An improved blind Gaussian source separation approach based on generalized Jaccard similarity

Xudan Fu
Xidian University
Jimin Ye (jmye@mail.xidian.edu.cn)
Xidian University
Jianwei E
Guangxi Minzu University

Research Article

Keywords: Blind source separation, Generalized Jaccard similarity, FSBLE, Imperialist competitive algorithm

Posted Date: August 4th, 2022

DOI: https://doi.org/10.21203/rs.3.rs-1725577/v2

An improved blind Gaussian source separation approach based on generalized Jaccard similarity

Xudan Fu[1], Jimin Ye[1*] and Jianwei E[2]

[1,1*]School of Mathematics and Statistics, Xidian University, Xi'an, 710071, China.
[2]School of Mathematics and Physics, Guangxi Minzu University, Nanning, 530006, China.

*Corresponding author(s). E-mail(s): jmye@mail.xidian.edu.cn;
Contributing authors: fuxudan_math@163.com;
jianwei_math@gxun.edu.cn;

Abstract

Blind source separation (BSS) consists of recovering the independent source signals from their linear mixtures with unknown mixing channel. The existing BSS approaches rely on the fundamental assumption: the source signals are non-Gaussian，this limited the use of BSS seriously. To overcome this problem and the weakness of cosine index in measuring the dynamic similarity of signals, this study proposes the fuzzy statistical behavior of local extremum (FSBLE) based on generalized Jaccard similarity as the measure of signal's similarity to implement the separation of source signals. In particular, the imperialist competition algorithm is introduced to minimize the cost function which jointly considers the stationarity factor describing the dynamical similarity of each source signal separately and the independency factor describing the dynamical similarity between source signals. Simulation experiments on synthetic nonlinear chaotic Gaussian data and ECG signals verify the effectiveness of the improved BSS approach and the relatively small cross-talking error and root mean square error (RMSE) indicate that the approach improves the accuracy of signal separation.

Keywords: Blind source separation, Generalized Jaccard similarity, FSBLE, Imperialist competitive algorithm

1 Introduction

Blind source separation (BSS) aims at estimating source signals from their linearly mixtures without any prior information about the source signals and transmission channel [1, 2]. Originally, the BSS was introduced in 1984 by two French researchers to understand the biological behavior of neural system [3, 4]. As a momentous technical means of signal processing and analysis, BSS has been widely utilized in communication system [5], image processing [6], speech recognition [7], mechanical fault diagnosis [8], biomedical engineering [9] and many other aspects.

With the development and application of BSS technology, various approaches have been developed for estimating the source signals, such as, independent component analysis (ICA) [10], nonlinear principal component analysis (NPCA) [11], sparse component analysis (SCA) [12], etcetera. Among which ICA has attracted extensive attention of many experts and scholars as which is based only on statistical independence among the component. The main algorithms of ICA include fast fixed-point algorithm (FastICA) [13], maximum likelihood estimation algorithm [14], informax algorithm [15], joint approximate diagonalization of eigenmatrices (JADE) [16], fourth-order blind identification (FOBI) [17], and the like.The main assumption of ICA is that the source signals are non-Gaussianity, which shows that ICA is incapable of separating Gaussian source signals. The specific reason lies in that the invariance of Gaussian subspace makes it impossible to obtain the axis orientation information utilized to separate the source signals [18].

Nonetheless, the observed signals are mixtures of Gaussian signals in many cases. Therefore, the separation of source signals need to be accomplished by considering other assumptions related to source signals and accordingly algorithm. Take ECG signal as an example, the work on nonlinear dynamics has indicated that the ECG signal is chaotic and has extensive nonlinear dynamic characteristics [9]. Based on this, it is a vitally important idea to relax the assumption of non-Gaussianity via considering the dynamic similarity between source signals to achieve the separation of source signals, which has certain research significance.

The quantification of dynamic similarity of signals is perceived as a challenging work in the BSS problem. Recently, Niknazar et al. [19] proposed the fuzzy statistical behavior of local extreme (FSBLE) method based on cosine similarity to measure the dynamic similarity in signals, which solved the problem of isolating the linear chaotic Gaussian source signals and applied it to epileptic seizure prediction. Whereas, the cosine similarity is unable to make a distinction among the similarity vectors, and suffers from the problem of partial information loss, which lead to the unsatisfactory separation effect of the algorithm [20]. For the purpose of surmounting this problem, this paper investigates the modified FSBLE based on generalized Jaccard similarity and introduced it into the BSS problem with Gaussian source signals to improve the separation performance of BSS approach. Particularly, due to the cost function in the proposed BSS algorithm is not differentiable, the imperialist competition algorithm with fast convergence and less computation time is selected to search for the optimal separation matrix.

The rest of this paper is described below. In Section 2, the FSBLE based on generalized Jaccard similarity is put forward. In Section 3, the modified BSS approach with Gaussian source signals is introduced. In Section 4, simulation experiments are conducted. In closing, the conclusion is given in Section 5.

2 FSBLE with generalized Jaccard similarity

In this Section, the cosine similarity and its existing problems in measuring the dynamic similarity of signals are introduced in subsection 2.1, the rationality and advantages of using the generalized Jaccard index to measure the similarity of signals are analyzed in subsection 2.2, and the FSBLE based on generalized Jaccard similarity is raised in subsection 2.3.

2.1 Cosine similarity

The previous studies have shown that the information of signal can be represented by a eigenvector [21]. From this, using the information of eigenvector to measure the dynamic similarity of signals is the key step of BSS method based on the dynamic characteristics of signals. Furthermore, selecting an appropriate distance to measure the similarity is momentous, because equipped with a relatively good distance indicator can reduce the time and processing cost. In terms of different distance measurement indexes, it is more reasonable to employ bounded distance as the measure of similarity of signals. Hitherto, the existing BSS approach utilizes the cosine distance to quantify the similarity of signals. The cosine similarity is given via the dot product and length of vectors, that is, given two attribute vectors $\mathbf{V}_1$ and $\mathbf{V}_2$, which is able to be represented by

$$\rho_1(\mathbf{V}_1, \mathbf{V}_2) = \frac{< \mathbf{V}_1, \mathbf{V}_2 >}{\mathbf{V}_1 \cdot \mathbf{V}_2}, \tag{1}$$

where $\mathbf{V}_1$ and $\mathbf{V}_2$ represent both eigenvectors about the information of signals, $< \cdot, \cdot >$ is the dot product operator between two vectors, and $|| \cdot ||$ denotes the Euclidean norms operator of $\mathbf{V}_1$ and $\mathbf{V}_2$.

In the light of the Eq. (1), the value of cosine similarity is always between 0 and 1 and will not be affected by the dimension of vector. Furthermore, it avoids the disadvantage of poor robustness about absolute distance. Whereas, cosine index exists the problem of information loss in measuring similarity [20], which results from that the cosine similarity is poor to distinguish the similarity vectors when employing it to measure the dynamic similarity of signals. Consequently, the signal separation performance is not ideal. For the sake of overcoming this problem, this paper proposes to utilize generalized Jaccard similarity, which will be introduced in the next subsection.

2.2 Generalized Jaccard similarity

Jaccard index, as a measure indicator, is utilized to quantify similarities and differences of limited sample sets. Let A and B be two sets, the Jaccard index is shown as the following ratio

$$J(A, B) = \frac{|A \bigcap B|}{|A \bigcup B|}, \tag{2}$$

where $\bigcap$ and $\bigcup$ denote the intersection and union of sets A and B, respectively, $| \cdot |$ represents the potential of the set, that is, the number of elements in the set. It has been extended to the fuzzy Jaccard index, that is, X_a and X_b are two vectors with n components, in which each component is fuzzied, the value of which ranges from 0 to 1, and then the fuzzy Jaccard index of X_a and X_b can be given by

$$J(X_a, X_b) = \frac{X_a \cdot X_b}{X_a \cdot X_a + X_b \cdot X_b - X_a \cdot X_b}. \tag{3}$$

According to this extended Jaccard index, a new measure of similarity in signals, which is called as generalized Jaccard similarity, is represented as follows

$$\rho_2(\mathbf{V}_1, \mathbf{V}_2) = \frac{< \mathbf{V}_1, \mathbf{V}_2 >}{< \mathbf{V}_1, \mathbf{V}_1 > + < \mathbf{V}_2, \mathbf{V}_2 > - < \mathbf{V}_1, \mathbf{V}_2 >}, \tag{4}$$

where all of the symbols have the same meaning as their counterparts in Eq. (1).

The proposed generalized Jaccard similarity $\rho_2(\mathbf{V}_1, \mathbf{V}_2)$ satisfies the following properties:

Proposition 1. (Normative) $\mathbf{V}_1 = \mathbf{V}_2$ if and only if $\rho_2(\mathbf{V}_1, \mathbf{V}_2) = 1$.

Proposition 2. (Reflexivity) $\rho_2(\mathbf{V}_1, \mathbf{V}_2) = \rho_2(\mathbf{V}_2, \mathbf{V}_1)$.

Proposition 3. (Scale invariance) $\rho_2(\mathbf{V}_1, \mathbf{V}_2) = \rho_2(\tilde{\mathbf{V}}_1, \tilde{\mathbf{V}}_2)$, where $\tilde{\mathbf{V}}_1 = \theta\mathbf{V}_1$, $\tilde{\mathbf{V}}_2 = \theta\mathbf{V}_2$, $\theta \neq 0$.

According to Eq. (4), the previous part of the denominator of generalized Jaccard similarity can be regarded as the arithmetic average of two vectors squared, and the latter part can be regarded as subtracting the same part of two vectors on the basis of the arithmetic average of vectors. Relying on the fact that arithmetic average can retain the original state of each component than geometric average better, which further highlights the difference of signals, increases the identification of signal similarity, makes the signal less confusing, and solves the problem of partial information loss of original signals that cosine similarity will encounter. In addition, the rationality of the generalized Jaccard similarity employed to quantify the dynamical similarity of signals is illustrated via propositions 1, 2, 3. On this account, the generalized Jaccard similarity can quantify the similarity between two eigenvectors containing the information of signal better. In what follows, we get down to describe the modified FSBLE based on generalized Jaccard similarity in detail.

2.3 Modified FSBLE

The FSBLE is a dynamical similarity measure which employs time and amplitude information of local extrema to describe the dynamic characteristics of signals [22, 23]. This technique is mainly segmented into three steps to carry out: find optimal amplitude and time difference interval via the local extreme of signal, use the subordinate function to implement fuzzy processing on signal data, and measure similarity based on the generalized Jaccard similarity with extracted signal information.

2.3.1 Find optimum amplitude and time distance segmentation

The local extremum of signal has the information of amplitude and global frequency, which retains the important characteristics of signal. Thereby, the local extremum of signal plays a vitally important role. In order to better extract the information of signals, the time difference of consecutive local ectremums are also adopted. After obtaining the amplitude and time difference of consecutive local

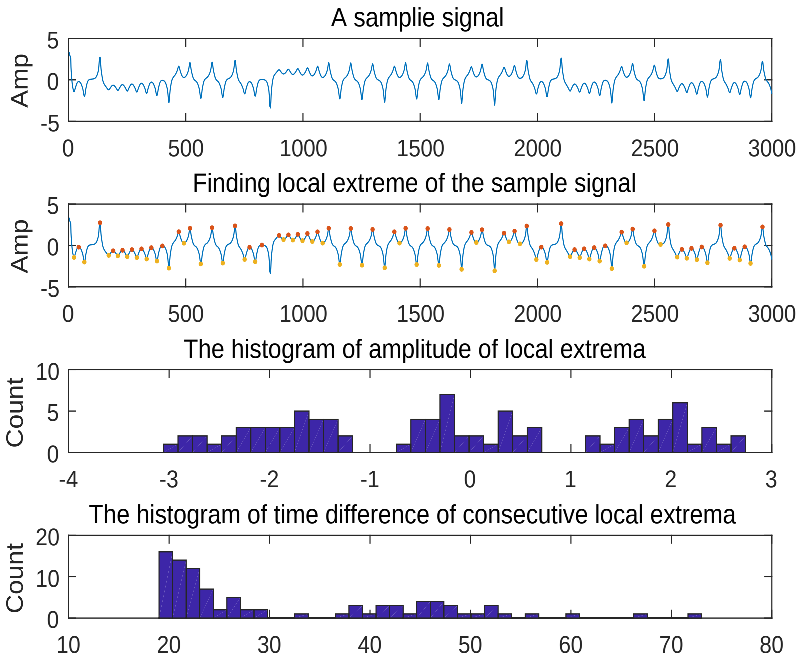


Fig. 1: The specific process diagram of 2.3.1

extremums, it is considered to divide into several intervals. To make the amount of local extremums on each interval identical, the histograms of amplitude and time difference are considered to be isolated into $O+1$ and $P+1$ fragments respectively, all of which possess the equal region. The values of O and P are employed to fuzzy the value of amplitude and time differences, which are selected in compliance with experience. The specific process is illustrated by Lorenz in Fig. 1.

2.3.2 Signal data fuzzy processing

On account of the acquired knowledge, statistical behavior of local extrema (SBLE) [23], symbolic aggregate approximation (SAX) [24], and their extension methods split the amplitude and time difference into intervals by means of the determined value of local extremum. Whereas, one problem with the above-mentioned methods is that it is highly sensitive to noise. Consequently, FSBLE is proposed, which isolates the amplitude and time difference into intervals via the fuzzy boundary of local extremum, surmounts the aforementioned problem, and thus improves the stability of the method. The $O+1$ and $P+1$ intervals of amplitude and time difference segregated in the previous stage are exploited to stipulate subordinate functions (SF) of fuzzifier and the type of subordinate function can be choosed by taking the histogram of values of amplitude and time difference of consecutive local extremums into consideration. It means that in order to maximize the extracted information, for the i-th local extremum, the matrix $\mathbf{B}_i$ is constructed by incorporating the subordinate function value of amplitude and time difference into every Li using the following equation,

$$\mathbf{B}_i = \begin{bmatrix} Li_{A(1),T(1)} & \cdots & Li_{A(1),T(P+1)} \\ \vdots & \ddots & \vdots \\ Li_{A(O+1),T(1)} & \cdots & Li_{A(O+1),T(P+1)} \end{bmatrix}, \tag{5}$$

where $Li_{A(o),T(p)} = F_{A(o)}(A(Li)) * F_{T(p)}(T(Li))$, in which $A(Li)$ is the amplitude of i-th local extrema and $T(Li)$ is its time difference with the $(i+1)$-th local extrema, $F_{A(o)}(\cdot)$ is the o-th subordinate function where $o = \{1,2,3,\cdots,O+1\}$, analogously, $F_{T(p)}(\cdot)$ is the p-th subordinate function where $p = \{1,2,3,\cdots,P+1\}$, as an aside, F refers to fuzzy subordinate function, in which triangular subordinate functions are widely adopted [25]. And what is more, each signal is converted into a matrix sequence $\mathbf{B}$ to facilitate further processing. The specific process refers to literature [22].

2.3.3 Measure similarity

As indicated above, each local extremum corresponds to a matrix, based on which the signal is converted to a sequence consisted of $n-1$ matrixes, in which n is the amount of local extremum. And then the statistical distribution of the defined pattern is exploited to extract the dynamic characteristics of signals. For each pattern, the following equation is employed to quantify belonging number of r sequential local extrema to possible amplitude and time difference intervals,

$$V_{(A(1),T(1))_1,\cdots,(A(r),T(r))_r} = \frac{1}{n-r}[\sum_{i=1}^{n-r}\mathbf{B}_i(A(1),T(1)) * \cdots * \mathbf{B}_{i+r}(A(r),T(r))]^{\frac{1}{n-r}}. \tag{6}$$

Specifically, once the value of r is selected, the amount of possible patterns of signals sequences corresponding to r is:

$$\sharp(r) = ((O+1)\cdot(P+1))^r. \tag{7}$$

For each signal $\mathbf{V}_R^{signal}$, it is constructed by changing r from 1 to R, where R denotes the maximum number of continuous local extremum defined.

$$\begin{aligned}\mathbf{V}_R^{signal} = \{&V_{(1,1)},\cdots,V_{(O+1,P+1)},V_{(1,1),(1,1)},\\ &V_{(1,1),(1,2)},\cdots,V_{(O+1,P+1),(O+1,P+1),\cdots}\},\end{aligned} \tag{8}$$

where $\mathbf{V}_R^{signal}$ possesses the dynamic features of local extremum of signal and is able to be utilized to quantify similarity.

It is well known that the utilization of bounded distance metrics can make the similarity values more comparable in different studies. At the same time, in compliance with the previous analyses of generalized Jaccard similarity and cosine similarity, employing generalized Jaccard index to quantify the similarity between $\mathbf{V}_R^{signal}$ is superior to that exploits cosine distance. Consequently, the generalized Jaccard index bounded by 0 and 1 is chosen as the measurement indicator to quantify dynamical similarity of signals, as shown below:

$$\rho_2(\mathbf{V}_R^1,\mathbf{V}_R^2) = \frac{<\mathbf{V}_R^1,\mathbf{V}_R^2>}{<\mathbf{V}_R^1,\mathbf{V}_R^1> + <\mathbf{V}_R^2,\mathbf{V}_R^2> - <\mathbf{V}_R^1,\mathbf{V}_R^2>}, \tag{9}$$

where the vectors $\mathbf{V}_R^1$ and $\mathbf{V}_R^2$ have been processed fuzzily via the preceding information.

3 Improved BSS approach

BSS is one of the most commonly methods in digital signal processing, which aims to separate the source signals under the situation of unknown transmission channel and source signals. The simplest model is a linear combination of the source signals and the hybrid matrix in the deterministic case (where the number of source signals and observed signals is equal). The specific mathematical expression is shown below:

$$\mathbf{X} = \mathbf{QS}, \tag{10}$$

where $\mathbf{Q}$ is a full rank mixed matrix, $\mathbf{S}$ is the source signals, and $\mathbf{X}$ is the mixed signals. For BSS, the ultimate goal is to recover the source signals via finding the separation matrix $\mathbf{W}$. Hence, it must be implemented by using

$$\hat{\mathbf{S}} = \mathbf{WX}, \tag{11}$$

where $\hat{\mathbf{S}}$ is the estimate of source signals $\mathbf{S}$.

Whereas, when the source signals is chaotic or nonlinear and there is no information about non-Gaussianity, the commonly used BSS approaches based on non-Gaussian assumption will be invalid. Consequently, taking some other hypotheses from observed signals into consideration is necessary to acquire the separated matrix **W**. According to this, the source signals are deemed to be chaotic and have stable dynamics, where ECG signal is one of the representatives. The following restrictions and requirements are able to be considered to obtain the appropriate matrix **W** for this problem.

(1) The dynamic similarity between each source signal and itself is the highest, which is called the signal dynamic stationarity, that is, the dynamic characteristics of source signal change with time is relatively static.

(2) The dynamic similarity between each source signal and other signals is the lowest, which is called the signal dynamic independence, that is, the maximum separability of source signal is satisfied.

For the sake of solving the problem of BSS based on the above-mentioned hypotheses, the method adopted is to find the separative matrix **W** which satisfies two assumptions above. Nonetheless, the premise of searching the separated matrix **W** is to quantify the dynamic similarity of signals, which means that it is crucially important to find an index to measure the similarity between signals. As described in subsection 2.3, the FSBLE based on generalized Jaccard similarity is taken as the measure of signal dynamic similarity in this study.

On the basis of aforementioned works, the problem can be transformed into an optimization problem based on the known dynamic information between signals, which mainly includes the following two factors:

1. Dynamic stationary factor: Hypothesis (1) is satisfied to maximize *StaFac* function.

2. Dynamic independent factor: Hypothesis (2) is satisfied to minimize *IndFac* function.

The ambition of the dynamic stationarity factor and the dynamic independent factor is the maximization of dynamic stability of each estimated source signal separately and dynamic independence between estimated source signals, in which the maximization of independence is able to be understood as the minimization of dynamic similarity between estimated source signals. Thereinto, to quantify the dynamic stationarity and independence of the source signals, each estimated source signal is segmented into d fragments, in which $d \geq 2$ is required to ensure that the following equation holds, *StaFac* and *IndFac* are calculated by

$$StaFac = \frac{1}{N_s} \sum_{i=1}^{N_s} \{ \frac{1}{d*(d-1)} \sum_{k=1}^{d} \sum_{l=1, l \neq k}^{d} \rho_2(\hat{\mathbf{S}}_i^k, \hat{\mathbf{S}}_i^l) \}, \tag{12}$$

$$IndFac = \frac{1}{N_s*(N_s-1)} \sum_{i=1}^{N_s} \sum_{j=1, j \neq i}^{N_s} \{ \frac{1}{d*(d-1)} \sum_{k=1}^{d} \sum_{l=1, l \neq k}^{d} \rho_2(\hat{\mathbf{S}}_i^k, \hat{\mathbf{S}}_j^l) \}, \tag{13}$$

where $\hat{\mathbf{S}}_i^k$ is the k-th fragment of the i-th estimated source signal and N_s is the number of source signals. The function *StaFac* calculates the average similarity of total corresponding segments in each estimated source signal and the function *IndFac* gives the dynamic similarity of whole corresponding segments between two different estimated source signals. Different cost functions can be defined via specific assumptions. In this problem, cost function must meet the maximization of *StaFac* and the minimization of *IndFac* simultaneously. According to the existing knowledge, the exponential cost function is beneficial to the cost calculation of additive model. Therefore, the exponential combination of dynamic stationary factor and dynamic independent factor is selected as the cost function, as follows:

$$CostFcn = e^{-StaFac} * e^{IndFac}. \tag{14}$$

By minimizing $CostFcn$, the dynamic stability and independence of source signals are satisfied, and thus the relatively optimal source signals are estimated. Generally speaking, as the corresponding prior knowledge about the parameter information of source signals and the characteristics of transmission channel is unknown, which leads to an uncertainty expansion factor on the inside. Accordingly, the separated source signals have major differences in amplitude, phase, and order, whereas, the waveforms of separated source signals are still consistent with the corresponding source signals. Meanwhile, $CostFcn$ is not differentiable due to FSBLE based on generalized Jaccard similarity. Hence, the current problem (minimize $CostFcn$) is incapable of being solved with derivative-based iterative methods, which is the conventional solution of many other BSS methods (such as *FastICA*, $Informax$, etcetera). Consequently, the imperialist competition algorithm in the metaheuristic search algorithms is utilized to minimize the $CostFcn$ via searching the value of matrix **W** [26, 27].

Algorithm 1 describes the main process of proposed BSS approach based on generalized Jaccard similarity in detail. Similar to the BSS approach based on cosine

Algorithm 1 Improved BSS approach based on generalized Jaccard similarity

Require: N_s observed signals, parameters (O,P,R) of FSBLE based generalized Jaccard similarity, d.

Ensure: Estimated matrix $\mathbf{W}$, N_s source signals.

1: Produce a group of matrix $\mathbf{W}$ randomly.

2: Use $\mathbf{W}$ to acquire the estimated source signals from the observed signals.

3: Measure the similarity between signals via using FSBLE based on generalized Jaccard similarity:

$$\rho_2(\mathbf{V}_R^1, \mathbf{V}_R^2) = \frac{< \mathbf{V}_R^1, \mathbf{V}_R^2 >}{< \mathbf{V}_R^1, \mathbf{V}_R^1 > + < \mathbf{V}_R^2, \mathbf{V}_R^2 > - < \mathbf{V}_R^1, \mathbf{V}_R^2 >}.$$

4: Calculate $StaFac$, $IndFac$, $CostFcn$.

5: Minimize the value of $CostFcn$ to find the best matrix $\mathbf{W}$ via the imperialist competition algorithm.

6: If the terminational condition is not met: produce a new group of matrix $\mathbf{W}$ and go to 3, else return the best matrix $\mathbf{W}$ and output source signals.

7: End.

similarity, the improved approach firstly applies generalized Jaccard index to BSS to obtain source signals and surmounts the problem of partial information loss in cosine similarity. Additionally, as depicted in the introduction, the performance of the improved BSS approach will outperform that based on cosine similarity, which will be shown in the next section.

4 Numerical experiment results and analyses

In this section, the experimental model is introduced in subsection 4.1, the ability of the improved BSS approach to estimate Gaussian source signals and ECG signals in practical experiments is proved in subsection 4.2 and subsection 4.3, and then the cross-talking error and RMSE are adopted to compare the BSS approach based on generalized Jaccard similarity with that based on cosine similarity in subsection 4.4. The experimental results will illustrate that the improved BSS approach is superior to the original one. The total experiments are performed in MATLAB 2017b on a 64-bit PC with an Intel(R) Core(TM) i7-6700HQ CPU (2.60GHz) and 16 GB of RAM.

4.1 The experimental model

In the following simulation experiments, we consider three classical models as objects in the field of chaotic systems: Lorenz [28], Rossler [29] and Mackey Glass [30], which are given by

$$\begin{aligned} \frac{dX_1}{dt} &= \sigma(Y_1 - X_1) \\ \frac{dY_1}{dt} &= \rho X_1 - Y_1 - X_1 Z_1, \\ \frac{dZ_1}{dt} &= X_1 Y_1 - \beta Z_1 \end{aligned} \tag{15}$$

$$\begin{aligned} \frac{dX_2}{dt} &= -\omega Y_2 - Z_2 \\ \frac{dY_2}{dt} &= \omega X_2 + \alpha Y_2 \qquad , \\ \frac{dZ_2}{dt} &= \eta + Z_2(X_2 - \gamma) \end{aligned} \tag{16}$$

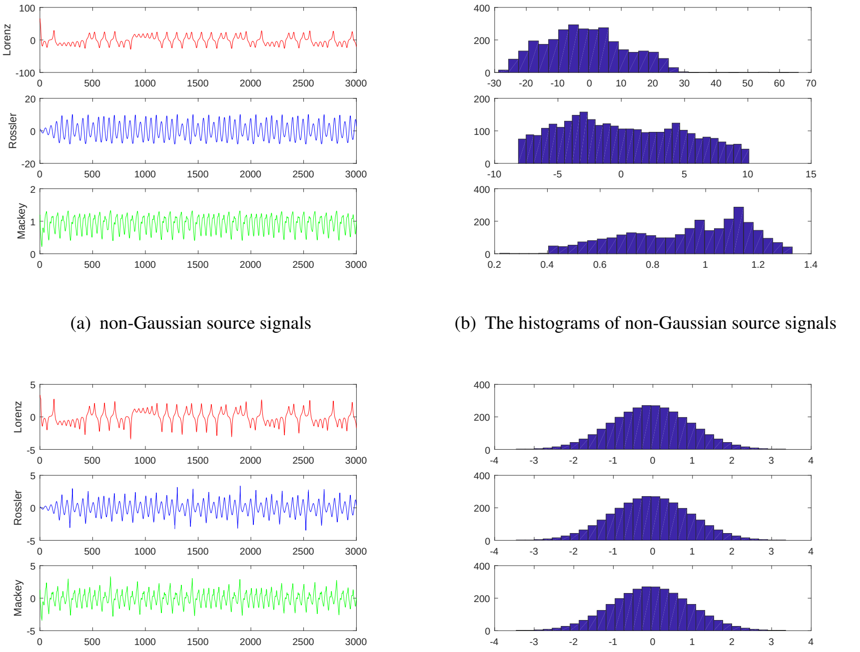
$$X_3(i+1) = X_3(i) + \frac{aX_3(i-r)}{1 + X_3(i-r)^c} - bX_3(i), \tag{17}$$

where σ, ρ and β denote Prandtl number, Rayleigh number and directional ratio respectively, ω signifies natural frequency, as the most typical example of time-delay chaotic system, in which r expresses time-delay number. In the following experiments, these parameters are set to $\sigma = -16$, $\rho = 45.92$, $\beta = 4$, $\omega = 1$, $\alpha = 0.2$, $\eta = 0.4$, $\gamma = 5.7$, $a = 0.2$, $b = 0.1$ and $c = 10$. Once the parameters are selected, the influence of different initial points on the dynamic similarity of signals can be almost ignored in chaotic system which results from that the modified FSBLE is used to measure the dynamic similarity between signals [22]. Then, the Runge-Kutta method is adopted to select the signals X_1, X_2, X_3 of these systems as basic source signals $\mathbf{S}$. The specific simulation experiment will be studied in the next subsection.

4.2 Evaluation on Gaussian source signals

As mentioned above, the signal $\mathbf{S}$ consisting of Lorenz, Mackey Glass and Rossler in the chaotic set is regarded as the basic source signals. Each basic source signal is non-Gaussian. For purpose of highlighting the capacity of the improved approach to separate Gaussian signals, the histogram matching technique is exploited to convert non-Gaussian source signals into Gaussian source signals. The converted Gaussian signals are utilized as the source signals and multiplied by the randomly generated matrix $\mathbf{Q}$ to obtain the mixed Gaussian signals.

$$\mathbf{Q} = \begin{bmatrix} 0.8147 & 0.9134 & 0.2785 \\ 0.9058 & 0.6324 & 0.5469 \\ 0.1270 & 0.0975 & 0.9575 \end{bmatrix}, \tag{18}$$



(a) non-Gaussian source signals

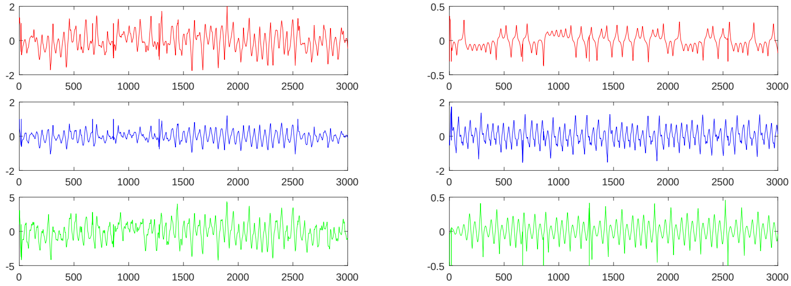
(b) The histograms of non-Gaussian source signals

(c) The transformed Gaussian source signals

(d) The histograms of transformed Gaussian source signals

Fig. 2: The source signals and corresponding histograms

As stated in the introduction, ICA approach will not be able to use the linear mixture of these Gaussian sources to acquire the source signals. Therefore, the improved BSS approach based on generalized Jaccard similarity is considered to carry out the separation of the observed signals in this part. Running the algorithm requires some initial parameters, such as the values of d, O, P and R, in which d represents that each estimated source signals are isolated into d fragments, O and P express that the corresponding histograms of amplitude and time difference are segmented into O and P sections respectively, and R denotes the number of sequential



(a) The mixed Gaussian signals.

(b) The separated Gaussian source signals.

Fig. 3: The separation result diagrams of Gaussian signal

local extrema. The values of these parameters will affect the results and computation time of the algorithm [22], they are empirically set as: $d = 2$, $O = 3$, $P = 3$, $R = 3$ in the simulated experiment. Since $CostFcn$ is not differentiable, the imperialist competition algorithm with fast convergence is exploited to seek the optimal solution via updating matrix $\mathbf{W}$, in which the actual amount of population, the amount of imperialists and the maximum amount of iterations are $npop = 150$, $nimp = 30$, and $maxdecades = 150$, respectively, and other parameters of which are set to $\tilde{\beta} = 2$, $\tilde{\gamma} = \pi/4$ and $\zeta = 0.1$ per the recommendations of this paper [31].

Note that three chaotic Gaussian signals are selected in the experiment to generate observed signals so as to certify the ability of the improved BSS approach to separate Gaussian source signals. Fig. 2 depicts the obtained result of Gaussian source signals, in which Fig. 2(a) expresses three non-Gaussian source signals, Fig. 2(b) expresses the corresponding histograms, Fig. 2(c) shows three Gaussian source signals after transformation, analogously, Fig. 2(d) shows the transformed histograms.

Fig. 3(a) depicts three observed Gaussian signals, which are generated via multiplying by a randomly generated matrix $\mathbf{Q}$ on the basis of Gaussian source signals. The elements of the matrix $\mathbf{Q}$ are uniformly distributed between 0 to 1 and the observed Gaussian signals is regarded as the simulated signals which will be executed by the improved BSS approach.

Fig. 3(b) shows the final separation result of the improved BSS approach, in which the imperialist competition algorithm is employed for searching the optimal solution of the $CostFcn$, and that is a common intelligent optimization algorithm.

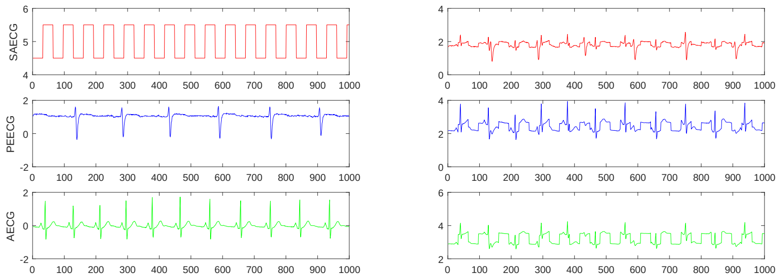
It can be seen by comparing Fig. 3(b) and Fig. 2(c) that the separated signals are mainly consistent with the source signals except some differences in sequence and amplitude. Consequently, it can be considered that the approach has successfully separated the source signals, which confirms the effectiveness of the improved BSS approach.

4.3 Evaluation on ECG signal

The separation of ECG signals is considered as a real-world application of the proposed method. This study select ECG signals in three different states as the source signals, and multiply it with the randomly matrix $\mathbf{U}$ to obtain the mixed ECG signal, where these signals of AEECG, PEECG and SAECG obtained from Apnea-ECG Database, Post-Ictal Heart Rate Oscillations in Partial Epilepsy and UCD Sleep Apnea Database, respectively.

$$\mathbf{U} = \begin{bmatrix} 0.2343 & 0.6153 & 0.2845 \\ 0.4647 & 0.1226 & 0.7357 \\ 0.6194 & 0.1238 & 0.4113 \end{bmatrix}. \tag{19}$$

Three ECG signals are selected in the experiment to certify the ability of the improved BSS approach to separate ECG signals. Fig. 4(a) depicts the original ECG signals, Fig. 4(b) depicts the mixed ECG signals, and which will be executed by the improved BSS approach.



(a) The original ECG signals.

(b) The mixed ECG signals.

Fig. 4: The diagrams of ECG signals

Fig. 5 shows the final separation result of the improved BSS approach. It can be seen that Fig. 5 has roughly the same waveforms as Fig. 4(a), which indicates that the approach has successfully separated the ECG signals. Based on this, this method can be also applied to the separation of biological signals.

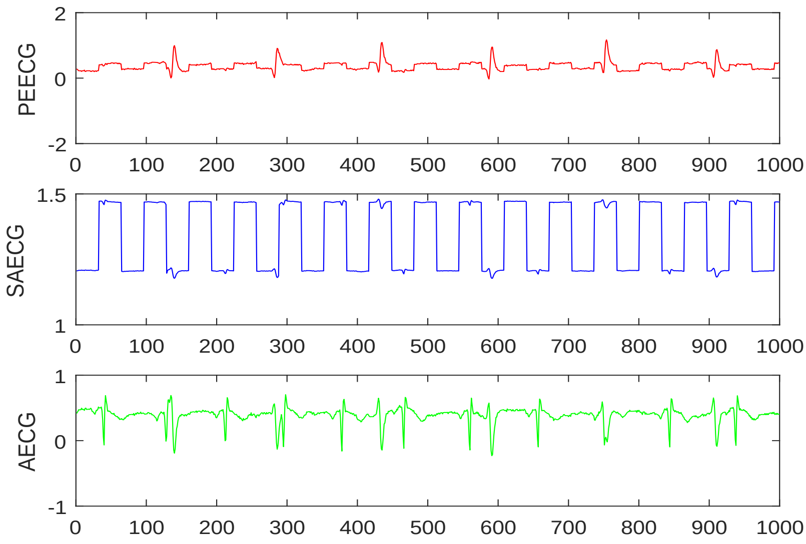


Fig. 5: The separated ECG signals.

4.4 Performance comparison

To evaluate the performance of the improved BSS approach based on generalized Jaccarcd similarity with that based on cosine similarity [19], cross-talking error [32] and RMSE are adopted as the measurement indexes, which have the following expressions respectively:

$$E_{ct} = \frac{1}{n}\{\sum_{p=1}^{n}(\sum_{q=1}^{n}\frac{|c_{pq}|}{\max_{l}|c_{pl}|}-1)+\sum_{q=1}^{n}(\sum_{p=1}^{n}\frac{|c_{pq}|}{\max_{l}|c_{lq}|}-1)\}, \tag{20}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(X_{sep,i}-X_{ori,i})^2}{n}}, \tag{21}$$

where $\mathbf{C} = \mathbf{WQ} = (\mathbf{C}_{pq})$ represents the transfer matrix of the mixing-separation composite system, $X_{sep,i}$ and $X_{ori,i}$ denote the i-th separated signal and the corresponding original signal respectively. If the source signals are separated well, the value of

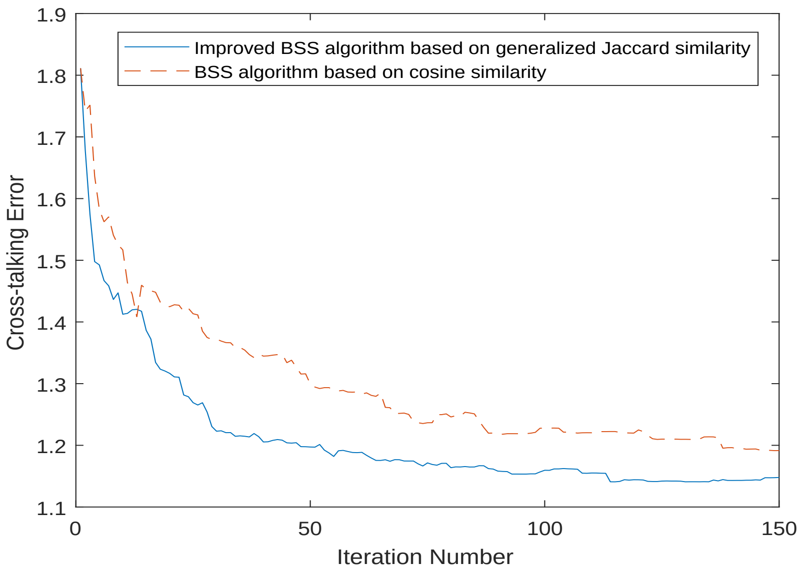


Fig. 6: Cross-talking error of the BSS approach

RMSE will be small and **C** will become a permutation matrix (although the elements may have different symbols). Only one element in each row or column of the permutation matrix is equal to 1, and all other elements are equal to 0. Obviously, the larger the value of cross-talking error and RMSE, the worse the separation performance of the approach.

Table 1: The comparison of RMSE between two BSS algorithm.

signals	RMSE of improved BSS	RMSE of BSS based on cosine similarity
Lorenz	0.7291	1.3393
Rossler	1.0135	1.3968
Mackey Glass	1.2730	1.2339

The curve of cross-talking error is depicted in Fig. 6. Wherein blue curve is the cross-talking error of the improved BSS approach based on generalized Jaccard similarity, red curve is that based on cosine similarity. The result of RMSE about two BSS algorithms is shown in Table 1. As can be seen from the simulated results in Fig. 6 and Table 1, the improved BSS approach based on generalized Jaccard similarity generally outperforms that based on cosine similarity under the criterions of the cross-talking error and RMSE, which confirms the superiority of the improved BSS approach.

5 Conclusion

In this paper, an improved BSS approach based on generalized Jaccard similarity is studied. Firstly, the generalized Jaccard index is proposed to solve the problem of information loss in cosine similarity, and the modified FSBLE employed to extract the dynamic characteristics of signals is utilized to quantify the dynamic similarity of signals. Secondly, the improved algorithm based on the dynamic characteristics of signals is applied to the blind separation of Gaussian signals, and the imperialist competition algorithm is selected to find the separation matrix on the condition that the cost function is not differentiable. Finally, a series of simulation experiments on the nonlinear chaotic model verified that the proposed algorithm can successfully separate Gaussian source signals. At the same time, the cross-talking error and root mean square error (RMSE) of the improved approach is relatively small, which indicates that the separation accuracy of the algorithm is improved. In addition, it can also be applied to a series of biological signals besides ECG signals.

Acknowledgments. This work is supported by the National Natural Science Foundation of China under Grant 61573014.

Declarations

- Conflict of interest: the authors declare that there is no conflict of interest.
- Data availability statement: all of the material is owned by the authors and no permissions are required.
- Authors contribution: Fu Xudan, Ye Jimin, and E Jianwei carried out the blind source separation of Gaussian signals based on generalized Jaccard similarity. Among them, Fu xudan put forward the idea of the research, participated in its design and coordination and drafted the manuscript, Ye Jimin participated in the construction of the framework of the paper, and helped to draft the manuscript, E Jianwei participated in the design of the experiment and helped to draft the manuscript. All authors reviewed the manuscript and approved the final draft.

References

[1] Feng, F., Kowalski, M.: Revisiting sparse ica from a synthesis point of view: Blind source separation for over and underdetermined mixtures. Signal Processing **152**, 165–177 (2018)

[2] Feng, F., Kowalski, M.: Sparsity and low-rank amplitude based blind source separation. In: 2017 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), pp. 571–575 (2017). IEEE

[3] Herault, J., Ans, B.: Reseaux de neurones a synapses modifiables : Decodage de messages sensoriels composites par une apprentissage non supervise et permanent. Comptes rendus des séances de l'Académie des sciences. Série III, Sciences de la vie **299**(13), 525–528 (1984)

[4] Herault, J., Jutten, C.: Space or time adaptive signal processing by neural network models. American Institute of Physics, 206–211 (1986)

[5] Khosravy, M., Gupta, N., Patel, N., Dey, N., Nitta, N., Babaguchi, N.: Probabilistic stone's blind source separation with application to channel estimation and multi-node identification in MIMO IoT green communication and multimedia systems. Computer Communications **157**, 423–433 (2020)

[6] Pó, G.: A robust digital image processing method for measuring the planar burr length at milling. Journal of Manufacturing Processes **80**, 706–717 (2022)

[7] Pawar, R.V., Jalnekar, R.M., Chitode, J.S.: Review of various stages in speaker recognition system, performance measures and recognition toolkits. Analog Integrated Circuits and Signal Processing **94**(2), 247–257 (2018)

[8] Wang, Z., Chen, J., Dong, G., Zhou, Y.: Constrained independent component analysis and its application to machine fault diagnosis. Mechanical systems and signal processing **25**(7), 2501–2512 (2011)

[9] Gupta, V., Mittal, M., Mittal, V.: R-peak detection based chaos analysis of ecg signal. Analog Integrated Circuits and Signal Processing **102**(3), 479–490 (2020)

[10] Feng, F., Kowalski, M.: An unified approach for blind source separation

using sparsity and decorrelation. In: 2015 23rd European Signal Processing Conference (EUSIPCO), pp. 1736–1740 (2015). IEEE

[11] Song, J., Li, B.: Nonlinear and additive principal component analysis for functional data. Journal of Multivariate Analysis **181**, 104675 (2021)

[12] Rahoma, A., Imtiaz, S., Ahmed, S.: Sparse principal component analysis using bootstrap method. Chemical Engineering Science, 116890 (2021)

[13] Yuan, L., Zhou, Z., Yuan, Y., Wu, S.: An improved fastica method for fetal ecg extraction. Computational and mathematical methods in medicine **2018** (2018)

[14] Li, M., Liu, X., Ding, F.: The filtering-based maximum likelihood iterative estimation algorithms for a special class of nonlinear systems with autoregressive moving average noise using the hierarchical identification principle. International Journal of Adaptive Control and Signal Processing **33**(7), 1189–1211 (2019)

[15] Chen, Y., Xue, S., Li, D., Geng, X.: The application of independent component analysis in removing the noise of eeg signal. In: 2021 6th International Conference on Smart Grid and Electrical Automation (ICSGEA), pp. 138–141 (2021). IEEE

[16] Rutledge, D.N., Bouveresse, D.J.-R.: Independent components analysis with the JADE algorithm. TrAC Trends in Analytical Chemistry **50**, 22–32 (2013)

[17] Li, B., Van Bever, G., Oja, H., Sabolová, R., Critchley, F.: Functional independent component analysis: an extension of the fourth-order blind identification. Unpublished manuscript (2019)

[18] Vignat, C., Plastino, A.: Scale invariance and related properties of q-gaussian systems. Physics Letters A **365**(5-6), 370–375 (2007)

[19] Niknazar, H., Nasrabadi, A.M., Shamsollahi, M.B.: A new blind source separation approach based on dynamical similarity and its application on epileptic seizure prediction. Signal Processing **183**, 108045 (2021)

[20] Jimenez, S., Gonzalez, F.A., Gelbukh, A.: Mathematical properties of soft cardinality: Enhancing jaccard, dice and cosine similarity measures with

element-wise distance. Information Sciences **367**, 373–389 (2016)

[21] Jiang, Y., Lan, T., Zhang, D.: A new representation and similarity measure of time series on data mining. In: 2009 International Conference on Computational Intelligence and Software Engineering, pp. 1–5 (2009). IEEE

[22] Niknazar, H., Nasrabadi, A.M., Shamsollahi, M.B.: A new similarity index for nonlinear signal analysis based on local extrema patterns. Physics Letters A **382**(5), 288–299 (2018)

[23] Niknazar, H., Nasrabadi, A.M.: Epileptic seizure prediction using a new similarity index for chaotic signals. International Journal of Bifurcation and Chaos **26**(11), 1650186 (2016)

[24] Lin, J., Keogh, E., Lonardi, S., Chiu, B.: A symbolic representation of time series, with implications for streaming algorithms. In: Proceedings of the 8th ACM SIGMOD Workshop on Research Issues in Data Mining and Knowledge Discovery, pp. 2–11 (2003)

[25] Juang, Y.-T., Chang, Y.-T., Huang, C.-P.: Design of fuzzy PID controllers using modified triangular membership functions. Information Sciences **178**(5), 1325–1333 (2008)

[26] Maheri, M.R., Talezadeh, M.: An enhanced imperialist competitive algorithm for optimum design of skeletal structures. Swarm and Evolutionary Computation **40**, 24–36 (2018)

[27] Peri, D.: Hybridization of the imperialist competitive algorithm and local search with application to ship design optimization. Computers & Industrial Engineering **137**, 106069 (2019)

[28] Moon, S., Baik, J.-J., Seo, J.M.: Chaos synchronization in generalized lorenz systems and an application to image encryption. Communications in Nonlinear Science and Numerical Simulation **96**, 105708 (2021)

[29] Liao, X., Yu, P.: Chaos control for the family of rössler systems using feedback controllers. Chaos, Solitons & Fractals **29**(1), 91–107 (2006)

[30] Junges, L., Gallas, J.A.: Intricate routes to chaos in the Mackey–Glass delayed

feedback system. Physics letters A **376**(30-31), 2109–2116 (2012)

[31] Atashpaz-Gargari, E., Lucas, C.: Imperialist competitive algorithm: An algorithm for optimization inspired by imperialistic competition. In: IEEE Congress on Evolutionary Computation, pp. 4661–4667 (2007)

[32] Ye, J., Jin, H., Zhang, Q.: Adaptive weighted orthogonal constrained algorithm for blind source separation. Digital Signal Processing **23**(2), 514–521 (2013)