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Obtaining Optical Solitons via Resonant Nonlinear Schrödinger's Equation for Optical Fiber Application

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ABSTRACT

Solitons which can be described as a localized wave form that maintain their shape after a collision with another soliton have become a very important phenomena in nonlinear optics due to their potential. They can be used as lossless information carriers in optical fibers due to their robustness arising from their particle grade stability upon a collision. Many scientists from various areas including electronic communication engineers have made solitons the main subject of study. Analytical solutions of nonlinear Schrödinger equation have a very important place in these studies. With the progress of nonlinear optics, some types of nonlinear Schrödinger equation have been derived for better understanding. Resonant nonlinear Schrödinger equation which is being used for describing nonlinear optical phenomena is a generic example for newly derived nonlinear Schrödinger equation. In this study, resonant nonlinear Schrödinger equation has been solved by using functional variable method and sixteen new soliton solutions have been obtained.

1 Introduction

In recent decades nonlinear phenomena attract much attentions not for only our desire to understand nonlinear dynamics of nature but for also their inevitability in explaining some events such as solitary waves. On the other hand, solitons which can be described as a localized wave form that maintain their shape after a collision with another soliton are the main study subject of many scientists from different disciplines. For instance, electronic communication engineers are very interested in solitons, because they can be used as lossless information carriers in optical fibers due to their robustness arising from their "particle grade stability" upon a collision. Or, ocean engineers are interested in solitons due to devastating effect caused by some of them.

Among a bunch of nonlinear equations fully integrable ones such as KdV[1], sine-Gordon[2], Klein Gordon[3], Ginzburg Landau[4] and nonlinear Schrödinger (NLSE)[7, 8] equations have been studied extensively due to their success on describing some complex phenomena. For instance, NLSE is very successful on applications of gravitational waves propagating over the

surface of deep water, Bose-einstein condensate, the Langmuir waves in hot plasmas, etc. In more general terms, NLSE is very successful in describing the evolution of slowly varying packets of quasi-monochromatic waves in weakly nonlinear media that have dispersion.

With the progress of nonlinear optics, some types of NLSE have been derived for better understanding. It can be said that the most famous one is the resonant nonlinear Schrödinger equation (rNLSE) which is an intermediate case (inter-modal dispersion) between focusing and defocusing. It has recently been understood that the rNLSE can be used not only to describe nonlinear optical phenomena but also to explain phenological approaches in many other fields such as cosmology[5] and cold plasma physics[6]. In this respect, it has attracted the attention of researchers in recent years.

In this study, the general form of (1+1)-dimensional resonant nonlinear Schrödinger's equation which is describing the the propagation of optical pulses in nonlinear optical fibers is considered as [7, 8]

$$iW_t + aW_{xx} + (b|W|^2 + c|W|^4)W + d\left(\frac{|W|_{xx}}{|W|}\right)W = 0, \quad i = \sqrt{-1} \quad (1.1)$$

where $W = W(x, t)$ is a complex function that denotes a normalized complex amplitude of the pulse packed in the optical fiber and a, b, c, d are nonzero constants.

2 Description of Functional Variable Method

In this part a brief description of functional variable method [9, 10, 11, 12] can be expressed. First of all an arbitrary nonlinear partial differential equation can be considered as follows

$$F\left(W, \frac{\partial W}{\partial t}, \frac{\partial W}{\partial x}, \frac{\partial^2 W}{\partial t^2}, \frac{\partial^2 W}{\partial x^2}, \dots\right) = 0. \quad (2.1)$$

Let us to determine the wave variable such as

$$W(x, t) = \Phi(\xi), \xi = x + \lambda t \quad (2.2)$$

where λ describes the wave number and can be obtained later. By using the wave variable and chain rule Eq.(2.1) changes into a nonlinear ordinary differential equation as follows.

$$Q(\Phi, \Phi_\xi, \Phi_{\xi\xi}, \Phi_{\xi\xi\xi}, \Phi_{\xi\xi\xi\xi}, \dots) = 0. \quad (2.3)$$

Then let us describe a functional variable such as

$$\Phi_\xi = F(\Phi) \quad (2.4)$$

transforming to unknown function W . Sequential derivatives of the unknown function Φ can be expressed as

$$\begin{aligned} \Phi_{\xi\xi} &= \frac{1}{2}(F^2)', \\ \Phi_{\xi\xi\xi} &= \frac{1}{2}(F^2)''\sqrt{F^2}, \\ \Phi_{\xi\xi\xi\xi} &= \frac{1}{2}[(F^2)'''F^2 + (F^2)''(F^2)'], \\ &\vdots \end{aligned} \quad (2.5)$$

where ''' prime indicates the derivative $\frac{d}{d\Phi}$. Subrogating the Eqs. (2.5) into Eq. (2.3), the ODE can be reduced in terms of Φ, F and the derivatives of F upon Φ as

$$R(\Phi, F, F', F'', F''', F^{(4)}, \dots) = 0. \quad (2.6)$$

After integrating Eq.(2.6), the result arises with respect to F . Putting this equality into (2.4), the appropriate solutions of the regarded problem can be obtained.

3 Analytical Solutions of the (1+1)-Dimensional Resonant Non-linear Schrödinger's Equation

Regard the complex wave transformation:

$$W(x, t) = \Phi(\xi)e^{i\Omega}, \xi = x + \lambda t, \Omega = -kx + \omega t + \theta. \quad (3.1)$$

Subrogating Eq.(3.1) into Eq.(1.1), led to the following expressions

$$(a + d)\Phi'' - (\omega + ak^2)\Phi + b\Phi^3 + c\Phi^5 = 0 \quad (3.2)$$

and

$$\lambda = 2ak.$$

Putting $\Phi'' = \frac{1}{2}(F^2)'$ into (3.2) concludes to the following equation

$$F(\Phi) = \Phi' = \pm \sqrt{\frac{\omega + ak^2}{a + d}\Phi^2 - \frac{b}{2(a + d)}\Phi^4 - \frac{c}{3(a + d)}\Phi^6} \quad (3.3)$$

We acquire the solutions of Eq. (1.1) by solving Eq. (3.3) as follows:

Case 1: If $\frac{\omega + ak^2}{a + d} > 0$ and $3b^2 + 16c(\omega + ak^2) > 0$, then Eq. (3.3) allows the following hyperbolic function solution

$$W_{1,2}(x, t) = \pm \sqrt{\frac{2(ak^2 + \omega) \operatorname{sech}\left(2\sqrt{\frac{ak^2 + \omega}{a + d}}\xi\right)}{\sqrt{\frac{1}{12}(16c(ak^2 + \omega) + 3b^2) + \frac{1}{2}b \operatorname{sech}\left(2\sqrt{\frac{ak^2 + \omega}{a + d}}\xi\right)}}} e^{i\Omega}.$$

Case 2: If $\frac{\omega + ak^2}{a + d} > 0$ and $3b^2 + 16c(\omega + ak^2) < 0$, then following hyperbolic function solution can be obtained

$$W_{3,4}(x, t) = \pm \sqrt{\frac{2(ak^2 + \omega) \operatorname{csch}\left(2\sqrt{\frac{ak^2 + \omega}{a + d}}\xi\right)}{\sqrt{-\frac{1}{12}(16c(ak^2 + \omega) + 3b^2) + \frac{1}{2}b \operatorname{csch}\left(2\sqrt{\frac{ak^2 + \omega}{a + d}}\xi\right)}}} e^{i\Omega}.$$

Case 3: If $\frac{\omega + ak^2}{a + d} > 0$ and $c = -\frac{3b^2}{16(ak^2 + \omega)}$, then by using Eq. (3.3) hyperbolic function solutions can be acquired

$$W_{5,6}(x, t) = \pm \sqrt{\frac{2(ak^2 + \omega) \left(\tanh\left(\sqrt{\frac{ak^2 + \omega}{a + d}}\xi\right) + 1\right)}{b}} e^{i\Omega},$$

$$W_{7,8}(x, t) = \pm \sqrt{\frac{2(ak^2 + \omega) \left(\coth\left(\sqrt{\frac{ak^2 + \omega}{a + d}}\xi\right) + 1\right)}{b}} e^{i\Omega}.$$

Case 4: If $\frac{\omega + ak^2}{a + d} < 0$ and $3b^2 + 16c(\omega + ak^2) > 0$, also taking Eq. (3.3), led to the following trigonometric function solutions

$$W_{9,10}(x, t) = \pm \sqrt{\frac{2(ak^2 + \omega) \sec\left(2\sqrt{-\frac{ak^2 + \omega}{a + d}}\xi\right)}{\sqrt{\frac{1}{12}(16c(ak^2 + \omega) + 3b^2) + \frac{1}{2}b \sec\left(2\sqrt{-\frac{ak^2 + \omega}{a + d}}\xi\right)}}} e^{i\Omega},$$

$$W_{11,12}(x, t) = \pm \sqrt{\frac{2(ak^2 + \omega) \csc\left(2\sqrt{-\frac{ak^2 + \omega}{a+d}}\xi\right)}{\sqrt{\frac{1}{12}(16c(ak^2 + \omega) + 3b^2) + \frac{1}{2}b \csc\left(2\sqrt{-\frac{ak^2 + \omega}{a+d}}\xi\right)}}} e^{i\Omega}.$$

Case 5: If $\frac{\omega + ak^2}{a+d} > 0$, $b = -4\sigma(ak^2 + \omega)$ and $c = -3(\sigma^2 - 1)(ak^2 + \omega)$, also considering Eq. (3.3) yields the following hyperbolic function solution

$$W_{13,14}(x, t) = \pm \frac{1}{\sqrt{\cosh\left(2\sqrt{\frac{ak^2 + \omega}{a+d}}\xi\right) - \sigma}} e^{i\Omega}$$

where $\sigma < 1$.

Case 6: If $\frac{\omega + ak^2}{a+d} > 0$, $b = -4\sigma(ak^2 + \omega)$ and $c = -3(\sigma^2 + 1)(ak^2 + \omega)$, and by using Eq. (3.3) gives the following hyperbolic function solution

$$W_{15,16}(x, t) = \pm \frac{1}{\sqrt{\sinh\left(2\sqrt{\frac{ak^2 + \omega}{a+d}}\xi\right) - \sigma}} e^{i\Omega}$$

where $\sigma < 1$.

4 Applications of Some Solutions

In order to see the spatial distribution of probabilities of the wave function for the obtained solutions of $W_3(x, t)$, $W_9(x, t)$, $W_{11}(x, t)$ and $W_{15}(x, t)$, modulus vs. location plots were given in figure 1. Kink solitons extended along the x dimension with periodical layout can be seen in figure 1(a) and (b). A single bell shaped soliton obtained from the solution $W_{11}(x, t)$ can be seen in figure 1(c). On the other hand, multiple bell shaped solitons extended along the x dimension with periodical layout can be seen in figure 1(d).

Figure 1: Probability distribution of the wave functions $W_3(x,t)(a)$, $W_9(x,t)(b)$, $W_{11}(x,t)(c)$ and $W_{15}(x,t)(d)$ for the parameters of $\omega = 0.1, a = 0.1, k = 0.001, d = -2, t = 0.1, \theta = 0$.

5 Conclusions

In this article authors obtained the optical soliton solutions of Schrödinger's Equation arising in optical fibers. For this aim authors used functional variable method as a tool. All the calculation are made by using computer software named Mathematica. Also graphical representations and some illustrative explanations of the solutions are given in Section 4. Authors think that the results can give scientists a chance to make further insight on the optical theory and also hope that this work will be very useful in better understanding the optical solitons.

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Figures

Figure 1

Probability distribution of the wave functions $W_3(x; t)$ (a), $W_9(x; t)$ (b), $W_{11}(x; t)$ (c) and $W_{15}(x; t)$ (d) for the parameters of $\omega = 0.1$; $a = 0.1$; $k = 0.001$; $d = -2$; $t = 0.1$; $\theta = 0$.