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## Research Article

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# NURBS Interpolation Algorithm to Minimize Chord Error in 5-Axis CNC Milling Operations of Turbine Blades

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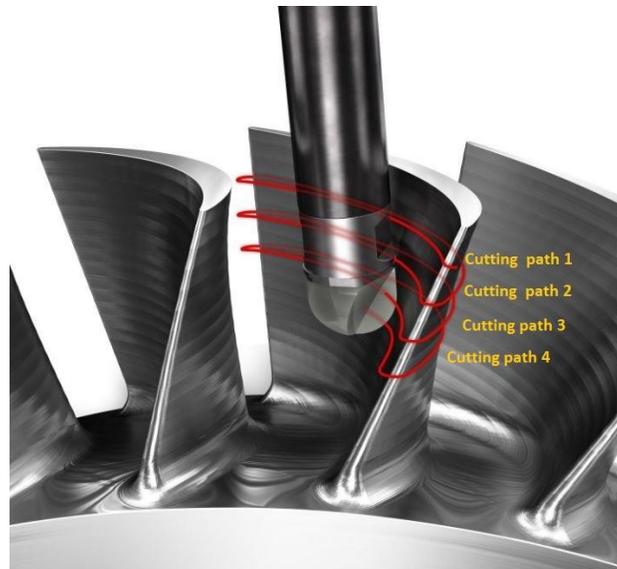
## Abstract:

5-Axis CNC machining operations are added to create complicated items with free form surfaces, such as turbine blades. Virtual machining systems are used in machining processes to improve the precision of manufactured products. Chord errors can be caused by cutting tool paths with a set feed rate in 5-Axis machining processes of turbine blades with free form surfaces. As a consequence, chord errors should be studied and eliminated in order to obtain the appropriate precision in machined products with free form surfaces. The research work proposed a virtual machining system to reduce chord error in 5-Axis milling of free form surfaces. To obtain the NURBS (Non-Uniform Rational B-Splines) profiles of cutting tool paths with abilities of control points modification along machining operations, the curved surface interpolation algorithm during turbine blade five axes milling processes is implemented. Then, a NURBS interpolation algorithm with restricted chord error and control of acceleration is developed in the study to minimize chord error in machined parts with free form surfaces. Finally, the sample turbine blade is machined by using the 5-Axis CNC milling and the chord error along machining paths are measured by using the CMM machine. As a consequence, the proposed virtual machining system can provide an efficient device for boosting the machined components accuracy with free form surfaces by employing 5-Axis CNC milling machines.

**Keywords:** NURBS interpolation, Chord error, 5-Axis CNC, Turbine blades, Accuracy of machined parts, Virtual machining

## 1- Introduction

In machining processes involving free form surfaces like as turbine blades, cutting tool pathways can cause chord errors, overcut, and increase the risk of cutting tool insert failure. There are concave surfaces and convex surfaces in the cutting tool paths of the turbine blades which can create the chord errors after machining operations as are shown in the figure 1.

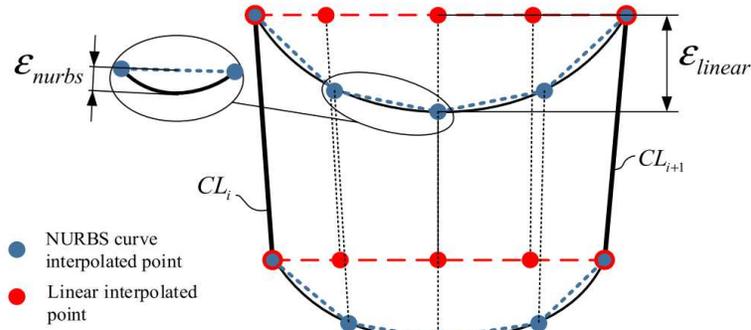


**Fig. 1.** The four cutting pathways used in turbine blade milling.

To produce machined components more precise, cutting procedures are modeled in virtual environments. In improving the precision of machined components, errors during milling can be examined and minimized in virtual environments.

The discrepancy between a predicted point and the real curve is known as radial error. Rounding errors in computer systems are the primary cause of radial error. Because of the fast improvement of microprocessors for high-precision applications, radial error is no longer a major concern [1]. The chord error is the maximum distance between a reference line (chord) that links two approximated places and the curve's corresponding arc. The chord error in curve pathways is the difference between the ideal arc section and the approximation segments.

It is not essential to split the curve down into little sections when using NURBS curve interpolation to transfer it to a CNC system. Unified mathematic model is an advantage of the NURBS curve interpolation. The technique not only expresses curve and surface flexibility, but also accurately represents conical surface curves and surfaces, resulting in a unified mathematical model for CAD/CAM systems. Also, flexible shape control is another advantage of the NURBS curve interpolation. To adjust the curve shape or surface, change the control vertex, weighted factor, or node value, which provides enough freedom for a variety of shapes creation. Furthermore, the NURBS curve interpolation has a powerful modeling capability which come with a set of robust methods for geometric shaping (such as node insertion, subdivision, and so on) that may be utilized for all areas of design, analysis, and processing. The accuracy of the NURBS interpolator is greater for replicating the shape of a designed CAD model when compared to a linear interpolator, because of its method for determining the next interpolation step. Instead of interpolating on a straight line connecting two positions of adjacent cutter, this approach pulls points directly from the NURBS curve. [2]. Figure 2 depicts NURBS route chord errors and linear pathway approximation systems in 5-axis CNC end milling. [2].



**Fig. 2.** The chord errors of Non - uniform rational and linear pathway interpolation methods in 5-axis CNC milling. [2].

In machining operations on free form surfaces, cutting tool pathways with a fixed feed rate can cause chord error and raise the chance of cutting tool insert failure. As a consequence, a virtual machining system that can reduce chord error by modifying the feed rate of cutting tool paths in free form surface machining processes can be a valuable tool for improving machined component precision.

To decrease chord error as well as contour error during milling operations of free form surfaces, iterative algorithm in feedrate scheduling system is developed by Liu et al. [3]. To decrease chord error and increase accuracy during machining operations of free form curves, the second-order estimation method is implemented by Du et al. [4]. Lee et al. provide feedrate optimization of NURBS interpolator for CNC machine tools to reduce chord error in manufactured products [5]. The least - square method's progressive and iterative approximations is used by He et al. [6] in order to decrease chord errors during free-form surface cutting procedures. To provide feed rate scheduling system with High-speed estimation of a micro-line tool path with a chord error restriction, advanced B-spline fitting scheme with abilities of knot vector and control points movement along machining paths is presented by Bi et al. [7]. To decrease chord errors during milling operations of sculptured surfaces, optimized cutting tool paths by using non-conventional genetic algorithm is developed by Fountas et al. [8]. To decrease chord length deviation during machining operations of aero-engine thin-walled blade, advanced method of machined blade profile errors is proposed by Feng et al. [9]. Advanced cutting tool path, curve fitting and velocity planning and optimization system is presented by Yang et al. [10] to decrease chord error during free-form surface cutting procedures. Peng et al. introduce an upgraded NURBS Interpolation system based on the modified Adams-Moulton technique for reducing chord error during milling operations of improvisational structures [11].

To provide desired chord error in machining operations of free form surfaces, Real-time Bezier interpolation algorithm is developed by Wei et al. [12]. To provide smooth cutting tool path during 5-Axis free-form surface cutting procedures, an advanced NURBS interpolation algorithm is described by Wang et al. [13]. To increase surface quality and provide tool routes which are smooth during 5-Axis milling operations of integral impeller, danded NURBS curve generation of cutter pathway is developed by Wei et al. [14]. To enhance motion stability during 5-Axis free-form surface machining procedures, dual NURBS Interpolator algorithm by considering the feed rate as well as cutting speed is developed by Ma et al. [15]. To compensate cutting tool radius during 3-Axis milling operations of complex surfaces, advanced NURBS modelling of cutting tool paths is developed by Wang et al. [16].

Soori et al. provide assessment and improvement of CNC machining in digital settings using virtual machining methods and processes [17-20]. Soori et al. provide a review of current developments in friction stir welding operations in order to examine and improve efficiency in the process of component manufacturing employing welding techniques [21]. Soori and Asamel investigated applications of simulated milling systems to reduce residual stress and deflection error throughout five-axis end milling of turbine blades [22]. Soori and Asmael created applications of virtual machining system in order to assess and reduce the cutting temperature during machining processes of hard to cut components [23]. Soori et al. proposed an improved virtual machining method to improve

surface properties throughout five-axis milling operations of turbine blades [24]. To decrease displacement error during five-axis milling procedures of impeller blades, Soori and Asmael invented virtual milling approaches [25]. Altintas and Merdol also describe a virtual machining system and application in order to achieve optimal milling conditions [26]. Altintas et al. describe a virtual correction of bending error utilizing a cutting tool pathways modification method to improve accuracy in ball end milling of elastic turbine blades [27]. Habibi et al. adjusted the position and direction of cutting tool pathways throughout milling of free form surfaces to enhance five-axis ball end milling precision [28].

According to prior published publications, the field of improving accuracy in 5 Axis CNC milling operations of turbine blades by reducing Chord error utilizing virtual machining systems has not been investigated. An improved virtual machining system is created in the study to enhance accuracy in 5-Axis machining processes of turbine blades. Throughout turbine blade milling processes in five axes CNC machine tools, curved surface interpolation is used to create NURBS profiles of cutting tool pathways with the ability to modify control points during machining processes. The free form surfaces of 5-Axis CNC milling operations are then analyzed using a NURBS interpolator method with restricted chord error and acceleration/deceleration control in order to reduce chord error. The suggested approach separates the NURBS curve into segments with geometric qualities and points that are comparable. The method breaks the curve into pieces and calculates the feedrate limitation for each segment in the first phase, confirming that both the highest circular path velocity and the maximum chord error tolerance limitations are fulfilled. The second phase is a speed-controlled interpolator that generates a tangential acceleration limited feedrate profile using the data from the first stage. Then, to reduce the chord error in machining operations of free form surfaces, final feedrates for each segment are computed using look-ahead analysis and the NURBS interpolator technique.

The prototype turbine blade is produced using 5-axis CNC machine, and the chord errors are then assessed using a CMM equipment to confirm the approach proposed in the study. Furthermore, virtual simulation of 5-Axis sample turbine blade CNC machining procedures is carried out in the study. Then, in order to obtain the chord errors in virtual environment, virtual produced part and CAD model of part are compared by using CAD software and chord error along desired points of machining paths are obtained. As a consequence, utilizing the study's created virtual machining procedures, The accuracy of turbine blade 5-axis machining operations can be enhanced..

## **2- The curved surfaces interpolation algorithm to generate NURBS profiles during 5-Axis milling operations**

In 5-Axis milling operations, the offset surface is created in order to get NURBS profiles. During milling, it's when a cutting tool radius causes the surface to be offset in the normal direction. The offset surface is divided by the pick feed direction to produce the cutting tool center curves. The cutting tool axis curve is then generated by shifting the curve of cutter center to the direction normal to the offset surface by the tool length. Between the curve of cutter center and the cutter axis is a ruled surface. The surface of cutter axis panned with the tool axis vectors shifted in the tool feed direction is equal to the ruled curve. Two ruled surface curves are converted into NURBS curves in this tool path development example which is shown in the Figure 3 [29].

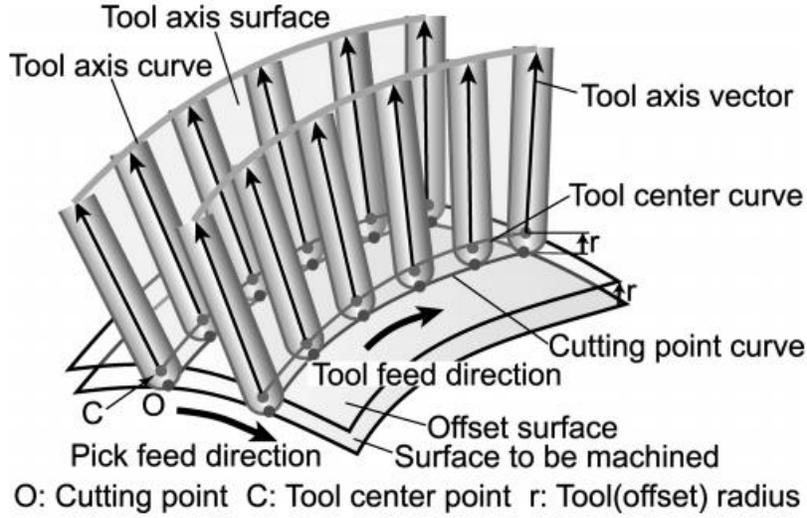


Fig. 3. Terminology in tool path generation [29].

The fundamental functions of B-Spline  $N_{i,m}(t)$  can be presented as [29].

$$N_{i,1}(t) = \begin{cases} 1 & (t_i \leq t < t_{i+1}) \\ 0 & (t < t_i, t_{i+1} \leq t) \end{cases}$$

$$N_{i,m}(t) = \frac{t-t_i}{t_{i+m-1}-t_i} N_{i,m-1}(t) + \frac{t_{i+m}-t}{t_{i+m}-t_{i+1}} N_{i+1,m-1}(t) \quad (1)$$

Where  $m$  is the basis functions degree and  $t_i$  are the knots spline parameter values.

As a consequence, regarding the basic functions, a NURBS curve can be created by  $N_{i,m}(t)$  and control points in coordinate space  $q_i(x_i, y_i, z_i)$  can be presented as [29],

$$p(t) \equiv (x(t), y(t), z(t)) = \frac{\sum_{i=0}^{n-1} N_{i,m}(t) \omega_i q_i(x_i, y_i, z_i)}{\sum_{i=0}^{n-1} N_{i,m}(t) \omega_i}, \quad t_{m-1} \leq t < t_n \quad (2)$$

Where  $n$  is the control points number and  $\omega_i$  are weights.

Moreover, a vector of homogenous control points  $Q_i = (\omega_i q_i(x_i, y_i, z_i), \omega_i)$ , ( $i = 0, 1, \dots, n-1$ ) and a knot vector  $T = [t_0 \dots t_{m+n+1}]$  are introduced.

When the new knot  $t'$  is inserted between  $K_{th}$  knot and  $k+1_{th}$  knot of the original knot vector  $T$ , a new knot vector  $T'$  can be presented as [29],

$$T' = [t' \dots t'_{m+n}] = [t_0 \dots t_k \quad t' \quad t_{k+1} \dots t_{m+n+1}]$$

$$t'_i = \begin{cases} t_i & (0 \leq i \leq k) \\ t' & (i = k+1) \\ t_{i-1} & (k+2 \leq i \leq m+n) \end{cases} \quad (3)$$

The newly established control points  $Q'_i$  should be calculated because of adding the control points numbers by inserting the new knots in the NURBS profile which can be expressed as [29],

$$Q'_i = (1 - \alpha_i) Q_{i-1} + \alpha_i Q_i$$

$$\alpha_i = \begin{cases} 1 & (i \leq k - m + 1) \\ \frac{t' - t_i}{t_{i+m-1} - t_i} = \frac{t'_i - t'}{t'_{i+m} - t'_i} & (k - m + 2 \leq i \leq k) \\ 0 & (k + 1 \leq i) \end{cases} \quad (4)$$

The NURBS curvature is a 4th order Bezier form with a 3rd order Curves which represents numerous segments. As a result,  $3i_{th}$  and  $3i + 1_{th}$  control points appear on the curve in a predictable pattern by making all NURBS curves 4th order Bezier form. This process of NURBS curve generation during 5-Axis CNC milling operations can consider the freely designate the cutting tool posture along cutting paths. The process of cutting tool axis surface transformation to NURBS is shown in the figure 4 [29].

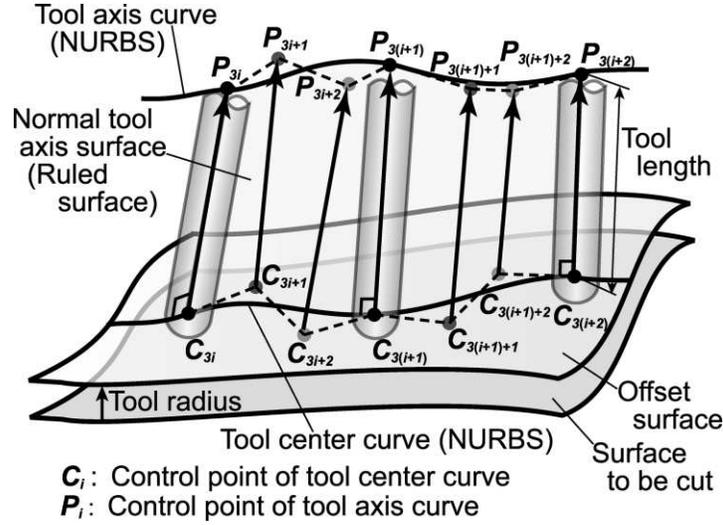


Fig. 4. Normal tool axis interpolation on a curved surface [29].

To provide modification and cutting tool path correction in the generated NURBS curve along machining paths, two parameters  $\vartheta$  and  $\varphi$  for The cutter displacement is evaluated when the tool axis surface is changed against the surface to be milled. The  $\theta$  is a degree of inclination with respect to the normal vectors, and  $\phi$  is a degree of rotations with respect to the normal vector. So, the axis curve's control points of the milling cutter as  $P_{3i}$  and  $P_{3(i+1)}$  can be modified by taking into account two variables  $\vartheta$  and  $\varphi$ , because the control points of the cutting tool center curve  $C_{3i}$  and  $C_{3(i+1)}$  also exist on the generated curve. The  $P'_{3i}$  and  $P'_{3i+1}$  are derived control points on the cutting tool axis curve. It is difficult to obtain the normal vectors and derivations at corresponding points when the  $3i + 1_{th}$  and  $3i + 2_{th}$  control points are not existed on the generated curve of cutting tool paths. As a result, movement process of  $P_{3i+1}$  and  $P_{3i+2}$  are not with the same method of  $P_{3i}$  and  $P_{3i+1}$ . So, a movement vector  $V_{3i}$  is introduced as the subtraction of  $P_{3i}$  and  $P'_{3i}$ . Also,  $V_{3i+1}$  is obtained from the  $P'_{3i+1}$ . Finally, vectors of movement for  $P_{3i+1}$  and  $P_{3i+2}$  are defined as the interpolation of the movement vectors in the following equations by using the movement vectors  $V_{3i}$  and  $V_{3i+1}$ . The  $V_{3i+1}$  and  $V_{3i+2}$  can be presented as [29],

$$V_{3i+1} = \frac{2}{3}V_{3i} + \frac{1}{3}V_{3(i+1)}$$

$$V_{3i+2} = \frac{1}{3}V_{3i} + \frac{2}{3}V_{3(i+1)} \quad (5)$$

By using these movement vector, control points  $P_{3i+1}$  and  $P_{3i+2}$  are moved to  $P'_{3i+1}$  and  $P'_{3i+2}$  respectively. The adjustment of the control points is enabled for all segments of the tool axes curve. As a result, the newly formed cutting tool axis surface is handled as the cutting tool pathway with the tool position given by the user. The process of surface shape modification by moving control points is shown in the figure 5 [29].

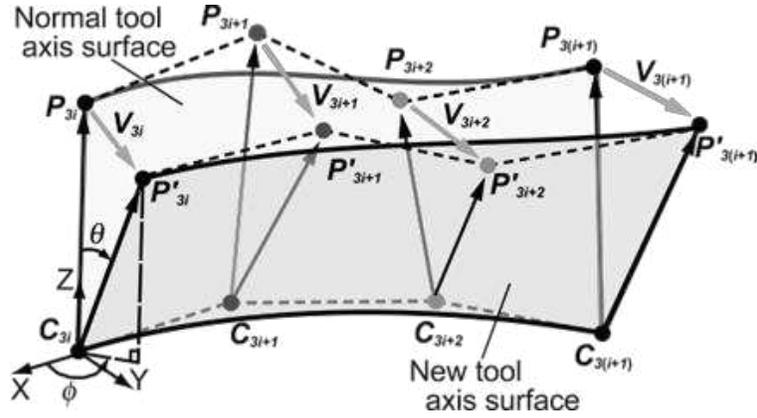


Fig. 5. The process of surface shape modification by moving control points [29].

### 3- The NURBS Interpolation algorithm to decrease the Chord errors

The chord error  $ER$  can be obtained as [1],

$$ER = \rho - \sqrt{\rho^2 - \left(\frac{vT}{2}\right)^2} \quad (6)$$

Where  $T$  is the each interpolation cycle's duration and  $v$  is Feedrate along machining paths. Also,  $\rho$  is the reciprocal of curvature's radius which can be presented as,

$$\rho = \frac{1}{K} \quad (7)$$

Where is  $K$  the curvature along machining paths. According to Eq. (6), the chord error increased as the curvature or feedrate rises. As a result, as the radius of curvature increases, the feedrate must be reduced to limit chord error. The maximum feedrate at which a chord error less than a given tolerance can be calculated as [30].

$$V_{chord} = \frac{2}{T} \sqrt{\rho^2 - (\rho - ER_{max})^2} \quad (8)$$

Where  $ER_{max}$  is maximum feedrate required to achieve a chord error within a certain tolerance. So, the centripetal acceleration is calculated as [30],

$$a_c = \frac{v^2}{\rho} \quad (9)$$

As the curvature and feedrate increases, the centripetal acceleration improves. Acceleration limits must be respected. Because, excessive acceleration causes mechanical vibrations, which decrease machining efficiency and cause deviation from the desired direction. Thus, From the Eq. (6), the maximum feedrate which permits the centripetal acceleration limits to be satisfied as the  $a_{c\ max}$  is obtained as [30],

$$V_{acc} = \sqrt{a_{c\ max} \cdot \rho} \quad (10)$$

The feedrate should not exceed the chord error and centripetal acceleration limitations as the Eq. (11) [30].

$$V_{max} = \min(V_{acc}, V_{chord}) \quad (11)$$

To minimize chord error, the study uses an improved NURBS interpolator technique with restricted chord error and control of acceleration and deceleration. To minimize rapid feedrate fluctuations, acceleration/deceleration control methods are utilized to regulate the tool's tangential acceleration. With the suggested NURBS interpolator, both the circular path acceleration limitation and tolerance of chord error can be met. Furthermore, while using control of acceleration and deceleration, the tangential acceleration limit is obeyed. The suggested approach separates the NURBS curve into segments with comparable geometric features and similar spots. The maximum feedrate for each segment is estimated before interpolation utilizing control of acceleration and deceleration and lookahead methods. The segmentation module is in responsibility of the first stage of the process, while the interpolation module is in responsibility of the second phase. In the first phase, the algorithm separates the curve into segments and estimates the feedrate limit for each segment, confirming that the circular path acceleration limitation and tolerance of chord error are both fulfilled.

The generated NURBS curve along cutting tool paths is broken down into curve segments by the segmentation module based on the curvature values encountered. The same segment is made up of consecutive points with values of curvature that are within the same range. The high curvature zones of the curve are divided from the low curvature zones of the curve by segments in this fashion. the segment's length; the value of parameter  $u$  corresponding to the segment's end; maximum feedrate obtained by using Eq. (8) and considering the segment's maximum curvature value; In the segmentation process, the ultimate feedrate of the segment is generated using a look-ahead algorithm that analyzes the subsequent segments. The algorithm used by the segmentation module for the NURBS curve of Eq. (2) to split the curve into sections by scanning the whole curve is depicted in Fig. 6 [31].

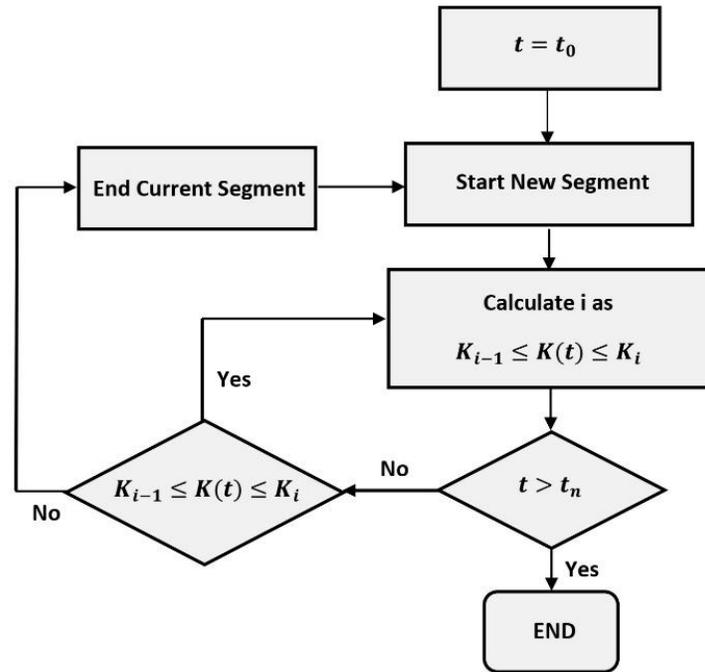


Fig. 6. Segmentation module of NURBS curve [31].

The second phase is a speed-controlled interpolator which uses the data from the first stage to build a tangential acceleration restricted feedrate profile. The maximum feedrate, volume, and final feedrate are used to construct a feedrate profile in the acceleration/deceleration controls before interpolation process. In terms of calculating curvature ranges, the first range includes all curvature values such that the maximum feedrate calculated using the Eq. (11) exceeds the commanded feedrate defined by program code. This method allows all segments in the first

range to be milled at the specified feedrate without violating the chord error tolerance or centripetal acceleration limitation. The values of curvature ( $K$ ) from 0 (a straight route that can be milled with no restrictions on feedrate) to  $K_1$ , which is the highest value of curvature as the Eq. (12) [31].

$$K_1 = \min\left(\frac{a_c}{V_{comm}^2}, \frac{8ER_{max}}{T^2V_{comm}^2+4ER_{max}}\right) \quad (12)$$

Where  $V_{comm}$  is the commanded federate,  $ER_{max}$  is maximum of feedrate which permits to achieve a chord error smaller than a desirable tolerance and  $T$  is the each interpolation cycle period and  $a_c$  is centripetal acceleration. The higher bounds of the ranges after that are derived by doubling the upper bound of the range before that, so the  $i$ th range bounds are  $K_{i-1}$  (lower bound) and  $K_i$  (upper bound), where  $K_i$  is,

$$K_i = \begin{cases} 0 & \text{if } i = 0 \\ \min\left(\frac{a_c}{V_{comm}^2}, \frac{8ER_{max}}{T^2V_{comm}^2+4ER_{max}}\right) 2^{i-1} & \text{if } i > 0 \end{cases} \quad (13)$$

The interpolation module is a speed-controlled interpolator that employs a second-order reduced Taylor series expansion which can be presented as [31],

$$t[K + 1] = t[K] + \frac{v}{\sigma}T + \frac{1}{\sigma}\left(a_t - \frac{x'x''+y'y''+z'z''}{\sigma^3}v^2\right)\frac{T^2}{2} \quad (14)$$

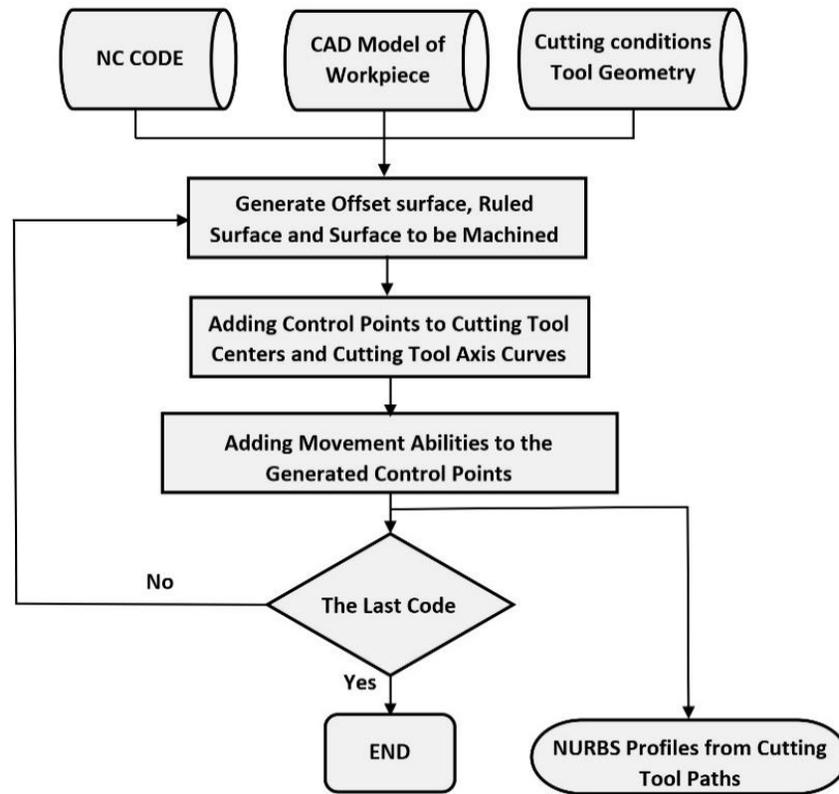
Where  $\sigma$  is the parametric speed which can be explained as,

$$\sigma = \frac{ds}{dt} = \sqrt{x'^2 + y'^2 + z'^2} \quad (15)$$

Where  $x$ ,  $y$  and  $z$  are the components of vector  $p(t)$  and their derivatives are considered with respect to  $t$ . As a consequence, the interpolator should take into consideration the required feedrate, the appropriate tangential acceleration, and the curve's first and second order derivatives while creating a new interpolated location.

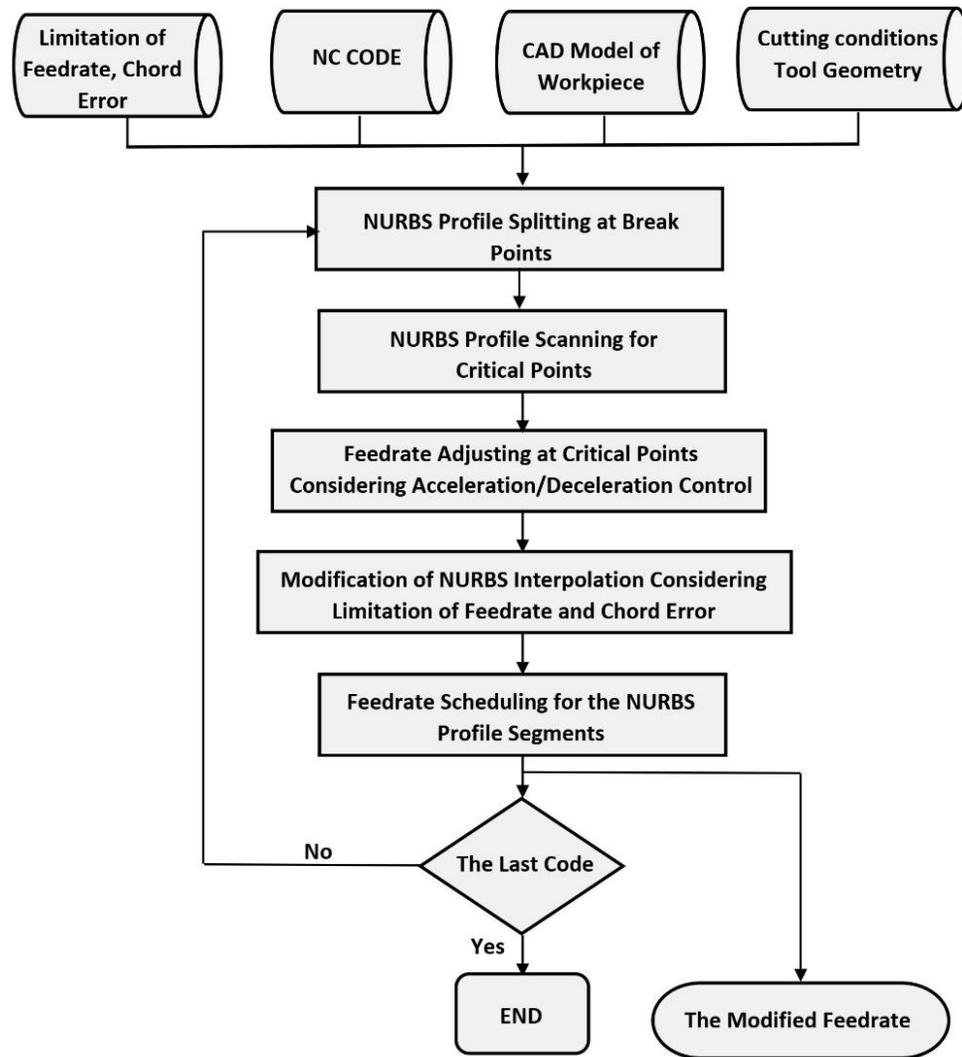
#### 4- Virtual machining system

The proposed virtual machining project in this research was established using the Visual Basic programming language. In the first step, the curved surface interpolation algorithm is implemented in order to obtain the NURBS profiles of cutting tool pathways during turbine blades 5-Axis milling. The obtained NURBS profiles can be modified by moving the control points of generated surfaces. The flowchart of curved surfaces interpolation algorithm to generate NURBS profiles in 5-Axis milling operations is described in the figure 7.



**Fig. 7.** The flowchart of curved surfaces interpolation algorithm to generate NURBS profiles in 5-Axis milling operations.

The approach creates a feedrate profile that allows for high machining speeds in low-curvature zones while dropping speed as curvature increases, while taking into account chord error and centripetal acceleration constraints. Limitations of feedrate as well as chord error, NC codes, Workpiece CAD design and cutting conditions are entered to the system. The method breaks the curve into segments and determines the feedrate limit for each section in the first phase, confirming that the chord error tolerance and maximum circular path acceleration restrictions are fulfilled. The second step is a interpolator with speed-controlled that uses the data from the first stage to generate a tangential acceleration restricted feedrate profile. Modification and cutting tool path correction in the generated NURBS curve along machining paths is the applied in order to provide the limitation of feedrate as well as chord error in the NURBS interpolation algorithm. The final feedrate of each segment is then determined using look-ahead analysis and the NURBS interpolator algorithm to reduce chord error in machining operations of free form surface. The flowchart of the virtual machining systems in chord error minimization using a NURBS interpolator algorithm is described in the figure 8.

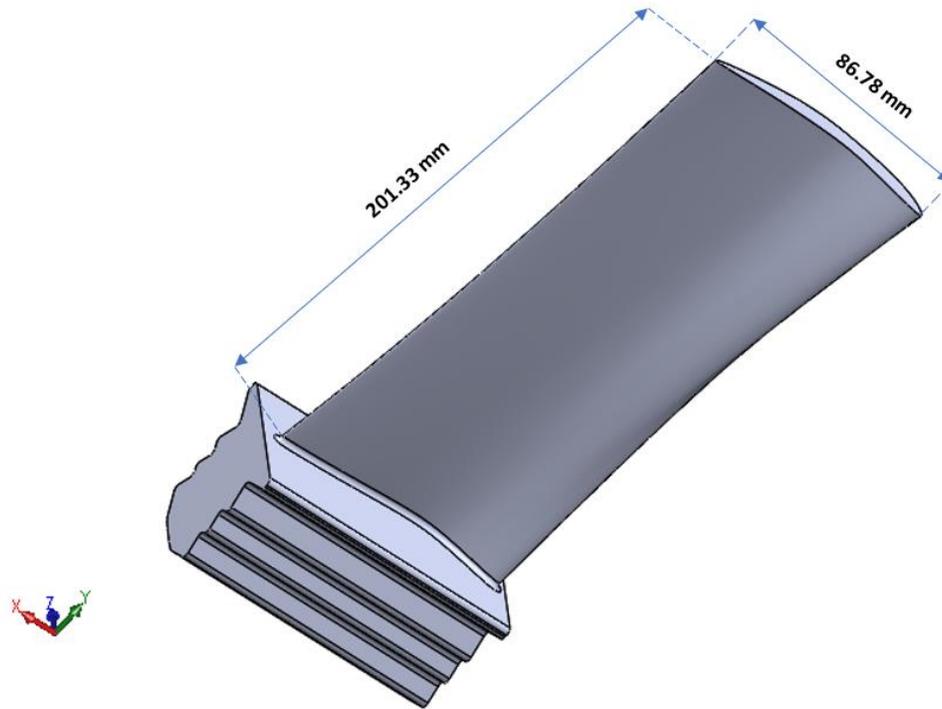


**Fig. 8.** The virtual machining techniques' flowchart in chord error minimization using a NURBS interpolator algorithm.

As a result, by using the NURBS interpolator algorithm, modified feed rate for the generated segments from free form surfaces regarding the tolerance of chord error and the limitation of centripetal acceleration are calculated in order to minimize the chord errors in the machined blisks. Ultimately, new machining routes are created based on the determined deflection error and changed feed rates at each cutting tool location along the machining pathways. Thus, the new machining paths regarding the deflection error compensation and chord error reduction can increase accuracy of machined blisk blades.

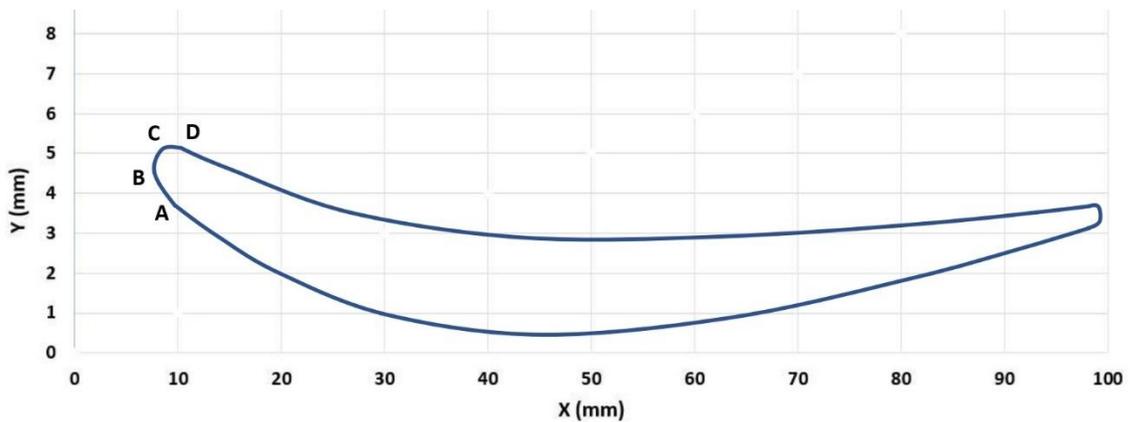
### 5- Simulation and validation

The Vericut CAM software is used to perform a 5-axis milling operation on a sample turbine blade in order to validate the effectiveness of the study's recommended approach. The used computer system in the simulation is with CPU Intel(R) Core (TM) i5-2450M 2.50 GHz and the operation system is Windows 10. The CAD model of sample turbine blade is shown in the figure 9.



**Fig. 9.** Sample turbine blade CAD model.

To measure the chord error on the corner of sample turbine blade, four curvature zone of A, B, C and D are selected on the section curve of blade. The section curve of sample turbine blade with the selected curvature zone of A, B, C and D is shown in the figure 10.



**Fig. 10.** Section curve of sample turbine blade with the selected curvature zone of A, B, C and D.

The parameters of generated NURBS curve on the selected curvature zone of A, B, C and D are as Control points (mm): (10, 3.5), (8, 4.3), (8.5, 5.2), (12, 5.1), Knot vector: [0, 0, 0, 0, 0.25, 0.5, 0.75, 1, 1, 1, 1] and Weight vector: [1, 1, 1, 1, 1, 1, 1, 1]. Figure 11 shows the developed NURBS curves and control points for the specified curvature zone of A, B, C, and D.

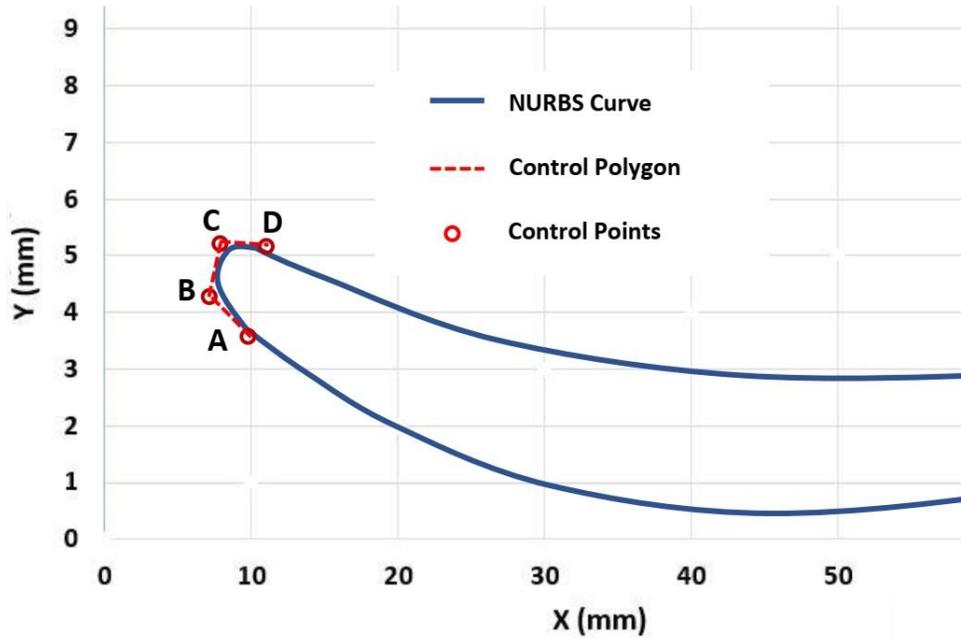


Fig. 11. Generated NURBS curves and control points to the selected curvature zone of A, B, C and D.

In order to produce the turbine blade in virtual environments, the 5-Axis CNC milling machine tool is simulated in the Vericut CAM software. The cutting tool that was utilized in the experiment is 4 fluted Tungsten carbide end-mill with 10 mm diameter and 30 mm length. The spindle speed is 10000 rpm, namely feed rate is 100 mm/s, Maximum chord error 0.001 mm and sampling time 1.4 s. The feed rate after NURBS interpolation algorithm for the selected curvature zone of A, B and C is shown in the figure 12.

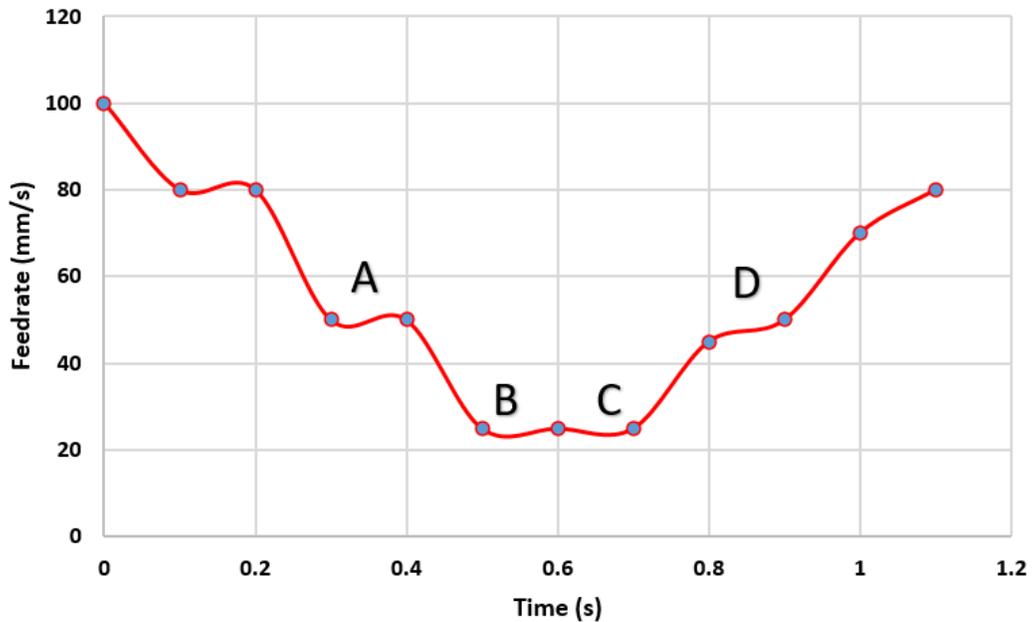


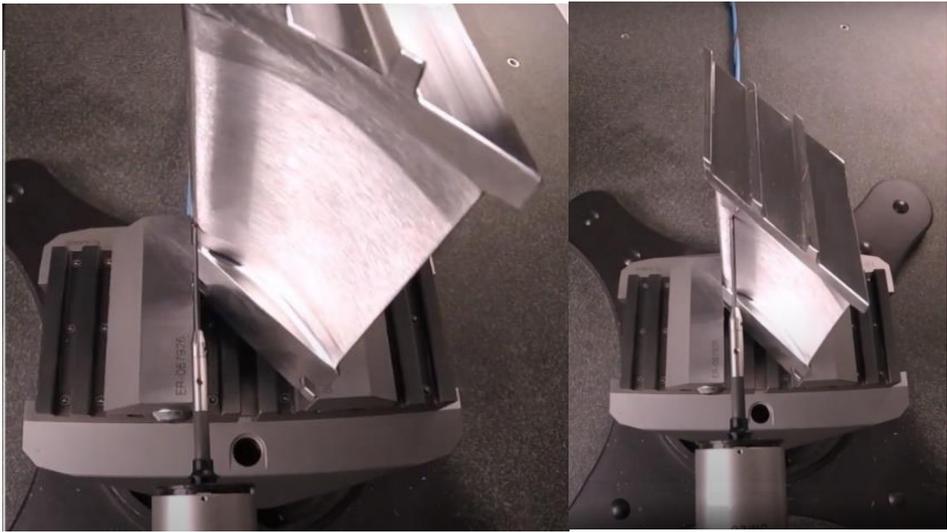
Fig. 12. Feed rate after NURBS interpolation algorithm for the selected curvature zone of A, B, C and D.

Figure 13 depicts the example turbine blade machining procedure utilizing a 5-axis CNC milling machine.



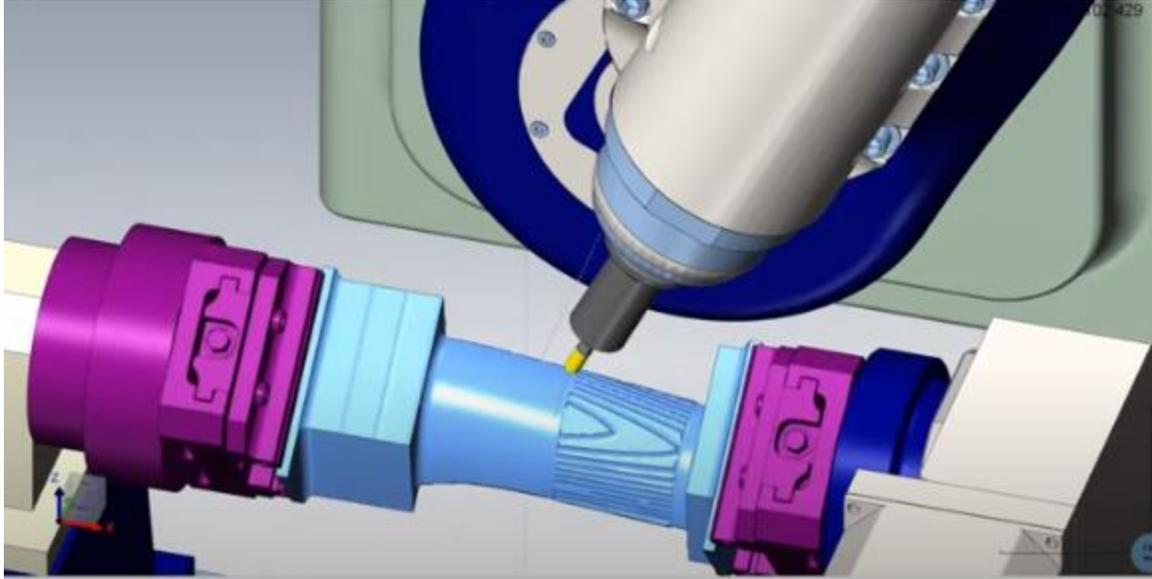
**Fig. 13.** The machining procedures of the sample turbine blade utilizing the 5-axis CNC milling.

The CMM equipment is employed to evaluate the chord inaccuracy in the manufactured sample turbine blade. The Renishaw SM25-2 probe is used to find the chord error of machined turbine blade. The process of chord error measurement utilizing a CMM device is shown in the figure 14.



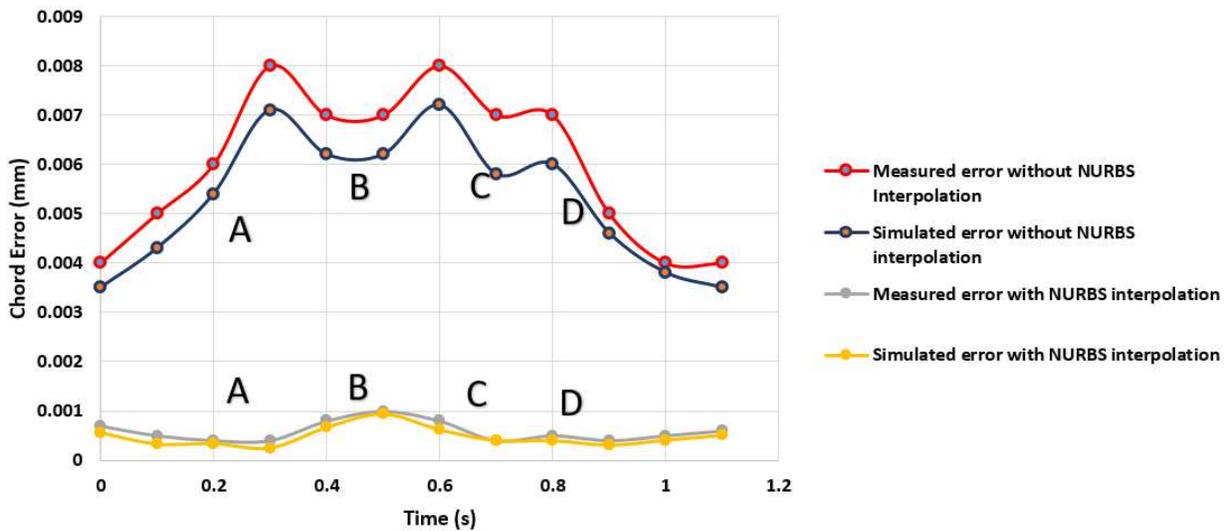
**Fig. 14.** The process of chord error measurement using the CMM machine.

The simulaed 5-Axis milling of sample turbine blade by using the Vericut CAM software is shown in the fihure 15.



**Fig. 15.** The simulated 5-Axis milling operation of sample turbine blade by using the Vericut CAM software.

The CAD software is used to measure the chord error for the selected curvature zone of A, B, C and D. As a result, the measured and simulated chord error before and after applying the proposed NURBS interpolation algorithm for the selected curvature zone of A, B, C and D are obtained as figure 16.



**Fig. 16.** The measured and simulated chord error before and after applying the proposed NURBS interpolation algorithm for the selected curvature zone of A, B, C and D.

Measured and predicted chord errors of 12 selected points along the curve length of machined sample turbine blade without and with NURBS interpolation algorithm is shown in tables 1.

No.	Without NURBS interpolation		With NURBS interpolation		Percentage of change	
	Measured Errors (mm)	Predicted Errors (mm)	Measured Errors (mm)	Predicted Errors (mm)	Measured Errors	Predicted Errors
Point 1	0.004	0.0035	0.0007	0.00056	82.5	84
Point 2	0.005	0.0043	0.0005	0.00033	90	92.3255
Point 3	0.006	0.0054	0.0004	0.00034	93.3333	93.7037
Point 4	0.008	0.0071	0.0004	0.00025	95	96.4788
Point 5	0.007	0.0062	0.0008	0.00067	88.5714	89.1935
Point 6	0.007	0.0062	0.001	0.00094	85.7142	84.8387
Point 7	0.008	0.0072	0.0008	0.00062	90	91.3888
Point 8	0.007	0.0058	0.0004	0.0004	94.2857	93.1034
Point 9	0.007	0.006	0.0005	0.0004	92.8571	93.3333
Point 10	0.005	0.0046	0.0004	0.00031	92	93.2608
Point 11	0.004	0.0038	0.0005	0.00041	87.5	89.2105
Point 12	0.004	0.0035	0.0006	0.00051	85	85.4285
<b>Average</b>	<b>0.006</b>	<b>0.0053</b>	<b>0.000583</b>	<b>0.000478</b>	<b>89.7301</b>	<b>91.1034</b>

**Table 1.** Measured and predicted chord errors of 12 selected points along the curve length of machined sample turbine blade without and with NURBS interpolation algorithm.

## 6- Conclusion

The implementation of virtual machining technologies in the accuracy improvements of turbine blade milling utilizing 5-Axis CNC machine tools is described in this study paper. The chord error in the 5-Axis machining operations of turbine blade is analyzed in the study in order to be minimized. To obtain the NURBS profiles of cutting tool paths with abilities of control points modification along machining operations, the curved surface interpolation during 5-Axis milling operations of turbine blades is implemented. The study employed a NURBS interpolator technique with limited chord error and control of acceleration and deceleration to reduce chord error. As a result, final feedrates for each segment are determined using the NURBS interpolation technique to reduce chord error in turbine blade manufacturing processes. Ultimately, the sample turbine blade is made utilizing 5-axis CNC machine tools, and the chord errors are assessed with a CMM machine to ensure that the methods described in the study is correct. Moreover, simulation of machining procedures in five axes cutting of the turbine blades is carried out in virtual environments in order to be measured by using the CAD software. So, a %89.7 and %91.1 reduction in measured and predicted chord error of four selected curvature zone along corner machining of turbine blade is obtained by using the proposed NURBS interpolation algorithm in the study. As a consequence, minimizing chord errors during 5-Axis machining processes can improve the precision of manufactured turbine blades. The look ahead as well as gauging error in 5-Axis machining operations of free form surfaces can also be analyzed in order to improve the precision and efficiency of the part-production process utilizing machining procedures. These are the authors' ideas for future research studies.

## Declarations

- Availability of data and materials

The used data and materials in the research works are available.

- Competing interests

There is no conflict of interest regarding the submitted manuscript

- Funding

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- Authors' contributions

All authors are contributed in the research works as idea development, data collection/analysis, paper writing and submission. All authors read and approved the final manuscript.

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