

# An Efficient Computational Numerical Approach For Nonlinear Mathematical Influenza Disease Modelling

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## Research Article

**Keywords:** Nonlinear mathematical influenza model, Diseased model, Levenberg-Marquardt Backpropagation, Reference databased, Neural networks, Numerical computing

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# An efficient computational numerical approach for nonlinear mathematical influenza disease modelling

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**Abstract:** In this study, an advanced computational numerical scheme based on the Levenberg-Marquardt backpropagation (LMB) neural network (NN) process, i.e., LMB-NN is presented for solving the nonlinear mathematical influenza disease model. The nonlinear mathematical influenza disease model depends on four categories named susceptible  $S(t)$ , infected  $I(t)$ , recovered  $R(t)$  and cross-immune individuals proportion  $C(t)$ . Six different cases of the nonlinear mathematical influenza disease model have been numerically treated using the LMB-NN process and the comparison of the results has been presented by using the reference data-based solutions designed based on the Adams results. The numerically obtained results of the nonlinear mathematical influenza disease model using the verification, testing, and training procedures are calculated using the LMB-NN process to reduce the functions of mean square error (MSE). For the correctness, competence, effectiveness, and efficiency of the LMB-NN process, the proportional and analysis methods are performed using the analysis of correlation, MSE results, error histograms and regression.

**Keywords:** Nonlinear mathematical influenza model; Diseased model; Levenberg-Marquardt Backpropagation; Reference databased; Neural networks; Numerical computing.

## 1. Introduction

There are many serious diseases caused by viruses, among them influenza is one of them that mainly attacks the upper respiratory parts, nose, bronchi, throat and sometimes affects the lungs. Influenza is not a common deathly disease and most people recover in 1–2 weeks without taking any medical cure. Influenza is a serious risk for those who are older age or those who are suffering serious medical illnesses like as diabetes, lung diseases, cancer, heart and kidney problems. In such people, the infection might lead to severe problems of primary illnesses, pneumonia as well as death. The influenza epidemic rate is reported 5–15% per year

of the population that is affected by the upper breathing tract infections. Around the world, yearly epidemics are reported between 3-5 million cases of severe illness and the number of deaths is reported around 250,000 and 500,000 [1]. Many mathematical epidemiology problems are demonstrated by the autonomous nonlinear ordinary differential system of equations, which indicates the assumptions of the model parameters are independent of time. In these systems, the variables signify subpopulations of recovered, infected, transmitted disease vectors, and susceptible.

Astuti et al [2] proposed a multi-step differential transform method to solve the influenza virus model with disease resistance. Erdem et al [3] give a mathematical analysis of a SIQR influenza model with imperfect quarantine. Alzahrani and Khan [4] introduced a numerical technique to solve a fractional epidemic influenza model. Sun et al [5] presented a multi-objective optimization model for patient allocation during a pandemic influenza outbreak. Ghanbari et al [6] give an analysis of two avian influenza epidemic models involving fractal-fractional derivatives with power and Mittag-Leffler memories. González-Parra et al [7] presented a fractional-order epidemic model for the simulation of outbreaks of influenza A. Schulze-Horsel et al [8] studied the infection dynamics and virus-induced apoptosis in cell culture-based influenza vaccine production. Tchuenche et al [9] discussed the impact of media coverage on the transmission dynamics of human influenza. Hovav et al [10] discussed a network flow model for inventory management and distribution of influenza vaccines through a healthcare supply chain. Patel et al [11] examined the optimal vaccination strategies for pandemic influenza using genetic algorithms. Kanyiri et al [12] presented the applications of optimal control to influenza pneumonia coinfection with antiviral resistance.

The human population influenza disease model is based on the four classes, susceptible class  $S(t)$ , infectious class  $I(t)$ , recovered class  $R(t)$  and cross-immune class  $C(t)$ . The mathematical form of the influenza disease nonlinear model is given as [13]:

$$\begin{cases} S'(t) = \mu - (\mu + \beta I(t))S(t) + \gamma C(t), & S(0) = r_1, \\ I'(t) = \beta(\sigma C(t) + S(t))I(t) - (\alpha + \mu)I(t), & I(0) = r_2, \\ R'(t) = \alpha I(t) - \beta(\sigma - 1)C(t)I(t) - (\delta + \mu)R(t), & R(0) = r_3, \\ C'(t) = \delta R(t) - (\beta I(t) + \gamma + \mu)C(t), & C(0) = r_4, \end{cases} \quad (1)$$

where the transmission/contact rate for susceptible to infected is  $\beta$ .  $r_1$ ,  $r_2$ ,  $r_3$  and  $r_4$  represent the initial conditions. The periods based on infected, cross-immune and infectious are denoted as  $\gamma^{-1}$ ,  $\alpha^{-1}$  and  $\delta^{-1}$ , respectively. The exposed cross-immune based individuals are denoted by  $\sigma$ , who are drafted in a unit time into the infectious subpopulation [14].

The aim of the present study is to solve the nonlinear mathematical influenza disease model using the Levenberg-Marquardt backpropagation (LMB) neural network (NN) process *i.e.*, LMB-NN. This method has never been reported to solve the nonlinear mathematical influenza disease model. Six different cases representing the nonlinear mathematical influenza disease model have been numerically treated using the LMB-NN scheme. The comparison is performed based on the reference dataset of Adams results using the LMB-NN scheme. Artificial neural networks have been extensively applied in most advance and recent applications of biological system model [15-19], doubly singular multi-fractional order Lane-Emden system [20], singular Thomas-Fermi model [21], model of heat conduction in human head [22], functional differential models [23-25] and higher order nonlinear singular models [26-29]. The novelty of the presented work is itemized as:

- A novel integrated design is presented using the intelligent computing scheme through the designed LMB-NN scheme to find the solutions of the nonlinear mathematical influenza disease model.
- The designed LMB-NN scheme is accessible from the reference Adams dataset for different transmission/contact rate values ( $\beta$ ) to solve the nonlinear mathematical influenza disease model.
- Closely matching of the results using the dataset of the Adams results improves the value and worth of the designed LMB-NN scheme for solving the nonlinear mathematical influenza disease model.
- The presentation through relative investigations of the metrics based on regression, error histograms (EHs), mean square error (MSE) and correlation enhance the proposed LMB-NN scheme for solving the nonlinear mathematical influenza disease model.

The remaining paper parts are described as the proposed solution of the designed LMB-NN scheme for solving the nonlinear mathematical influenza disease model in Sect 2. The methodology based on the designed LMB-NN scheme for solving the nonlinear mathematical influenza disease model with crucial explanation is narrated in Sect 3. The concluding declarations with latent linked investigations together with the statement of future research are given in the final Sect.

## 2. Methodology

The proposed LMB-NN scheme is provided in two steps for the numerical results of the nonlinear mathematical influenza disease model.

- Necessary explanations are provided to make the data based on LMB-NN scheme.
- Execution procedures approve the LMB-NN scheme for solving the nonlinear mathematical influenza disease model.

The proposed LMB-NN scheme is a combination of the NN multi-layer structure together with the optimization LMB process is given in Fig 1. Suggested methodology for the single neuron-based system is provided in Fig 2. The proposed LMB-NN scheme is applied using the ‘nftool’ in the ‘Matlab’ software for the proper parts of testing data, hidden neurons, learning schemes and validation data.

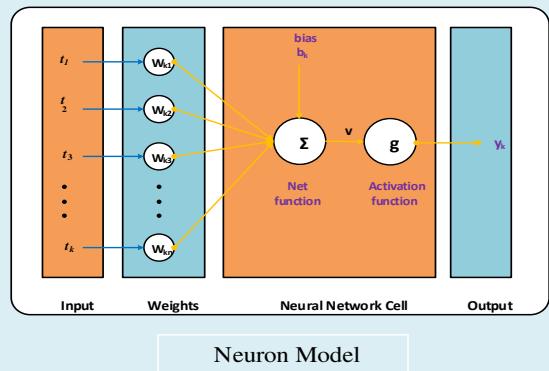
## 1. Methodology

### Designed Database

Designed dataset based Adams results for solving the influenza mathematical disease model

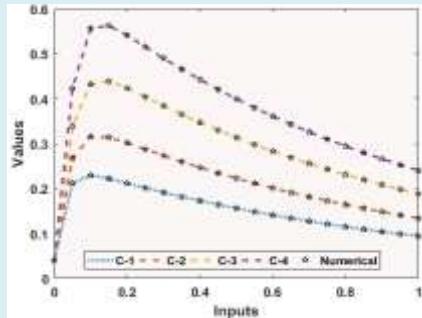
### Intelligent computing

Multi-layer structure of designed LMB based NN model

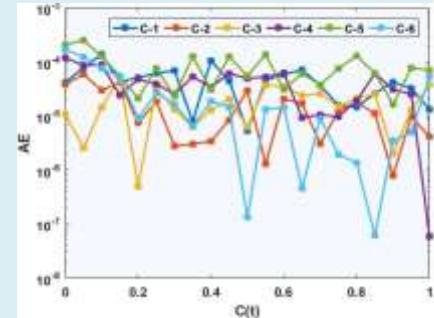


## 2. Results Simulations

Overlapping of results based on LMB based NN approach along with the reference solutions using the Adams dataset for solving the influenza mathematical disease model

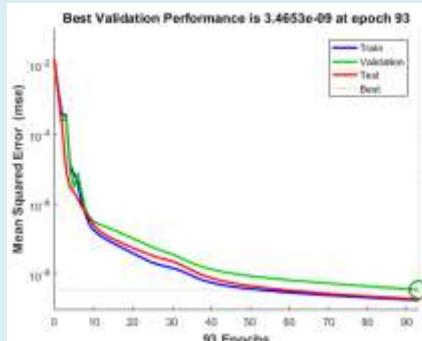


Result Comparison

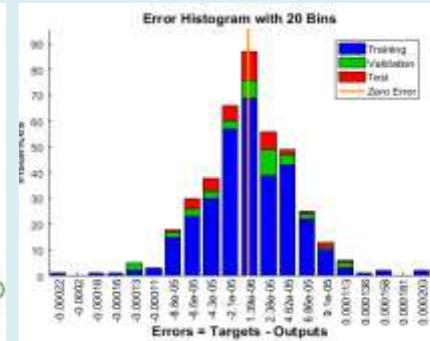


AE

Results of approximate LMB based NN scheme together with the EHs, Regression, MSE, State transition, Fitness plots to solve the influenza mathematical disease model

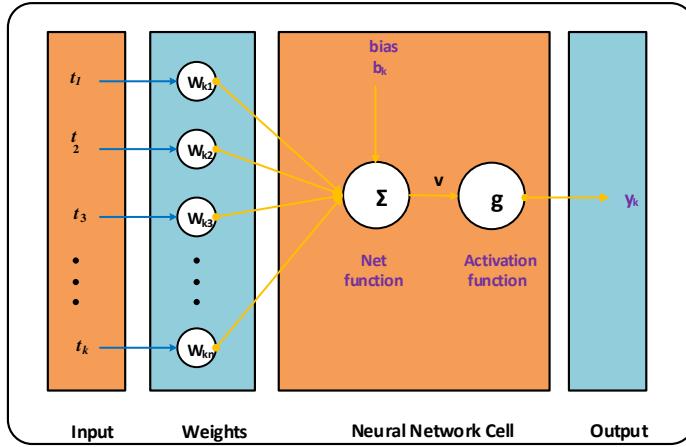


Performance



EHS

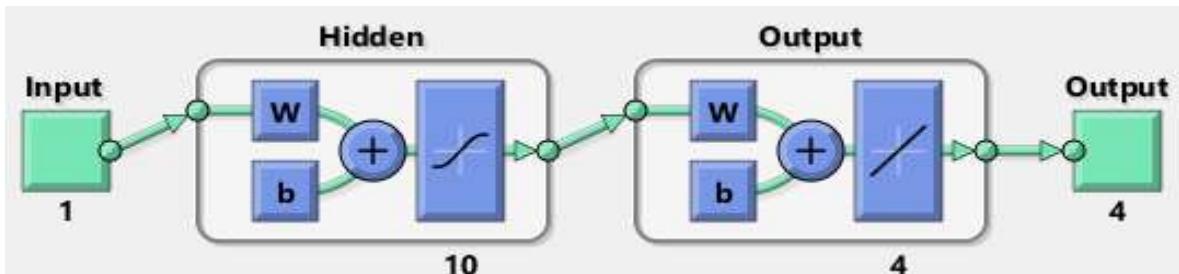
**Fig 1:** Workflow diagram of the proposed LMB neural network approach to solve the mathematical influenza disease model



**Fig 2:** A proposed model based on the single neuron.

### 3. Numerical results with analysis

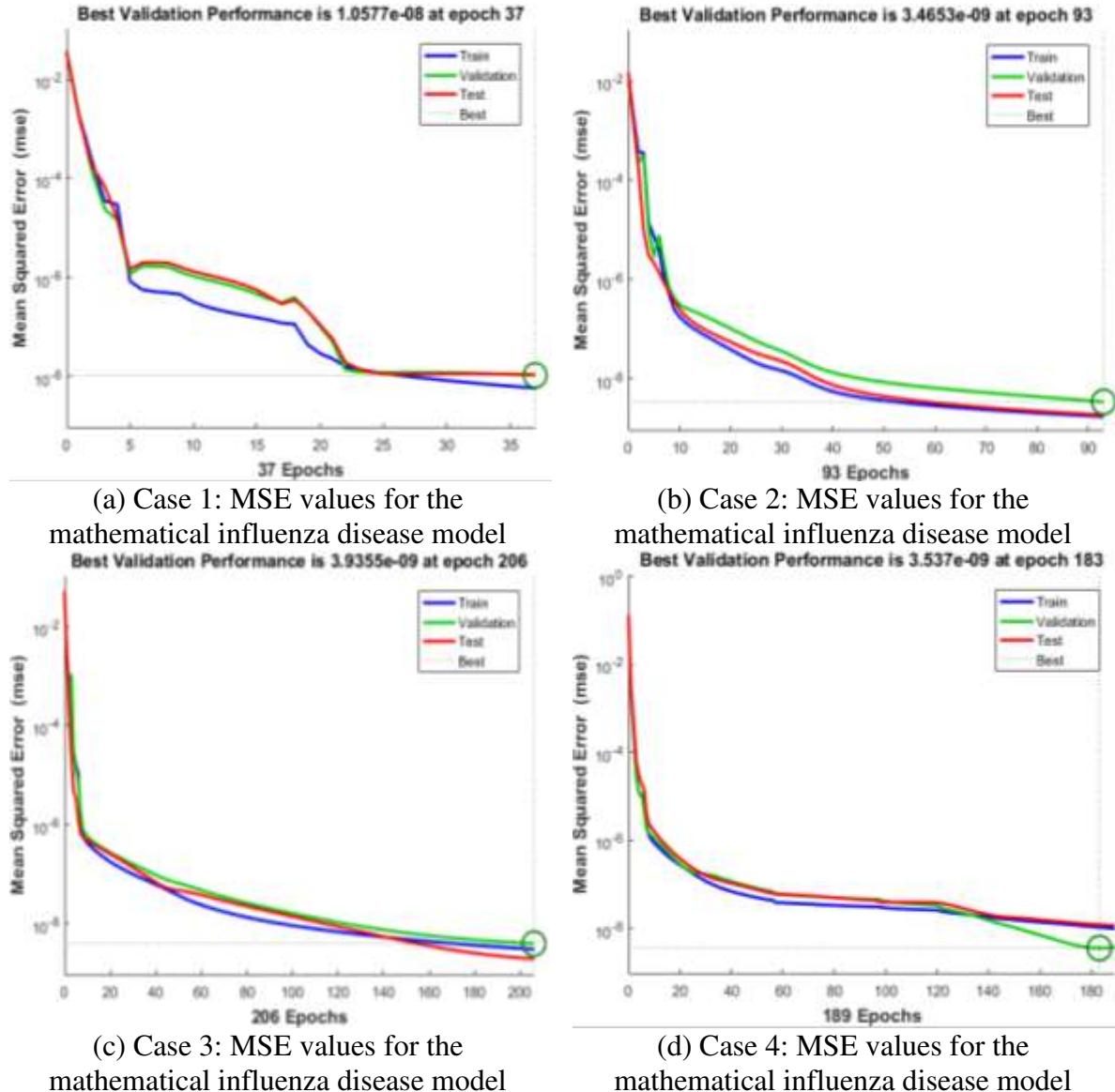
This section numerically presents the solution of six cases of the nonlinear mathematical influenza disease model using the proposed LMB-NN approach. These cases based on the nonlinear mathematical influenza disease model are presented by taking the values of  $\beta$  are 50, 70, 90, 110, 130 and 150. Whereas the values of the other are  $\mu = 0.02$ ,  $\gamma = 0.5$ ,  $\sigma = 0.05$ ,  $\alpha = 73$ ,  $\mu = 0.02$ , and  $\delta = 1$ . The initial conditions are  $r_1 = 0.8$ ,  $r_2 = 0.1$ ,  $r_3 = 0.04$  and  $r_4 = 0.06$ . The obtained results using the proposed LMB-NN approach in the interval [0, 1] with step size 0.01 for the nonlinear mathematical influenza disease model with 10 neurons, together with the training data is 80%, testing and validation data is 10%. The proposed LMB-NN approach is provided in Fig. 3, whereas, the designed LMB based SNN scheme is performed to solve each case of the nonlinear mathematical influenza disease model.

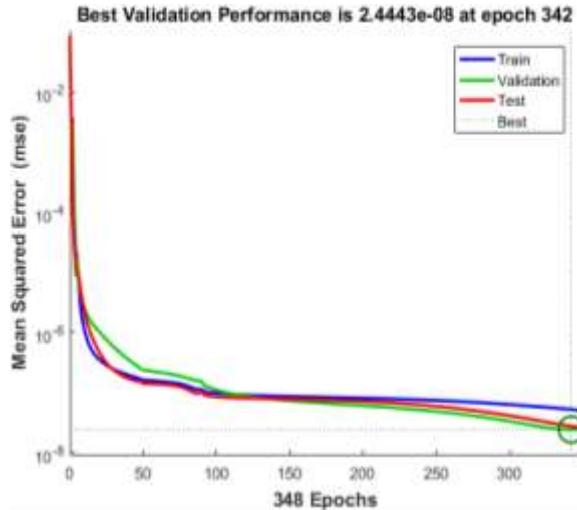


**Fig 3:** Proposed LMB based NN scheme to solve the mathematical influenza disease model

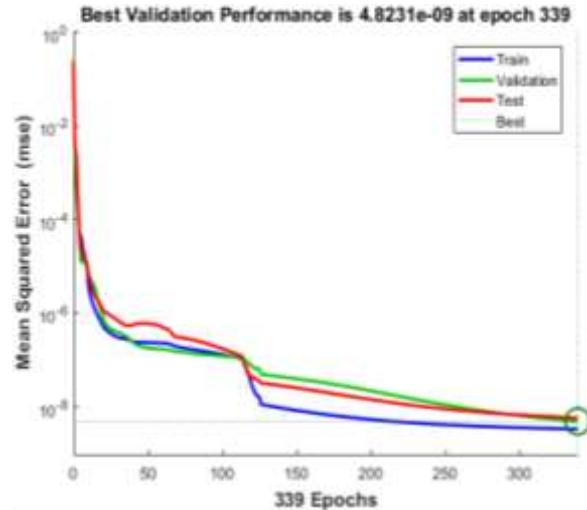
Figs 4-20 shows the plots of the proposed LMB-NN approach for solving the nonlinear mathematical influenza disease model. The achieved numerical outcomes of each case of the nonlinear mathematical influenza disease model using transition and performance states are provided in Figs 4 and 5. The MSE values are calculated for the training state, testing stat, best curve and validation are provided to solve each case of the nonlinear mathematical influenza disease model are given in Fig 4. The best performance values for each case are given at epoch 37, 93, 206, 183, 342 and 339 lie around  $1.057 \times 10^{-8}$ ,  $3.465 \times 10^{-9}$ ,  $3.935 \times 10^{-9}$ ,  $3.537 \times 10^{-9}$ ,  $2.444 \times 10^{-8}$  and  $4.823 \times 10^{-9}$ , respectively. Fig. 5 shows the gradient values with the step size, i.e., Mu of the LMB-NN approach for solving the nonlinear mathematical influenza disease model are  $[9.430 \times 10^{-8}, 9.959 \times 10^{-8}, 9.883 \times 10^{-8}, 3.695 \times 10^{-7}, 2.498 \times 10^{-7}$  and  $9.996 \times 10^{-8}$ . These plotted values show the exactness and the convergence of the LMB-NN approach for solving all the cases of the nonlinear mathematical influenza disease model. Figs 6-11 shows the fitting curves for solving the nonlinear mathematical influenza disease model that shows the obtained results comparability through the designed LMB-NN approach using

the reference-based data set of the Adams results. The maximum error is plotted using the validation, testing, and training inputs through LMB-NN approach lie around  $10^{-7}$  to  $10^{-8}$  for solving the nonlinear mathematical influenza disease model. The EHs are plotted in Fig. 12, whereas the regression plots are drawn in Figs 13 to 18 for each case of the nonlinear mathematical influenza disease model. The co-relation values are implemented to authenticate the regression analysis. It is seen that the correlation values ( $R$ ) for the nonlinear mathematical influenza disease model lie around 1 that indicates the perfect model form. The validation, training, testing plots signify the correctness of the designed LMB-NN approach for solving the nonlinear mathematical influenza disease model.



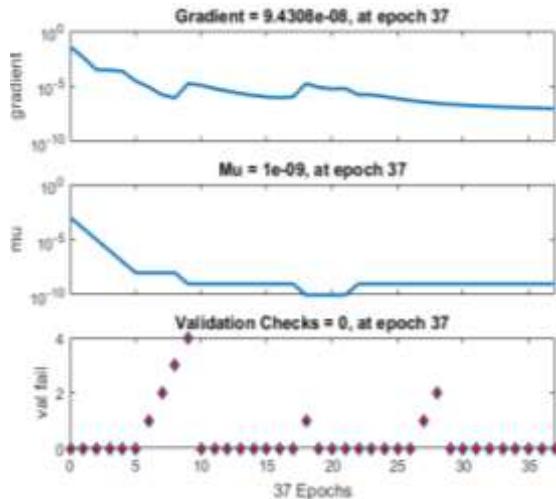


(e) Case 5: MSE values for the mathematical influenza disease model

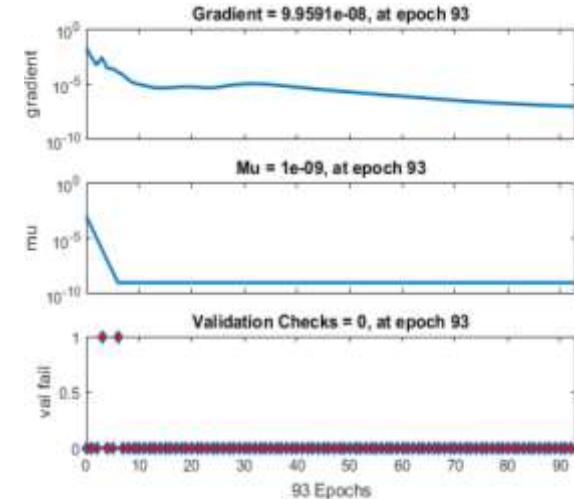


(f) Case 6: MSE values for the mathematical influenza disease model

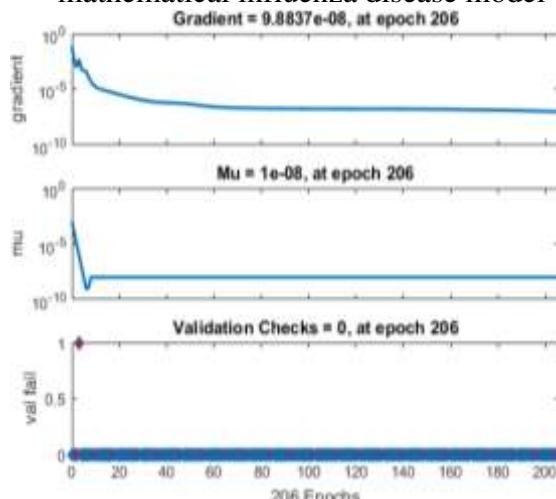
**Fig 4:** MSE performance for the proposed LMB based NN scheme to solve the mathematical influenza disease model.



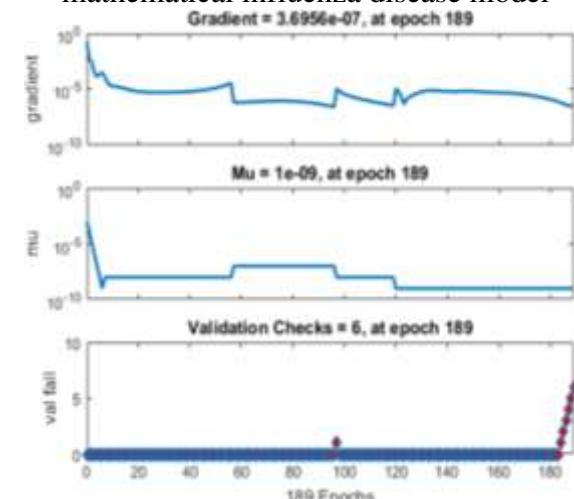
(a) Case 1: State transition for the mathematical influenza disease model



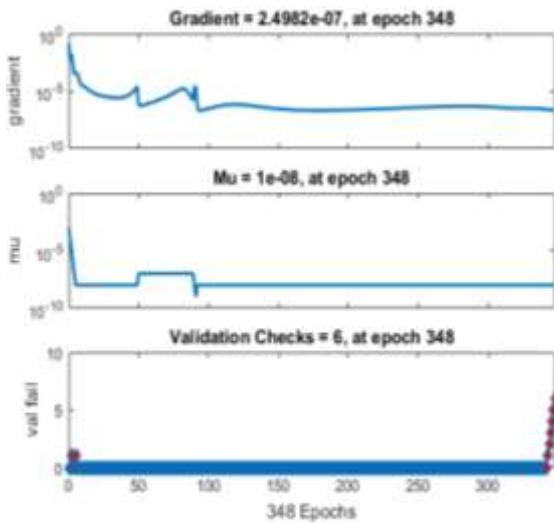
(b) Case 2: State transition for the mathematical influenza disease model



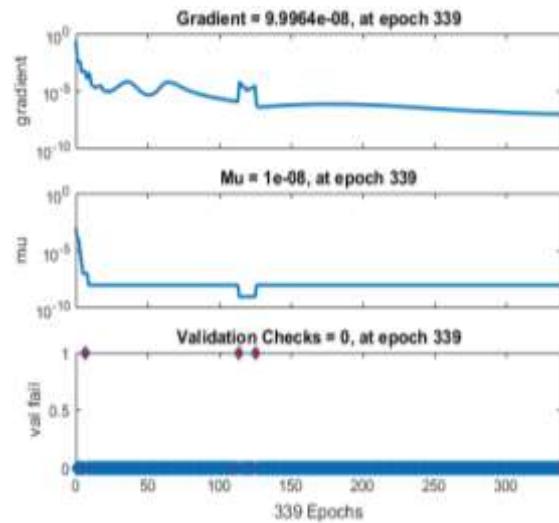
(c) Case 3: State transition for the mathematical influenza disease model



(d) Case 4: State transition for the mathematical influenza disease model

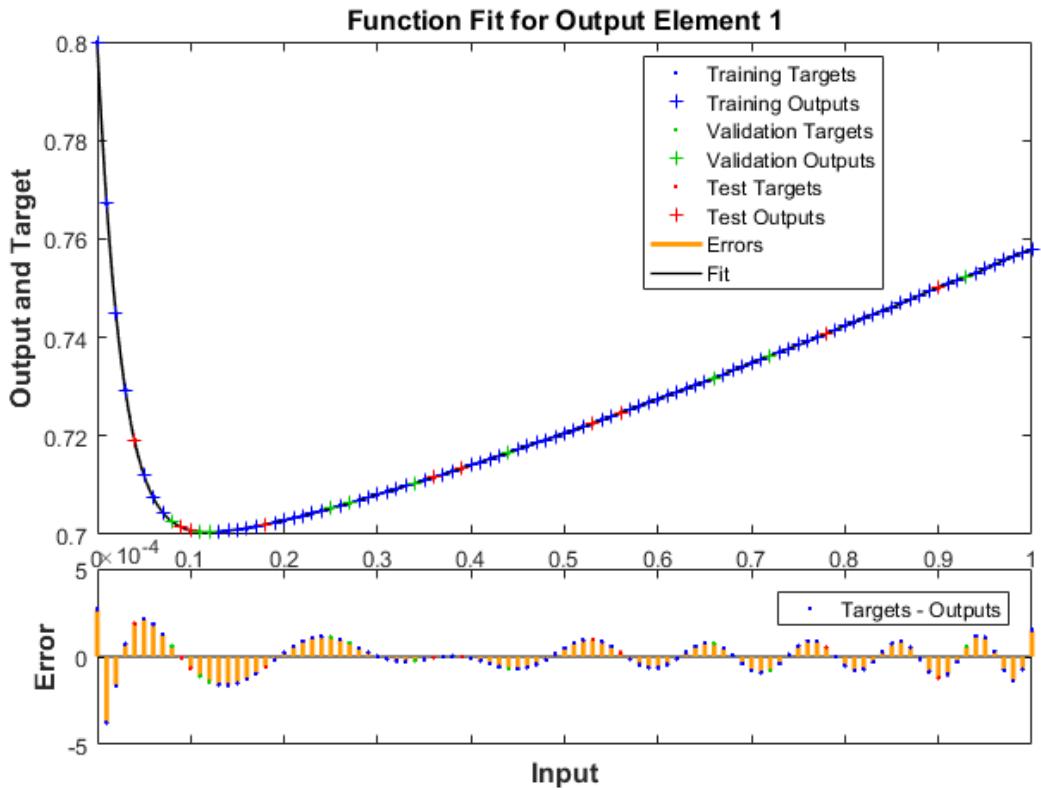


(e) Case 5: State transition for the mathematical influenza disease model

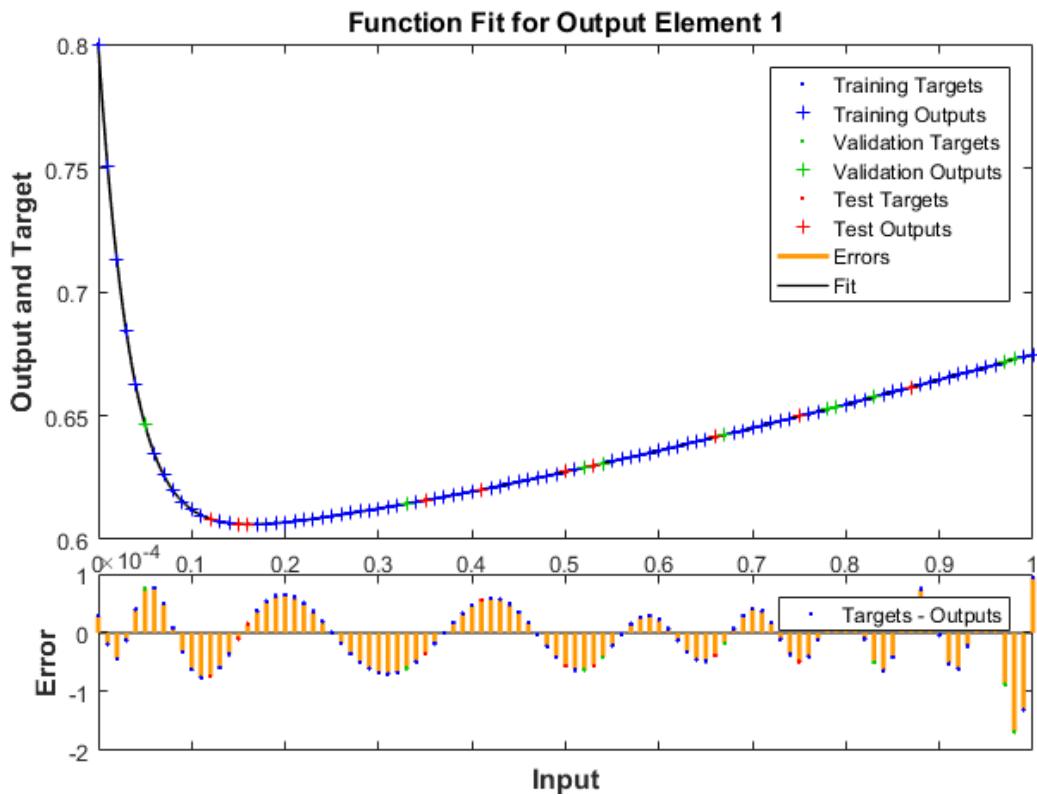


(f) Case 6: State transition for the mathematical influenza disease model

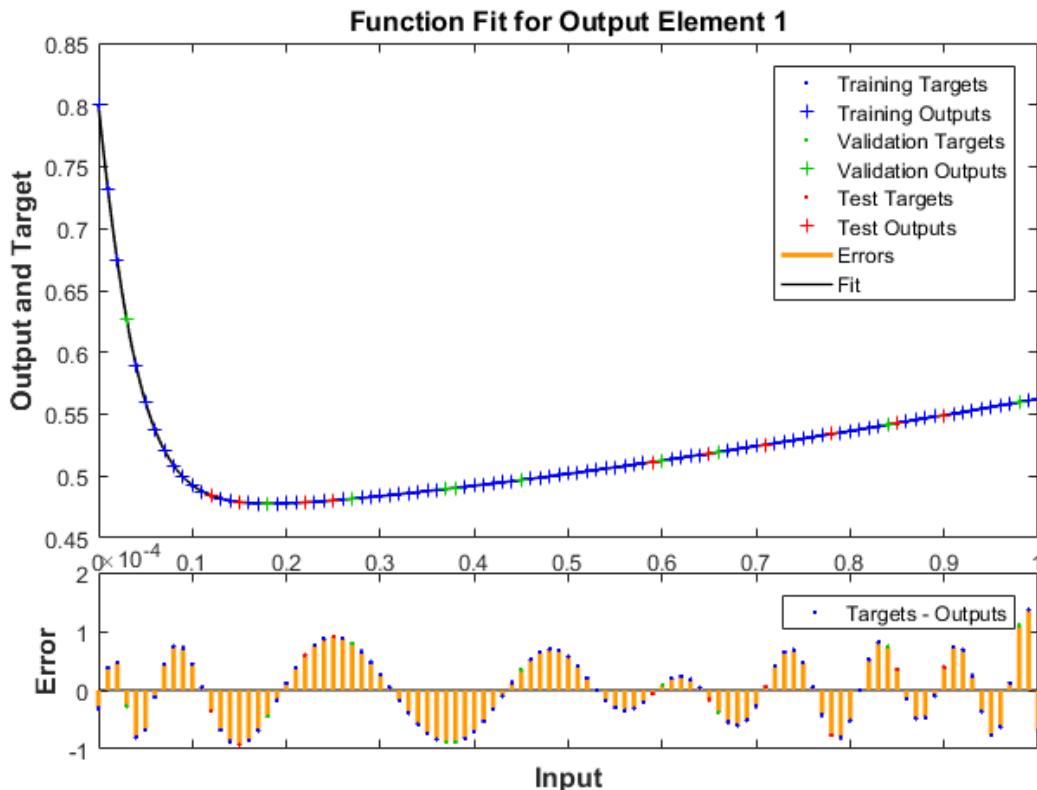
**Fig 5:** State transition values for the LMB based NN scheme to solve the mathematical influenza disease model.



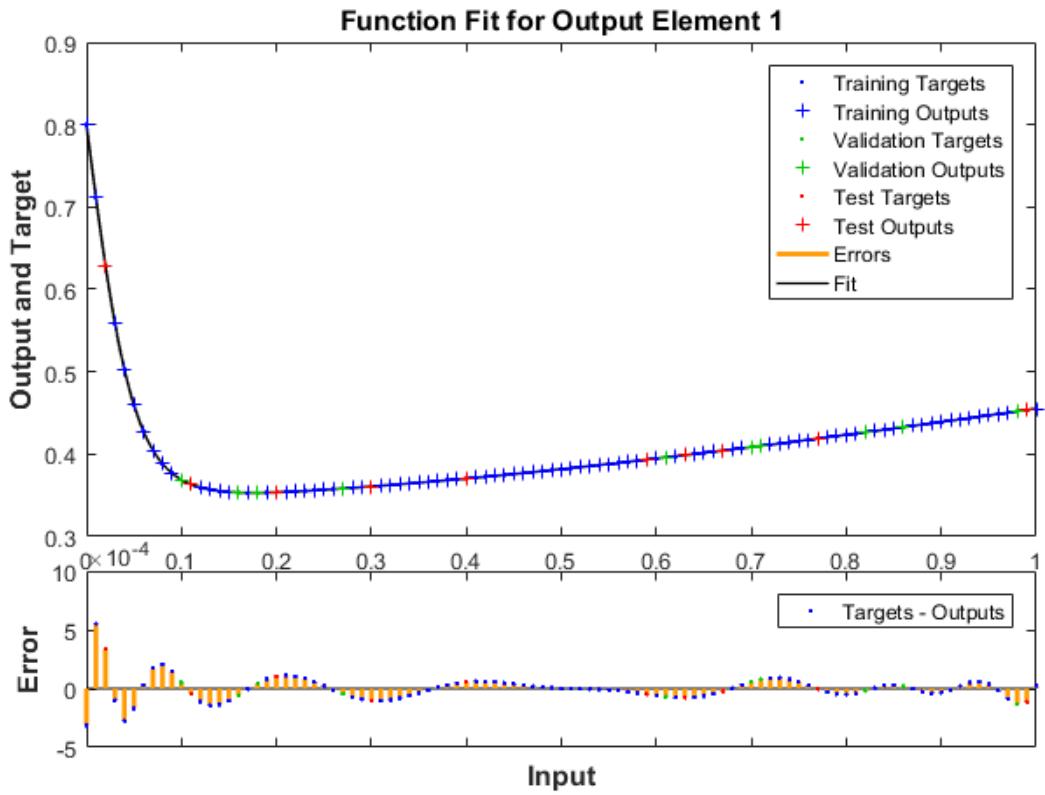
**Fig 6:** Comparison of the LMB based NN scheme to solve the mathematical influenza disease model of Case 1



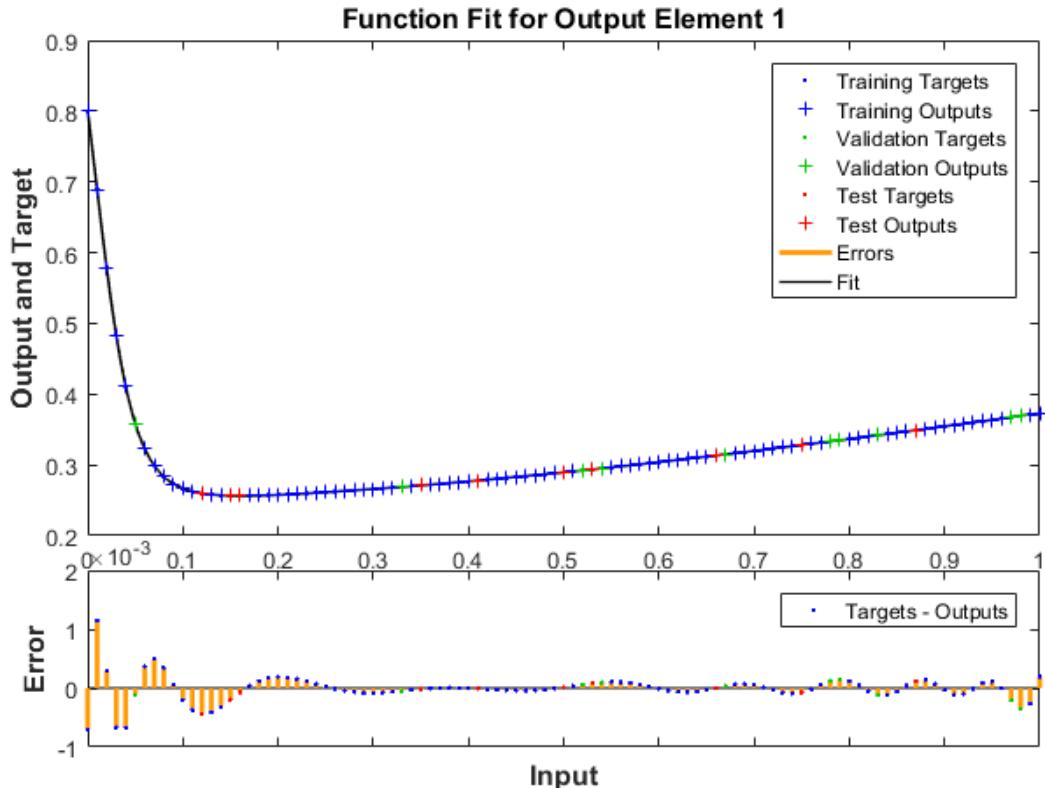
**Fig 7:** Comparison of the LMB based NN scheme to solve the mathematical influenza disease model of Case 2.



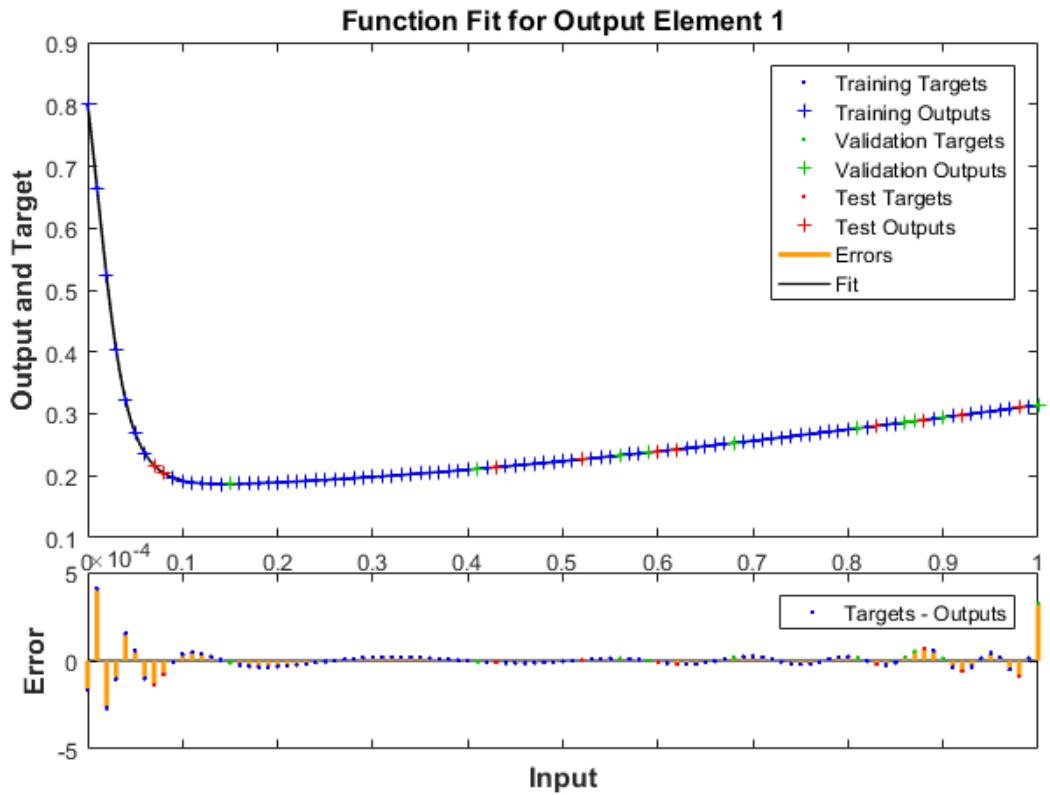
**Fig 8:** Comparison of the LMB based NN scheme to solve the mathematical influenza disease model of Case 3



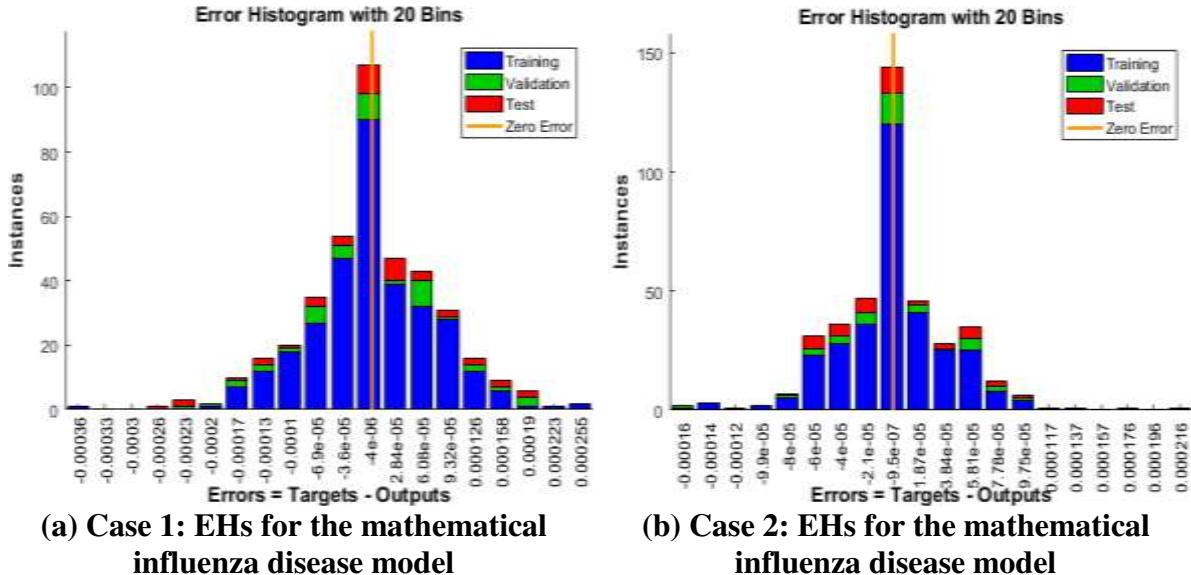
**Fig 9:** Comparison of the LMB based NN scheme to solve the mathematical influenza disease model of Case 4.

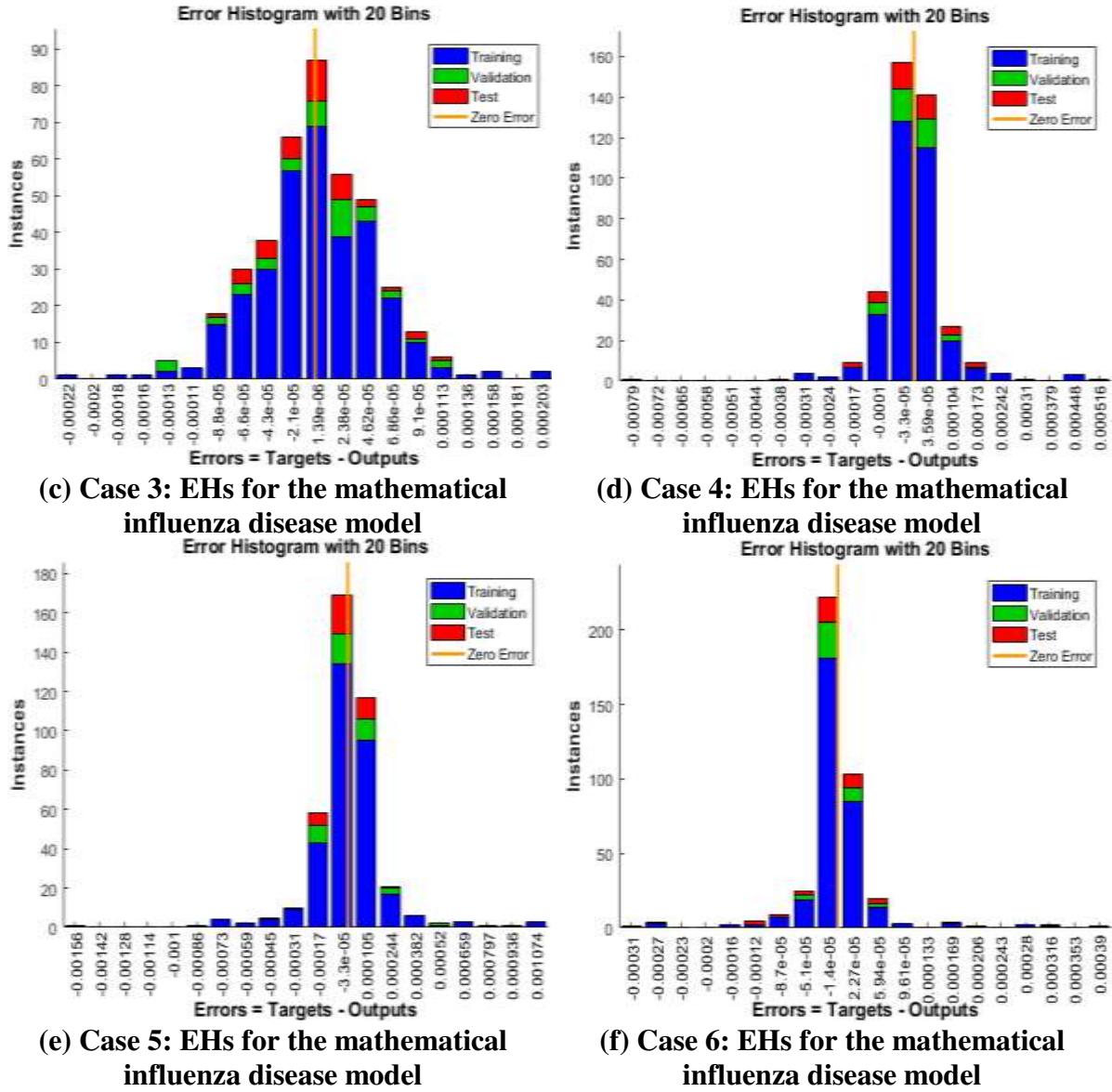


**Fig 10:** Comparison of the LMB based NN scheme to solve the mathematical influenza disease model of Case 5.

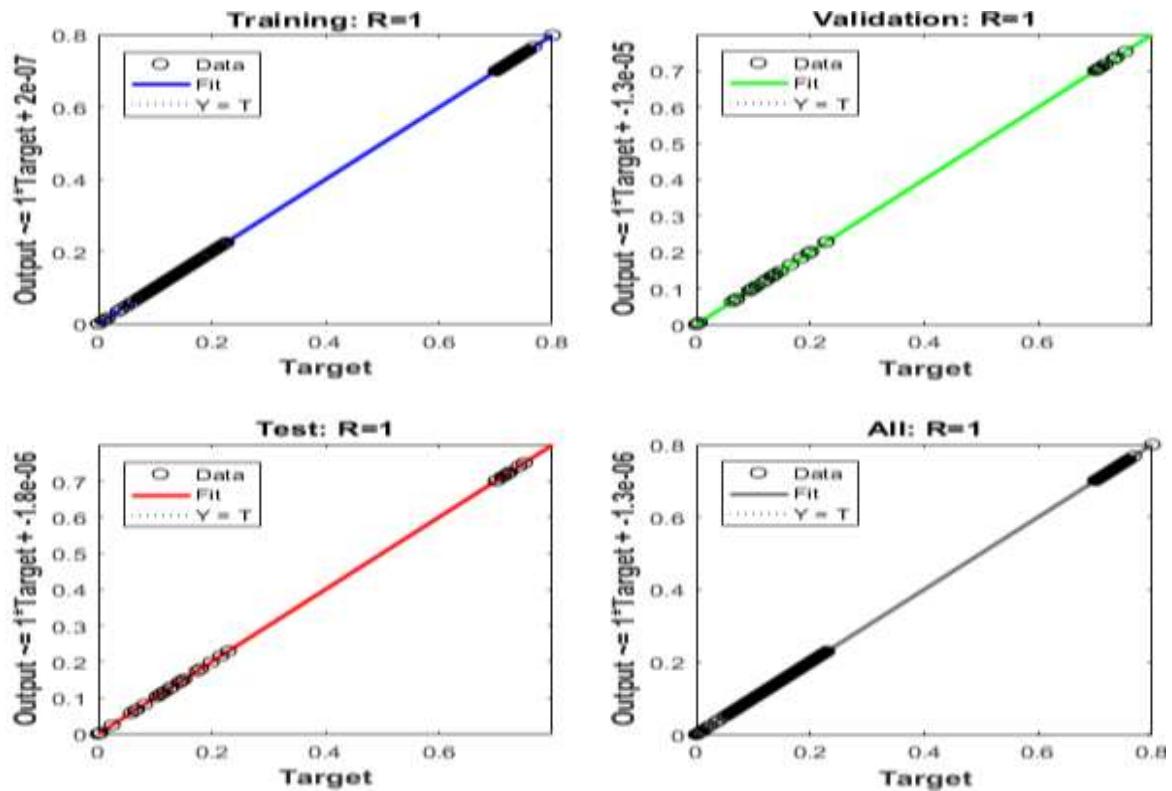


**Fig 11:** Comparison of the LMB based NN scheme to solve the mathematical influenza disease model of Case 6

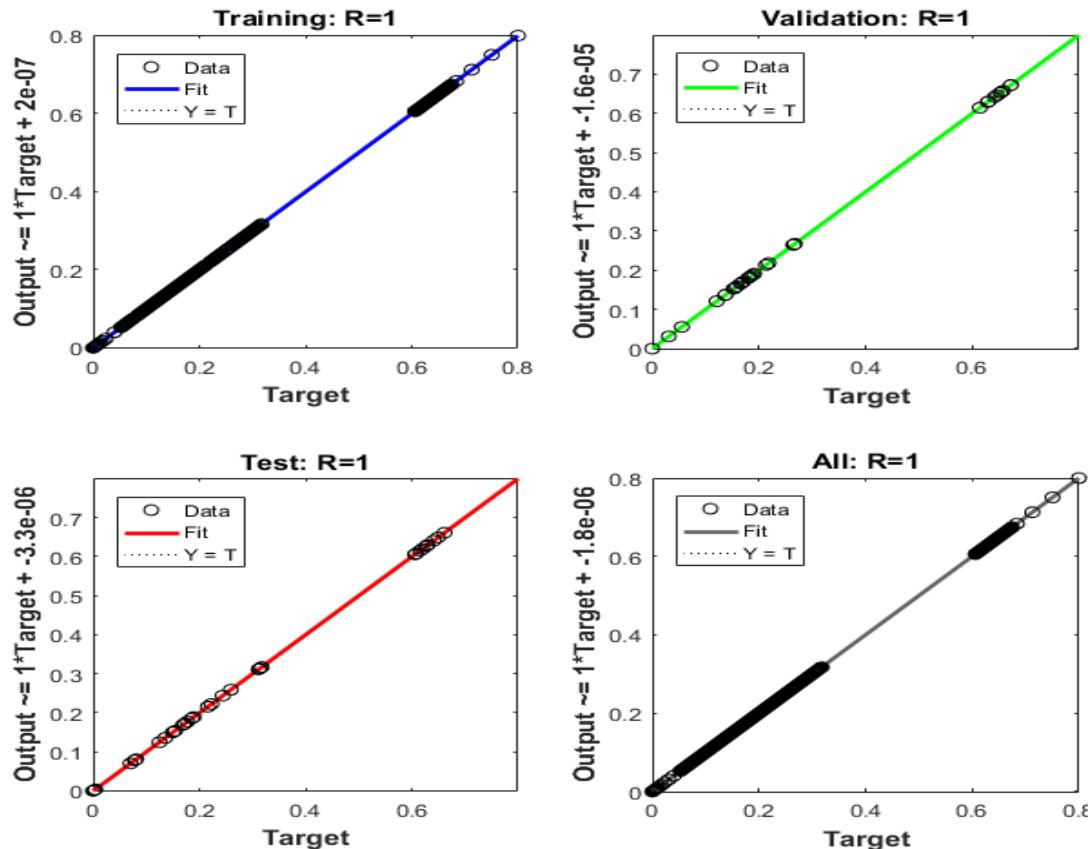




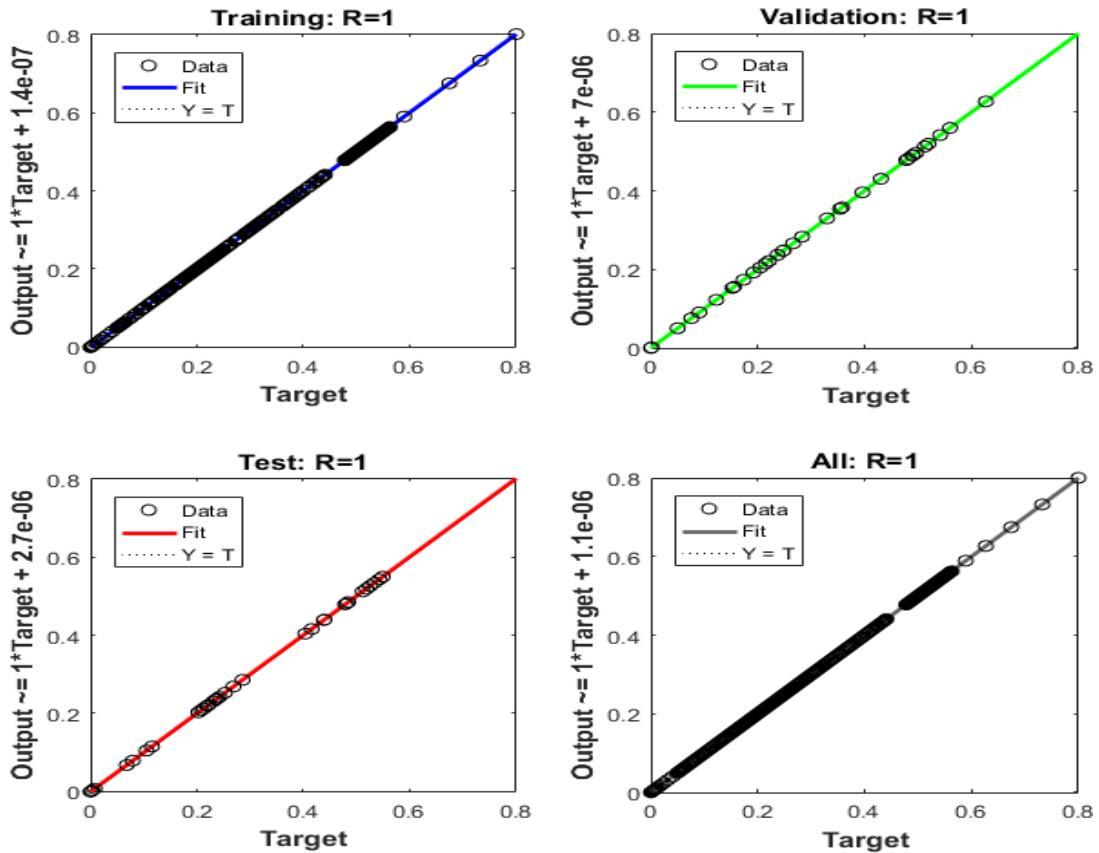
**Fig 12:** EHs for the LMB based NN scheme to solve the mathematical influenza disease model.



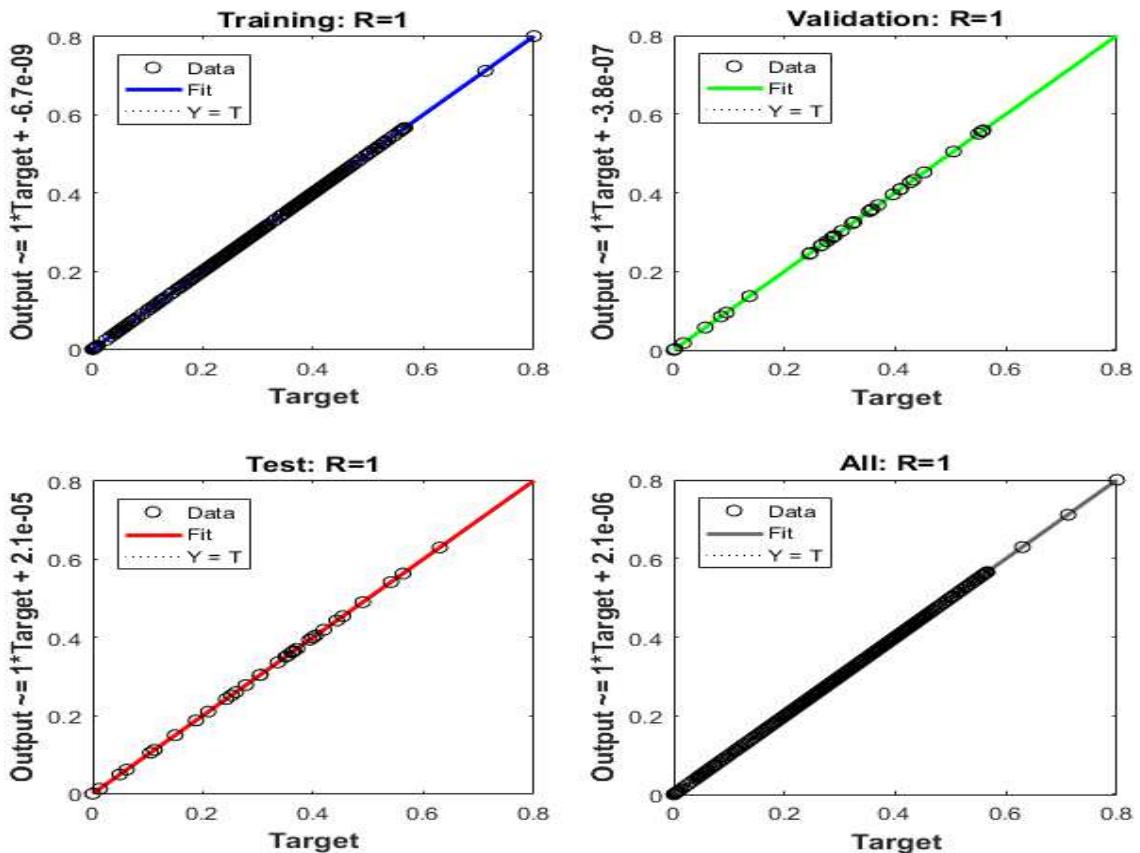
**Fig 13:** Regression of case-1 for the designed LMB based NN scheme to solve the mathematical influenza disease model.



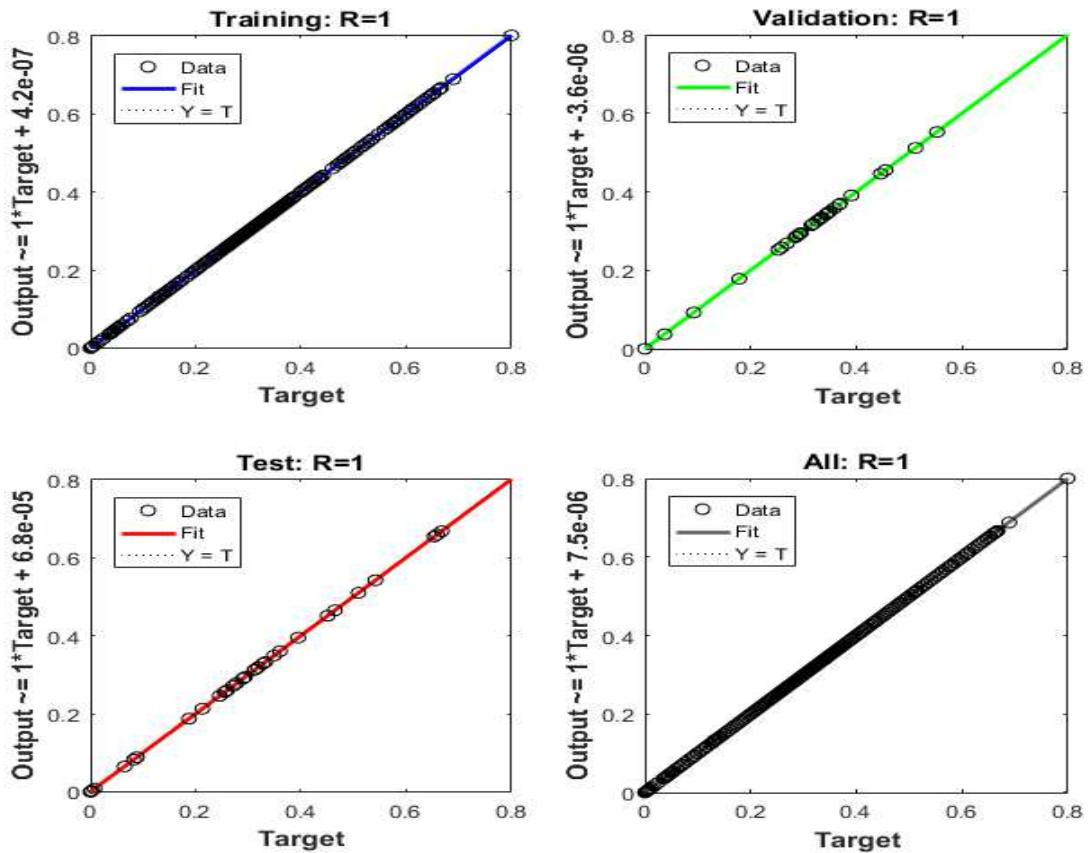
**Fig 14:** Regression of case-2 for the designed LMB based NN scheme to solve the mathematical influenza disease model.



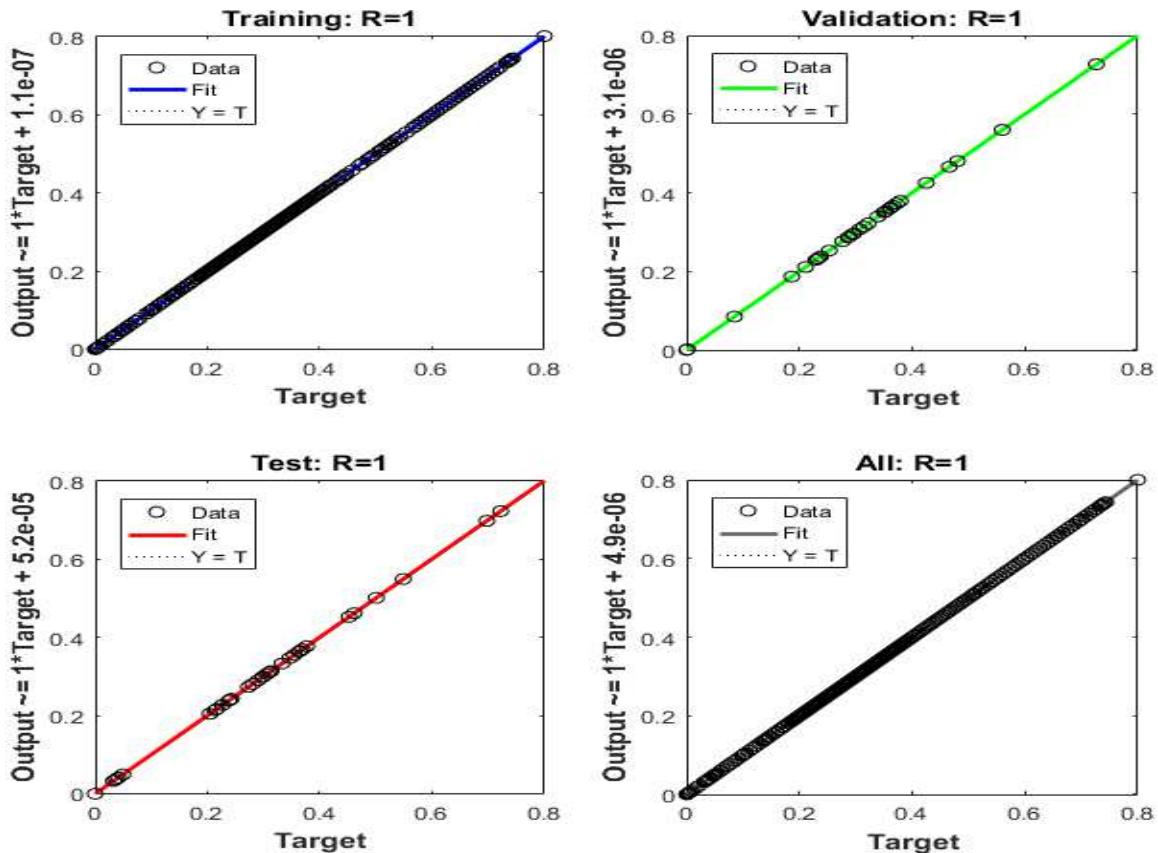
**Fig 15:** Regression of case-3 for the designed LMB based NN scheme to solve the mathematical influenza disease model.



**Fig 16:** Regression of case-4 for the designed LMB based NN scheme to solve the mathematical influenza disease model.

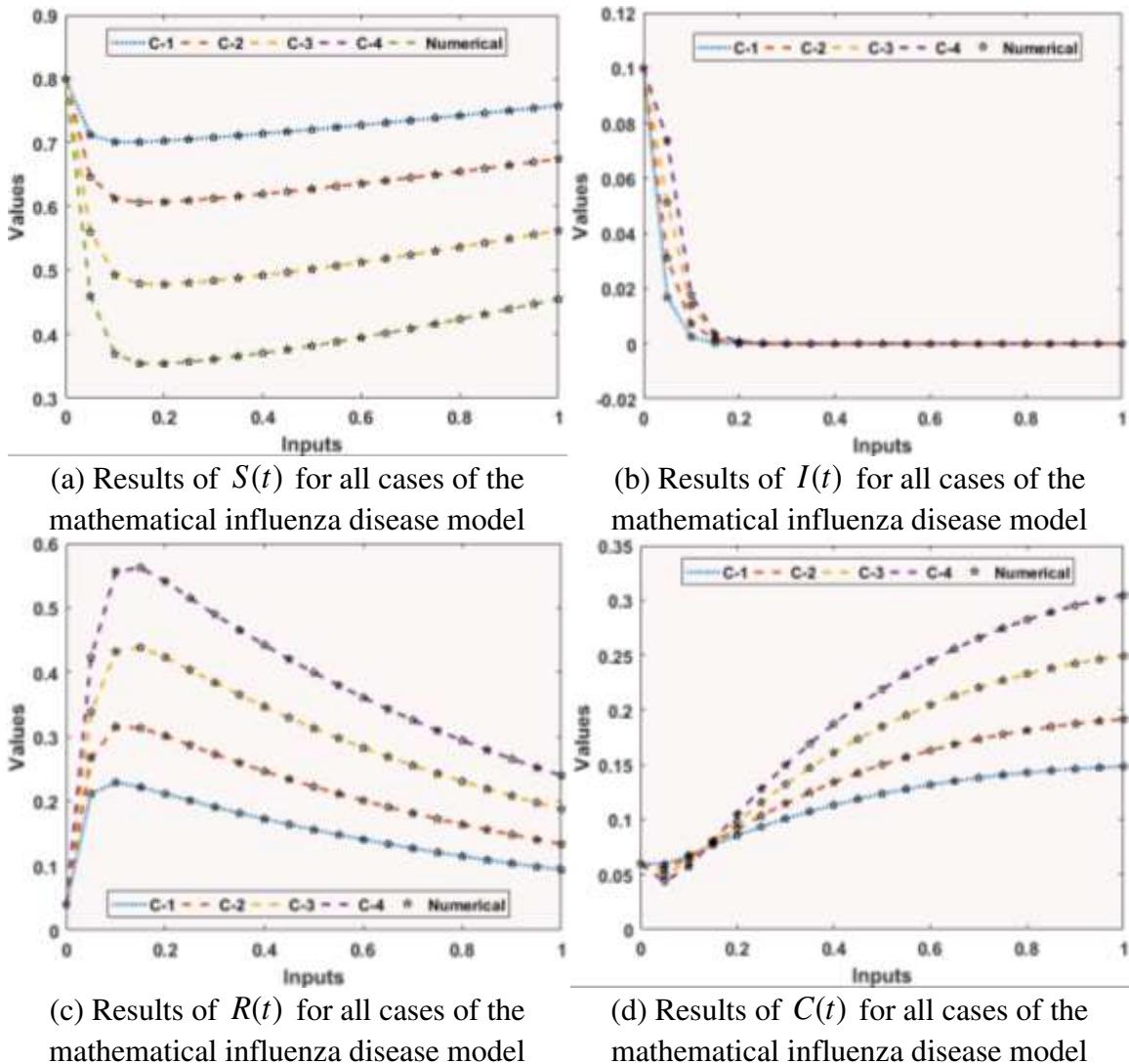


**Fig 17:** Regression of case-5 for the designed LMB based NN scheme to solve the mathematical influenza disease model.

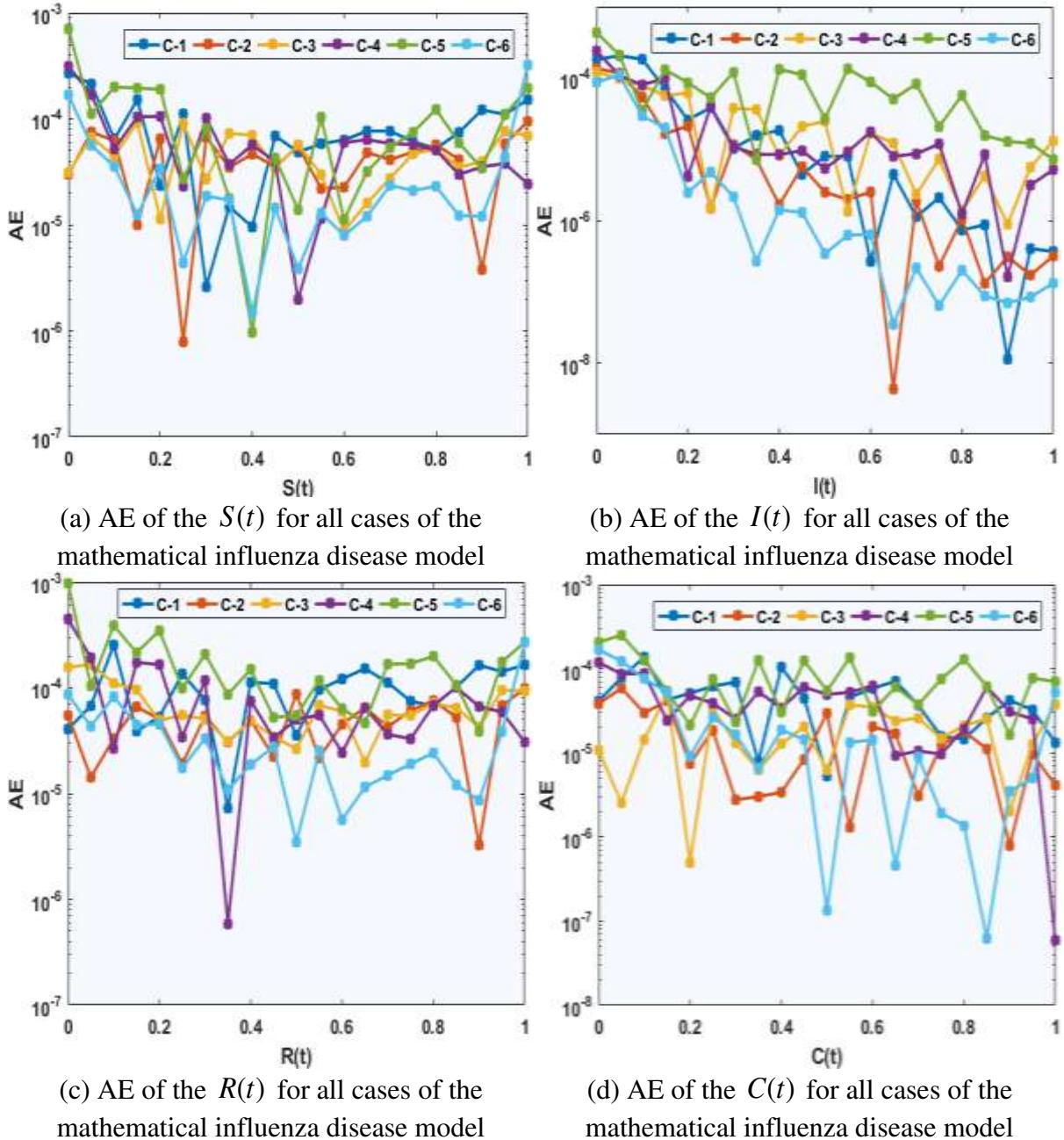


**Fig 18:** Regression of case-6 for the designed LMB based NN scheme to solve the mathematical influenza disease model.

Fig 19 represents the comparison of the outcomes through the designed LMB-NN approach using the dataset based on the Adams numerical results for each form of the nonlinear mathematical influenza disease model. The results overlapping prove the validations of the designed LMB-NN approach for solving the nonlinear mathematical influenza disease model. These performance-based values demonstrate the exactness of the designed LMB-NN approach. Fig 20 depicts the AE plots for each case of the nonlinear mathematical influenza disease model. The AE values for the parameters  $S(t)$ ,  $I(t)$ ,  $R(t)$  and  $C(t)$  lie around  $[10^{-04}, 10^{-06}]$ ,  $[10^{-04}, 10^{-08}]$ ,  $[10^{-03}, 10^{-05}]$  and  $[10^{-03}, 10^{-06}]$  for each case of the nonlinear mathematical influenza disease model, respectively. These numerical results verify the accurateness and the exactness of the designed LMB-NN approach to solve each case of the mathematical influenza disease model.



**Fig 19:** Comparison of the results of LMB based NN scheme for solving each case of the mathematical influenza disease model.



**Fig 20:** AE values using LMB-NN scheme to solve each case of the nonlinear mathematical influenza disease model.

## 5. Conclusions

The presented work is associated to design a biological nonlinear mathematical influenza disease model using the numerical computing integrated Levenberg-Marquardt backpropagation (LMB) approach and the supervised neural networks (NN). The hidden neurons are used 10 and the training data is used 80 %, while the validation/testing data is used 10% data to adjust the proposed LMB-NN approach to solve each case of the nonlinear mathematical influenza disease model. To check the excellence, brilliance, perfection and exactness of the LMB-NN approach, the overlapping of the obtained results from the designed LMB-NN scheme with the reference databased Adams results are implemented. The values of the performance using the mean square error and convergence for training, best curve, authentication and testing are provided for all cases of the nonlinear mathematical influenza

disease model. The performance through co-relation is accomplished for the authentication of the regression process. The gradient values along with the step size are performed for each case of the nonlinear mathematical influenza disease model. Moreover, the correctness and precision are calculated using the graphical as well as numerical conformations of the convergence plots, error-histograms, MSE indexes and regression dynamics, respectively.

In future, the proposed LMB-NN scheme can be explored for the fluid dynamic models, biological nonlinear models, prediction differential models and singular higher-order models [30-38].

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Not applicable.

### Compliance with ethical standards

#### Conflict of interest

The authors also declare that there is no conflict of interest.

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# Figures

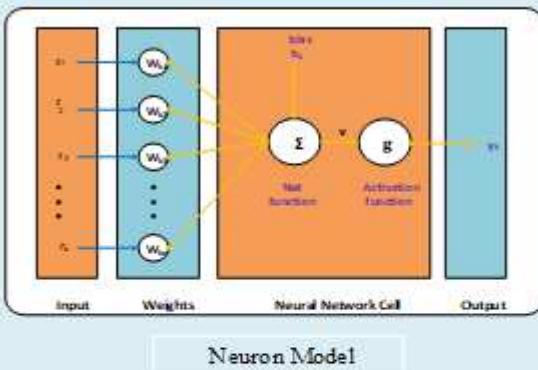
## 1. Methodology

### Designed Database

Designed dataset based Adams results for solving the influenza mathematical disease model

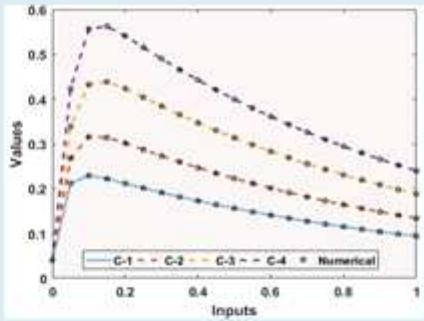
### Intelligent computing

Multi-layer structure of designed LMB based NN model

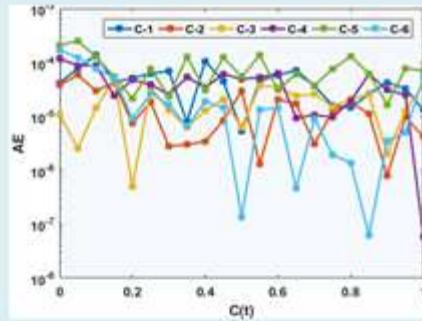


## 2. Results Simulations

Overlapping of results based on LMB based NN approach along with the reference solutions using the Adams dataset for solving the influenza mathematical disease model

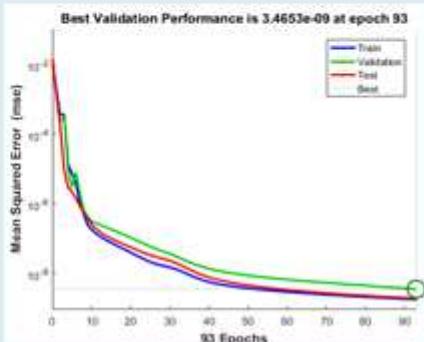


Result Comparison

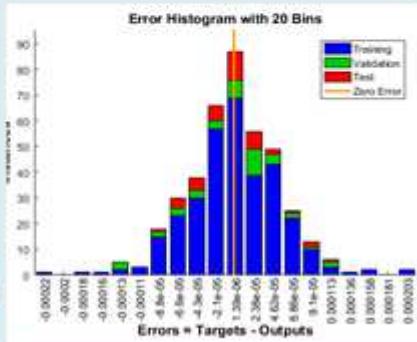


AE

Results of approximate LMB based NN scheme together with the EHs, Regression, MSE, State transition, Fitness plots to solve the influenza mathematical disease model



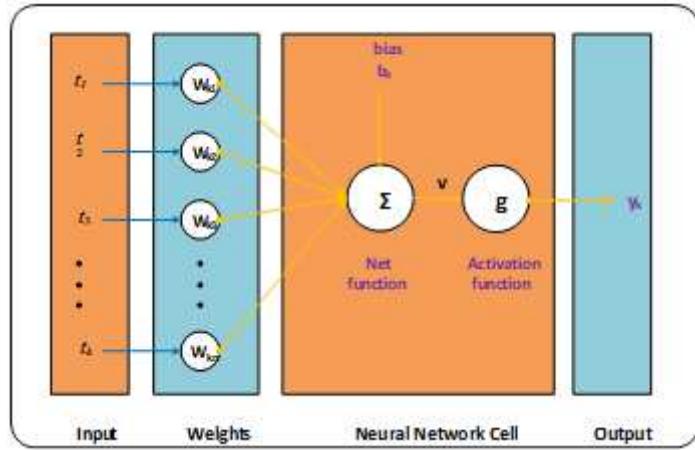
Performance



EHS

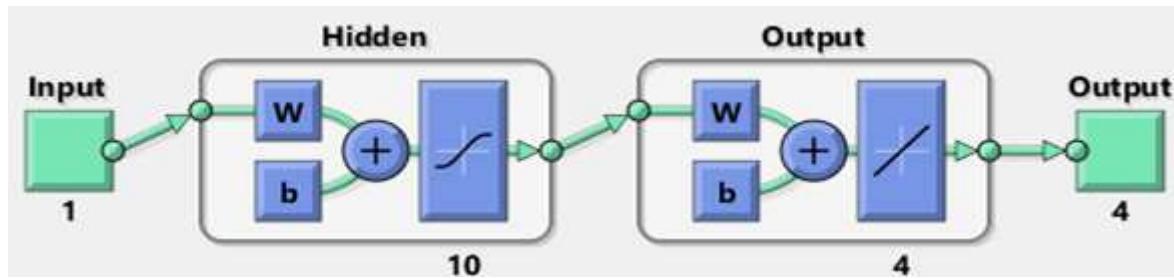
Figure 1

Workflow diagram of the proposed LMB neural network approach to solve the mathematical influenza disease model



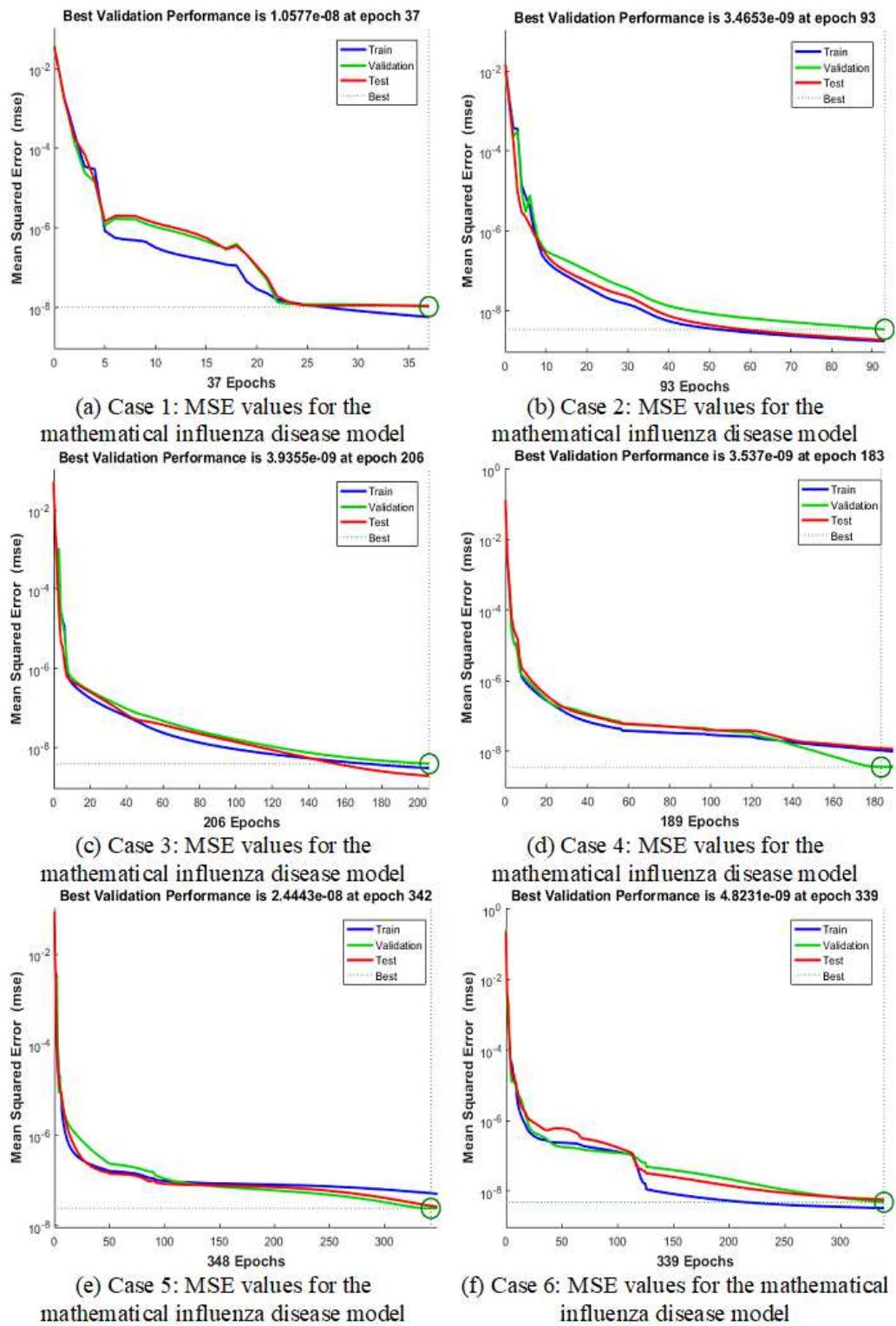
**Figure 2**

A proposed model based on the single neuron.



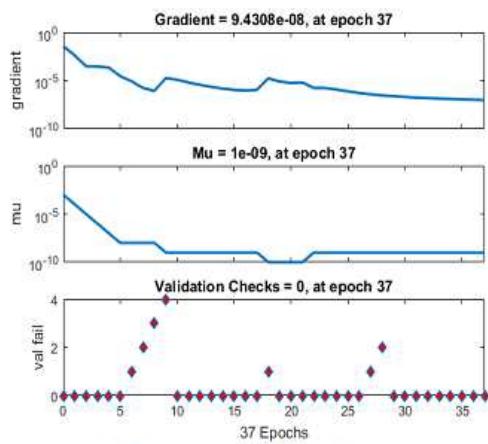
**Figure 3**

Proposed LMB based NN scheme to solve the mathematical influenza disease model

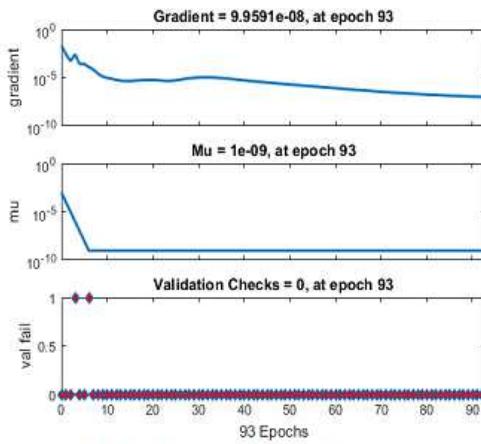


**Figure 4**

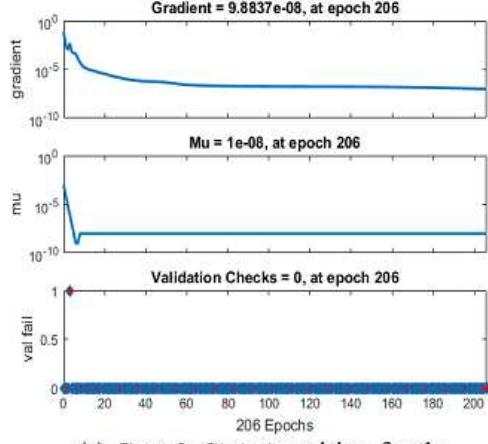
MSE performance for the proposed LMB based NN scheme to solve the mathematical influenza disease model.



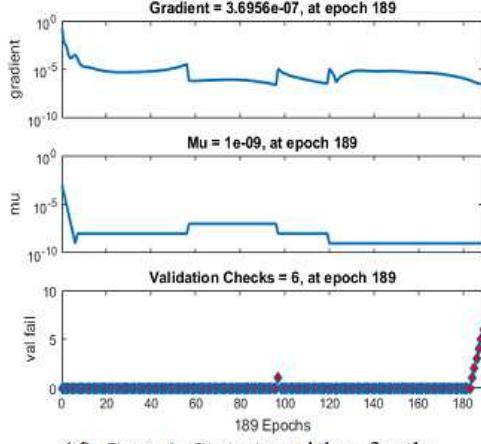
(a) Case 1: State transition for the mathematical influenza disease model



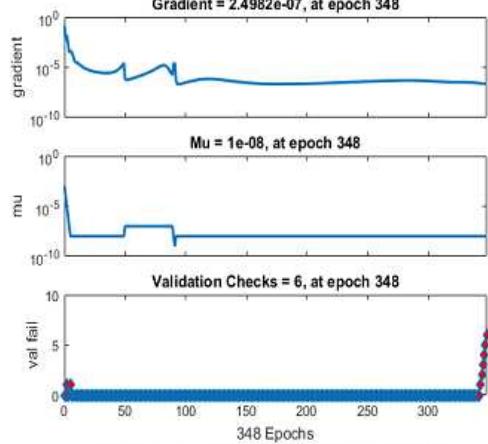
(b) Case 2: State transition for the mathematical influenza disease model



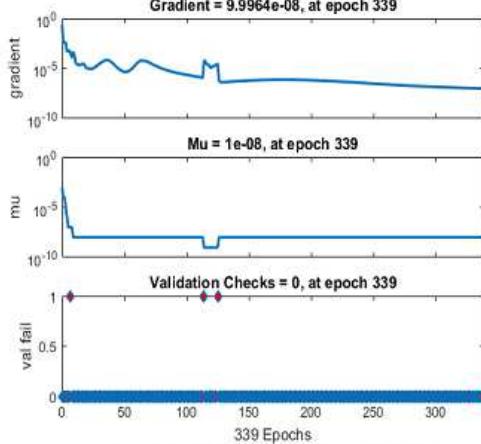
(c) Case 3: State transition for the mathematical influenza disease model



(d) Case 4: State transition for the mathematical influenza disease model



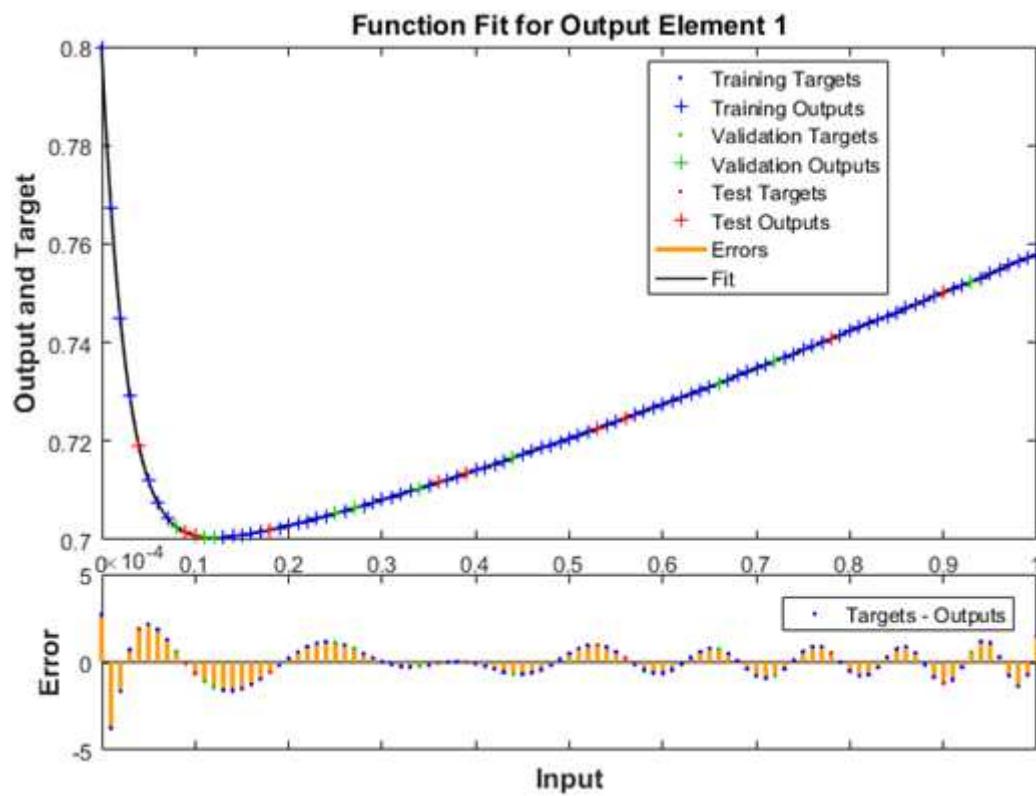
(e) Case 5: State transition for the mathematical influenza disease model



(f) Case 6: State transition for the mathematical influenza disease model

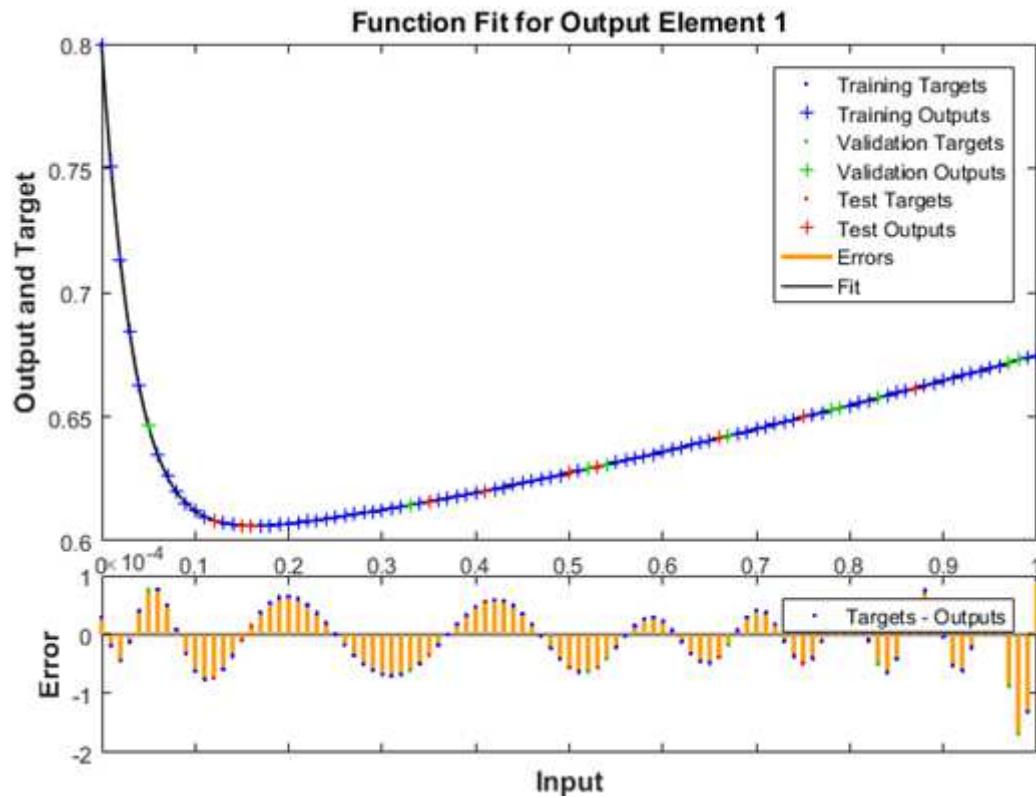
**Figure 5**

State transition values for the LMB based NN scheme to solve the mathematical influenza disease model.



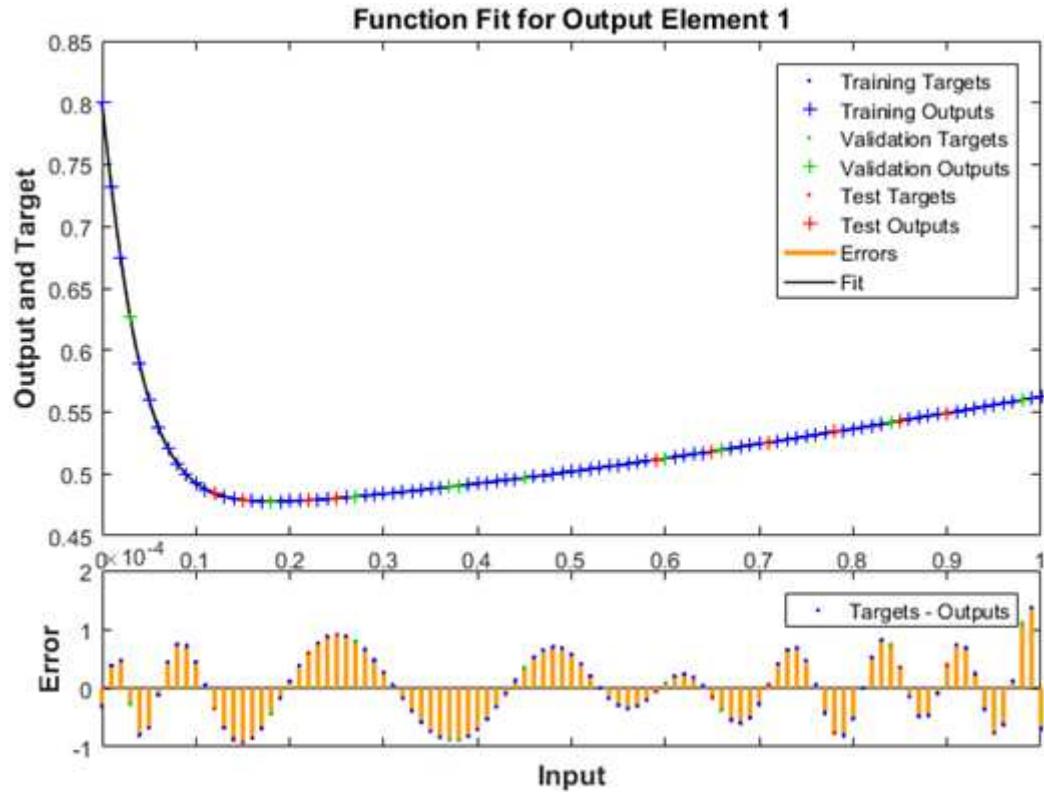
**Figure 6**

Comparison of the LMB based NN scheme to solve the mathematical influenza disease model of Case 1



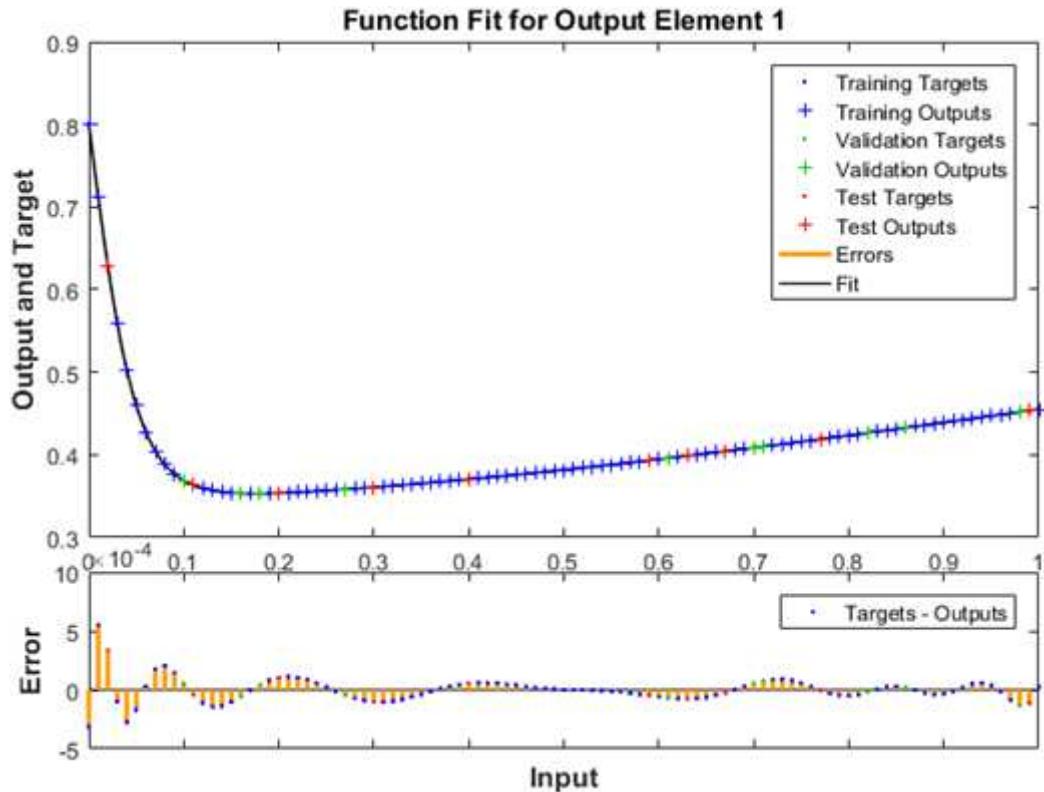
**Figure 7**

Comparison of the LMB based NN scheme to solve the mathematical influenza disease model of Case 2.



**Figure 8**

Comparison of the LMB based NN scheme to solve the mathematical influenza disease model of Case 3



**Figure 9**

Comparison of the LMB based NN scheme to solve the mathematical influenza disease model of Case 4.

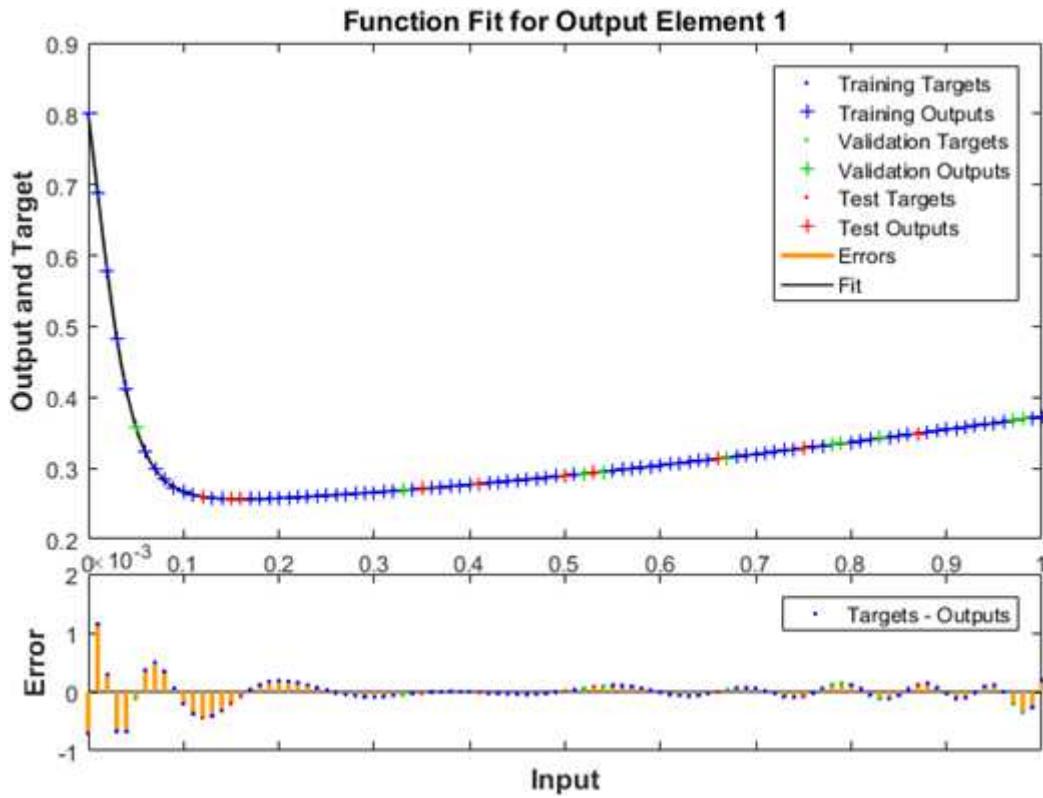


Figure 10

Comparison of the LMB based NN scheme to solve the mathematical influenza disease model of Case 5.

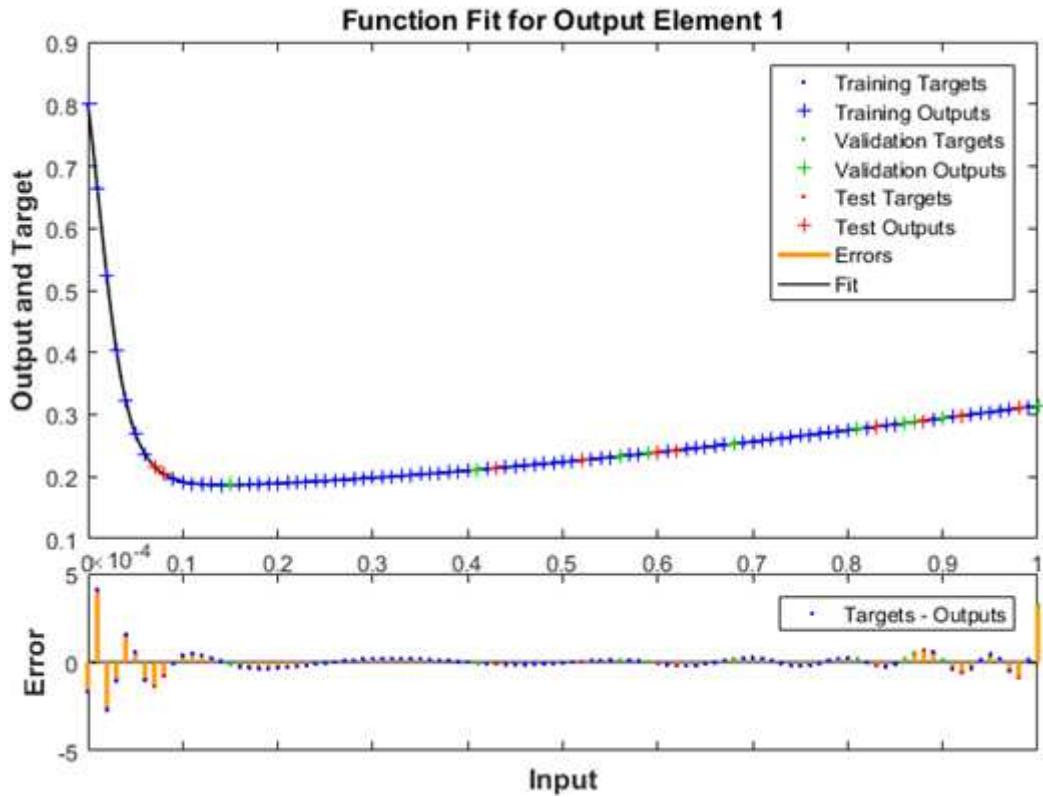
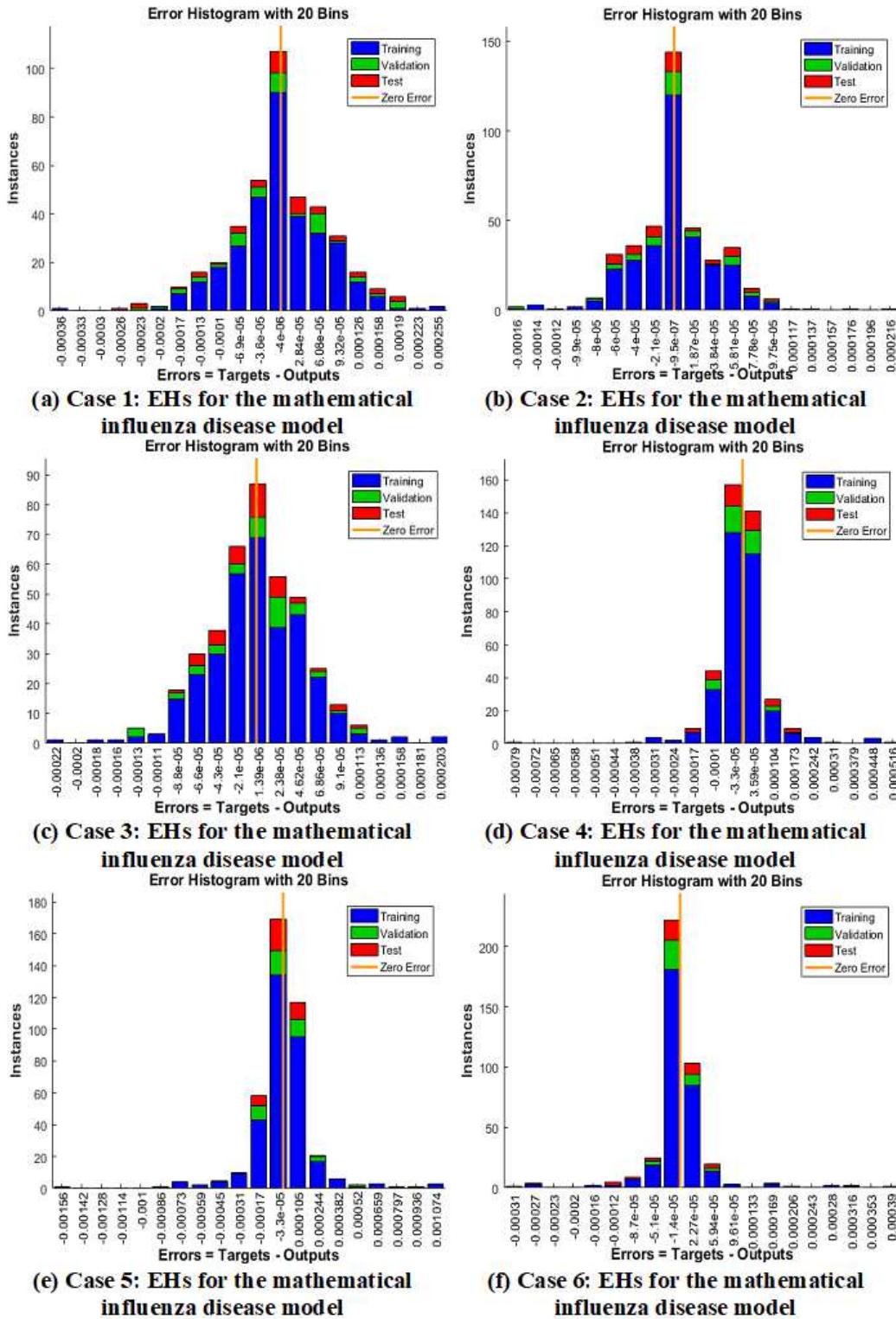


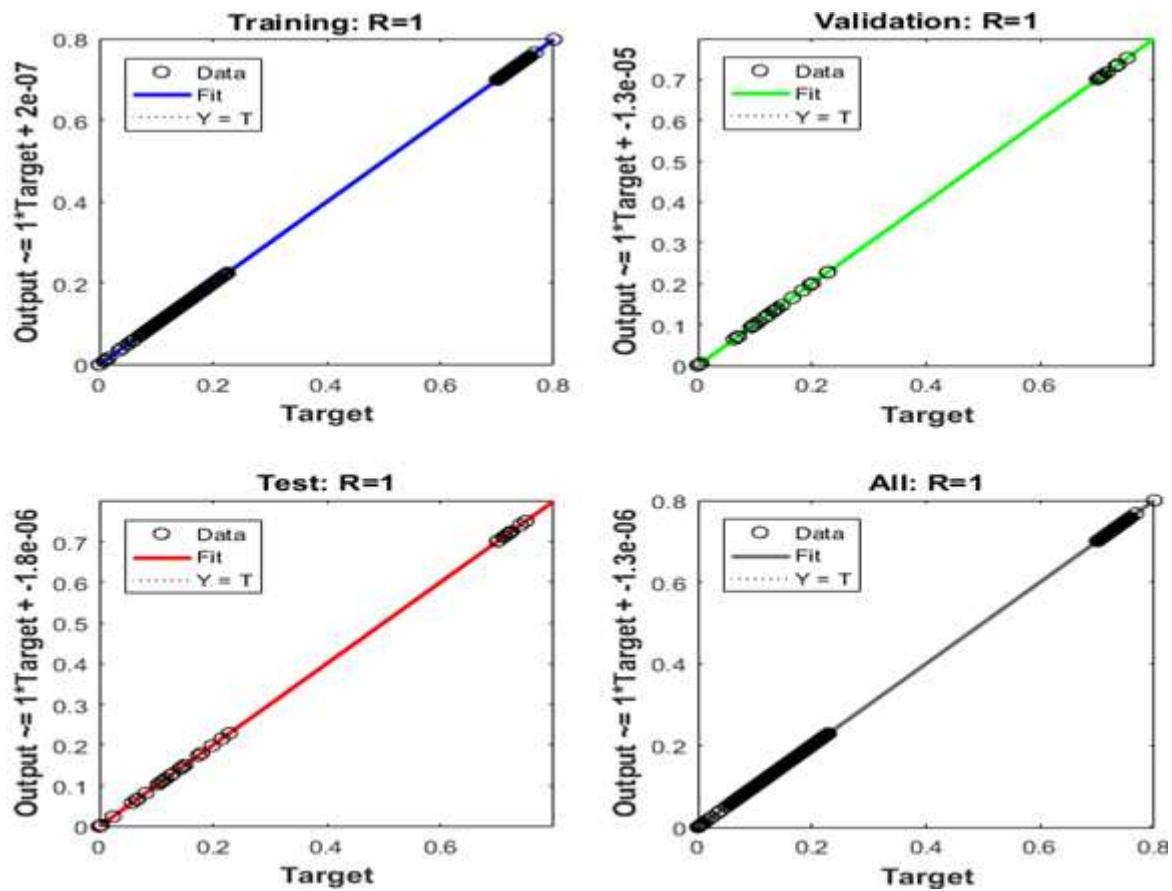
Figure 11

## Comparison of the LMB based NN scheme to solve the mathematical influenza disease model of Case 6



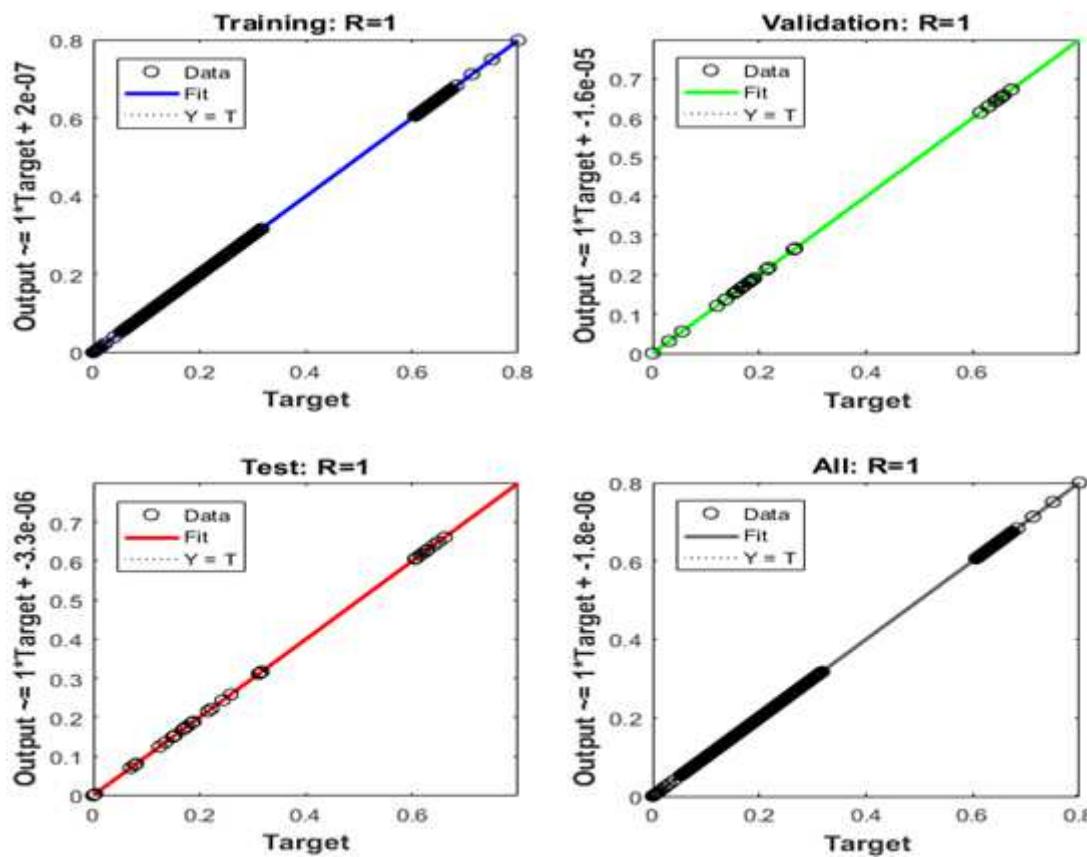
**Figure 12**

EHs for the LMB based NN scheme to solve the mathematical influenza disease model.



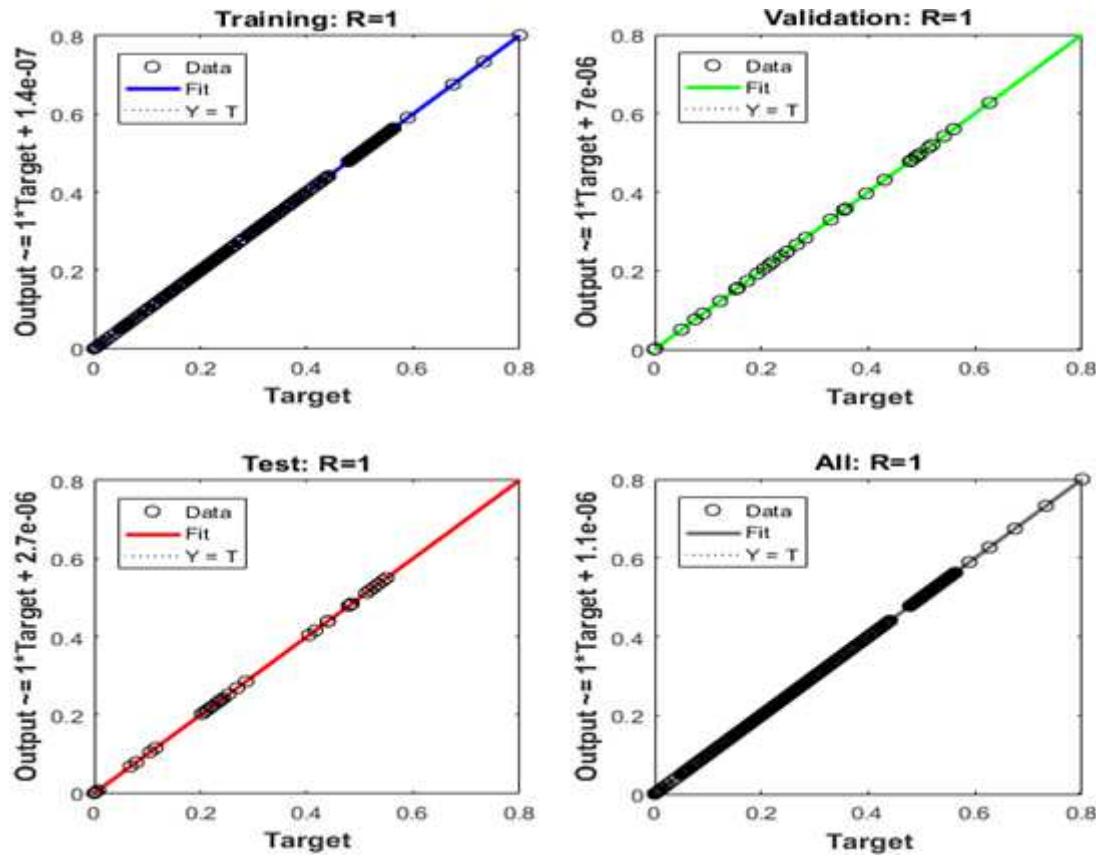
**Figure 13**

Regression of case-1 for the designed LMB based NN scheme to solve the mathematical influenza disease model.



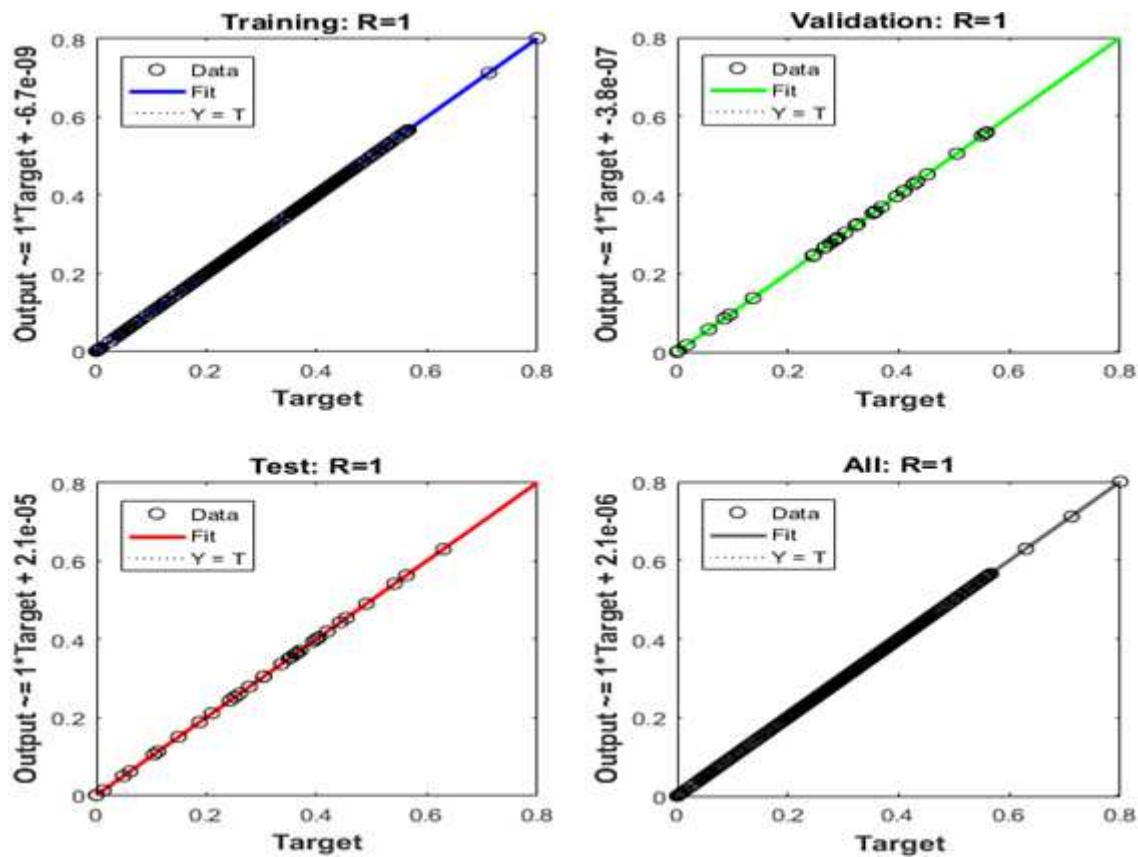
**Figure 14**

Regression of case-2 for the designed LMB based NN scheme to solve the mathematical influenza disease model.



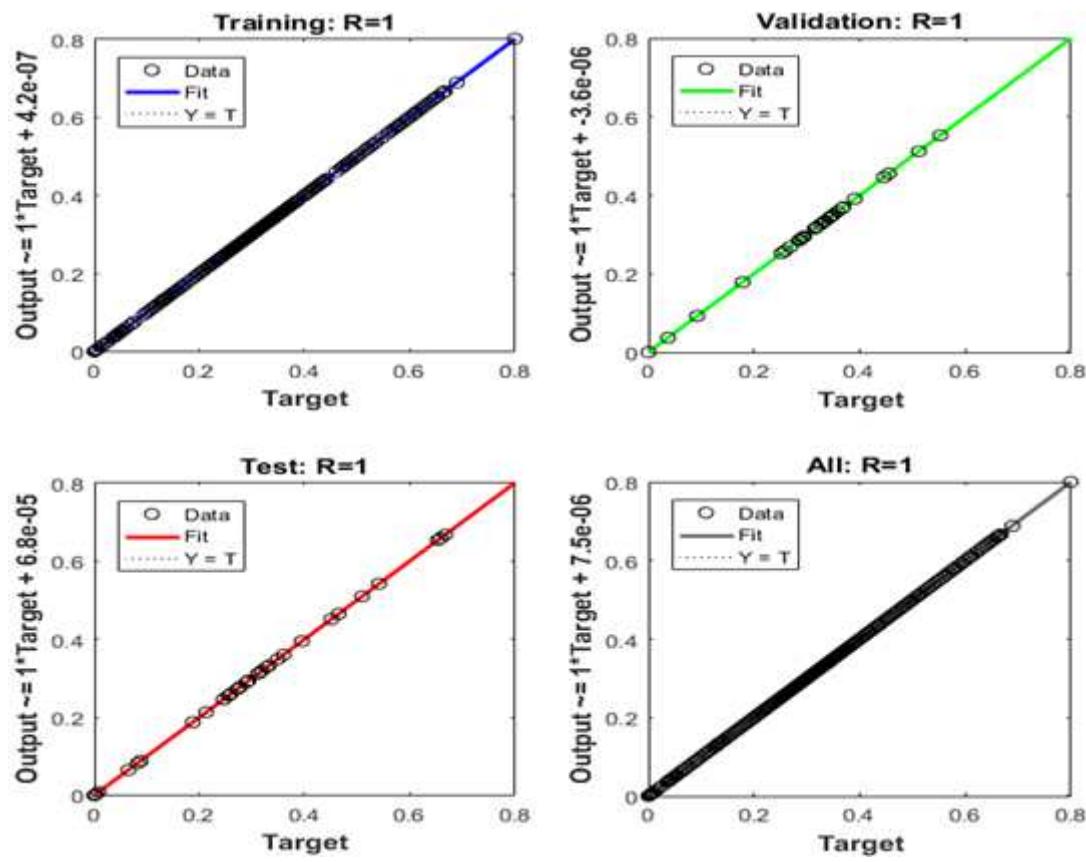
**Figure 15**

Regression of case-3 for the designed LMB based NN scheme to solve the mathematical influenza disease model.



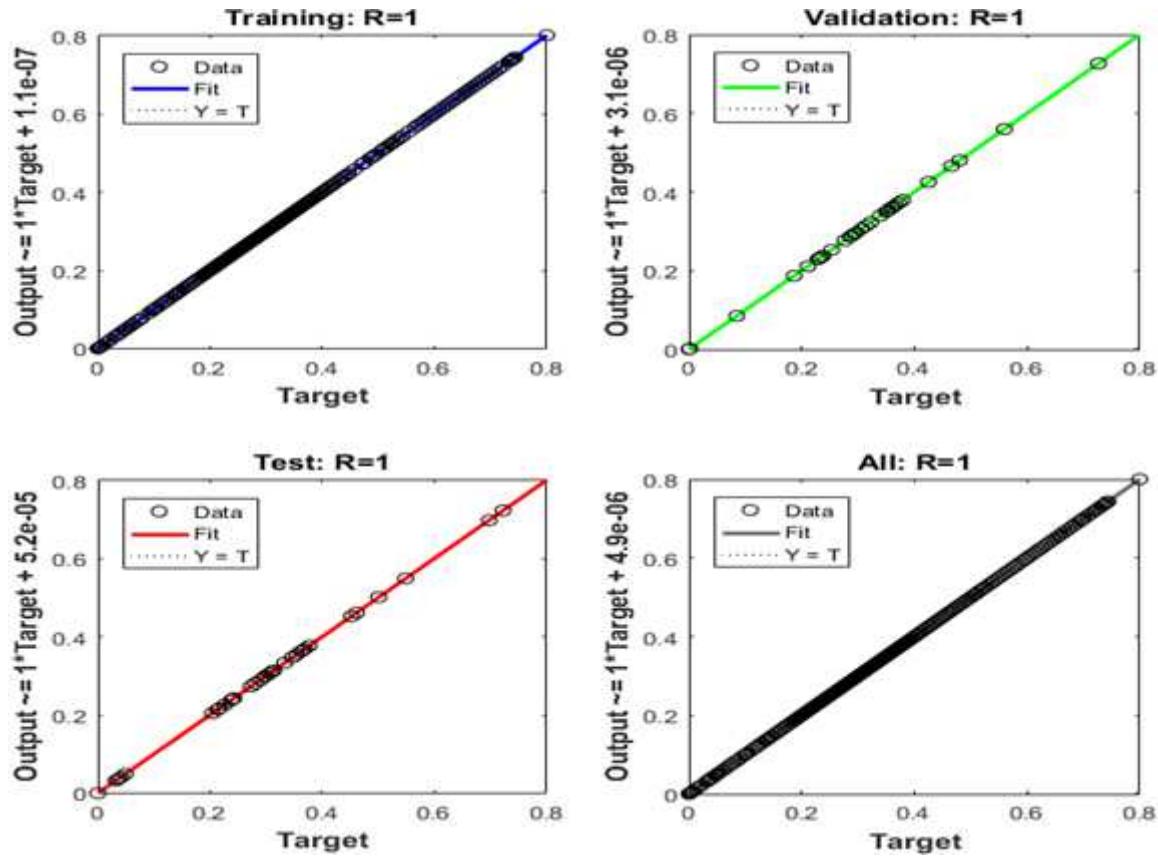
**Figure 16**

Regression of case-4 for the designed LMB based NN scheme to solve the mathematical influenza disease model.



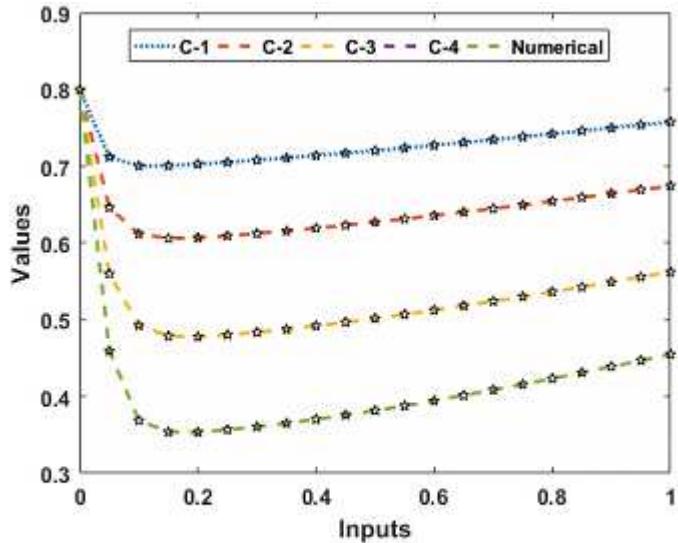
**Figure 17**

Regression of case-5 for the designed LMB based NN scheme to solve the mathematical influenza disease model.

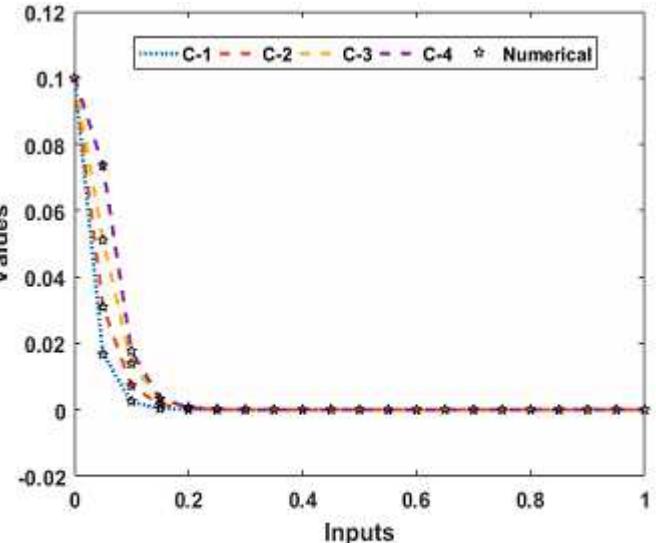


**Figure 18**

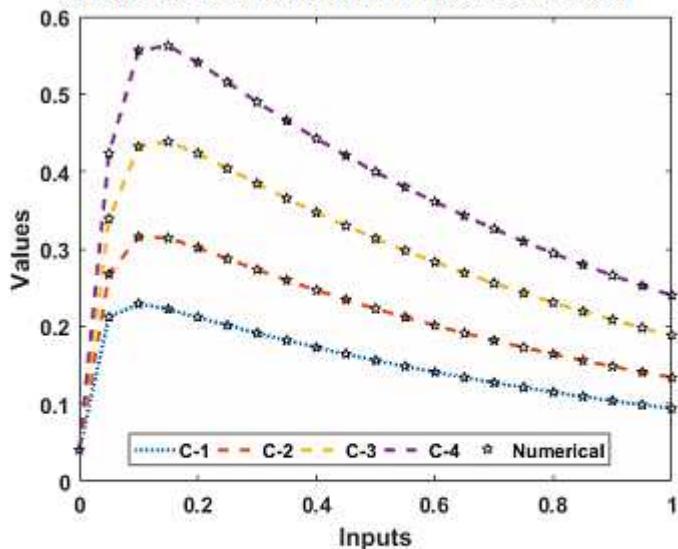
Regression of case-6 for the designed LMB based NN scheme to solve the mathematical influenza disease model.



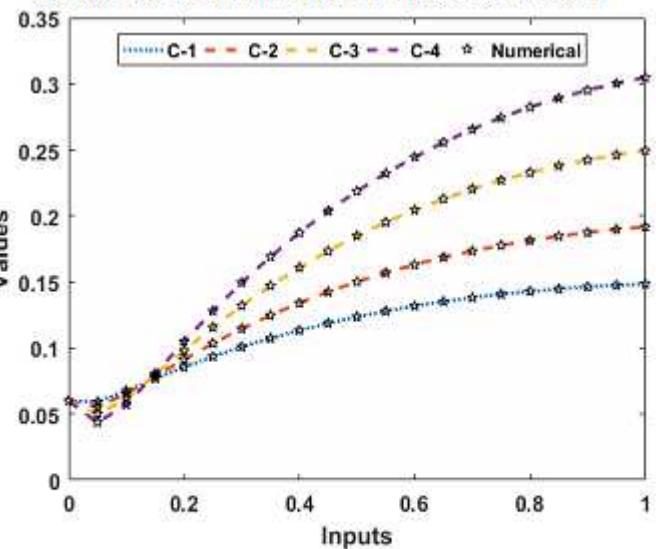
(a) Results of  $S(t)$  for all cases of the mathematical influenza disease model



(b) Results of  $I(t)$  for all cases of the mathematical influenza disease model



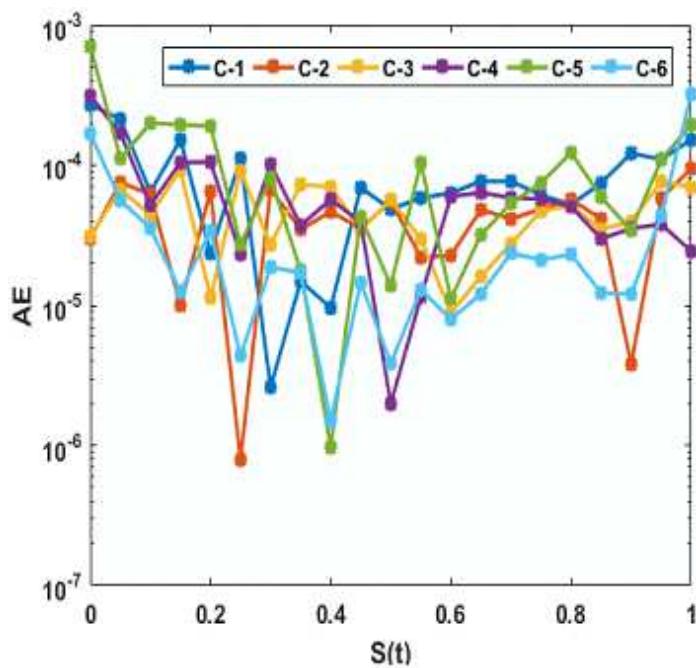
(c) Results of  $R(t)$  for all cases of the mathematical influenza disease model



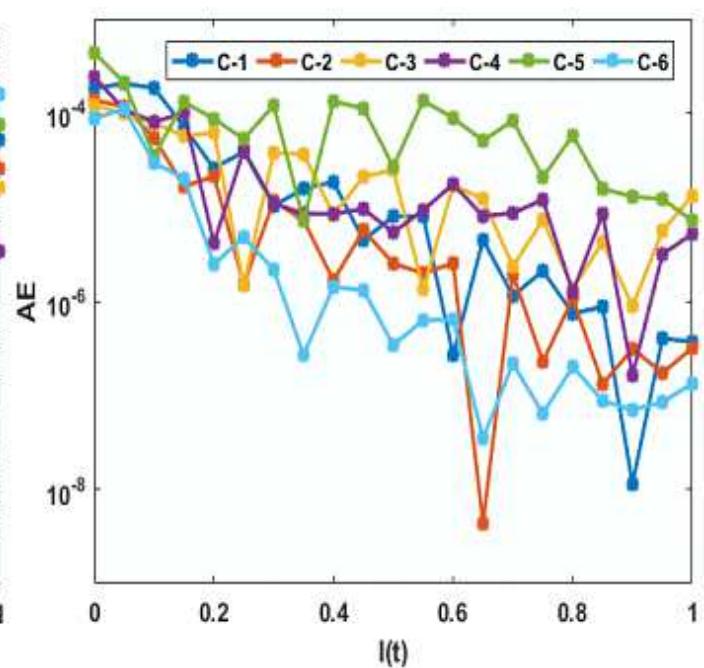
(d) Results of  $C(t)$  for all cases of the mathematical influenza disease model

**Figure 19**

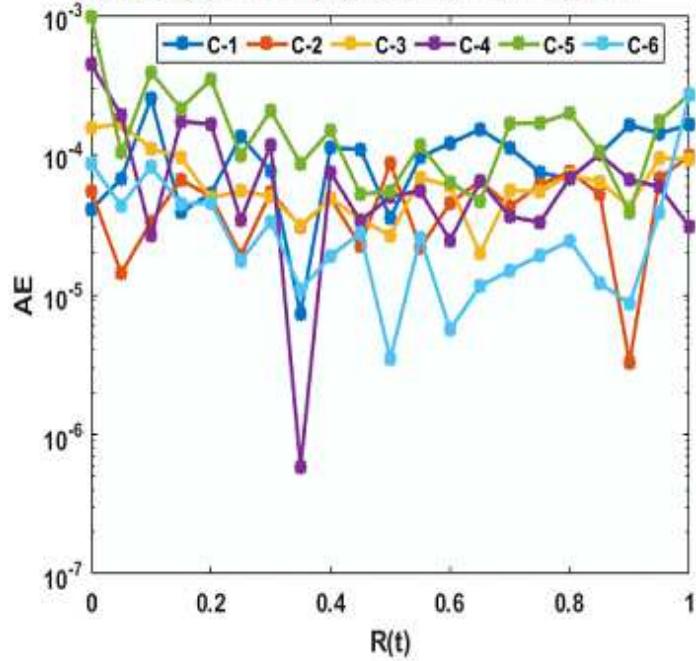
Comparison of the results of LMB based NN scheme for solving each case of the mathematical influenza disease model.



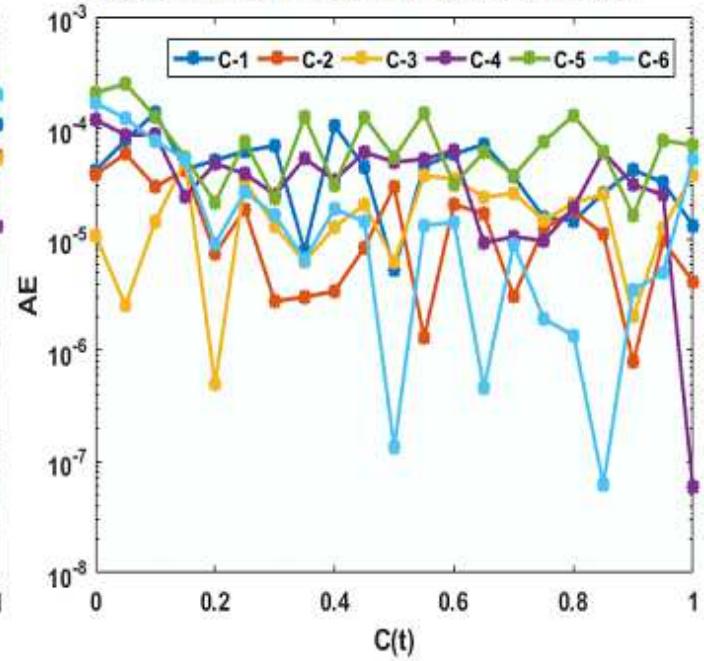
(a) AE of the  $S(t)$  for all cases of the mathematical influenza disease model



(b) AE of the  $I(t)$  for all cases of the mathematical influenza disease model



(c) AE of the  $R(t)$  for all cases of the mathematical influenza disease model



(d) AE of the  $C(t)$  for all cases of the mathematical influenza disease model

**Figure 20**

AE values using LMB-NN scheme to solve each case of the nonlinear mathematical influenza disease model.