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On Collapse of Quantum State on Measurement

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Abstract:

In the context of the century-long debate on quantum measurement problem, the current work proposes a model that describes the process of collapse of state by quantum interaction, which resolves the controversies of the framework of quantum mechanics and describes the entire domain of quantum-to-classical world including the weak measurement and partial collapse. ‘Measurement’, being the process of physically interacting with a system in order to extracting information from it, is theorized in the current model by synthesizing the quantum interaction between system and measuring apparatus with the information entropy of such process. The model assumes Schrödinger equation to be the only guiding equation for all physical systems including the measuring apparatus, and does not presuppose ‘superposition principle’, rather derives it theoretically from the formulation. The superposed state is shown to be independent of the choice of measurement operator (observable) or basis states (pointers) of the measuring apparatus. Most interestingly, the current model explains the non-observance of ‘superposition principle’ by classical systems as the classical limit of such quantum description of measurement. Along with solving the quantum measurement problem, the work also explains weak measurement and partial collapse, which can be further extended to investigate such several emerging critical phenomena.

Keywords: Quantum measurement problem; Collapse of state; Superposition principle; Information entropy.

The concept of ‘Measurement’ for quantum objects has been a subject of century-long debate [1]. The central idea regarding such quantum measurement is that the quantum state of a system, while performing a single measurement of an observable on it, collapses into one of the eigenstates of the observable, which is a definite state of the measuring apparatus; and measured value of the observable is statistical average of the results of a large number of such measurement runs [2]. The quantum system is believed to be in a linear superposition of all such eigenstates of the observable, which, however, collapses on measurement irreversibly and randomly to any one of them. On the other hand, the measuring apparatus being a macroscopic object is supposed to be in a definite state and not a superposed one. Such model has been observed to be consistent and in full agreement with all the quantum phenomena in practice, however, led to a number of contradictory issues from theoretical perspective [3]. This raises, first and foremost, the question why superposition principle is not valid for macro-objects like the measuring apparatus which should belong to the classical limits of quantum conditions. This in turn poses the question, where is the boundary line between the quantum and classical world? Secondly, since it is a subjective choice to measure an observable of a system, how the system remains in a linear superposition of the eigenstates of that very observable in a predetermined way? Or does it simultaneously exist in the superposition of eigenstates of all the possible observables? Thirdly, the evolution of a quantum system is guided by Schrödinger equation in a so-called deterministic manner; however, probability comes into play when a measurement is performed and then the system is guided by Born rule. Quantum theory, as per Copenhagen interpretation [4], has not provided any specific dynamics for such transition.

Numerous attempts, such as decoherence, many-worlds interpretation, Bohmian model and the latest GRW and other spontaneous collapse models, have been made to resolve such seemingly contradictory issues [3, 5-6]. Such theories are proposed on the basis of two requirements:

- 1) The system (supposed to be quantum) and the measuring apparatus (supposed to be classical) both are subsystems of the inseparable world and thus must be described by the same equation of motion including the measurement process; and
- 2) If Schrödinger equation is incapable to inclusively explain such measurement process, then either it does not provide a complete description of the nature and needs additional

variables, or otherwise, a modified equation; and/or during measurement, the system jumps to one of the eigenstates being guided by some other equation(s) of motion which is(are) still known.

Thus, such efforts have been in search of either ‘hidden variables’ or entirely new law(s) of motion with quantum mechanics as its limiting/special case. However, no general consensus has been developed to reach any conclusion till date.

In this context, the current work proposes a model that redefines the measurement scheme that can lead to resolve the so-called “quantum measurement problem” in general. Such model does not presuppose ‘superposition principle’ as an ontological description of a quantum state, rather, explores the general process of quantum measurement which inevitably leads to the ‘superposition principle’. The model considers Schrödinger equation to be the only guiding equation for all physical systems, be it the system subjected to measurement or be it the systems belonging to measuring apparatus.

Any measurement process, in general, is carried out by probing a system (i.e. by interaction between the system and measuring apparatus) and extracting information from it objectively. In practice, different measurement processes may be very diverse, case specific and complex. However, they can all be conceptualized in the current measurement scheme elaborated as follows:

Nature of interaction: If two isolated systems are exposed before each other to interact, there can be two kinds of possible interactions: 1) the initial state (i.e. isolated state before the interaction starts) of one of the systems damps out in time during interaction; and 2) no such damping occurs. Only the former type of interaction, say Type-I, can be utilized for measurement; and call the other one Type-II. There is a third possibility that the systems, even if exposed before each other, do not interact without any external influence, which are termed in the current work to be ‘mutually non-interacting’.

Measuring apparatus: For measurement of an observable, the corresponding pointers of a measuring apparatus are a number of ‘mutually non-interacting’ physical systems, may be real (e.g. a RGB photon detector) or conceptual (spin-up and spin-down systems for Stern-Gerlach

experiment) being in several different states, each of which can interact with the system subjected to measurement in of Type-I manner to damp its initial state. The relevant operator mathematically represents the particular interaction between system and pointers corresponding to the observable and must commute with the Hamiltonian of the pointers; it may or may not commute with the Hamiltonian of the system subjected to measurement.

Measurement process: It may be described by probing the system subjected to measurement by measuring apparatus in order to extract information from it objectively.

Probing: When the measuring apparatus consisting of the pointers is entirely exposed before the system subjected to measurement in a given state (i.e. pre-select state), it selects to interact with one of the pointers of measuring apparatus in a statistical probabilistic manner depending on their relative strength of interactions. It is imperative to mention that, for such ‘mutually non-interacting’ pointers, the system subjected to measurement cannot but select only one of them to interact in Type-I manner [for detail see [Appendix-I](#)]. Once the selection is done, the state (i.e. post-select state) of the system on which measurement is being performed suffers a transition, or the so-called ‘collapse’, to that of the selected pointer of measuring apparatus due to Type-I nature of interaction.

Extraction of information: In general, to perform a measurement, one needs to probe the system subjected to measurement with each of the pointers of measuring apparatus and record the results and average out to get the final measurement value. Physically such probing implies that each of the pointers be exposed (to interact) to the system a large number of times in its pre-select state, and after selection, each measurement run is considered to be complete once the post-select state of the system collapses to the corresponding state of the pointer. Such several runs of the measurement can be done, either by re-preparing the system in its pre-select state after each measurement run is complete, or by preparing a number of identical replicas of the system in the pre-select state. One now only needs to find the relative weights of the ‘grades’ assigned to each of the pointers that correspond to measurement value of the observable and calculate the ‘expectation value’. It will be shown in this work that objectivity of information extracted in an entire measurement process necessitates such relative weights to be equal to the

corresponding probabilities of collapse of the post-selected state of the system into the respective pointer states in time.

Objectivity of information: The probing process consists of two consecutive probabilities: first, the probability that the system ‘subjected to measurement in pre-select state selects a particular pointer among all the pointers of measuring apparatus statistically based on interaction strength, and second, after selecting one pointer, the probability that its post-select state collapses to the pointer state in time. Now, for the result of a quantum measurement to be objective in general, the entropy of information for such physical process involving two conditional probabilities must be independent of any particular system in any particular initial state, and thus also of its energy, which requires such entropy of information to be extremized in terms of system energy.

On the basis of such generalized measurement scheme, the measurement process for a quantum system can be described as follows:

Suppose an isolated physical system ‘ S ’ in a definite state $|\psi\rangle$ with energy ε_s is described by Schrödinger equation,

$$i\hbar \frac{d}{dt}|\psi\rangle = \varepsilon_s |\psi\rangle \quad (1)$$

and, an observable ‘ O ’ of the system is to be measured. Then formal quantum mechanics considers a particular choice of a set of non-degenerate orthonormal eigenstates $\{|\varphi_j\rangle, j = 1, 2, \dots\}$ of the operator Θ (i.e. $\Theta|\varphi_j\rangle = O_j|\varphi_j\rangle$) that corresponds to the measurement of observable ‘ O ’. According to the current model, in order to develop a corresponding measuring apparatus for measurement of ‘ O ’, firstly, one needs to prepare a set of ‘mutually non-interacting’ physical systems ‘ I_j ’ (real or conceptual) in definite states $|\zeta_j\rangle = |\varphi_j\rangle e^{-\left(\frac{i}{\hbar}\varepsilon_j t\right)}$ with energy values ε_j , respectively, to be used as pointers of the measuring apparatus. Thus, all of such physical systems (i.e. pointers) are also described by the same Schrödinger equation as,

$$i\hbar \frac{d}{dt}|\zeta_j\rangle = \varepsilon_j |\zeta_j\rangle \quad (2)$$

Therefore, irrespective of the fact whether the operator Θ commutes with the Hamiltonian of system ‘ S ’ or not, it must commute with the Hamiltonian of the pointers ‘ I_j ’. Secondly, another necessity for the set of such physical systems to perform as pointers of measuring apparatus is that all of them must interact with the system ‘ S ’ only in Type-I manner, as mentioned earlier, in order to damp its post-select state. In fact, mathematically the operator Θ and its eigenvalues O_j are related to such particular type of physical interactions. When these conditions are fulfilled, each individual of the set of such specific physical systems may be assigned ‘grades’ by choosing appropriate standards of units corresponding to the values of observable ‘ O ’ (i.e. eigenvalues O_j of the operator Θ); and such systems ‘ I_j ’ can then be considered as the ‘pointer’s of measuring apparatus.

Suppose, in a measurement run, the system ‘ S ’ selects ‘ I_n ’ among all ‘ I_j ’s, and in the corresponding post-select states $|\psi(t)\rangle$ of and $|\zeta_n(t)\rangle$, respectively, ‘ S ’ and ‘ I_n ’ start interacting at time $t = 0$. Thus the equations (1) and (2) are modified for such post-select states respectively by [for detail see [Appendix-II](#)],

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = \varepsilon_s |\psi(t)\rangle + \tau_n |\zeta_n(t)\rangle \quad (3)$$

$$i\hbar \frac{d}{dt} |\zeta_n(t)\rangle = \varepsilon_n |\zeta_n(t)\rangle + \tau_n^* |\psi(t)\rangle \quad (4)$$

where τ_n , a complex function in general, represents the interaction between ‘ S ’ and ‘ I_n ’, which was zero at their isolated or initial states. Therefore, the probability that ‘ I_n ’ out of all ‘ I_j ’s is selected for interaction by the system ‘ S ’ is given by the statistical definition as,

$$p_n = \frac{|\tau_n|^2}{\sum_j |\tau_j|^2} \quad (5)$$

It is worthy to mention at this point that Eq. (3) and (4), not individually, but together represent Schrödinger equation for the combined system ‘ $S+I_n$ ’ (i.e. may be written as

$$i\hbar \frac{d}{dt} \begin{pmatrix} |\psi(t)\rangle \\ |\zeta_n(t)\rangle \end{pmatrix} = \begin{pmatrix} \varepsilon_s & \tau_n \\ \tau_n^* & \varepsilon_n \end{pmatrix} \begin{pmatrix} |\psi(t)\rangle \\ |\zeta_n(t)\rangle \end{pmatrix} \text{ which is in corroboration with the assumption that}$$

Schrödinger equation is the only guiding equation of motion for both, the system as well as the pointers. It is also imperative to mention that such post-select states of the system and the pointer are not ‘pure’ states and evidently dependent on each other, however, can be separated conceptually [for detail see [Appendix-II](#)].

Considering the Green’s function (G_n) for ‘ I_n ’, the solution of Eq. (4) is given by,

$$|\zeta_n(t)\rangle = |\varphi_n\rangle e^{-\left(\frac{i}{\hbar}\varepsilon_n t\right)} + \int dt_1 G_n(t, t_1) \tau_n^* |\psi(t_1)\rangle \quad (6)$$

where, $\left(i\hbar \frac{d}{dt} - \varepsilon_n\right) G_n(t, t_1) = \delta(t - t_1)$. Since a phase $[e^{-\frac{i}{\hbar}\varepsilon_n(t-t_1)}]$ does not change any result on multiplying with delta function $[\delta(t - t_1)]$, the Green’s function is given by $G_n(t, t_1) = -\frac{i}{\hbar} \theta(t - t_1) e^{-\left(\frac{i}{\hbar}\varepsilon_n(t-t_1)\right)}$. Putting the solution $|\zeta_n(t)\rangle$ given in (6) using such Green’s function into Eq. (3), it is obtained to be,

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = \varepsilon_s |\psi(t)\rangle + \int dt_1 \tau G_n(t, t_1) \tau_n^* |\psi(t_1)\rangle + \tau_n |\varphi_n\rangle e^{-\left(\frac{i}{\hbar}\varepsilon_n t\right)} \quad (7)$$

At this point, the requirement for the nature of interaction between the system and pointer of the measuring apparatus, i.e. of Type-I as mentioned earlier, is described as follows:

It is apparent that before the interaction starts the state of isolated system ‘ S ’ evolves in time as unitary process. However, once it starts interacting in Type-I manner with the measuring apparatus, the time evolution operator ($T(t, t_0)$) of the system given by $|\psi(t)\rangle = T(t, t_0) |\psi(t_0)\rangle$ must have the property,

$$T^+(t, t_0) T(t, t_0) \neq 1 \quad (8)$$

since the probability of finding ‘S’ at a time no longer remains the same as in its initial post-select state due to the process of collapse of such state while interacting. Therefore, the time evolution operator of the system ‘S’ only (a sub-part of the total system ‘S+I_n’) during such interaction may be considered to be $T(t, t_0) = e^{-\left(\frac{i}{\hbar}(t-t_0)\Xi\right)}$ where Ξ has a non-zero imaginary part (thus cannot be considered as the conventional Hamiltonian). Nevertheless, the system and the pointer together (‘S+I_n’) evolves unitarily according to Schrödinger equation, the time evolution operator being $T_{\text{total}}(t, t_0) \equiv e^{-\left(\frac{i}{\hbar}\begin{pmatrix} \varepsilon_S & \tau_n \\ \tau_n^* & \varepsilon_n \end{pmatrix}(t-t_0)\right)}$ with $T_{\text{total}}^\dagger T_{\text{total}} = 1$. Thus considering such Type-I interaction, Eq. (7) leads to,

$$\left(i\hbar \frac{d}{dt} - [\varepsilon_S - \tau_n(\Xi - \varepsilon_n)^{-1}\tau_n^*]\right) |\psi(t)\rangle = \tau_n |\varphi_n\rangle e^{-\left(\frac{i}{\hbar}\varepsilon_n t\right)} \quad (9)$$

where Ξ is considered to be independent of time. The solution of Eq. (9) is obtained to be

$$|\psi(t)\rangle = |\psi(0)\rangle e^{-\frac{i}{\hbar}(\varepsilon_S - \tau_n(\Xi - \varepsilon_n)^{-1}\tau_n^*)t} + \left(\frac{\tau_n}{\varepsilon_n - \varepsilon_S + \tau_n(\Xi - \varepsilon_n)^{-1}\tau_n^*}\right) |\varphi_n\rangle e^{-\left(\frac{i}{\hbar}\varepsilon_n t\right)} \quad (10)$$

The condition for collapse is that the first term of such net solution, i.e. mathematically the complementary function and physically the initial state of the system, damps out in time, which poses the restriction on imaginary part of the term ‘ $\tau_n(\Xi - \varepsilon_n)^{-1}\tau_n^*$ ’ to be positive. The second term, i.e. the particular integral, gives rise to the steady-state solution indicating the selected pointer state (or the n^{th} eigenstate of the operator Θ in conventional framework of quantum

mechanics). The coefficient $\left(\frac{\tau_n}{\varepsilon_n - \varepsilon_S + \tau_n(\Xi - \varepsilon_n)^{-1}\tau_n^*}\right)$ of the steady state solution thus

represents the probability amplitude of collapse of the post-select state of ‘S’ into that of ‘I_n’. Hence, such probability amplitude must be equal to unity if $\varepsilon_S = \varepsilon_n$, which results to

$\Xi = \varepsilon_n + \tau_n^*$, and thereby simplifies the expression of such coefficient to $\left(\frac{\tau_n}{\varepsilon_n - \varepsilon_S + \tau_n}\right)$. It is

interesting to note at this point that Eq. (9) represents a single measurement run which is

equivalent to $\Theta|\psi\rangle = O_n|\varphi_n\rangle$ and therefore $\Theta \equiv \left(i\hbar \frac{d}{dt} - [\mathcal{E}_S - \tau_n] \right)$ represents a general theoretical form of an operator in quantum measurement (‘ $\equiv$ ’ is used since the dimension in general is not identical to that of the observable and the expression may need to be multiplied by a constant along with an ‘ $i = \sqrt{-1}$ ’ in order to make it Hermitian). Different operators ‘ Θ ’ are characterized by different kinds of interactions defined by ‘ τ ’. Thus considering the real and imaginary parts of such interaction term as $\tau_n = \tau_n^R + i\tau_n^I$, the probability that the post-select state $|\psi(t)\rangle$ of the system while interacting in Type-I manner with the pointer state $|\zeta_n(t)\rangle$ collapses to it in time is obtained to be,

$$P_n = \frac{\tau_n^{R^2} + \tau_n^{I^2}}{(\mathcal{E}_S - \mathcal{E}_n - \tau_n^R)^2 + \tau_n^{I^2}} \quad (11)$$

which is maximum at $\mathcal{E}_S = \mathcal{E}_n + \tau_n^R$. A zero value of the real part τ_n^R of interaction term (i.e. $\mathcal{E}_S = \mathcal{E}_n$), therefore, indicates the capability of measuring apparatus of ‘accurately’ measuring the observable and thus it can be considered as the instrumental error. However, even if it is zero, if $\mathcal{E}_S \neq \mathcal{E}_n$, then there will be a lack of ‘precision’ due to the imaginary part of interaction (τ_n^I) which is a measure of the inherent uncertainty. It is to be noted that this is in complete agreement with the ‘Copenhagen Interpretation’ of quantum theory interpreting the inaccuracy in a measurement follows from the uncertainty relations as well as due to the difficulty of the experiment [4]. Therefore, to deal with the indeterminacy inherent in nature, such instrumental error is presently assumed to be zero, i.e. $\tau_n^R = 0$ and $\Xi = \mathcal{E}_n - i\tau_n^I$, for further advancement of the model. The rate of collapse in such interaction can be obtained from the time evolution

operator as $T^+(t,0)T(t,0) = e^{-\left(\frac{2\tau_n^I}{\hbar}t\right)}$. It can be seen that the essential difference of the current work from conventional ‘Copenhagen Interpretation’ of quantum measurement is that the process of collapse is not a discrete ‘jump’, but ‘asymptotic’ in time with the rate of collapse depending on the strength of interaction. For instance, if the interaction is of the order of ‘eV’ in energy unit, the time constant for collapse ($\frac{\hbar}{2\tau_n^I}$) is of the order of 10^{-15} s and thus may be

considered as mere ‘jump’ in practice. However, the collapse process can be much slower if the interaction is significantly weak and such case can interpret the partial collapse and/or the measurement of ‘weak value’ [7-10]. Very recently a few experiments have shown that the state of a system can evolve coherently and continuously during such so-called ‘jump’ transitions [11-12].

As mentioned earlier, the entire probing process consists of two consecutive probabilities: first the probability (p_n) that the system ‘S’ in pre-select state selects a particular pointer ‘ I_n ’ statistically among all the ‘ I_j ’s for interaction given by (5), and second, if it selects to interact with ‘ I_n ’, then the probability (P_n) that its post-select state $|\psi(t)\rangle$ collapses to $|\varphi_n\rangle$ in time as given by (11). Therefore, for the purpose of extracting information from such physical process involving conditional probability, the entropy of information can be written as [13],

$$S = -\sum_n p_n \ln(P_n) \quad (12)$$

Now, as explained earlier, the objectivity in extraction of information requires $\frac{\partial S}{\partial \varepsilon_S} = 0$. Thus,

considering the instrumental broadening to be zero for ideal case (i.e. $\tau_n^R = 0$), it results to,

$$\sum_n \left(\tau_n^{I^2} / [(\varepsilon_S - \varepsilon_n)^2 + \tau_n^{I^2}] \right) (\varepsilon_S - \varepsilon_n) = 0, \text{ which can be rewritten as,}$$

$$\varepsilon_S = \frac{\sum_n \left(\tau_n^{I^2} / [(\varepsilon_S - \varepsilon_n)^2 + \tau_n^{I^2}] \right) \varepsilon_n}{\sum_n \left(\tau_n^{I^2} / [(\varepsilon_S - \varepsilon_n)^2 + \tau_n^{I^2}] \right)} = \frac{\sum_n P_n \varepsilon_n}{\sum_n P_n} \quad (13)$$

It is interesting to note that Eq. (13) indicates that the energy of the system (ε_S) will be equal to the statistical ‘expectation value’ of energies of all the pointers (ε_n) of the measuring apparatus, i.e. $\varepsilon_S = \langle \varepsilon_n \rangle$, if the probabilities of post-select state of the system to collapse into different pointer states (as given by (11) with $\tau_n^R = 0$) are regarded as the relative weights of the corresponding ‘grades’ for averaging. This result corroborates quite well with the conventional ontological description of ‘superposition principle’ of quantum mechanics and the principal

essence of ‘Copenhagen Interpretation’. Therefore, the current model of measurement scheme and process of collapse provides the information about the system, that, the initial state ($|\psi\rangle$) of system ‘S’ (with energy ε_s) can be expressed as a ‘linear superposition’ of the eigenstates ($|\zeta_j\rangle$) of an operator ‘ Θ ’, which can be considered as the pointers ‘ I_j ’s of a measuring apparatus. The respective ‘coefficients’ of superposition are identical to the non-statistical probability amplitudes of collapses of the post-select states into the corresponding pointer states, if it would be subjected to such measurement, and are given by,

$$|\psi\rangle = \sum_j \left(\frac{i\tau_j^I}{\varepsilon_j - \varepsilon_s + i\tau_j^I} \right) |\zeta_j\rangle \quad (14)$$

and the measured value of the observable will be its statistical expectation value accordingly. Interestingly, RHS of the expression given in (13) is identical to ε_s independent of the observable measured, pointer energy as well as the interaction terms, i.e. the choice of ‘basis set’ of eigenstates of any arbitrary observable; and therefore, the same is also valid for the expression of superposition given in (14). In a similar manner, any pointer state can also be expressed as a superposition of eigenstates of any such arbitrary observable, which upholds that there is no restriction(s) on the pointer states of measuring apparatus (conventionally considered as classical object) to follow the ‘superposition principle’ in general.

Further, beyond conventional formalism of quantum mechanics, (14) represents the analytical expression of coefficients of superposing states as a function of the system energy, pointer energy values of the measuring apparatus and their interactions. It is apparent that such result endorses that the quantum probability which is norm of such coefficients originates from physical interaction of quantum type and thus distinguishably different from the conventional statistical probability. Finally, it resolves the seemingly contradictory issue that a classical system does not exhibit superposition principle as follows:

A classical system can be distinguished from a quantum object by the fact that the disturbance created by a measurement process on the former one is negligible. Thus at classical limit, the energy of interaction (τ_j^I) between the system and pointers of measuring apparatus is negligibly

small compared to the order of their energy values ($\varepsilon_s, \varepsilon_j$); and correspondingly, for classical objects, the superposition principle of (14) results to,

$$|\psi\rangle = \sum_j \lim_{\tau_j^I \rightarrow 0} \left(\frac{i\tau_j^I}{\varepsilon_j - \varepsilon_s + i\tau_j^I} \right) |\zeta_j\rangle = \begin{cases} |\zeta_j\rangle & \text{for } \varepsilon_s = \varepsilon_j \\ 0 & \text{for } \varepsilon_s \neq \varepsilon_j \end{cases} \quad (15)$$

which corroborates to the non-observance of ‘superposition principle’ in classical reality.

Fig.1. The plot of probability ($\langle\psi|\psi\rangle$) of the system under measurement with and its energy (ε_s) and the strength of interaction with the pointers (τ^I). The pointer states are considered to be of energies $\varepsilon_n = [0.10, 0.25, 0.50, 0.75]$ of each order (powers of 10 in eV) of energies.

The overall description of the measurement phenomenon as described in the current model is illustrated in Fig. 1 that encompasses the entire domain of quantum to classical world including the weak measurement region. The figure represents how the probability of a system ($\langle\psi|\psi\rangle$) varies with the order its energy (ε_s) along with the strength of interaction (τ^I) with the pointers. It is apparent from the plot of Fig. 1, that for lower energy range (i.e. comparable to interaction

strength), the probability is distributed into the pointer states with broadened intensity around them indicating superposition of such states thereby defining the quantum domain. However, with increasing energy (w.r.t. interaction strength), it is observed to be highly sharpened, discretely at the pointer states only, implying no-superposition as expected in classical reality. On the other hand, the region of interaction strength below the order of μeV (in energy unit) of Fig. 1 represents the weak measurement and partial collapse phenomena considering collapse time of the order of $\geq \text{'ns'}$.

In conclusion, it can be said that, ‘measurement’, as an aim, implies some information to be extracted from a system objectively, whereas ‘measurement’, as a process, involves a physical interaction with such system. Therefore, theorization of ‘measurement’ requires to synthesizing these two features, i.e. information and interaction, which is presented in the current work for interpretation of quantum measurement scheme which is in utter consonance with the classical limit. The work may provide a guide to find out answers to many questions regarding the century-long debate on the formalism of quantum mechanics including the so-called ‘measurement problem’ and can be further extended to study weak measurement, partial collapse and other such emerging critical phenomena.

Appendix-I

In order to show that the system subjected to measurement cannot select more than one pointers to interact simultaneously, let us assume hypothetically that the system interacts in type-I manner with more than one pointers. In such case, Eq. (3) would be given by,

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = \varepsilon_s |\psi(t)\rangle + \sum_j \tau_j |\zeta_j(t)\rangle \quad (\text{A-I-1})$$

which, instead of Eq. (9), would lead to

$$\left(i\hbar \frac{d}{dt} - [\varepsilon_s - \sum_j \tau_j] \right) |\psi(t)\rangle = \sum_n \tau_n |\varphi_n\rangle e^{-\left(\frac{i}{\hbar} \varepsilon_n t\right)} \quad (\text{A-I-2})$$

It is imperative to note that τ 's in the RHS of Eq. (A-I-2) represent the inflow of interaction flux to the system from the corresponding pointers, whereas τ 's in LHS of such equation contributes to the corresponding outflow. Thus, the steady state of the system would be given by,

$$|\psi(t)\rangle_{S_t} = \int dt_1 G_S(t, t_1) |S_P(t_1)\rangle \quad (\text{A-I-3})$$

where $|S_P(t)\rangle = \sum_n \tau_n |\varphi_n\rangle e^{-\left(\frac{i}{\hbar} \varepsilon_n t\right)}$ and $G_S(t, t_1) = -\frac{i}{\hbar} \theta(t - t_1) e^{-\left(\frac{i}{\hbar} [\varepsilon_S - \sum_j \tau_j](t - t_1)\right)}$. Hence, considering

$\Sigma_P(t_1, t_2) = \langle S_P(t_2) | S_P(t_1) \rangle$, it is obtained to be,

$$\langle \psi(t') | \psi(t) \rangle_{S_t} = \int dt_1 \int dt_2 G_S(t, t_1) \Sigma_P(t_1, t_2) G_S^*(t_2, t') \quad (\text{A-I-4})$$

Thus, the two-time flux $\Phi(t, t') = \left(\frac{d}{dt} - \frac{d}{dt'} \right) \langle \psi(t') | \psi(t) \rangle_{S_t}$ results to be

$$\begin{aligned} \Phi(t, t') = & \frac{1}{\hbar} \int dt_1 \int dt_2 \left[\sum_j i(\tau_j - \tau_j^*) G_S(t, t_1) \Sigma_P(t_1, t_2) G_S^*(t_2, t') \right] \\ & - \frac{i}{\hbar} \int d\bar{t} \left[\Sigma_P(t, \bar{t}) G_S^*(\bar{t}, t') - G_S(t, \bar{t}) \Sigma_P(\bar{t}, t') \right] \end{aligned} \quad (\text{A-I-5})$$

It is apparent that the first term in RHS of Eq. (A-I-5) represents the total outward flux ($\Phi_{out}(t, t')$) from system to all the pointers (note the term $\sum_j i(\tau_j - \tau_j^*)$) while its second term indicates the total inward flux ($\Phi_{in}(t, t')$) accordingly from all the pointers to the system (note the term Σ_P). Neglecting the instrumental error, considering $\tau_n = i\tau_n^I$ and putting the expressions of G_S and Σ_P , such inward and outward fluxes corresponding to a particular pointer, suppose the n^{th} one, are respectively obtained to be,

$$\Phi_{in}^n(t, t') = \frac{2\sum_k \tau_k^I}{\hbar} \left[\frac{(\tau_n^I)^2}{(\varepsilon_n - \varepsilon_S)^2 + (\sum_j \tau_j^I)^2} \right] e^{-\left(\frac{i}{\hbar} \varepsilon_n (t - t')\right)} \quad (\text{A-I-6})$$

and

$$\Phi_{Out}^n(t, t') = -\frac{2\tau_n^I}{\hbar} \sum_m \left[\frac{(\tau_m^I)^2}{(\varepsilon_m - \varepsilon_s)^2 + (\sum_j \tau_j^I)^2} \right] e^{-\left(\frac{i}{\hbar} \varepsilon_m (t-t')\right)} \quad (\text{A-I-7})$$

It is observed from (A-I-6) and (A-I-7) that, if the system interacts with more than one pointers simultaneously (as assumed hypothetically), the net interaction flux between the system and any one of the pointers at steady state is not zero, i.e. $\Phi_{In}^n(t, t') + \Phi_{Out}^n(t, t') \neq 0$. However, sum of such net interaction fluxes for all the interacting pointers is zero, i.e. $\sum_n \Phi_{In}^n(t, t') + \Phi_{Out}^n(t, t') = 0$. It implies that at steady state, net inflow of interaction fluxes from some of the interacting pointers to the system is equal to the net outflow of interaction fluxes to the rest of the interacting pointers. Such case is equivalent to ‘mutual interaction’ between the pointers with the system to be pseudo-present between them (which therefore cannot be considered as “external influence” as mentioned earlier), which is in apparent contradiction to the basic assumption of ‘mutually non-interacting’ property of the pointers. However, if the system subjected to measurement selects only one pointer, the sums on all τ ’s in (A-I-6) and (A-I-7) are removed, which leads to the net steady state interaction flux to be zero, i.e. $\Phi_{In}^n(t, t') + \Phi_{Out}^n(t, t') = 0$.

Appendix-II

After the system starts interacting with a pointer, the total Hamiltonian of ‘system+pointer’ can be represented as,

$$\hat{H} = \varepsilon_s |\psi\rangle\langle\psi| + \varepsilon_p |\zeta\rangle\langle\zeta| + \tau |\zeta\rangle\langle\psi| + \tau^* |\psi\rangle\langle\zeta| \quad (\text{A-II-1})$$

It is worthy to mention that the combined state $|\psi, \zeta\rangle$ could be represented in tensor product of basis states of the two corresponding Hilbert spaces; however, since the ‘superposition principle’ is not presupposed in the current work, such approach is not adopted. Also, reduced density matrix of the system or the pointer by partial trace cannot be considered for the same reason. Therefore, the states $|\psi\rangle$ and $|\zeta\rangle$ which are localized to the system and the pointer, respectively, although not separable but dependent on each other, can be obtained by considering the corresponding local projection operators,

$$\begin{cases} \hat{\rho}_\psi = |\psi\rangle\langle\psi, \zeta| \\ \hat{\rho}_\zeta = |\zeta\rangle\langle\psi, \zeta| \end{cases} \quad (\text{A-II-2})$$

where, $\hat{\rho}_\psi|\psi, \zeta\rangle = |\psi\rangle$ and $\hat{\rho}_\zeta|\psi, \zeta\rangle = |\zeta\rangle$. It is imperative to note that since the physical boundaries of system and pointer are well-defined, the expectation values of such local projection operators, i.e. $\langle\hat{\rho}_\psi\rangle$ and $\langle\hat{\rho}_\zeta\rangle$, are constant. Thus, $i\hbar \frac{d}{dt}\langle\hat{\rho}_\psi\rangle = 0$, which leads to

$$i\hbar \frac{\partial}{\partial t} \hat{\rho}_\psi = [\hat{H}, \hat{\rho}_\psi] \quad (\text{A-II-3})$$

$$\begin{aligned} &\Rightarrow \left[i\hbar \frac{d}{dt} |\psi\rangle \right] \langle\psi, \zeta| + |\psi\rangle \left[i\hbar \frac{d}{dt} \langle\psi, \zeta| \right] = [\hat{H}, \hat{\rho}_\psi] \\ &\Rightarrow \left[i\hbar \frac{d}{dt} |\psi\rangle \right] \langle\psi, \zeta| - \hat{\rho}_\psi \hat{H} = [\hat{H}, \hat{\rho}_\psi] \\ &\Rightarrow \left[i\hbar \frac{d}{dt} |\psi\rangle \right] \langle\psi, \zeta| = \hat{H} |\psi\rangle \langle\psi, \zeta| \end{aligned} \quad (\text{A-II-4})$$

Now it is always possible to find one of the arbitrarily many states, $|\zeta_\perp\rangle$, normal to the state $|\zeta\rangle$, and to obtain the inner-product of Eq. (A-II-4) with $|\psi, \zeta\rangle |\zeta_\perp\rangle$, which since (A-II-1) results to,

$$\left[i\hbar \frac{d}{dt} |\psi\rangle \right] |\zeta_\perp\rangle = [\varepsilon_s |\psi\rangle + \tau |\zeta\rangle] |\zeta_\perp\rangle \quad (\text{A-II-5})$$

Since the state $|\zeta_\perp\rangle$ in Eq. (A-II-5) is arbitrary, it finally leads to Eq. (3), and similarly Eq. (4).

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Figures

Figure 1

The plot of probability of the system under measurement with its energy and the strength of interaction with the pointers. The pointer states are considered to be of energies 0.10, 0.25, 0.50, 0.75 of each order (powers of 10 in eV) of energies.