

Exponential or Pareto: towards an evolution of equal opportunity

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Pareto distribution and exponential distribution are related by the generalized Pareto distribution¹⁻³, which has been proposed to describe the income structure of the total population⁴⁻⁵. The underlying mechanism for driving the Pareto distribution has been known as the Matthew effect of income accumulation⁶⁻¹⁰. Today, the Pareto distribution has been observed universally in the richest class¹¹⁻¹⁴ (1%~3% of populations); however, this distribution could dominate a larger proportion of populations when the investigation dated back to Renaissance Europe¹⁵, Hungarian medieval society¹⁶, and ancient Egypt¹⁷. By contrast, the underlying mechanism for driving exponential income distribution is due to the equal opportunity of market competition¹⁸⁻²⁰, which radically differs from the Matthew effect. Here, we empirically find that, during the last 40 years, the income structures of different market-economy countries uniformly exhibit a two-class pattern, in which the great majority of populations obeys an exponential distribution and only the remaining (richest) part follows a Pareto distribution. In particular, we empirically show how the income structure in China evolved to an exponential distribution after the market-oriented economic reformation. The finding of a larger proportion of populations evolving to an exponential income distribution may reveal a potential trend of human civilization towards equal opportunity.

Through time, roughly speaking, we humans have explored four social structures, from hunter-gatherer societies, slave-based societies, feudal hierarchical societies, to various types and levels of democratic societies. At the early stage of human society, humans' behaviors were dominated by "The law of the jungle", which causes the Matthew effect in material allocations. Nowadays, it has been known that the Matthew effect leads to the Pareto distribution of income⁶⁻¹⁰. At the end of the 19th century, the economist Vilfredo Pareto discovered that²¹ the rich part of populations in several European countries, kingdoms and cities for the 15th-19th centuries obeys a Pareto distribution. Later, Pareto's finding was confirmed by investigating market-economy countries in the 20th and 21st centuries^{11-14, 22}. In particular, there had been evidence showing that

the income structure of an ancient Egypt city in the 14th century BC followed a Pareto distribution¹⁷. Because slaves had no property right of personal income, this evidence implies that slave-based societies might undergo a Pareto distribution of the total population. As the time came to the end of the 20th century, it had been found that the income structures of market-economy countries around the world uniformly exhibited a two-class pattern^{19-20, 23-33}: the majority of populations obeyed an exponential distribution and only the remaining (richest) part followed a Pareto distribution. Empirical evidence had covered more than 66 countries¹⁹, ranging from Europe to Latin America, North America, and Asia. Recently, Blanchet et al⁴⁻⁵ proposed to use the generalized Pareto distribution to describe the income structure of the total population, where the generalized Pareto distribution is defined as follows¹⁻³:

$$F_{\xi}(t \geq x) = \left(1 + \xi \frac{x-\mu}{\theta}\right)^{-1/\xi}, \quad (1)$$

where $\xi > 0$, x denotes the income level, and $F_{\xi}(t \geq x)$ denotes the fraction of population with the income higher than x .

The generalized Pareto distribution is a fairly general family which includes the Pareto distribution and the exponential distribution as two special cases¹⁻³. To see this, by ordering $\mu = \theta/\xi$, equation (1) yields the Pareto distribution:

$$F_{\xi}(t \geq x) = (x/\mu)^{-1/\xi}, \quad (2)$$

where the Pareto exponent is denoted by $1/\xi$, which measures the degree of income inequality¹⁴, that is, a larger Pareto exponent is associated with a lower income inequality. Intuitively, a lower income inequality should correspond to a more equal-opportunity society. Therefore, we anticipate that, as the Pareto exponent $1/\xi \rightarrow \infty$ (or $\xi \rightarrow 0$), equation (1) yields a distribution being close to equal opportunity:

$$F_0(t \geq x) = \lim_{\xi \rightarrow 0} \left(1 + \xi \frac{x-\mu}{\theta}\right)^{-1/\xi} = e^{-\frac{x-\mu}{\theta}}, \quad (3)$$

which is an exponential distribution.

Theoretically, it has been shown that¹⁸⁻²⁰ the exponential income distribution (3) emerges spontaneously in an idealized market economy with equal-opportunity, which is a decentralized self-organizing economic network, just as described by the Arrow-

Debreu general equilibrium model³⁴. Here, we briefly introduce such an economic network. To this end, we consider an “ N -agent non-cooperative game”, where are N agents, each of whom operates a firm, and each agent’s status is equal. In this economic network, all of these firms pursue maximum profit, and all of these agents exchange with each other to obtain maximum satisfaction. Arrow and Debreu prove that³⁴, for each agent, if there is free of cost for information transmission and transaction, then all of these agents will reach consensus on resource allocation (so-called the “general equilibrium”). Tao further proves that^{18,35}, if the property right of each firm is endowed to a different agent, then these consensus lead to the exponential income distribution (3). However, information transmission and transaction would take additional cost. Therefore, neoclassical economists³⁶ have to assume an imaginary “Walrasian auctioneer”, who has an advanced technology to eliminate the cost for information transmission and transaction, to selflessly assist all agents in reaching exchanges without any transaction cost. Unfortunately, when an agent has the potential to become a Walrasian auctioneer, motivated by selfishness it may instead take advantage of technology to acquire more incomes, undermining equal opportunity of market competition. For example, the internet business platforms (such as Amazon and Alibaba) could have the potential to become Walrasian auctioneers; however, all of them eventually become oligopolies benefiting from the Matthew effect. This explains why the Pareto distribution is observed in the top income class in a market economy. Although the internet technology did not eliminate beneficiaries of the Matthew effect, with rise of information technology, the block-chain is further proposed to play the role of a Walrasian auctioneer. This implies that there seems to be an intrinsic motivation²⁰ for driving a human society to achieve an equal-opportunity economic network. Therefore, it may be conjectured that a larger proportion of populations in a market-economy country is evolving towards to an exponential income distribution.

Here, we employ the household data from four market-economy countries to verify that a larger proportion of populations is evolving towards the exponential income distribution (3). The four countries include three typical developed economy (United Kingdom, United States, and Canada) and one typical developing economy (China). In

particular, because the China is a special sample that has undergone the transition from a planned economy to a market economy, it is more interesting to check if there is a transition towards an exponential income distribution after the market-oriented economic reformation. We first demonstrate that it is more reasonable to analyze the household income data by using the cumulative distribution function (rather than the density distribution function). In particular, when one employs the density distribution function to analyze the household income data, the results may be inconsistent. In Figure 1, we use the household income data from the United Kingdom in 2010 to show this inconsistency: *The shape of the density distribution of income may depend on the bin width of the histogram*. For example, when the bin width is denoted by 4000 GBP, Figure 1a shows a right-skewed density distribution with a peak, which is close to a Log-normal distribution or a Gamma distribution. Unfortunately, when the bin width is increased to 8000 GBP, as shown in Figure 1b, the shape of density distribution begins to deviate from a Log-normal distribution or a Gamma distribution. In particular, when the bin width is increased to 12000 GBP, the shape of density distribution turns to a monotonic function, see Figure 1c. This means that, for the same data, density distribution may lead to inconsistent graphs. However, when one uses the cumulative distribution function to describe the household income data, there are no such inconsistencies. Figure 1d shows that the great majority of populations in the United Kingdom obeys the exponential income distribution (3), which is a cumulative distribution function. Henceforth, we always employ the cumulative distribution function to analyze the household income data.

[Insert Figure 1 here]

Furthermore, Figure 2 shows that the income structures of China (in 2015), United Kingdom (in 2017), Canada (in 2018) uniformly exhibit a two-class pattern in which the great majority of populations obeys the exponential income distribution (3) and only the remaining (richest) part follows a Pareto distribution. Figure 2 also shows that the great majority of populations in the United States (in 2019) follows the exponential

income distribution (3). The dataset is gathered from the most recent years. The empirical observation supports our conjecture that a larger proportion of populations in a market-economy country is evolving towards to an exponential income distribution.

[Insert Figure 2 here]

Here, we particularly employ the household income data in China from 1978 to 2000 to demonstrate that the exponential income distribution (3) emerges spontaneously in a market economy, as argued by Tao¹⁸. This period of time covers the main stages of China's market-oriented economic reformation. In Figure 3, we empirically show that the income structure in China gradually evolved to an exact exponential distribution after the 1990s. By contrast, the income structure in China around the 1990s only approximately obeyed the exponential distribution, while the income structures in 1978 and 1980 radically differed from the exponential distribution. This is strong evidence that the exponential income distribution (3) emerges spontaneously in a market economy.

[Insert Figure 3 here]

The evolution of human society seems to reveal a trend from hunter-gatherer societies, slave-based societies, feudal hierarchical societies, to various types and levels of democratic societies. Along with historical advancement, human societies are getting close to an equal-opportunity economic network. This is reflected as that a larger proportion of populations is evolving towards to an exponential income distribution, while the Pareto distribution is squeezed into a fairly small proportion. By contrast, the Pareto distribution dominated a larger proportion of populations in the period of advocating the law of the jungle (e.g., slave-based societies), which causes the Matthew effect in material allocations. Today, with the continuous rise of digital interconnection, instantaneous communications, media, and blog-based echo chambers, our societies have been promoted to a more democratic and equal-opportunity structure. This

structure may be further improved by the rise of artificial intelligence and block-chain technology.

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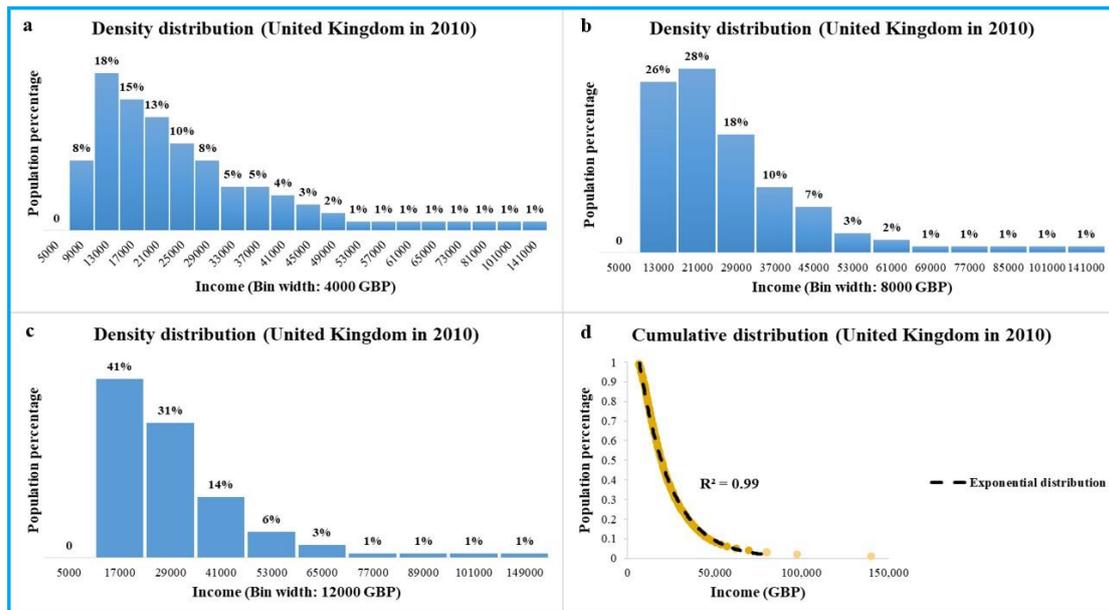


Figure 1. | Density distribution may lead to inconsistent graphs¹. **a:** When the bin width is denoted by 4000 GBP, the histogram of density distribution is a right-skewed graph with a peak, which is close to a Log-normal distribution or a Gamma distribution. **b:** When the bin width is increased to 8000 GBP, the histogram of density distribution begins to deviate from a Log-normal distribution or a Gamma distribution. **c:** When the bin width is increased to 12000 GBP, the histogram of density distribution turns to a monotonic function. **d:** The cumulative distribution of income exhibits an exponential function.

¹ Data resource: <https://www.gov.uk/government/statistics/percentile-points-from-1-to-99-for-total-income-before-and-after-tax>

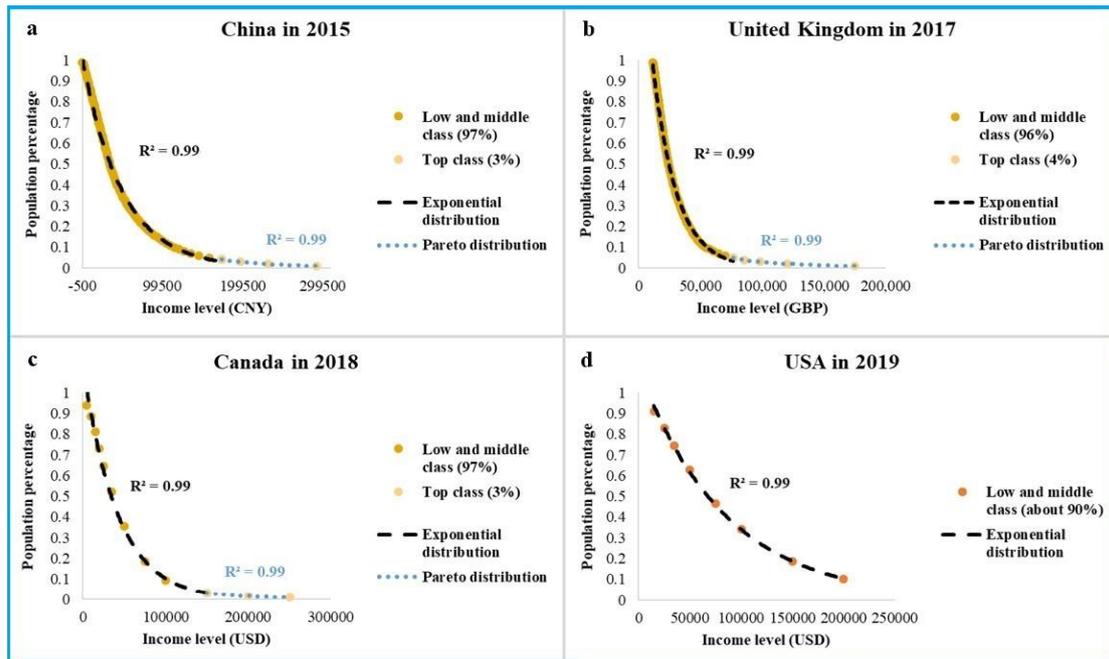


Figure 2. | A larger proportion of populations evolving towards to an exponential income distribution. **a:** The low and middle income classes (97% of populations) in China obeys an exponential income distribution². **b:** The low and middle income classes (96% of populations) in the United Kingdom obeys an exponential income distribution³. **c:** The low and middle income classes (97% of populations) in Canada obeys an exponential income distribution⁴. **d:** The low and middle income classes (more than 90% of populations) in the United States obeys an exponential income distribution⁵.

² Data resource for China: <http://wid.world/data/>

³ Data resource for the United Kingdom: <https://www.gov.uk/government/statistics/percentile-points-from-1-to-99-for-total-income-before-and-after-tax>

⁴ Data resource for Canada: <https://www150.statcan.gc.ca/t1/tb11/en/tv.action?pid=1110000801>

⁵ Data resource for the United States: <https://www.statista.com/statistics/203183/percentage-distribution-of-household-income-in-the-us/>

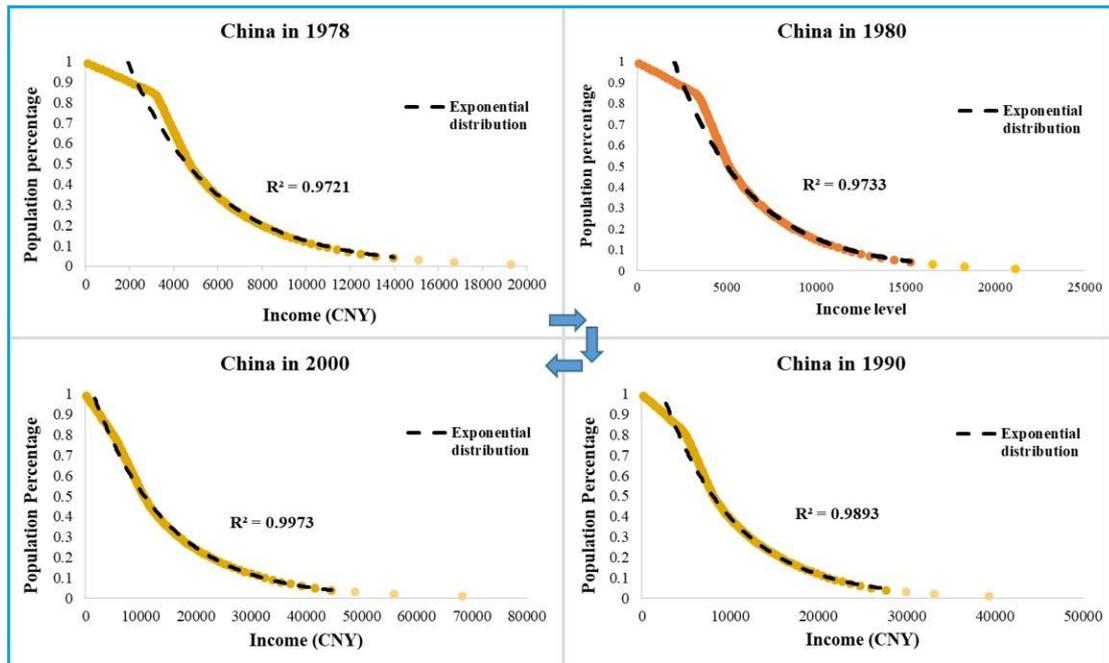


Figure 3. | The income structure in China gradually evolved to an exponential distribution after the market-oriented economic reformation⁶.

⁶ Data resource for China: <http://wid.world/data/>