A Novel Method for Designing Substitution Boxes Based on Mobius Group

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A Novel Method for Designing Substitution Boxes Based on Mobius Group

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Abstract

It is as of now realized that the use of web in the present period is expanding very quickly, numerous users are sharing open and private data over web/internet. They need to ensure security of information. Encryption is one of the most significant viewpoint which is helpful to verify secret data. In encryption, cryptography assume a significant role. There are many algorithms accessible to shield information from unapproved access. In this work, we will use the strategy for cryptography to build up an algorithm which is increasingly viable for correspondence and information trade. In this study, we use a procedure to assemble new substitution boxes (S-boxes) parallel to coset diagram for the action of Mobius group [1] on projective line $PL(F_7)$ over $GF(2^8)$. Afterward, the analyses of constructed S-boxes are performed on the basis of algebraic and statistical analysis.

Keywords: Substitution box (S-box), Galois Field $GF(2^8)$, Mobius Group, Nonlinearity, Linear and Differential Probability, Majority Logic Creation (MLC).

1. Introduction

The term cryptography means the encryption of e-mails, encrypted access to websites, smart cards for banking applications. Cryptography is tightly tied up with modern electronic communication. However, cryptography is an old business. Encryption of a message ensures that the meaning is concealed in it so that someone who reads the message cannot understand any of it unless people crack the message [2]. The substitution box plays a key role in maintaining safe communication. Shannon proposed the idea of a substitution box in 1949 [3]. Substitution box plays a major role in creating uncertainty in data. Several algorithms are available in the literature for construction S-boxes, such as “Data Encryption Standard (DES), Advanced Encryption Standard (AES), Affine Power Affine (APA), Gray, Skipjack, Xyi, and Residue Prime (RP)” S-boxes. A brief description of some of these algorithms is given. The National Bureau Standards (NBS) requested the American company in 1974 to create a cryptosystem that could be used by the government as a standard in unclassified U.S. applications. A system named 'LUCIFER' was designed by IBM. In 1977, this system was altered and rearranged to become DES. DES was made readily available on a speedy processor. This made it better suited for use in large networks of communication. The complete DES program was made public at the time of its launch. DES has been incredibly popular, and remains the most commonly used cryptosystem ever. The data encryption standard has been treated as a standard for symmetrical key encryption, but the biggest downside was that the key length was very small and can easily be hacked [4]. To overcome this problem, in 1997, the

National Institute of Standards and Technology (NIST) launched a contest to find a new symmetric cryptographic algorithm to replace the DES, which will be called the Advanced Encryption Standard. In October 2000, NIST finally revealed that "Rijndael" is the Advanced Encryption Standard (AES) algorithm, its software of two Belgian cryptographers, "Joan Daemen" and "Vincent Rijmen." AES is the most widely used symmetric key algorithm at present [5].

Different researchers have adopted various techniques for constructing S-boxes [6, 7, 8, 9], and a variety of methods have been developed to evaluate the statistical and algebraic structure of S-boxes. [10, 11, 12, 13]. In this research, we use a technique for constructing new S-boxes using coset diagram for Mobius group $M = \langle x, y: x^2 = y^6 = 1 \rangle$ [1, 14, 15,] on $PL(F_7)$. After the action of group on $PL(F_7)$ we then applied this on Galois Field $GF(2^8)$ to obtained new S-boxes. Afterward, the analyses of constructed S-boxes is performed on the basis of algebraic and statistical analysis.

This document is organized as follows: The construction process of the new S-boxes is described in Section 2. Analyses of the proposed S-boxes are investigated in section 3. These analyses include nonlinearity (NL), strict avalanche criterion (SAC), bit independence criterion (BIC), linear approximation probability (LAP), and differential approximation probability (DAP). We analyze the protection of the proposed S-boxes in section 4 with the majority logic criterion (MLC). The conclusion of our proposed work is shown in section 5.

2. Construction Process of Substitution Box

For the manufacturing of novel S-boxes, we shape an adjacency matrix relating to the coset diagram for the action of Mobius group on $PL(F_7)$. The Mobius group is defined as $M = \langle x, y: x^2 = y^6 = 1 \rangle$. For the action of Mobius group M on $PL(F_7)$ we use the following linear fractional transformation $(t)x = -1/3t$ and $(t)y = -1/3(t+1)$ [1, 14, 15]. PL is the projective line over finite field F_7 , i.e. $PL(F_7) = F_7 \cup \infty = \{0, 1, 2, 3, 4, 5, 6, \infty\}$. This action yields a permutation group generated by $\bar{x}$ and $\bar{y}$.

$$\bar{x} = (0 \ \infty) (1 \ 2) (3) (4) (5 \ 6)$$

$$\bar{y} = (0 \ 2 \ 3 \ 4 \ 6 \ \infty) (1) (5)$$

We draw coset diagram by using these permutation. Here $\bar{x}$ has order 2 so the generator $\bar{x}$ is denoted by black edges and bulky black dots denotes the static points of $\bar{x}$ and $\bar{y}$ has order 6, so the generator $\bar{y}$ is represented by blue color which is hexagon whose vertices permuted anticlockwise and the static points of $\bar{y}$ are indicated by bulky blue dots on the coset diagram as displayed in figure.

Figure 1: Coset diagram for modular group on $PL(F_7)$

Presently we structure an adjacency matrix from the above coset graph, for instance assemble an adjacency matrix related to the above coset diagram with the vertices labeled by 0, 1, 2, 3, 4, 5, 6, ∞ . When we perform an action of adjacency matrix on $GF(2^8)$ we do not obtained our desire results. To obtain a unique result we have developed a specific transformation $S(t_n) = At_n + t_{n+128(mod256)}$ named as “Affine Transformation”. This transformation give us different S-boxes having different results. The obtained S-box is shown in Table 1.

Table 1: Proposed S-box in the form of 16x16 matrix

54	19	108	80	227	250	24	73	1	222	87	119	229	59	113	247
18	16	28	197	40	82	216	173	84	220	63	212	137	208	35	136
122	75	23	254	64	51	37	69	4	65	138	225	121	50	102	150
228	199	226	126	191	190	215	168	158	145	5	182	201	106	92	245
213	99	81	169	234	180	79	45	142	85	202	6	238	13	200	98
53	120	198	123	217	179	88	152	161	189	230	232	111	31	104	43
167	131	196	205	195	139	26	103	146	151	96	171	233	32	224	61
204	9	177	255	242	8	67	125	101	153	52	149	3	109	155	83
22	129	38	164	127	134	172	124	170	62	236	251	154	194	10	184
133	249	156	235	132	185	160	71	105	188	33	143	12	86	14	78
221	192	239	17	207	57	203	248	30	140	74	46	187	91	128	244
39	68	243	181	7	159	118	165	90	246	60	107	117	240	148	29
147	97	110	94	253	241	162	176	93	76	47	218	174	70	25	163
58	214	183	175	252	77	42	186	2	15	178	211	21	44	55	166
157	114	141	209	34	237	193	95	48	56	112	72	219	36	144	0
49	223	116	231	100	130	115	89	210	41	206	135	66	20	11	27

3. Algebraic Analysis

In this segment, we narrate our S-boxes with some generally used evaluation techniques includes nonlinearity, strict avalanche criterion, differential approximation probability, bit independent criterion, linear approximation probability.

3.1 Nonlinearity (NL)

Pieprzyk and Finkelstein first introduced the concept of nonlinearity in 1988 [16]. The nonlinearity of a Boolean function can be described as the distance between the function and the set of affine functions. We may say in another sentence that nonlinearity is the number of bits that should be changed to achieve the nearest affine function in a Boolean function's truth table.

The average value for nonlinearity is 120. The nonlinearity over the $GF(2^8)$ of the S-box can be defined as,

$$N(f) = 2^{n-1} - 2^{\frac{n}{2}-1}$$

The highest nonlinearity is 110, the minimum nonlinearity is 106, and average nonlinearity is 107.25. Table 2 shows that the non-linearity of the proposed S-box is almost equal to that of the famous S-boxes when compared with various well-known S-boxes, and much better than that of the most popular S-boxes.

Table 2: Non-linearity of the various S-boxes.

S-boxes	f_0	f_1	f_2	f_3	f_4	f_5	f_6	f_7	Avg
Proposed S-box	108	110	108	106	108	106	106	106	107.25
Skipjack S-box [17]	104	104	108	108	108	104	104	106	105.75
Xyi S-box [18]	106	104	104	106	104	106	104	106	105
Residue Prime [19]	94	100	104	104	102	100	98	94	99.5
Tang et al. [20]	100	103	104	104	105	105	106	109	104.5
Chen et al. [21]	100	102	103	104	106	106	106	108	104.3
Belazi et al. [22]	106	106	106	104	108	102	106	104	105.25
Alkhalidi et al. [23]	108	104	106	106	102	98	104	108	104
Khan et al. [24]	102	108	106	102	106	106	106	98	104.25

3.2 Strict Avalanche Criterion (SAC)

The strict avalanche criterion depends on changes in input and output bit rates. An S-box meets strict avalanche criterion when a single bit shifts the input, resulting in a shift of half the output bits, an S-box meets the strict avalanche criterion [25].

Table 3: The suggested S-box's strict avalanche criterion.

0.5468	0.5312	0.4687	0.5156	0.5000	0.5000	0.4531	0.5000
0.5000	0.4843	0.5156	0.4531	0.5000	0.5312	0.4843	0.5468
0.5312	0.5156	0.5156	0.5000	0.5312	0.4843	0.5000	0.5000
0.4843	0.5000	0.5000	0.5000	0.4687	0.4843	0.5468	0.5312
0.5156	0.4531	0.5312	0.5312	0.5156	0.5000	0.4375	0.5312
0.4531	0.4687	0.5312	0.4843	0.5312	0.5468	0.5000	0.4375
0.4843	0.5468	0.5156	0.5312	0.5000	0.5000	0.4843	0.4531
0.5000	0.5000	0.5468	0.4843	0.5312	0.4375	0.5156	0.4843

Table 3 shows the outcomes of suggested S-box. The suggested S-box's strict avalanche criterion analysis is, the maximum value is 0.5468, the minimum is 0.4375 and the average value is 0.5017.

3.3 Bit Independent Criterion (BIC)

The bit independent criterion for the first time introduced by "Webstar and Tavares" in [26], which is a desirable cryptographic property. It is used to analyze nonlinear transformation behavior by altering input bits which change at the level of output bits. This implies that, for a

given vector set generated by the complement of a single plaintext bit, the entire avalanche variable must be pair-independent [25]. Table 4 shows the BIC output of the proposed S-box for nonlinearity, and Table 5 shows the SAC of the BIC. It can be observed that the BIC of the proposed S-box has an average value 107.

Table No 4: Non-linearity of bit independence criterion of the proposed S-box.

----	102	108	106	110	106	104	106
102	----	110	106	106	106	108	110
108	110	----	110	110	108	110	106
106	106	110	----	106	104	108	104
110	106	110	106	----	110	110	106
106	106	108	104	110	----	106	104
104	108	110	108	110	106	----	106
106	110	106	104	106	104	106	----

Table No 5: Strict avalanche creation of bit independent criterion.

----	0.4980	0.4980	0.5097	0.4980	0.5019	0.4882	0.4941
0.4980	----	0.4824	0.5195	0.4941	0.5156	0.5136	0.4863
0.4980	0.4824	----	0.4746	0.5019	0.5136	0.4804	0.5058
0.5097	0.5195	0.4746	----	0.4824	0.4960	0.4980	0.4824
0.4980	0.4941	0.5019	0.4824	----	0.5039	0.5039	0.4785
0.5019	0.5156	0.5136	0.4960	0.5039	----	0.5234	0.5039
0.4882	0.5136	0.4804	0.4980	0.5039	0.5234	----	0.4941
0.4941	0.4863	0.5058	0.4824	0.4785	0.5039	0.4941	----

3.4 Differential Approximation Probability (DAP)

Throughout the encryption of private information, personal data or documents, the substitution box is called the non-linear constituent. The substitution box recommends differential consistency in optimal situations. Δx is known to be a differential input and Δy may be indicated as a differential in output. An input differential Δx uniquely map to an output differential Δy .

The differential approximation probability of a given S-boxes i.e. DP is defined as,

$$DP^s(\Delta x \rightarrow \Delta y) = \left[\frac{\#\{x \in X | S(x) \oplus S(x \oplus \Delta x) = \Delta y\}}{2^m} \right]$$

As shown in Table 6, the average value of the differential approximation probability for the designed S-box is 6.

Table 6: Differential approximation probability of the planned S-box.

0.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0
4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	6.0	4.0	4.0
4.0	6.0	4.0	6.0	4.0	4.0	6.0	4.0	4.0	4.0	4.0	4.0	4.0	6.0	6.0	4.0
4.0	4.0	4.0	4.0	4.0	4.0	6.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0
4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0
4.0	4.0	4.0	4.0	4.0	4.0	6.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0
6.0	4.0	6.0	6.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0
4.0	4.0	4.0	4.0	6.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	6.0
4.0	4.0	4.0	4.0	6.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0
4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	6.0	4.0	4.0	4.0
4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	6.0	4.0	6.0	4.0
4.0	4.0	4.0	6.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	6.0	4.0
4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0
4.0	4.0	4.0	4.0	6.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	6.0	4.0	4.0
4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0
6.0	4.0	4.0	4.0	4.0	4.0	4.0	6.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0

3.5 Linear Approximation Probability (LAP)

It is regarded as the greatest value of an event disparity. The input bits and output bits will be given the same uniformity. The i^{th} bit is independently measured at the input stage and its results are determined in the output bits. In [27], the linear probability for a given S-box is defined as

$$LP = \max_{\tau x, \tau y} \left| \frac{|\{x | x. \tau x = S(x). \tau y\}|}{2^n} - \frac{1}{2} \right|$$

Here 2^n displays the components of the built S-box $\tau x, \tau y$, the two marks used for the parity of the input bits and the output bits. X contains all possible inputs. The newly proposed S-box's linear approximation probability is 0.1093, which is better than the Xyi and Residue prime S-boxes.

Complete Comparison

Table 7 affords a complete assessment of our proposed S-boxes's NL, SAC, BIC, DAP and LAP with different familiar S-boxes. Figure 2 suggests a graphical illustration of NL, SAC, BIC, DAP and LAP of our recommended S-box with some famous S-boxes.

Table No 7: Complete evaluation of our suggested S-box with few famous S-boxes.

S-boxes	NL	SAC	BIC	DAP	LAP
Proposed S-box	107.25	0.501	107	6	0.109
Ref. [28]	107.75	0.497	105	8	0.125
Ref. [29]	109.25	0.501	104	8	0.093
Ref. [30]	106.75	0.503	100	10	0.132
Ref. [31]	106.75	0.494	98	10	0.125
Ref. [32]	106.75	0.407	98	10	0.132
Ref. [33]	105.25	0.499	104	10	0.132
Ref. [34]	104.75	0.495	103	12	0.132
Ref. [35]	105.87	0.497	103.67	10	0.128
Ref. [36]	105.25	0.694	104.64	10	0.132
Ref. [37]	100	0.481	101.93	16	0.179
Ref. [38]	107	0.501	104.2	10	0.148
Ref. [39]	106.75	0.503	103.64	12	0.148
Ref. [40]	106	0.506	103	12	0.144
Ref. [41]	102.3	0.483	101.57	14	0.167
Ref. [41]	106.25	0.499	103.8	10	0.125
Ref. [43]	106.5	0.504	104.14	10	0.132

Figure 3: Graphically representation of NL, SAC, BIC, DAP, LAP of our suggested S-box with few famous S-boxes

4. Majority Logic Criterion (MLC)

Right now, we're presenting a few valuable statistical analysis of the brand new and a few well-known S-boxes. We use the majority logic creation (MLC) in order to assess the efficacy of the suggested S-box in image encryption applications. Because of the rapid developments in digital image processing, it is very essential to secure computerized data from various

attacks. By the method of encryption, a distortion in the image is generated; the statistical properties must therefore be addressed. In order to obtain it, a variety of analyses including entropy, contrast, correlation, energy and homogeneity are used.

By means of entropy, the quantity of uncertainty is measured. The degree of entropy in an image is associated with the object arrangement. To measure randomness, entropy is used. Contrast analysis tests the capacity of an image to distinguish objects. On the contrary, a high level of security is required to guarantee solid encryption. Encrypted /scrambled image have higher contrast levels in view of randomness. Effectiveness of S-boxes images are assessed by contrast. The contrast value is zero in the case of a constant image. The statistical characteristics of an S-box can be test through correlation analysis. Via this test, the similarities between the pixel patterns of plain and encoded images are determined. In three distinct ways correlation analysis is carried out. The vertical, the horizontal and the diagonal formats are appointed for this reason. Energy analysis determines the energy measurement of a scrambled image while processed by different S-boxes. This move returns the total of the square elements in the GLCM. The proximity of the distribution of elements in the Grey Level Co-Occurrence Matrix (GLCM) to the GLCM diagonal can be calculated using homogeneity analysis.

The consequences of these research are shown in Table 8 in evaluation with the other well-known S-boxes. The original Lena, Pepper, Baboon and Cameraman images are shown in Figure 4. The result of image encryption with the suggested S-box is shown in Figure 5. In Figures 6, 7, 8, 9, histograms of the original and encrypted images of Lena, Pepper, Baboon and Cameraman are displayed.

Table 8: Comparison of statistical analysis of new S-box with other familiar S-boxes.

Images	Entropy	Contrast	Correlation	Energy	Homogeneity
Plain image a	7.4545	0.4046	0.9114	0.1203	0.8762
Plain image b	7.5989	0.4618	0.9200	0.9995	0.8692
Plain image c	7.2572	0.9298	0.6980	0.0832	0.7324
Plain image d	7.0911	0.6165	0.9189	0.1771	0.8840
Encrypted image a	7.4545	8.3900	0.1564	0.0194	0.4824
Encrypted image b	7.5989	9.0854	0.0571	0.0178	0.4619
Encrypted image c	7.2572	9.7589	0.0159	0.0168	0.4061
Encrypted image d	7.0909	9.6683	0.1597	0.0208	0.4963
AES S-box [44]	7.2531	7.5509	0.0554	0.0202	0.4662
APA S-box	7.2264	8.1195	0.1473	0.0183	0.4676
Gray S-box [45]	7.2301	7.5283	0.0586	0.0203	0.4623

Figure 6: Histogram of (a) plan and (b) encrypted image of Lena

(a)

(b)

Figure 7: Histogram of (a) plan and (b) encrypted image of Pepper

(a)

(b)

Figure 8: Histogram of (a) plan and (b) encrypted image of Baboon

(a)

(b)

Figure 9: Histogram of plan and encrypted image of Cameraman

5. Conclusion

In this research, we use a methodology to create new S-boxes relating to coset diagram for the action of Mobius group on projective line $PL(F_7)$ over $GF(2^8)$ to obtained new S-boxes. Next, we analyzed the strength of our planned S-boxes with other notable established S-boxes existing in literature. To calculate the power of our proposed S-boxes, we use algebraic and statistical tests. The algebraic test involves nonlinearity, bit independence criterion, strict avalanche criterion, differential approximation probability, linear approximation probability. In the statistical test, we apply the Majority Logic Criterion (MLC) to assess the efficacy of the proposed S-box in application for image encryption. Statistical tests are “entropy, contrast, correlation, energy and homogeneity”. As our proposed S-box fulfill all the requirements of the analyses, and is taken into consideration as strong S-box for the secure communication. In the statistical test, we follow the Majority Logic Criterion (MLC) to decide the performance of the proposed S-box in in application for image encryption. Entropy, contrast, correlation, energy and homogeneity are statistical tests. As our proposed S-box fulfill all of the analytical conditions, it's far taken into consideration an effective S-box for secure communication.

6. Declaration Statement

I would like to submit the manuscript entitled “*A Novel Method for Designing Substitution Boxes Based on Mobius Group*” by Bilal Arshad, Nasir Siddiqui, Zamir Hussain to be considered for publication as an original article in the Wireless Personal Communications. We declare that this manuscript is original, has not been published before and is not currently being considered for publication elsewhere.

6.1 Funding

There has been no significant financial support for this work.

6.2 Conflicts of Interest/Competing Interests

We know of no conflicts of interest associated with this publication.

6.3 Availability of Data and Material (data transparency)

The data that support the findings of this study are available from the corresponding author, Mr. Bilal Arshad, upon reasonable request.

6.4 Code Availability

Due to the nature of this research, participants of this study did not agree for their code to be shared publicly, so supporting code is not available.

6.5 Author's Contributions

All authors have made equally contributions to the writing of this research paper.

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Figures

Figure 1

Coset diagram for modular group on $\mathbb{P}^1(\mathbb{Z}/7\mathbb{Z})$

Figure 2

Graphically representation of NL, SAC, BIC, DAP, LAP of our suggested S-box with few famous S-boxes

Figure 3
Original images of Lena, Pepper, Baboon, Cameraman

Figure 4
Encrypted images of Lena, Pepper, Baboon, Cameraman by proposed S-box

Figure 5

Histogram of (a) plan and (b) encrypted image of Lena

Figure 6

Histogram of (a) plan and (b) encrypted image of Pepper

Figure 7

Histogram of (a) plan and (b) encrypted image of Baboon

(a)

(b)

Figure 8

Histogram of plan and encrypted image of Cameraman