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Effect of the turbulent biological tissues on the propagation properties of Coherent Laguerre-Gaussian beams

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Abstract

In this paper, the effects of turbulent biological tissues (TBT) on the propagation properties of the coherent Laguerre-Gaussian (CLG) beams are studied. Based on the turbulence theory and using the power spectrum refractive-index model, the expression formulae of the average irradiance intensity and spreading properties of a CLG beam propagating in TBT are derived. The influence of propagation distance, beam orders, wavelengths and tissue turbulence parameters are then investigated numerically. It found that, the central dark zone of the circular/elliptical LG beams rises more rapidly as the propagation distance and the structural constant of the refractive index of the biological tissue increase and the beams become eventually more like Gaussian beams in the far-field under the influence of the turbulence biological tissues. Also, the numerical results proved that the effective beam spot radius increases as turbulence, wavelength, and propagation distance are increasing. Ultimately, the beams become circular under the influence of the turbulence of the biological tissue. As found that the effective beam spot radius along the x -axis becomes equal to that of the y -axis in high TBT which explain why an elliptical LG beam is converted into a circular one in higher structural constant of the turbulent tissue. Moreover, our results show that, the influence of the beam order m slightly greater than that of l on the beam spreading.

Keywords: Coherent Laguerre-Gaussian beams; Turbulent biological tissue; Irradiance distribution; Power spectrum refractive index; Effective beam spot radius.

1. Introduction

In the past years, the propagations of the light beams affected by a turbulent medium have been extensively studied due to their interesting applications (Andrews and Phillips 1998; Baykal 2005). These studies have been included several types of laser beams through various turbulence media such as Laguerre–Gaussian, flat-topped Gaussian, Hollow-Gaussian, Bessel–Laguerre–Gaussian and multi-Gaussian beams (Zhong et al. 2011;

Khannous and Belafhal 2016; Ez-zariy et al. 2016; Boufalah et al. 2018; Dong et al. 2019). In addition, many studies of this domain are recently published (Hricha et al. 2020; Li et al. 2020; Khandelwal 2020; Yang and Gao 2020). On the same context, it found that the biological tissue having a turbulent medium properties similar to the atmosphere and the ocean (Gökçe and Baykal 2018). In other words, the laser beam propagation interacts with the biological tissues in different forms such as reflection, refraction, interference, scattering, attenuation, polarisation, scintillation, etc. (Martelli et al. 2009; Korotkova 2014). Consequently, the light beams propagating in tissues have attracted increasing interest by researchers for several years, due to their potential applications in medical optics such as medical imagination and diagnosis (de Boer et al. 1997; Hitzenberger et al. 2001; Gao et al. 2006; Gao and Korotkova 2007).

In 1996, Schmitt and Kumar proposed a model of power spectrum refractive-index variations in biological turbulent tissue that was well fitted with Kolmogorov spectrum model in atmospheric turbulence (Schmitt and Kumar 1996). On the basis of this power spectrum model, many different optical beams are reported to describe their propagation properties in TBT. For instance, polarization, scintillation and spreading properties of optical beams propagating in TBT have been investigated (Gao et al. 2006; Gao and Korotkova 2007; Jin et al. 2016; Wu et al. 2016; Baykal et al. 2017; Yu and Zhang 2018; Arpali et al. 2020). Similar studies have been performed on the optical coherence tomography for disease diagnosis (de Boer et al. 1997; Hitzenberger et al. 2001). Gökçe and co-authors have investigated the liver tissue effects on the transmission for two different beams (Gökçe and Baykal 2018; Gökçe et al. 2020). As a significant result, they have proved that abnormalities such as cancer and tumor in a liver tissue can be diagnosed.

On the other hand, optical vortex beams have always attracted much attention due to their wide applications (Molina-Terriza et al. 2001; Yin et al. 2003; Gibson et al. 2004) and there are many methods to produce optical vortex family experimentally (Strohaber et al. 2007, Tokizane et al. 2009; Bekshaev et al. 2012). In this regard, through any biological tissue, the treatment of circular and elliptical of hollow and anomalous hollow Gaussian beams (Saad and Belafhal 2017; Lu et al. 2016) and the statistical properties of stochastic vortex beams (Luo et al. 2014) have been investigated. Other surveys have studied the propagation of vortex beams in biological tissues (Ni et al. 2019; Duan et al. 2020). However, the light field with the vortex formal of LG mode propagating in a turbulent tissue, to our knowledge, is not considered anywhere. The advantage of this mode is that a Laguerre polynomial, modulated by a Gaussian envelope and its properties has been widely suggested

in several studies of optical light fields and due to the possibility of its realization as a vortex beam.

In this work, we consider the CLG beams propagating through a turbulence of a biological tissue. These beams have high propagation stability and can carry the orbital angular momentum (Gbur and Tyson 2008; Chen et al. 2017; Li et al. 2018 a, b; Khandelwal 2020). We derive the analytical formulae using a new development integrals recently published by our research group (Belafhal et al. 2020). The present article is organized as the following: In Section 2, we define the electric field distribution of CLG beam in the cylindrical coordinates system at the source plane. The analytical calculations of the beam propagating through the turbulence of biological tissues, at a received plane are performed in Section 3. The formulae of the effective beam are derived in Section 4. In Section 5, some numerical examples are investigated to simulate the main formulas derived in the previous sections. Finally, a simple conclusion is outlined in the end of the paper.

2. Expression formula of CLG beam at the source plane

The electric field distribution of CLG beam in the cylindrical coordinates system at $z=0$ is expressed as (Takenaka et al. 1985; Zauderer 1986)

$$E(\rho, \theta, z=0) = E_0 \left(\frac{\sqrt{2}}{\omega_0} \rho \right)^l L_m^l \left(\frac{2\rho^2}{\omega_0^2} \right) \exp \left(-\frac{\rho^2}{\omega_0^2} \right) \exp(il\theta), \quad (1)$$

where E_0 is a constant (it can set $E_0=1$), ρ and θ are the radial and azimuthal coordinates, respectively, $L_m^l(\cdot)$ denotes the Laguerre polynomial with beam orders m and l , and ω_0 is the beam width of the fundamental Gaussian mode. By use of the following relation (Kimel and Elias 1993)

$$\rho^l L_m^l(\rho^2) e^{il\theta} = \frac{(-1)^m}{2^{2m+l} m!} \sum_{n=0}^m \sum_{t=0}^l i^t \binom{m}{n} \binom{l}{t} H_{2n+l-t}(x) H_{2m-2n+t}(y), \quad (2)$$

with $H_n(\cdot)$ is the Hermite polynomial and $\binom{m}{n}$ is the binomial coefficient, the field distribution indicated in Eq. (1) can be written as a superposition of series Laguerre-Gaussian modes and can be expressed as

$$E(x, y, z=0) = \frac{(-1)^m}{2^{2m+l} m!} \sum_{n=0}^m \sum_{t=0}^l i^t \binom{m}{n} \binom{l}{t} H_{2n+l-t}(\beta_x x) H_{2m-2n+t}(\beta_y y) \exp \left(-\frac{x^2 + y^2}{\omega_{0x} \omega_{0y}} \right), \quad (3)$$

where $\beta_x = \frac{\sqrt{2}}{\omega_{0x}}$ and, $\beta_y = \frac{\sqrt{2}}{\omega_{0y}}$ with ω_{0x} and ω_{0y} are the beam waist of the CLG beam in x and y directions, respectively. It is noted that, Eq. (3) represents the electric field of an elliptical LG beam that is symmetrical around $x = 0$. While, for $\omega_{0x} = \omega_{0y}$ the beam becomes circularly symmetrical. Fig.1 displays an example of the irradiance intensity distributions of a circular LG beam in the source plane for different beam orders l and m with $\omega_{0x} = \omega_{0y} = 2\mu\text{m}$. One finds from Fig.1 that, with zero orders, the CLG beam will be reduced to a fundamental Gaussian beam (see Fig. 1 (a)) whereas for the nonzero orders, the beam has a central dark spot surrounded by a number of the bright rings. As can be seen from Fig. 1, when the values of beam orders l and m vary, the central dark spot area, maximum intensity, and number of the bright rings ($m+1$) vary.

Figure 1: Irradiance intensity distribution of a circular LG beam at the source plane ($z=0$) with $\omega_{0x} = \omega_{0y} = 2\mu\text{m}$: (a) $l = m = 0$, (b) $l = m = 1$ and (c) $l = m = 4$.

3. Analytical formulae of CLG beam in TBT

According to the extended Huygens-Fresnel principle, upon propagation from the source plane to any receiver plane with $z > 0$ and by the paraxial approximation, the CLG field takes the form (Andrews and Phillips 2005; Qu et al. 2010; Duan et al. 2020)

$$\begin{aligned} \langle I(\rho_x, \rho_y, z) \rangle &= \frac{k^2}{4\pi^2 z^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E(x_1, y_1, 0) E^*(x_2, y_2, 0) \exp \left[-\frac{ik}{2z} (x_1 - \rho_x)^2 - \frac{ik}{2z} (y_1 - \rho_y)^2 \right] \\ &\quad \times \exp \left[\frac{ik}{2z} (x_2 - \rho_x)^2 + \frac{ik}{2z} (y_2 - \rho_y)^2 \right] \langle \exp[\psi(x_1, y_1, \rho_x, \rho_y) + \psi^*(x_2, y_2, \rho_x, \rho_y)] \rangle dx_1 dy_1 dx_2 dy_2, \end{aligned} \quad (4)$$

where

$$\langle \exp[\psi(x_1, y_1, \rho_x, \rho_y) + \psi^*(x_2, y_2, \rho_x, \rho_y)] \rangle = \exp \left[-\frac{1}{\rho_0^2} (x_1 - x_2)^2 - \frac{1}{\rho_0^2} (y_1 - y_2)^2 \right], \quad (5)$$

with

$$|\rho_0| = 0.22(C_n^2 k^2 z)^{-1/2}, \quad (6a)$$

and

$$C_n^2 = \frac{\langle \delta n^2 \rangle}{L_0(2 - \varsigma)}. \quad (6b)$$

In Eqs. (4) and (5), x_1, x_2, y_1, y_2 and ρ_x, ρ_y are the position parameters at the source and output planes, $\langle \cdot \rangle$ denotes an ensemble average along the turbulent media and ρ_0 is the coherence length of a spherical wave propagating through the tissue turbulence. In Eq. (6b), C_n^2 is the structural constant of the refractive index of the biological tissue, δn^2 is the variance of the refractive index, L_0 denotes the external scale of the refractive-index size and ς is the fractal standard of the tissue.

By substituting Eq. (3) into Eq. (4), the average irradiance distribution of CLG beams propagating in the turbulent biological tissue can be written as the follows

$$\begin{aligned} \langle I(\rho_x, \rho_y, z) \rangle &= \frac{k^2}{4\pi^2 z^2} \frac{1}{2^{4m+2l} (m!)^2} \sum_{n=0}^m \sum_{t=0}^l \sum_{h=0}^m \sum_{s=0}^l \binom{m}{n} \binom{l}{t} \binom{m}{h} \binom{l}{s} i^t (-i)^s \\ &\int_{-\infty-\infty}^{+\infty+\infty} \int_{-\infty-\infty}^{+\infty+\infty} H_{2n+l-t}(\beta_x x_1) \exp \left[-a_x x_1^2 + 2 \left(u_x + \frac{x_2}{\rho_0^2} \right) x_1 \right] H_{2h+l-s}(\beta_x x_2) \exp \left(-a_x^* x_2^2 - \frac{ik\rho_x}{z} x_2 \right) dx_1 dx_2 \\ &\int_{-\infty-\infty}^{+\infty+\infty} \int_{-\infty-\infty}^{+\infty+\infty} H_{2m-2n+t}(\beta_y y_1) \exp \left[-a_y y_1^2 + 2 \left(u_y + \frac{y_2}{\rho_0^2} \right) y_1 \right] H_{2m-2h+s}(\beta_y y_2) \exp \left(-a_y^* y_2^2 - \frac{ik\rho_y}{z} y_2 \right) dy_1 dy_2, \end{aligned} \quad (7)$$

with

$$a_x = \frac{1}{\omega_{0x}^2} + \frac{ik}{2z} + \frac{1}{\rho_0^2}, \quad (8a)$$

$$a_y = \frac{1}{\omega_{0y}^2} + \frac{ik}{2z} + \frac{1}{\rho_0^2}, \quad (8b)$$

$$u_x = \frac{ik\rho_x}{2z}, \quad (8c)$$

and

$$u_y = \frac{ik\rho_y}{2z}. \quad (8d)$$

Using the following integrals and expansion formulae (Belafhal et al. 2020; Erdelyi et al. 1954; Abramowitz 1970)

$$\int_{-\infty}^{+\infty} H_m(\alpha x) e^{-px^2+2qx} dx = e^{\frac{q^2}{p}} \sqrt{\frac{\pi}{p}} \left[1 - (\alpha^2/p) \right]^{\frac{m}{2}} H_m \left[\frac{\alpha q}{p \sqrt{1 - (\alpha^2/p)}} \right], \quad (9a)$$

$$H_n(x+y) = \frac{1}{2^{n/2}} \sum_{k=0}^n \binom{n}{k} H_k(\sqrt{2}x) H_{n-k}(\sqrt{2}y), \quad (9b)$$

$$H_n(x) = \sum_{k=0}^{\lfloor n/2 \rfloor} (-1)^k \frac{n!}{k!(n-2k)!} (2x)^{n-2k}, \quad (9c)$$

$$\int_{-\infty}^{+\infty} x^l H_m(\alpha x) e^{-px^2+2qx} dx = \frac{e^{q^2/p}}{2^l} \sqrt{\frac{\pi}{q}} \sum_{k=0}^{\lfloor m/2 \rfloor} \frac{(-1)^k m!}{k!(m-2k)!} \left(\frac{\alpha}{i\sqrt{p}} \right)^{m+l-2k} H_{m+l-2k} \left(\frac{iq}{\sqrt{p}} \right), \quad (9d)$$

and

$$\int_{-\infty}^{+\infty} x^l \exp[-(x-y)^2] dx = \sqrt{\pi} (2i)^{-l} H_l(iy), \quad (9e)$$

and after tedious algebraic calculations over x_1, x_2, y_1, y_2 , Eq. (7) is reduced to

$$\begin{aligned} \langle I(\rho_x, \rho_y, z) \rangle &= \frac{k^2}{4z^2} \frac{1}{2^{s\left(\frac{m+l}{2}\right)} (m!)^2} \frac{1}{\sqrt{a_x a_y b_x b_y}} \exp\left(\frac{u_x^2}{a_x} + \frac{\alpha_x^2}{b_x}\right) \exp\left(\frac{u_y^2}{a_y} + \frac{\alpha_y^2}{b_y}\right) \\ &\times \sum_{n=0}^m \sum_{l=0}^l \sum_{h=0}^m \sum_{s=0}^l \sum_{\mu=0}^{2n+l-t} \sum_{\sigma=0}^{2m-2n+t} \sum_{f_1=0}^{\lfloor \mu/2 \rfloor} \sum_{g_1=0}^{\lfloor (2h+l-s)/2 \rfloor} \sum_{f_2=0}^{\lfloor \sigma/2 \rfloor} \sum_{g_2=0}^{\lfloor (2m-2h+s)/2 \rfloor} \binom{m}{n} \binom{l}{t} \binom{m}{h} \binom{l}{s} \binom{2n+l-t}{\mu} \binom{2m-2n+t}{\sigma} \\ &\times \frac{i^l (-i)^s (-1)^{f_1+g_1+f_2+g_2}}{2^{\mu+\sigma-2f_1-2f_2}} \frac{\mu!}{f_1!(\mu-2f_1)!} \frac{(2h+l-s)!}{g_1!(2h+l-s-2g_1)!} \frac{\sigma!}{f_2!(\sigma-2f_2)!} \frac{(2m-2h+s)!}{g_2!(2m-2h+s-2g_2)!} \\ &\times \left(1 - \frac{\beta_x^2}{a_x}\right)^{\frac{2n+l-t}{2}} \left(1 - \frac{\beta_y^2}{a_y}\right)^{\frac{2m-2n+t}{2}} \left(\frac{2\sqrt{2}\gamma_x}{\rho_0^2}\right)^{\mu-2f_1} \left(\frac{2\sqrt{2}\gamma_y}{\rho_0^2}\right)^{\sigma-2f_2} \left(\frac{\beta_x}{i\sqrt{b_x}}\right)^{2h+l-s+\mu-2f_1-2g_1} \left(\frac{\beta_y}{i\sqrt{b_y}}\right)^{2m-2h+s+\sigma-2f_2-2g_2} \\ &\times H_{2n+l-t-\mu}(\sqrt{2}\gamma_x u_x) H_{2h+l-s+\mu-2f_1-2g_1}\left(\frac{i\alpha_x}{\sqrt{b_x}}\right) H_{2m-2n+t-\sigma}(\sqrt{2}\gamma_y u_y) H_{2m-2h+s+\sigma-2f_2-2g_2}\left(\frac{i\alpha_y}{\sqrt{b_y}}\right), \end{aligned} \quad (10)$$

where

$$\gamma_x = \frac{\beta_x}{a_x \sqrt{1 - \frac{\beta_x^2}{a_x}}}, \quad (11a)$$

$$\gamma_y = \frac{\beta_y}{a_y \sqrt{1 - \frac{\beta_y^2}{a_y}}}, \quad (11b)$$

$$b_x = a_x^* - \frac{1}{a_x \rho_0^4}, \quad (11c)$$

$$b_y = a_y^* - \frac{1}{a_y \rho_0^4}, \quad (11d)$$

$$\alpha_x = u_x \left(\frac{1}{a_x \rho_0^2} - 1 \right), \quad (11e)$$

and

$$\alpha_y = u_y \left(\frac{1}{a_y \rho_0^2} - 1 \right). \quad (11f)$$

Eq. (10) represents the theoretical results of the CLG beams propagating in TBT with $C_n^2 > 0$. This equation is the main result of the present work. Note that when $C_n^2 = 0$, Eq. (10) reduces to the propagation expression of a CLG beams in free space.

4. Square root of the beam spot radius of CLG beam in TBT

To understand the characterization of the dispersion properties of the CLG beams, we further derived the square root formula to investigate the effective beam spot radius of the considered beam. The effective beam spot radius of a CLG beam in TBT is defined as (Lu et al. 2016)

$$W_r(z) = \sqrt{\frac{2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} r^2 \langle I(x, y, z) \rangle dx dy}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle I(x, y, z) \rangle dx dy}}, \text{ with } r = (x, y). \quad (12)$$

On substituting from Eq. (10) into Eq. (12), one obtains the following expressions for the effective beam spot radius of the CLG beam after propagation

$$W_x(z) = \sqrt{\frac{A_x(z)}{A_z(z)}}, \quad (13a)$$

and

$$W_y(z) = \sqrt{\frac{A_y(z)}{A_z(z)}}. \quad (13b)$$

The quantities $W_x(z)$ and $W_y(z)$ are the effective beam spot radius along the x- and y-axes, with

$$\begin{aligned}
A_x(z) = & \frac{k^2}{2z^2} \frac{1}{2^{5(m+0.5l)} (m!)^2} \frac{\pi}{\sqrt{a_x a_y b_x b_y}} \\
& \times \sum_{n=0}^m \sum_{t=0}^l \sum_{h=0}^m \sum_{s=0}^l \sum_{\mu=0}^{2n+l-t} \sum_{\sigma=0}^{2m-2n+t} \sum_{f_1=0}^{\lfloor \mu/2 \rfloor} \sum_{g_1=0}^{\lfloor (2h+l-s)/2 \rfloor} \sum_{f_2=0}^{\lfloor \sigma/2 \rfloor} \sum_{g_2=0}^{\lfloor (2m-2h+s)/2 \rfloor} \sum_{c_1=0}^{\lfloor (2n+l-t-\mu)/2 \rfloor} \sum_{c_2=0}^{\lfloor (2h+l-s+\mu-2f_1-2g_1)/2 \rfloor} \\
& \times \sum_{c_3=0}^{\lfloor (2m-2n+t-\sigma)/2 \rfloor} \sum_{c_4=0}^{\lfloor (2m-2h+s+\sigma-2f_2-2g_2)/2 \rfloor} \binom{m}{n} \binom{l}{t} \binom{m}{h} \binom{l}{s} \binom{2n+l-t}{\mu} \binom{2m-2n+t}{\sigma} i^t (-i)^s \\
& \times \frac{(-1)^{f_1+f_2+g_1+g_2+c_1+c_2+c_3+c_4} (2i)^{2f_1+2f_2+2g_1+2g_2+2c_1+2c_2+2c_3+2c_4-4m-2l-2}}{2^{\mu+\sigma-2f_1-2f_2}} \left(1 - \frac{\beta_x^2}{a_x}\right)^{\frac{2n+l-t}{2}} \left(1 - \frac{\beta_y^2}{a_y}\right)^{\frac{2m-2n+t}{2}} \\
& \times \frac{\mu!}{f_1! (\mu-2f_1)!} \frac{(2h+l-s)!}{g_1! (2h+l-s-2g_1)!} \frac{\sigma!}{f_2! (\sigma-2f_2)!} \frac{(2m-2h+s)!}{g_2! (2m-2h+s-2g_2)!} \\
& \times \frac{(2n+l-t-\mu)!}{c_1! (2n+l-t-\mu-2c_1)!} \frac{(2h+l-s+\mu-2f_1-2g_1)!}{c_2! (2h+l-s+\mu-2f_1-2g_1-2c_2)!} \left(\frac{2\sqrt{2}\gamma_x}{\rho_0^2}\right)^{\mu-2f_1} \\
& \times \frac{(2m-2n+t-\sigma)!}{c_3! (2m-2n+t-\sigma-2c_3)!} \frac{(2m-2h+s+\sigma-2f_2-2g_2)!}{c_4! (2m-2h+s+\sigma-2f_2-2g_2-2c_4)!} \left(\frac{2\sqrt{2}\gamma_y}{\rho_0^2}\right)^{\sigma-2f_2} \\
& \times \left(\frac{\beta_x}{i\sqrt{b_x}}\right)^{2h+l-s+\mu-2f_1-2g_1} \left(\frac{\beta_y}{i\sqrt{b_y}}\right)^{2m-2h+s+\sigma-2f_2-2g_2} \left(\frac{\sqrt{2}ik\gamma_x}{z}\right)^{2n+l-t-\mu-2c_1} \\
& \times \left(\frac{\sqrt{2}ik\gamma_y}{z}\right)^{2m-2n+t-\sigma-2c_3} (2iq_x)^{2h+l-s+\mu-2f_1-2g_1-2c_2} (2iq_y)^{2m-2h+s+\sigma-2f_2-2g_2-2c_4} \\
& \times \left(\frac{1}{\sqrt{\delta_x^2 + q_x^2}}\right)^{2n+2l+2h-t-s-2f_1-2g_1-2c_1-2c_2+3} \left(\frac{1}{\sqrt{\delta_y^2 + q_y^2}}\right)^{2m-2n+t+2m-2h+s-2f_2-2g_2-2c_3-2c_4+1} \\
& \times H_{2n+l-t+2h+l-s-2f_1-2g_1-2c_1-2c_2+2}(0) H_{4m-2h-2n+t+s-2f_2-2g_2-2c_3-2c_4}(0),
\end{aligned} \tag{14}$$

$$\begin{aligned}
A_y(z) = & \frac{k^2}{2z^2} \frac{1}{2^{5(m+0.5l)} (m!)^2} \frac{\pi}{\sqrt{a_x a_y b_x b_y}} \\
& \times \sum_{n=0}^m \sum_{t=0}^l \sum_{h=0}^m \sum_{s=0}^l \sum_{\mu=0}^{2n+l-t} \sum_{\sigma=0}^{2m-2n+t} \sum_{f_1=0}^{\lfloor \mu/2 \rfloor} \sum_{g_1=0}^{\lfloor (2h+l-s)/2 \rfloor} \sum_{f_2=0}^{\lfloor \sigma/2 \rfloor} \sum_{g_2=0}^{\lfloor (2m-2h+s)/2 \rfloor} \sum_{c_1=0}^{\lfloor (2n+l-t-\mu)/2 \rfloor} \sum_{c_2=0}^{\lfloor (2h+l-s+\mu-2f_1-2g_1)/2 \rfloor} \\
& \times \sum_{c_3=0}^{\lfloor (2m-2n+t-\sigma)/2 \rfloor} \sum_{c_4=0}^{\lfloor (2m-2h+s+\sigma-2f_2-2g_2)/2 \rfloor} \binom{m}{n} \binom{l}{t} \binom{m}{h} \binom{l}{s} \binom{2n+l-t}{\mu} \binom{2m-2n+t}{\sigma} i^t (-i)^s \\
& \times \frac{(-1)^{f_1+f_2+g_1+g_2+c_1+c_2+c_3+c_4} (2i)^{2f_1+2f_2+2g_1+2g_2+2c_1+2c_2+2c_3+2c_4-4m-2l-2}}{2^{\mu+\sigma-2f_1-2f_2}} \\
& \times \frac{\mu!}{f_1! (\mu-2f_1)!} \frac{(2h+l-s)!}{g_1! (2h+l-s-2g_1)!} \frac{\sigma!}{f_2! (\sigma-2f_2)!} \frac{(2m-2h+s)!}{g_2! (2m-2h+s-2g_2)!} \\
& \times \frac{(2n+l-t-\mu)!}{c_1! (2n+l-t-\mu-2c_1)!} \frac{(2h+l-s+\mu-2f_1-2g_1)!}{c_2! (2h+l-s+\mu-2f_1-2g_1-2c_2)!} \left(\frac{2\sqrt{2}\gamma_x}{\rho_0^2}\right)^{\mu-2f_1} \\
& \times \frac{(2m-2n+t-\sigma)!}{c_3! (2m-2n+t-\sigma-2c_3)!} \frac{(2m-2h+s+\sigma-2f_2-2g_2)!}{c_4! (2m-2h+s+\sigma-2f_2-2g_2-2c_4)!} \left(\frac{2\sqrt{2}\gamma_y}{\rho_0^2}\right)^{\sigma-2f_2} \\
& \times \left(1 - \frac{\beta_x^2}{a_x}\right)^{\frac{2n+l-t}{2}} \left(1 - \frac{\beta_y^2}{a_y}\right)^{\frac{2m-2n+t}{2}} \left(\frac{\beta_x}{i\sqrt{b_x}}\right)^{2h+l-s+\mu-2f_1-2g_1} \left(\frac{\beta_y}{i\sqrt{b_y}}\right)^{2m-2h+s+\sigma-2f_2-2g_2} \\
& \times \left(\frac{\sqrt{2}ik\gamma_x}{z}\right)^{2n+l-t-\mu-2c_1} \left(\frac{\sqrt{2}ik\gamma_y}{z}\right)^{2m-2n+t-\sigma-2c_3} (2iq_x)^{2h+l-s+\mu-2f_1-2g_1-2c_2} (2iq_y)^{2m-2h+s+\sigma-2f_2-2g_2-2c_4} \\
& \times \left(\frac{1}{\sqrt{\delta_x^2 + q_x^2}}\right)^{2n+2l+2h-t-s-2f_1-2g_1-2c_1-2c_2+1} \left(\frac{1}{\sqrt{\delta_y^2 + q_y^2}}\right)^{2m-2n+t+2m-2h+s-2f_2-2g_2-2c_3-2c_4+3} \\
& \times H_{2n+l-t+2h+l-s-2f_1-2g_1-2c_1-2c_2}(0) H_{4m-2h-2n+t+s-2f_2-2g_2-2c_3-2c_4+2}(0),
\end{aligned} \tag{15}$$

and

$$\begin{aligned}
A_z(z) = & \frac{k^2}{4z^2} \frac{1}{2^{5(m+0.5l)}} \frac{\pi}{(m!)^2 \sqrt{a_x a_y b_x b_y}} \\
& \times \sum_{n=0}^m \sum_{t=0}^l \sum_{h=0}^m \sum_{s=0}^l \sum_{\mu=0}^{2n+l-t} \sum_{\sigma=0}^{2m-2n+t} \sum_{f_1=0}^{\lfloor \mu/2 \rfloor} \sum_{g_1=0}^{\lfloor (2h+l-s)/2 \rfloor} \sum_{f_2=0}^{\lfloor \sigma/2 \rfloor} \sum_{g_2=0}^{\lfloor (2m-2h+s)/2 \rfloor} \sum_{c_1=0}^{\lfloor (2n+l-t-\mu)/2 \rfloor} \sum_{c_2=0}^{\lfloor (2h+l-s+\mu-2f_1-2g_1)/2 \rfloor} \\
& \times \sum_{c_3=0}^{\lfloor (2m-2n+t-\sigma)/2 \rfloor} \sum_{c_4=0}^{\lfloor (2m-2h+s+\sigma-2f_2-2g_2)/2 \rfloor} \binom{m}{n} \binom{l}{t} \binom{m}{h} \binom{l}{s} \binom{2n+l-t}{\mu} \binom{2m-2n+t}{\sigma} i^t (-i)^s \\
& \times \frac{(-1)^{f_1+f_2+g_1+g_2+c_1+c_2+c_3+c_4} (2i)^{2f_1+2f_2+2g_1+2g_2+2c_1+2c_2+2c_3+2c_4-4m-2l-2}}{2^{\mu+\sigma-2f_1-2f_2}} \\
& \times \frac{\mu!}{f_1!(\mu-2f_1)!} \frac{(2h+l-s)!}{g_1!(2h+l-s-2g_1)!} \frac{\sigma!}{f_2!(\sigma-2f_2)!} \frac{(2m-2h+s)!}{g_2!(2m-2h+s-2g_2)!} \\
& \times \frac{(2n+l-t-\mu)!}{c_1!(2n+l-t-\mu-2c_1)!} \frac{(2h+l-s+\mu-2f_1-2g_1)!}{c_2!(2h+l-s+\mu-2f_1-2g_1-2c_2)!} \left(\frac{2\sqrt{2}\gamma_x}{\rho_0^2} \right)^{\mu-2f_1} \\
& \times \frac{(2m-2n+t-\sigma)!}{c_3!(2m-2n+t-\sigma-2c_3)!} \frac{(2m-2h+s+\sigma-2f_2-2g_2)!}{c_4!(2m-2h+s+\sigma-2f_2-2g_2-2c_4)!} \left(\frac{2\sqrt{2}\gamma_y}{\rho_0^2} \right)^{\sigma-2f_2} \\
& \times \left(1 - \frac{\beta_x^2}{a_x} \right)^{\frac{2n+l-t}{2}} \left(1 - \frac{\beta_y^2}{a_y} \right)^{\frac{2m-2n+t}{2}} \left(\frac{\beta_x}{i\sqrt{b_x}} \right)^{2h+l-s+\mu-2f_1-2g_1} \left(\frac{\beta_y}{i\sqrt{b_y}} \right)^{2m-2h+s+\sigma-2f_2-2g_2} \\
& \times \left(\frac{\sqrt{2}ik\gamma_x}{z} \right)^{2n+l-t-\mu-2c_1} \left(\frac{\sqrt{2}ik\gamma_y}{z} \right)^{2m-2n+t-\sigma-2c_3} (2iq_x)^{2h+l-s+\mu-2f_1-2g_1-2c_2} (2iq_y)^{2m-2h+s+\sigma-2f_2-2g_2-2c_4} \\
& \times \left(\frac{1}{\sqrt{\delta_x^2 + q_x^2}} \right)^{2n+2l+2h-t-s-2f_1-2g_1-2c_1-2c_2+1} \left(\frac{1}{\sqrt{\delta_y^2 + q_y^2}} \right)^{2m-2n+t+2m-2h+s-2f_2-2g_2-2c_3-2c_4+1} \\
& \times H_{2n+l-t+2h+l-s-2f_1-2g_1-2c_1-2c_2}(0) H_{4m-2h-2n+t+s-2f_2-2g_2-2c_3-2c_4}(0).
\end{aligned} \tag{16}$$

The quantity $A_z(z)$ in Eq. (13) denotes to the average irradiance at the received plane.

5. Numerical results and dissection

In this section, some numerical examples are performed to illustrate the theoretical results given by Eqs. (10) and (13). Fig. 2 represents the cross line ($y=0$) and corresponding contour graphs of the normalized irradiance distribution in the two dimensional and its corresponding contour graph at the receiver plane of the circular LG beam in TBT at different propagation distances. The calculation parameters are set to be: $\lambda=632.8 \text{ nm}$, $m = l = 1$, $\omega_x = \omega_y = 2\mu\text{m}$, and $C_n^2 = 5 \times 10^{-7} \mu\text{m}^{-2/3}$. It is noted that the outer beams appear different curves for a slightly increase with propagation distance. It means that the beams no longer keep their profile shapes during the propagation. For example, as the propagation distance increases to, about $z=50\mu\text{m}$, the main hollow part of the circular LG beam rises more rapidly.

Figure 2: Normalized irradiance distributions at cross line $y=0$ and corresponding contour graphs for a circular LG beam in TBT at several propagation distances z , with $m = l = 1$, $\omega_x = \omega_y = 2 \mu\text{m}$, $\lambda = 632.8 \text{ nm}$ and $C_n^2 = 5 \times 10^{-7} \mu\text{m}^{-2/3}$.

Further, the beam has a flat-topped profile at $z=400\mu\text{m}$, and with sufficiently large propagation distance (from about $z > 1500\mu\text{m}$), the beam profile becomes more focused and it is similar to the conventional Gaussian profile. To sum up, the propagation distance plays an important role in the beam shaping properties as it propagates in biological tissues.

Fig. 3 displays the contour graph of the irradiance distribution of an elliptical LG beam propagating in TBT, at several propagation distances: $z = 1, 50, 100, 1000, 2000$ and $5000 \mu\text{m}$, with $\omega_x = 5\mu\text{m}$, $\omega_y = 2\mu\text{m}$ and the other parameters are the same as in Fig.2.

Figure 3: Contour graph of irradiance distributions for an elliptical LG beam in TBT at several propagation distances z with $m = l = 1$, $\omega_x = 5\mu\text{m}$, $\omega_y = 2\mu\text{m}$, $\lambda = 32.8 \text{ nm}$ and $C_n^2 = 5 \times 10^{-7} \mu\text{m}^{-2/3}$.

We can see from this figure that, the elliptical LG profile can keep its original shape during propagation; only at the small propagation distances (about up to $z=10\mu\text{m}$) during its propagation in tissues. That means that the pattern of the beam depends strongly on the propagation distance. Also, it can be clearly observed from Fig. 3 that the first bright ring of the elliptical LG beam separates into two main lobes with increasing distance from the source (about at $z = 100\mu\text{m}$). Furthermore, we can observe, at large values of z ($z \geq 5000\mu\text{m}$), the beam will gradually convert into a Gaussian-like beam. Thus, we can deduce from Figs. 2 and

3 that, in the far field, the circular and the elliptical LG beam in TBT tend to a circular Gaussian beam.

Fig. 4 presents the cross line ($y=0$) normalized irradiance distribution of a circular LG beam propagating in TBT at a fixed propagation distance $z=100\text{ }\mu\text{m}$ for different values of the structural constant C_n^2 , and beam order m . The other parameters are set as $\lambda=632.8\text{ nm}$, $\omega_x = \omega_y = 2\text{ }\mu\text{m}$ and $l=1$.

Figure 4: Normalized irradiance distributions at cross line $y=0$ of a circular LG beam in turbulent biological tissues at $z = 100\text{ }\mu\text{m}$, with $l=1$, $\lambda = 632.8\text{ nm}$, $\omega_x = \omega_y = 2\text{ }\mu\text{m}$, for different values of m and C_n^2 :
 (a) $C_n^2 = 3 \times 10^{-7} \text{ }\mu\text{m}^{-2/3}$, (b) $C_n^2 = 7 \times 10^{-7} \text{ }\mu\text{m}^{-2/3}$, (c) $C_n^2 = 1 \times 10^{-6} \text{ }\mu\text{m}^{-2/3}$ and (d) $C_n^2 = 5 \times 10^{-5} \text{ }\mu\text{m}^{-2/3}$.

It is found that the effect of the beam order m becomes gradually unimportant when C_n^2 more increases, until it reaches a high value ($C_n^2 = 5 \times 10^{-5} \text{ }\mu\text{m}^{-2/3}$), the effect of m almost vanishes. Also, it is clearly seen that, circular LG beam exhibits minimal oscillation as C_n^2 increases which means that the influence of the turbulent tissue on the irradiance distribution

of the beam is strong. It is also observed that the propagation behaviour of a circular LG beam in biological tissues is exactly similar to the Gaussian beam in strongly tissue turbulence (see Fig. 4 (d)).

Similarly, Fig. 5 displays the normalized irradiance at cross line $y=0$ of the coherent circular LG beam with $m=1$ and for different beam orders l . The other parameters are chosen to be similar to those for Fig. 4. In general, it can be observed that the two parameters l and m have almost the same effect on the LG beam propagation in TBT. However, with more attention to the curve details of both figures, would show that the beam order l has an effect lower than that of m , on the behaviour of the beam, as seen from Figs. 4 (a), (b) and 5. (a), (b). In addition, from Figs. 4 and 5, one can be found that the effect of the orders m and l on the beam profiles gradually decreases with the increase in C_n^2 .

Figure 5: Normalized irradiance distributions at cross line $y=0$ of a circular LG beam in turbulent biological tissues for varying l and $m=1$ with $z = 100\mu\text{m}$, $\lambda = 632.8\text{ nm}$, $\omega_x = \omega_y = 2\mu\text{m}$ and for four values of C_n^2 :

(a) $C_n^2 = 3 \times 10^{-7} \mu\text{m}^{-2/3}$, (b) $C_n^2 = 7 \times 10^{-7} \mu\text{m}^{-2/3}$, (c) $C_n^2 = 1 \times 10^{-6} \mu\text{m}^{-2/3}$ and (d) $C_n^2 = 5 \times 10^{-5} \mu\text{m}^{-2/3}$.

On the other hand, based on our analytical expressions given by Eqs. (13), (14) and (16), numerical calculations for the square root beam width are simulated to investigate the effective beam width of the considered beam against the changes in the tissue turbulence and the laser beam parameters. Fig. 6 shows the effective beam spot radius of an elliptical LG beam on propagation distance z in biological tissues for different values of the structural constant C_n^2 with $m=l=1$, $\lambda = 632.8\text{nm}$, $\omega_{0x} = 5\mu\text{m}$ and $\omega_{0y} = 2\mu\text{m}$. In this figure, it can be inferred that, for a propagation distance given, the beam spot radius obviously reduces when C_n^2 decreases, which is clear from the curves of the free space cases of both Fig. 6. This result implies that the quality of the beam obviously depends on the tissue types (Air-tissue, cornea, tendon, cartilage, nerve, bone, tooth, etc.). As seen from the curves in this figure, $W_x(z)$ becomes equal to $W_y(z)$ in biological tissues at the far field. This observation is consistent with the discussed results of previous Figs. 2 and 3 and with the results of Refs. (Lu et al 2016; Saad et al. 2017).

Figure 6: Effective beam spot radius $W_x(z)$ and $W_y(z)$ of an elliptical LG beam on propagation distances z in turbulent biological tissues for different values of C_n^2 with $m=l=1$, $\lambda = 632.8\text{nm}$, $\omega_{0x} = 5\mu\text{m}$ and $\omega_{0y} = 2\mu\text{m}$: (a) $W_x(z)$ and (b) $W_y(z)$.

Fig. 7 presents the effective beam width $W_x(z)$ of a circular LG beam versus the propagation distance z in biological tissues for different values of wavelength. From this figure, we can easily observe the dependence of the effective beam spot radius $W_x(z)$ of circular LG beam on the wavelength and the propagation distance in TBT, where the examiner can find that, the large wavelength beam ($\lambda = 1550\text{nm}$) corresponds to the minimum

effective spot $W_x(z)$ (dash curve). For all the propagation distances, the effective beam expanding with the increasing of z that is entirely predictable.

Figure 7: Effective beam spot radius $W_x(z)$ of the circular LG beam on propagation distances z in turbulent biological tissues for different values of the wavelength with: $\omega_x = \omega_y = 2\mu m$, and $C_n^2 = 5 \times 10^{-6} \mu m^{-2/3}$.

Fig. 8 illustrates the effective beam spot radius $W_x(z)$ of a circular LG beam on propagation distance z in turbulent biological tissues with $C_n^2 = 5 \times 10^{-6} \mu m^{-2/3}$, $\lambda = 632.8nm$, $\omega_{0x} = 2\mu m$ and: $\omega_{0y} = 2\mu m$ (a) for different values of m with $l = 1$; and (b) for different values of l with $m = 1$.

Figure 8: Effective beam spot radius $W_x(z)$ of a circular LG beam on propagation distances z in turbulent biological tissues with $C_n^2 = 5 \times 10^{-6} \mu m^{-2/3}$, $\lambda = 632.8nm$, $\omega_{0x} = 2\mu m$ and $\omega_{0y} = 2\mu m$: for different (a) m with $l = 1$, and (b) l with $m = 1$.

We note from Fig. 8 that, the effective beam increases with the increasing of the beam orders m and l . However, the effect of the varying of m on the effective beam $W_x(z)$ is greater than that of l . Finally, from all the figures concerning the effective beam spot radius, one notes that the root square of the beam spot radius has larger values for longer propagation distances, stronger structural constant of the biological tissue, and higher wavelengths and beam orders.

6. Conclusion

In this work, we have employed a convenient way to derive the analytical formulae for the propagation and the effective beam spot radius of a CLG beam in TBT based on the turbulence theories using the power spectrum refractive-index model. Depending on these formulae, we have discussed numerically, the irradiance distribution and the expanding properties of the CLG beams in the turbulent biological tissue, respectively. The numerical results of the propagation part demonstrated that the irradiance distribution profile of the coherent circular /elliptical LG beams eventually becomes like the Gaussian-distribution profile, in the far-field and under the powerful of tissue turbulence. Another aspect, the simulation of the effective beam part proved that the effective beam spot radius increases with the increasing of the propagation distance, wavelength, the structural constant C_n^2 and both beam orders. Our numerical results of this part also show that the effective beam spot radius $W_x(z)$ becomes equal to that of $W_y(z)$ in biological tissues. That is the reason why the beam profile of an elliptical beam propagating in TBT will eventually become circular. Finally, it is found that the beam order l has a relatively weak effect on the effective beam than that of m . In general, our numerical result demonstrated that the average intensity and effective spot radius of CLG beams in TBT are more sensitive to turbulent biological tissue and propagation distance parameters what makes us believe that the CLG beams more suitable to interact with biological tissues.

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Figures

Figure 1

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Figure 2

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Figure 3

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