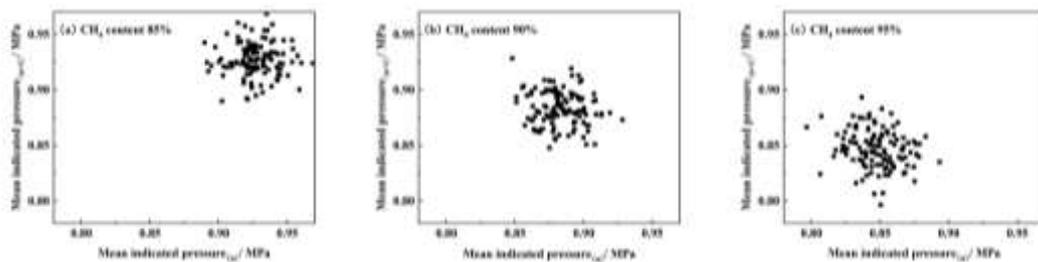
content is 95%, the area where the return mapping points are densely distributed is the circular

429

range of 0.52 to 0.61 MPa. The scattered points outside the circular range increase, the certainty

430

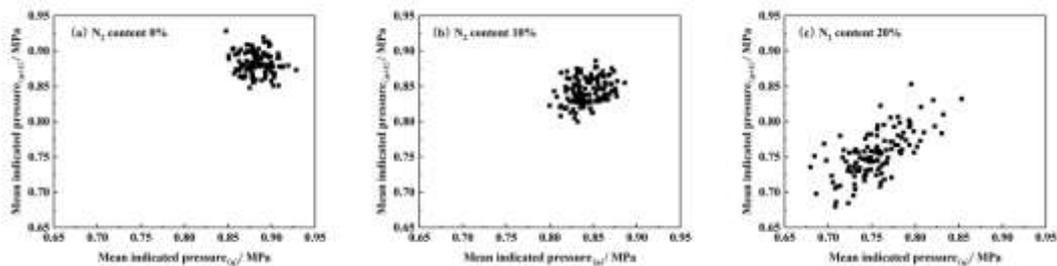
of the combustion cycle variation decreases, and the combustion stability decreases.



431

432 **Fig. 15.** The return mapping of the average indicated pressure time series of the calibration conditions

433 **Fig. 15** shows the return map of the average indicated pressure when the engine is running
434 under the calibrated condition and the CH₄ content changes. It can be seen from the figure that
435 when the CH₄ content is 85%, the mapping points are mainly distributed in the upper right
436 corner of the graphic area, the average indicating pressure is relatively large, and the
437 distribution area is concentrated in the range of 0.90~0.96MPa. Compared with the maximum
438 torque condition, the distribution area is increased by 0.03MPa, and the certainty of the
439 combustion cycle change is reduced; when the CH₄ content in shale gas is 90%, the mapping
440 point moves to the lower left corner, and the densely distributed area is 0.85~0.92MPa, some
441 scattered points are randomly distributed outside the area, and the combustion stability
442 decreases; when the content is 95%, the scattered points move further to the lower left corner,
443 and the densely distributed area is about 0.81~0.89MPa; when the CH₄ content in shale gas
444 increases, the dynamic characteristics of the combustion cycle change process gradually change
445 from determinism to a situation where randomness and determinism coexist. The process of
446 coexistence of randomness and determinism appears as a chaotic process.

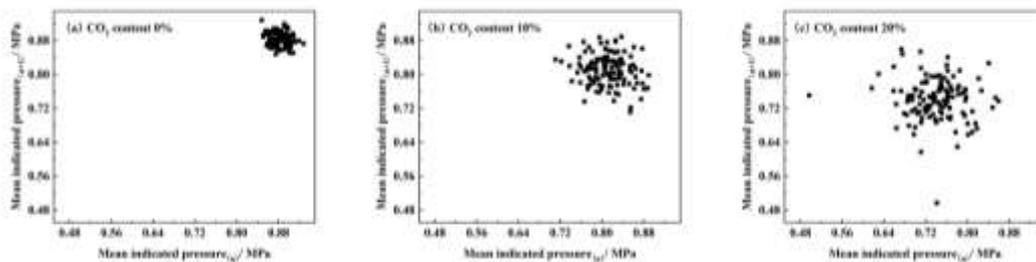


447

448 **Fig. 16.** The return mapping of the N₂ content change average indicating pressure time series

449 **Fig. 16** is the return map of the average indicated pressure time series when the engine is

450 running under the calibrated condition and the N_2 content changes. It can be seen from the
 451 figure that when the shale gas component contains N_2 , the mapping point moves to the lower
 452 left corner, the average indicated pressure decreases, the heat release rate decreases, and the
 453 combustion rate decreases. The scattered points are mainly distributed in the range of 0.80~
 454 0.88MPa. The distribution area increases, the relationship between the average indicated
 455 pressures weakens, and the combustion certainty decreases; when the N_2 content is 20%, the
 456 distribution area of the mapping points is in a belt shape, and the distribution range is
 457 significantly increased. In the lower left corner of the distribution area, there are several cycles
 458 adjacent to the average indicating low pressure, and shale gas heat is not completely released,
 459 which is a partial combustion cycle.



460

461 **Fig. 17.** Return mapping of CO_2 content change average indicating pressure time series

462 **Fig. 17** shows the return map of the average indicated pressure time series when the engine
 463 is running under the calibration condition and the CO_2 content changes. It can be seen from the
 464 figure that when the shale gas component contains CO_2 , the distribution area of the scattered
 465 points is mainly concentrated in the range of 0.72~0.89MPa. Compared with the N_2 content
 466 of 10%, the average indicated pressure decreases and the combustion rate decreases, resulting
 467 in a significant increase in the distribution range of scattered points, increasing to 0.09MPa,
 468 reducing the certainty of combustion, increasing cycle fluctuations, and increasing partial

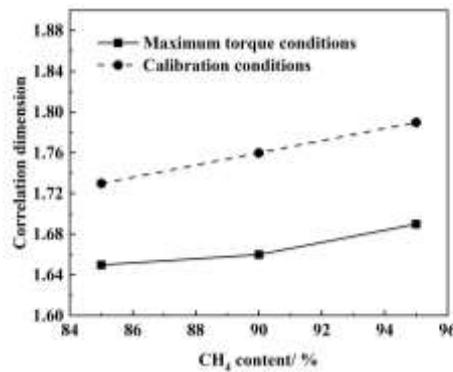
469 combustion cycles; When the CO₂ content is 20%, the distribution range of the mapping points
470 is further increased, and the area with relatively dense scatter points is concentrated in the range
471 of 0.66~0.88MPa. Outside the dense area, the number of irregular scattered points increases,
472 and the certainty of combustion further decreases. Part of the combustion cycle increases, there
473 are several cycles distributed on the left side of the dense area, and the adjacent average
474 indicating pressure is obviously smaller than the value of the dense area, indicating that the
475 combustion is not proceeding, and it is a misfire cycle.

476 4.4. Chaotic characteristics of the combustion process

477 (1) Association dimension

478 **Fig. 18** shows the correlation dimension of the reconstructed phase space trajectory when the
479 shale gas engine is operated under the maximum torque condition and the calibration condition,
480 and the CH₄ content changes. It can be seen from the figure that the engine is operating under
481 the maximum torque condition and when the CH₄ content in shale gas is 85%, the correlation
482 dimension of the phase space trajectory is 1.65. When the CH₄ content increases, the correlation
483 dimension increases, indicating that the complexity of the combustion system attractor
484 increases, and the combustion rate decreases, which leads to an increase in heat release time,
485 an increase in cycle variation coefficient, and a decrease in combustion stability. When the
486 content is 95%, the correlation dimension increases to 1.69; when the shale gas engine is
487 operating under the calibration conditions, the correlation dimension of the phase space
488 trajectory increases significantly. When the CH₄ content is 85%, the correlation dimension is
489 1.73, an increase of 0.08, It shows that the complexity of the attractor is obviously increased,

490 the distribution range of the phase space trajectory is increased, the unpredictability of the
491 overall working state of the engine is increased, and the combustion stability is decreased. The
492 main reason is that the engine speed increases, the charging efficiency in the cylinder decreases,
493 and the flame propagation is difficult; similar to the maximum torque condition, during the
494 calibration condition, with the increase of the CH₄ content in the shale gas, the correlation
495 dimension increases linearly. When the volume fraction is 95%, the correlation dimension
496 reaches 1.79.



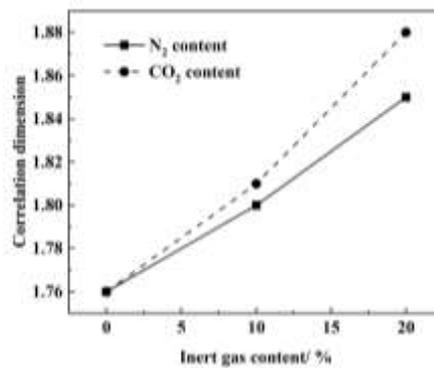
497

498

Fig. 18. Correlation dimension under different CH₄ content

499 **Fig. 19** shows the correlation dimension of reconstructed phase space trajectory with
500 different contents of N₂ and CO₂ in shale gas when the engine is running under calibration
501 conditions. It can be seen from the figure that when the shale gas component does not contain
502 inert gas, the correlation dimension of the phase space trajectory is 1.76. When the N₂ content
503 is 10%, the flame propagation speed of shale gas decreases and the combustion stability
504 decreases. The complexity of the phase space trajectory attractor increases, the distribution
505 range of the trajectory increases, the correlation dimension increases to 1.80, and when the N₂
506 content increases to 20%, the correlation dimension further increases, and the increase is
507 increased by 0.01 to 1.85. The distribution range of the phase space trajectory increases, and

508 the combustion stability decreases; when the shale gas contains 10% CO₂, the correlation
509 dimension of the phase space trajectory increases to 1.81, which is higher than when the N₂
510 content is 10%. It is greater than 0.01, indicating that CO₂ has a stronger inhibitory effect on
511 combustion than N₂, the combustion rate decreases more, the complexity of the phase space
512 trajectory is increased, and the unpredictability of the combustion process is enhanced. When
513 the CO₂ content is 20%, the correlation dimension increases significantly, reaching 1.88,
514 indicating that when the shale gas contains 20% CO₂, the difficulty of flame propagation has
515 increased significantly, the burning rate has decreased significantly, the unpredictability of the
516 phase space trajectory has increased, and the combustion stability has decreased significantly
517 [45-46].



518

519

Fig. 19. Correlation dimension under different contents of inert gas

520 (2) The largest Lyapunov exponent

521 **Fig. 20** shows the maximum Lyapunov exponent of the time series of the in-cylinder pressure
522 when the shale gas engine is operating under the maximum torque and calibration conditions,
523 and the CH₄ content changes.

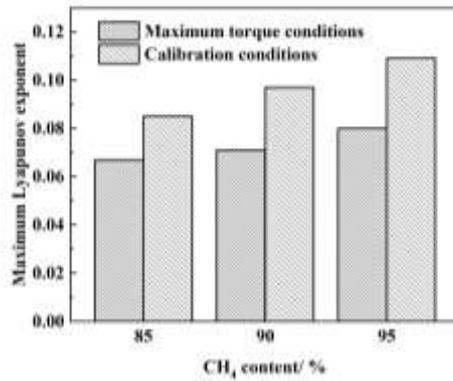


Fig. 20. The largest Lyapunov exponent under different CH₄ content

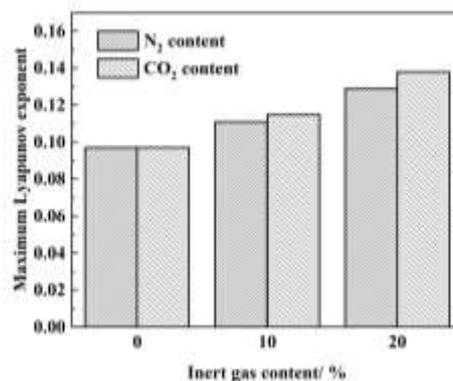
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525

526 It can be seen from the figure that when the engine is operating at the maximum torque, the
 527 maximum Lyapunov index is 0.067 when the CH₄ content is 85%. With the increase of CH₄
 528 content, the index increases, indicating that when the CH₄ content in shale gas increases, the
 529 periodicity of the cylinder pressure time series decreases, the degree of chaos increases, the
 530 volume fraction increases to 95%, and the index increases by 0.013 to 0.080; when the shale
 531 gas engine is running under the calibration condition, when the CH₄ content is 85%, the
 532 maximum Lyapunov exponent reaches 0.085, which is 0.018 more than the maximum torque
 533 condition, and the combustion stability of the engine decreases; In the calibration condition,
 534 with the increase of CH₄ content, the maximum Lyapunov exponent increases. When the CH₄
 535 content is 90% and 95%, the index increases by 0.012 respectively. Compared with the
 536 maximum torque operating condition, the exponential increase rate increases, indicating that
 537 when the shale gas engine speed increases, the time series of the in-cylinder pressure is more
 538 sensitive to the increase of CH₄ content, the cycle changes are large, and the combustion
 539 stability is poor.

540 **Fig. 21** shows the maximum Lyapunov exponent of the cylinder pressure time series with
 541 different contents of N₂ and CO₂ in shale gas when the engine is running under the calibrated

542 conditions. It can be seen from the figure that when the shale gas does not contain inert gas, the
543 maximum Lyapunov exponent is 0.097. As the N₂ content increases, the index gradually
544 increases, the chaotic degree of the cylinder pressure time series increases, the periodicity
545 decreases, and the combustion is stable. When the content is 20%, the index reaches 0.129, an
546 increase of 32.9%; when the CO₂ content in shale gas is 10%, the maximum Lyapunov index
547 is 0.115, which is an increase of 0.004 compared to when the N₂ content is 10%. It shows that
548 under the same volume fraction, the in-cylinder pressure time series is more sensitive to CO₂
549 content, and the combustion is more unstable. When the content is 20%, the maximum
550 Lyapunov exponent is 0.138, the increase is the largest, and the cycle change increases
551 significantly.



552

553 **Fig. 21.** The maximum Lyapunov exponent under different contents of inert gas

554 **5. Conclusion**

555 Using methods such as phase space reconstruction, Poincaré mapping, and return mapping,
556 researches have been carried out on the nonlinear dynamic characteristics of shale gas engine
557 combustion cycle changes. The research shows that:

558 (1) During the working process of a shale gas engine, the pressure in the cylinder shows a

559 quasi-periodical state in the entire phase space, and the working process has a certain chaotic
560 law; the phase space trajectory has a higher degree of coincidence during the intake and exhaust
561 phases, the pressure in the cylinder fluctuates greatly in the combustion phase, the trajectory is
562 arc-shaped, and the arrangement is more dispersed; As the content of CH₄, N₂ and CO₂ in shale
563 gas increases, the combustion cycle changes increase, the trajectory of the arc segment is
564 gradually dispersed, and the randomness of the engine working process increases.

565 (2) The phase space trajectory has a monotonically increasing distribution of the Poincaré
566 mapping points on the ΣXY_+ section. When the shale gas engine has a small cycle change, the
567 scattered points are distributed in an elliptical shape, and the distribution range is small, and the
568 engine working process has a strong periodicity, with the increase of the combustion cycle, the
569 linear relationship of the scattered points gradually increases. At the lower left corner of the
570 distribution area, some of the scattered points rise upward, and the randomness of the
571 combustion process increases.

572 (3) The return mapping points of the engine combustion characteristic parameters are
573 distributed in a cluster. When the CH₄ content in shale gas increases, the distribution range of
574 the average indicated pressure return mapping points increases, indicating that the randomness
575 of the engine working process increases and the stability of the combustion process decreases;
576 with the increase of N₂ and CO₂ content, on the left and lower sides of the scatter-intensive area,
577 several cycles of mapping points are randomly distributed, indicating that abnormal combustion
578 such as partial combustion or misfire occurs during engine combustion. The uncertainty of the
579 process increases.

580 (4) With the increase of engine speed, the correlation dimension and the maximum Lyapunov

581 exponent increase, the randomness of the combustion process increases, and the chaotic
582 characteristics of the engine working process are obvious; the time series of the cylinder
583 pressure is more sensitive to the content of inert gas. With the increase of N₂ and CO₂ content
584 in the gas, the correlation dimension and the maximum Lyapunov exponent increase
585 significantly, the complexity of the phase space trajectory increases, and the chaotic
586 characteristics become more obvious.

587 **Declaration of Competing Interest**

588 The authors declare that they have no known competing financial interests or personal
589 relationships that could have appeared to influence the work reported in this paper.

590

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Figures

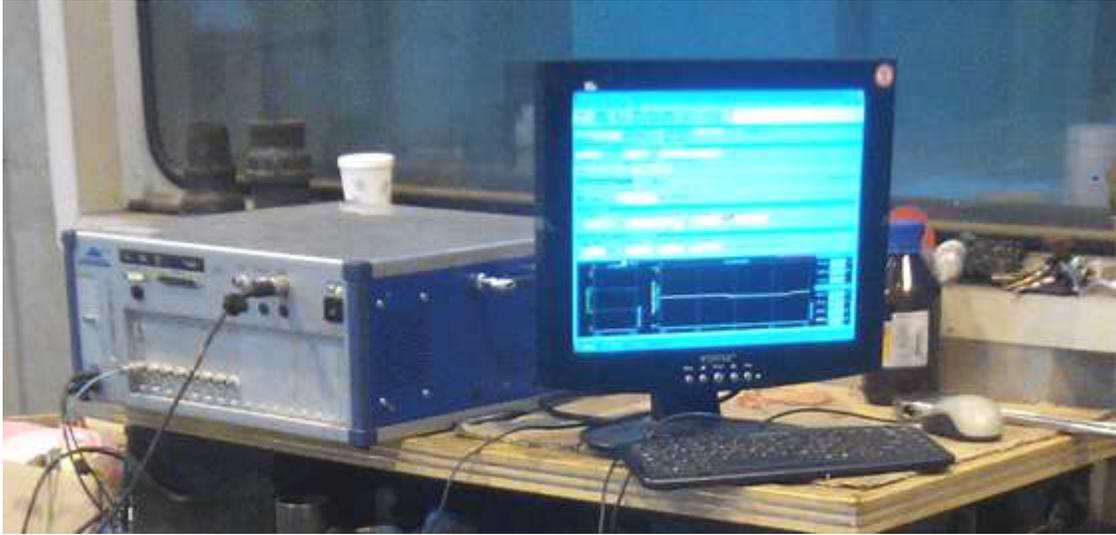


Figure 1

DEWE800 combustion analyzer

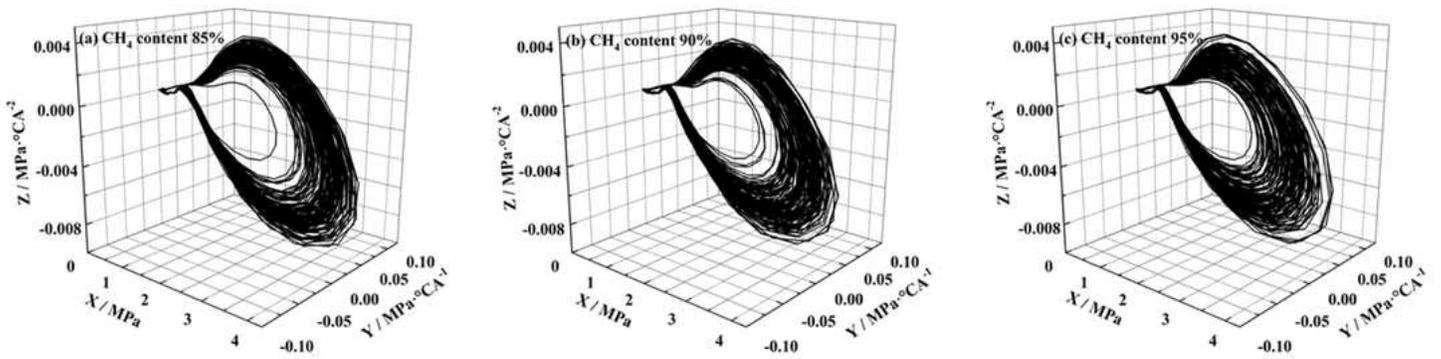


Figure 2

The phase space of the combustion process in the cylinder under the maximum torque condition

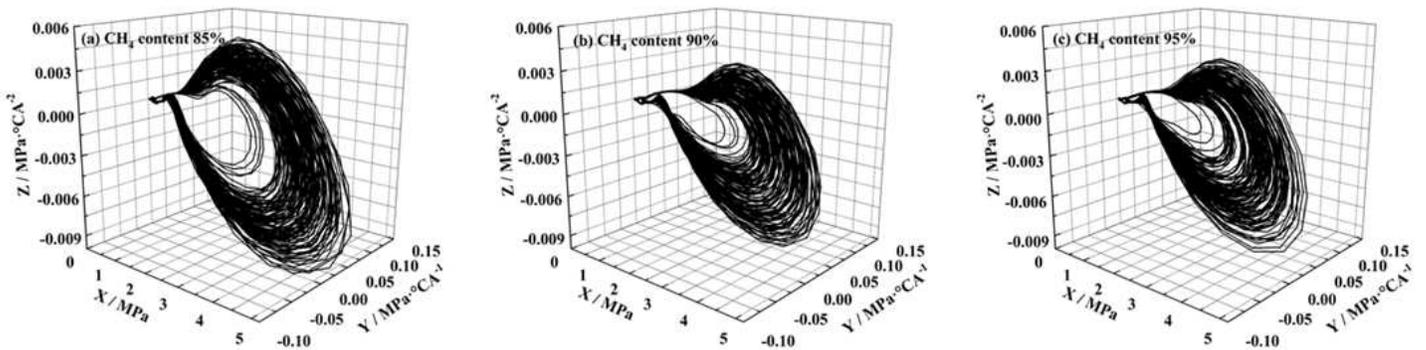


Figure 3

The phase space of the combustion process in the cylinder under calibration conditions

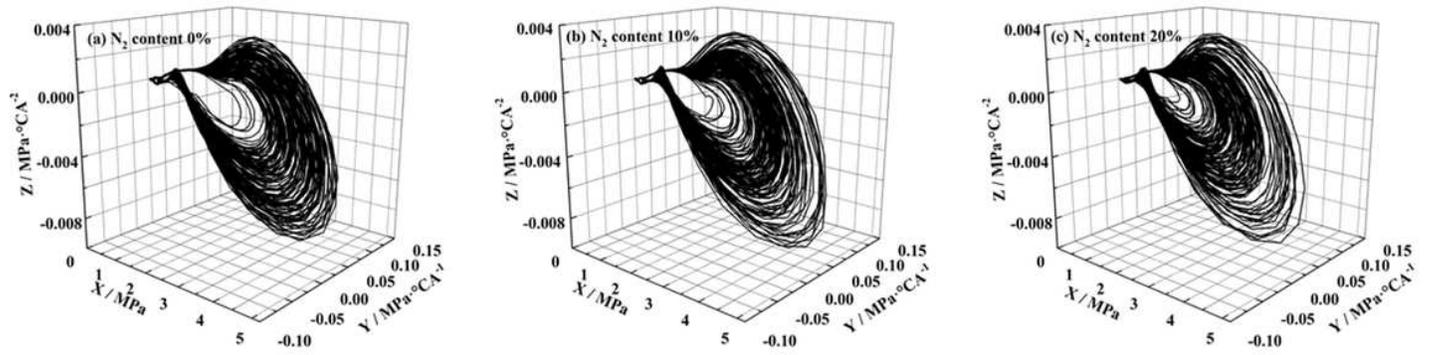


Figure 4

The phase space of the combustion process in the cylinder when the N2 content changes

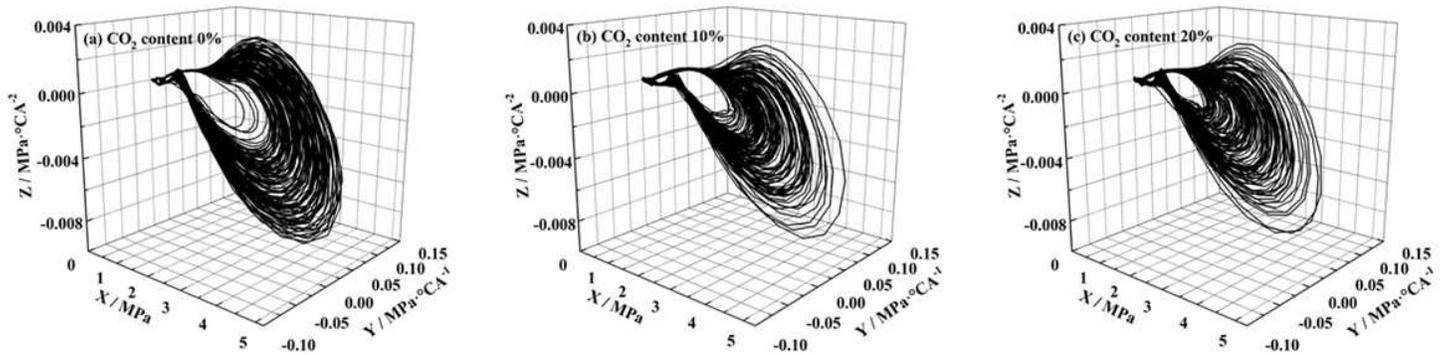


Figure 5

The phase space of the combustion process in the cylinder when the CO2 content changes

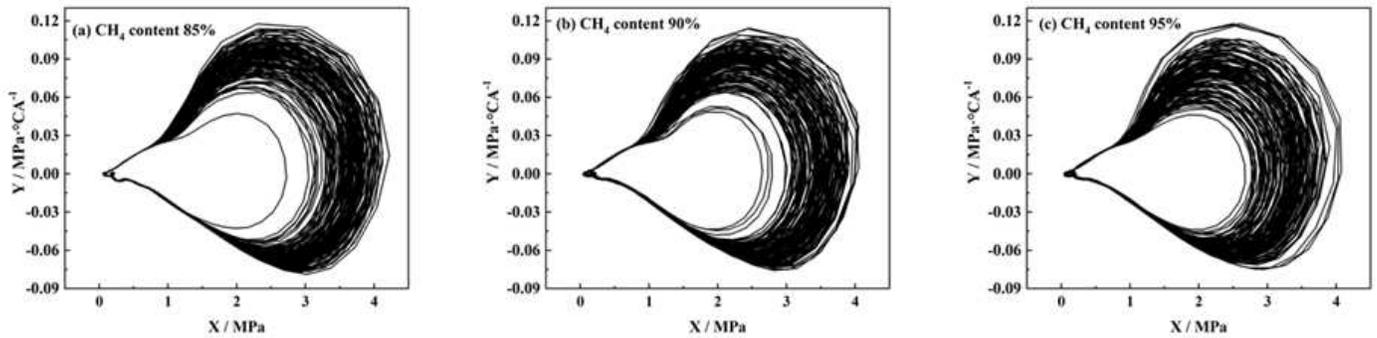


Figure 6

Projection of phase space trajectory on XY plane for maximum torque condition

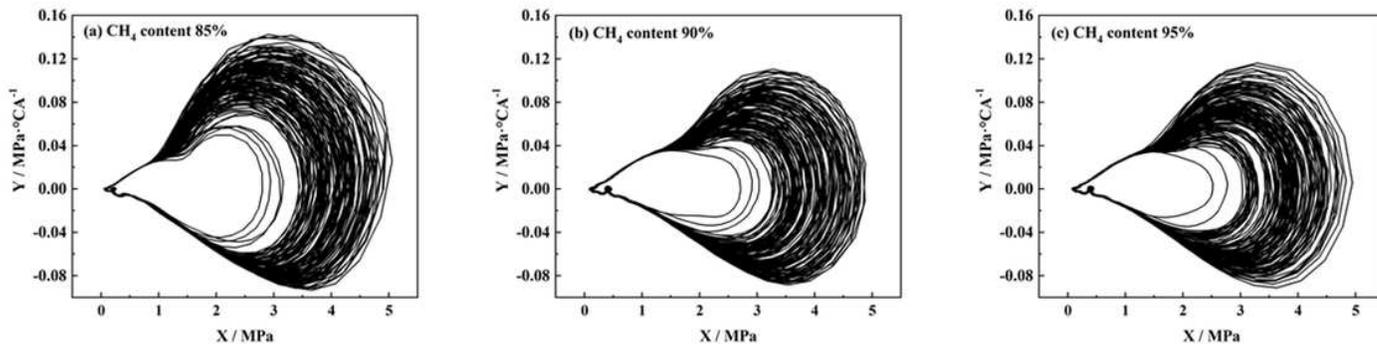


Figure 7

The projection of the phase space trajectory on the XY plane of the calibration condition

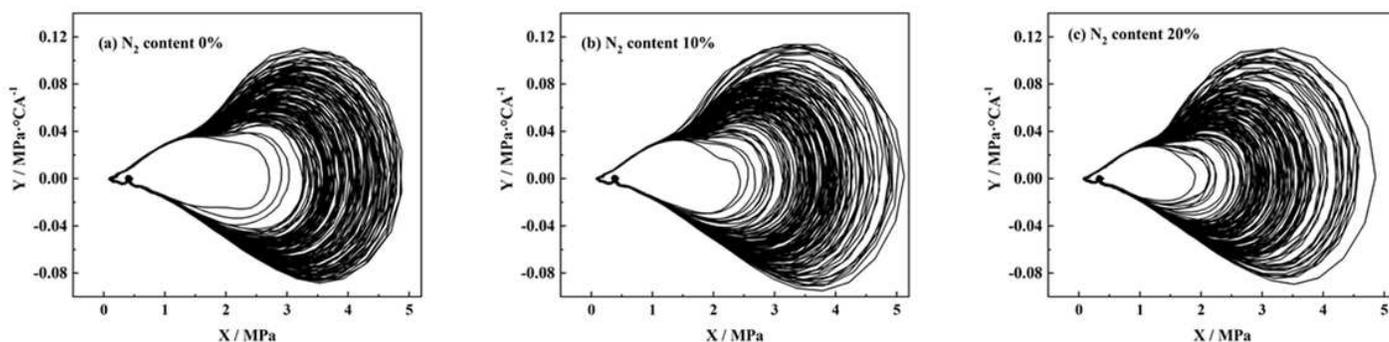


Figure 8

Projection of phase space trajectory on XY plane when N2 content changes

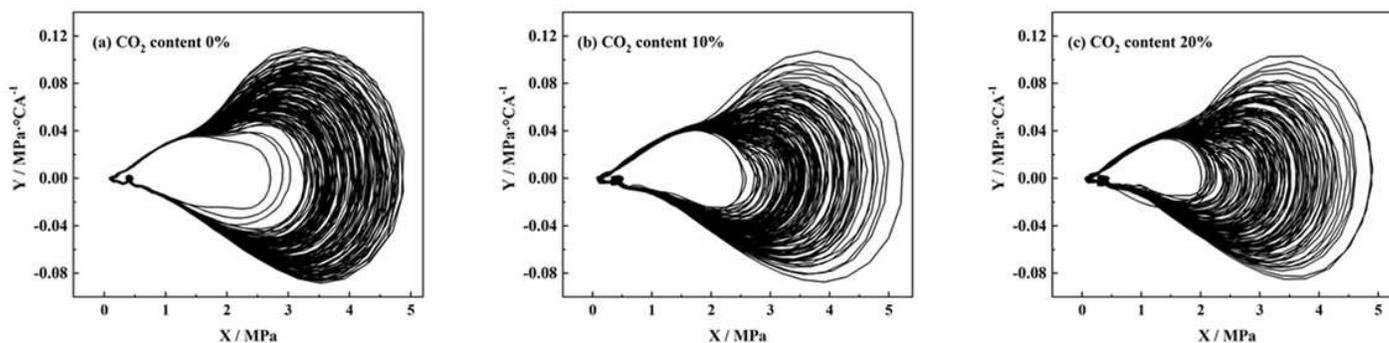


Figure 9

Projection of phase space trajectory on XY plane when CO2 content changes

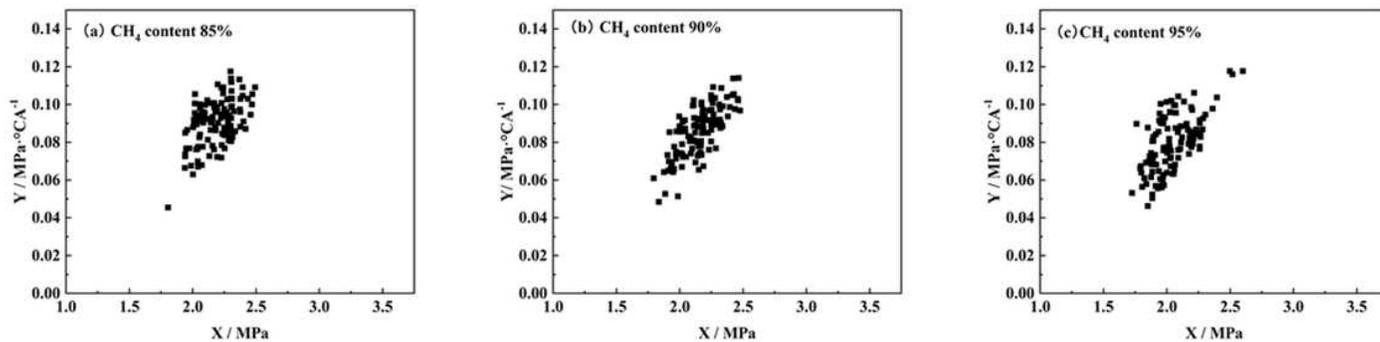


Figure 10

Poincaré mapping of the phase space of the maximum torque condition on the section Σ_{XY+}

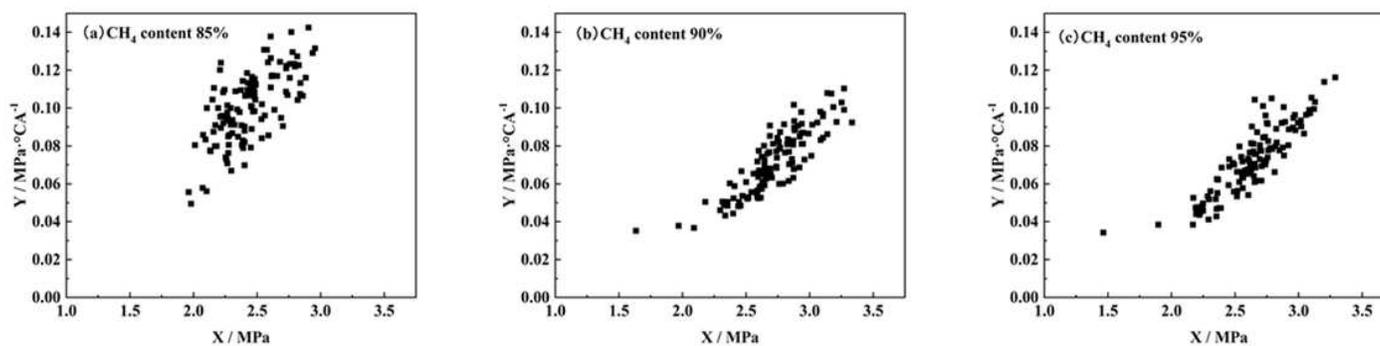


Figure 11

The Poincaré mapping of the phase space of the calibrated condition on the section Σ_{XY+}

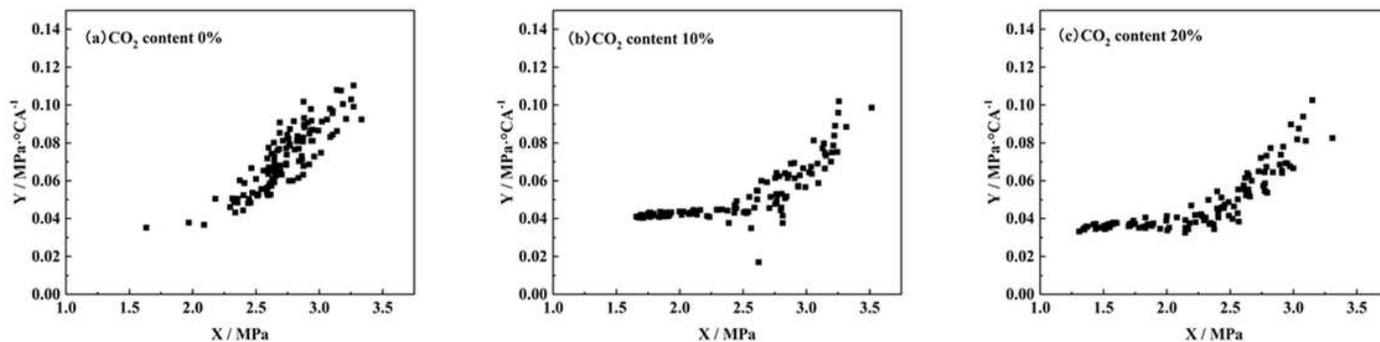


Figure 13

Poincaré mapping of the phase space on the section Σ_{XY+} when the CO₂ content changes

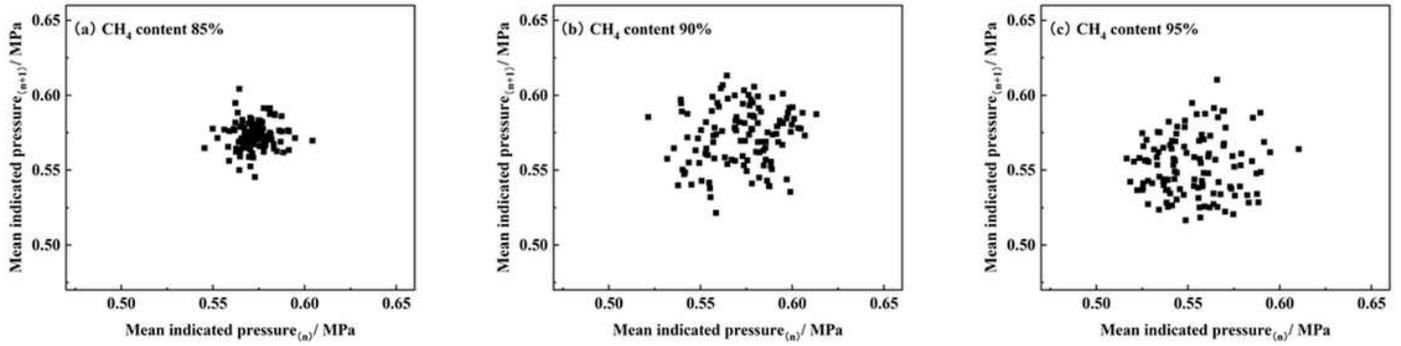


Figure 14

The return map of the average indicated pressure time series under the maximum torque condition

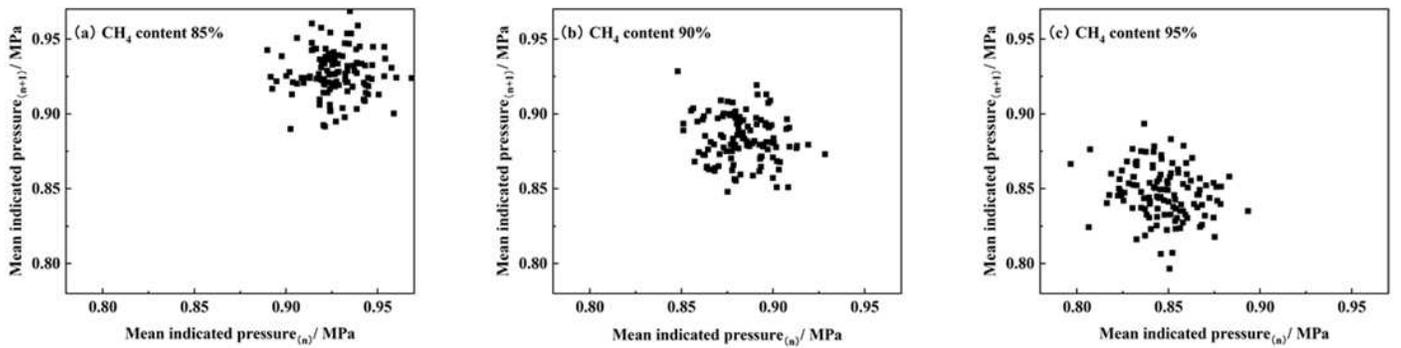


Figure 15

The return mapping of the average indicated pressure time series of the calibration conditions

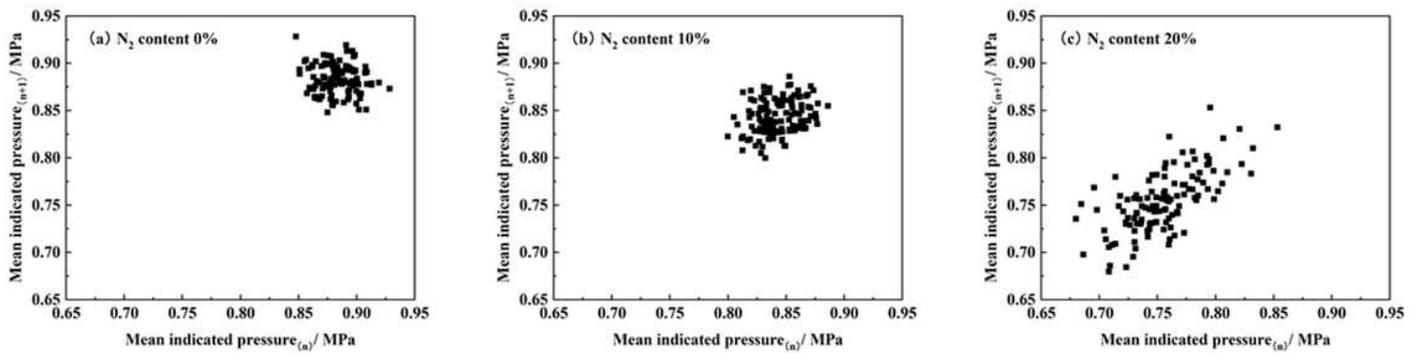


Figure 16

The return mapping of the N2 content change average indicating pressure time series

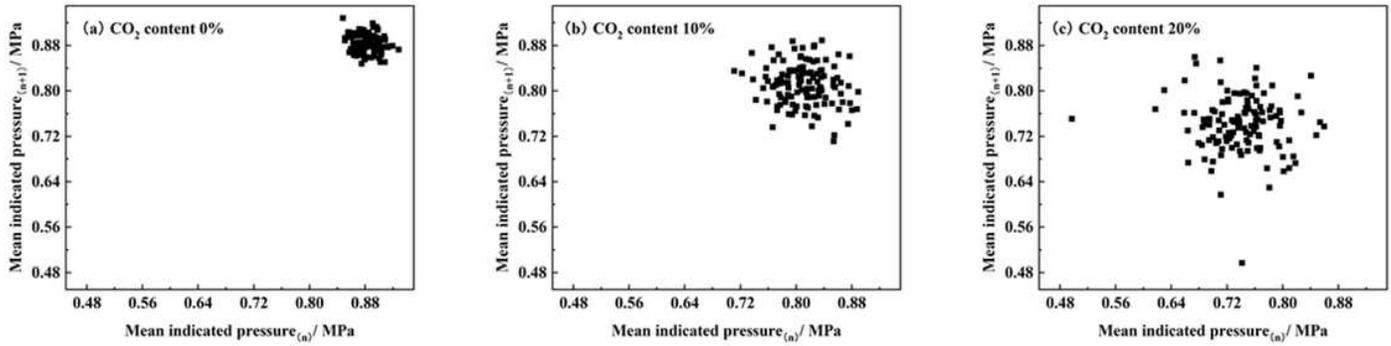


Figure 17

Return mapping of CO₂ content change average indicating pressure time series

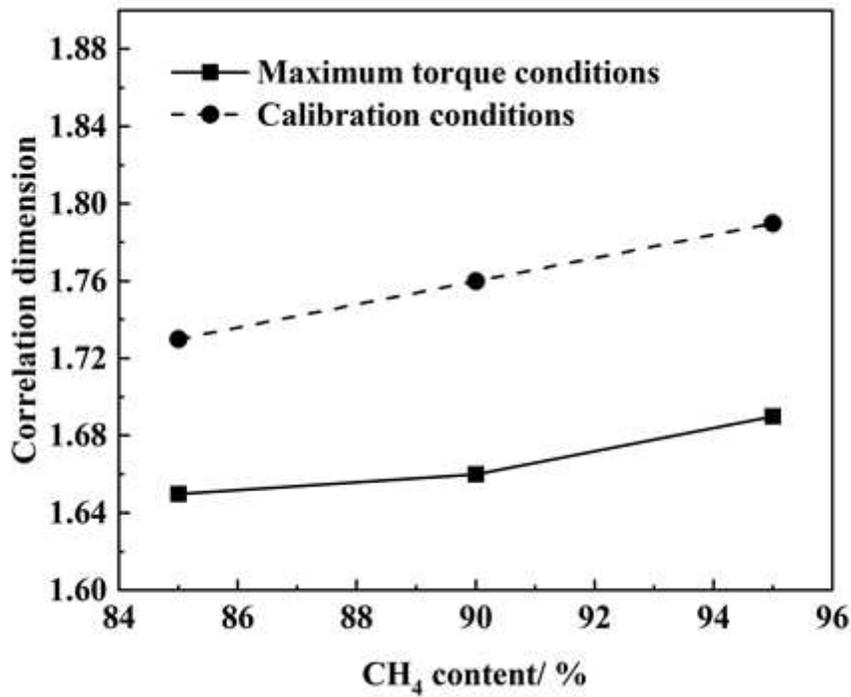


Figure 18

Correlation dimension under different CH₄ content

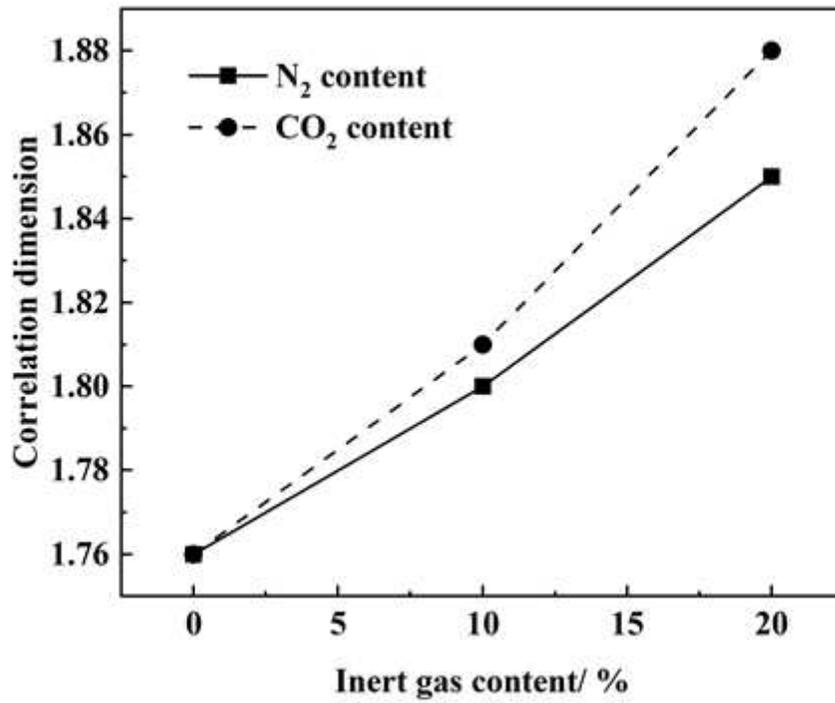


Figure 19

Correlation dimension under different contents of inert gas

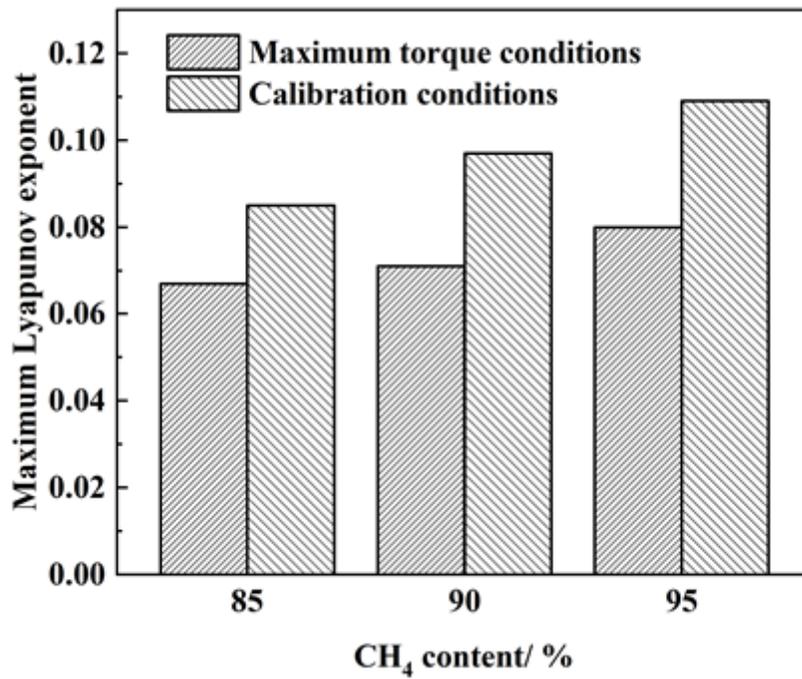


Figure 20

The largest Lyapunov exponent under different CH₄ content

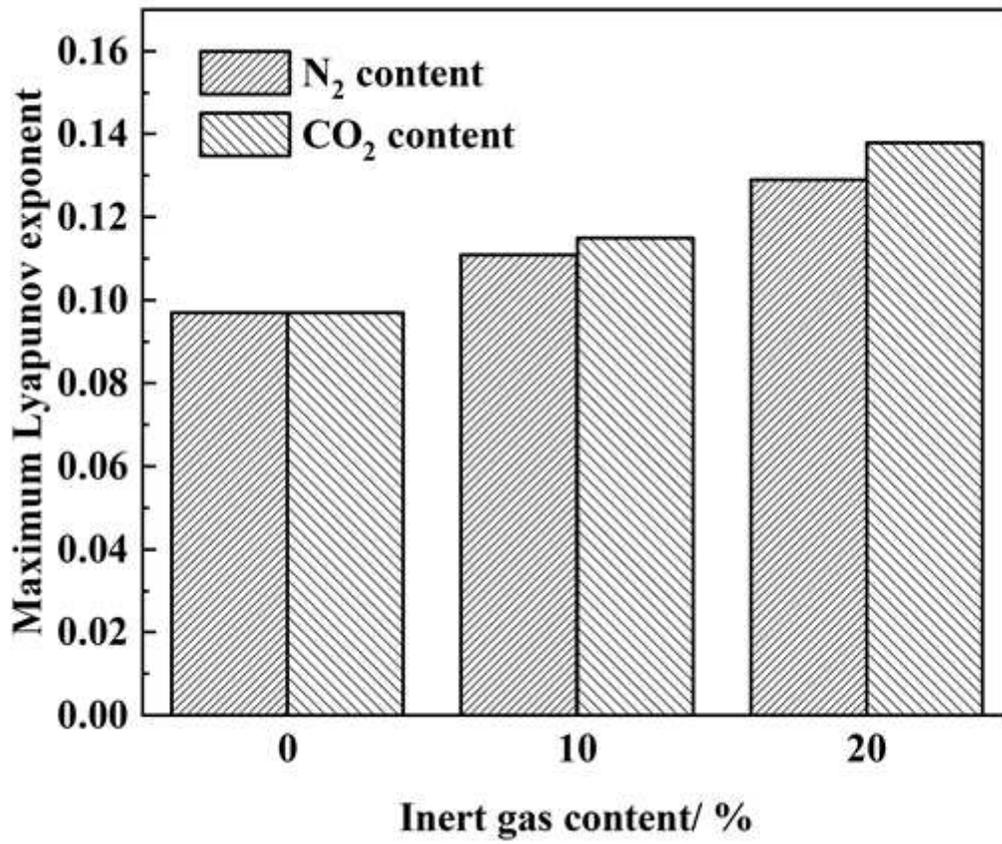


Figure 21

The maximum Lyapunov exponent under different contents of inert gas