Near-resonant dynamics, period doubling and chaos of a 3-DOF vibro-impact system

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Abstract

A mechanical system composed of two weakly coupled oscillators under harmonic excitation is considered. Its main part is a vibro-impact unit composed of a linear oscillator with an internally colliding small block. This block is coupled with the secondary part being a damped linear oscillator. The mathematical model of the system has been presented in a non-dimensional form. The analytical studies are restricted to the case of a periodic steady-state motion with two symmetric impacts per cycle near 1:1 resonance. The multiple scales method combined with the sawtooth-function-based modelling of the non-smooth dynamics is employed. The approximate analytical solutions allow for stability analysis of the periodic motions. Moreover, the frequency-response curves and force-response curves with stable and unstable branches are determined, and the interplay between various model parameters is investigated. The theoretical predictions related to the motion amplitude and the range of stability of the periodic steady-state response is verified via a series of numerical experiments and computation of Lyapunov exponents.

Full Text

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Figures

Figure 1

The analyzed vibro-impact system

Figure 2

Description of the SIM of the system: a) the sawtooth function, b) the relation $B(C)$, c) the relation $\rho(\kappa)$

Figure 3

Geometric restrictions on C : a) degenerate (bold black) and non-degenerate (solid and dashed blue) solutions for $\kappa = 0.75$, b) the maximal value of C as a function of κ

Figure 4

[See Manuscript PDF file for full caption] Restrictions on C related to the stability of periodic motions.

Figure 5

Restrictions on C related to the stability of periodic motions: stability regions and stability boundaries for varying κ and ϵ

(a)

(b)

Figure 6

SIM of the system and different stability regions: a) the regions of stable fixed points (SFP) and stable periodic motions (SPM), b) the stable branch (bold) and two fixed points

Figure 7

[See Manuscript PDF file for full caption] Analytical solutions corresponding to the stable fixed point

Figure 8

[See Manuscript PDF file for full caption] Fixed points on the SIM of the system with stable (bold solid) and unstable (dashed) branches

Figure 9

[See Manuscript PDF file for full caption] Frequency-response curves of the system

Figure 10

[See Manuscript PDF file for full caption]

Figure 11

The effect of selected pairs of parameters on existence and stability of periodic solutions: no solutions (white), at least one stable solution (blue), one or two unstable solutions (gray)

Figure 12

[See Manuscript PDF file for full caption] Numerical solutions - time histories and phase portraits

Figure 13

[See Manuscript PDF file for full caption]

Figure 14

[See Manuscript PDF file for full caption] Phase portraits of periodic and chaotic solutions

Figure 15

[See Manuscript PDF file for full caption] Comparison of the analytical and numerical results