Realization of a Novel Chaotic System Using Coupling Dual Chaotic System

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Realization of a novel chaotic system using coupling dual chaotic system

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Abstract

This paper establishes coupling between two various chaotic systems for Lorenz and Rössler circuits. The x-dynamics of Lorenz circuit was coupled numerically with the x dynamics of Rössler circuit. As a result of the optical coupling between these two chaotic systems, it has been observed an exceptional variation in the time series and attractors, which exhibits a novel behavior, leading to a promises method for controlling the chaotic systems. However, performing fast Fourier transforms (FFT) of chaotic dynamics before and after coupling showed an increase in the bandwidth of the Rössler system after its coupling with the Lorenz system, which, in turn increases the possibility of using this system for secure and confidential optical communications.

Keywords: Nonlinear dynamics, Chaos, Lorenz circuit, Rössler circuit, optical communications.

Introduction

The nonlinear dynamics is an important branch in science in which inclusive researchers have been performed in the last decades [1]. Basically, this branch is an interdisciplinary of science that deals with the systems that can be represented using mathematical nonlinear equations. The nonlinear dynamics study has substantial importance in technology and science due to nonlinearity in the most engineering and natural systems [2]. The nonlinear systems exhibit a wide variety of phenomena such as pattern formation, self-sustained oscillations and chaos [3].

The study of nonlinear dynamics has improved after the invention of the deterministic chaos phenomenon by Lorenz in 1963 [4]. Chaos may be defined as the phenomenon of the irregular fluctuations of the system outputs obtained from models represented by deterministic equations. The dynamical system is said to be chaotic when it has three important properties: infinite recurrence, boundedness and sensitive dependence on the initial conditions [5]. Chaos phenomenon can be found in a various disciplines, such as engineering, physics, economics, biology, chemistry [6], steganography [7] and encryption [8]. Recently, the concept of dual synchronization of two different pairs of chaotic dynamical systems has been investigated and used experimentally in communication applications [9].

In this work, a novel chaotic system scheme is presented by coupling two kinds of hyper chaotic systems (Lorenz and Rössler circuit) and studying the effect of the x-dynamics of Lorenz circuit on the x-dynamics of Rössler circuit. The data are analyzed numerically using MATLAB (9.2) to investigate the effect of the coupling on the Rössler circuit using the time series, attractor, and the fast Fourier transformation.

Lorenz and Rössler circuits

The Lorenz model is an essential computational approach in the nonlinear dynamics studies [6]. This model consists of a system of three ordinary differential equations which have two nonlinear terms (x_1y_1) and (x_1z_1) [6]:

$$\begin{aligned}\dot{x}_1 &= \sigma(y_1 - x_1) \\ \dot{y}_1 &= rx_1 - y_1 - x_1z_1 \\ \dot{z}_1 &= x_1y_1 - \beta z_1\end{aligned}\tag{1}$$

The parameters σ , r , and $\beta > 0$, and their values are 10, 28, and 8/3 respectively. This model is explicit in the sense that both of attractor size and time scale depends on some parameters each of which has inverse time dimension.

In order to have the simplest attractor with a chaotic behavior without a property of symmetry, a Rössler model was suggested also. The Rössler model is a system includes three non-linear ordinary differential equations, which represent a continuous-time nonlinear system that shows a chaotic behavior associated with the fractal characteristics of the attractor [10].

$$\begin{aligned}\dot{x}_2 &= -y_2 - z_2 \\ \dot{y}_2 &= x_2 + ay_2 \\ \dot{z}_2 &= b + z_2(x_2 - c)\end{aligned}\tag{2}$$

Where a , b , and c are constants. The Rössler model is simpler than the Lorenz system because it only exhibits a single spiral. In this model there is only a single nonlinear term (z_2x_2), and it is chaotic when $a=b=0.2$, $c=5.7$ [10].

In order to attain a novel chaotic scheme, two chaotic systems are connected. The two systems; Lorenz and Rössler are coupled where the output of the variable x-dynamics for Lorenz circuit is the input of variable of x-dynamics of Rössler circuit. The main components

of both Rössler system and Lorenz system are not identical. Consequently the output signal of Lorenz system will enter to Rössler system which in turn will be affected by any variation that occurs in Lorenz circuit. The result of the coupling of Lorenz and Rössler system will be as follows:

$$\begin{aligned}\dot{x}_2 &= -y_2 - z_2 + \sigma(y_1 - x_1) \\ \dot{y}_2 &= x_2 + ay_2 \\ \dot{z}_2 &= b + z_2(x_2 - c)\end{aligned}\quad (3)$$

Results and discussion

Fourth-order Runge–Kutta method was used to solve the systems of differential equations in all numerical simulations with step of $\Delta=0.0078$. Fig. 1 represents the variation of the x_1 -dynamic with time at the initial conditions are taken as ($x_1=y_1=z_1=0.01$) according to x -dynamics of time series of Lorenz system. The range of x_1 -dynamics are $[-20$ to $22]$.

The time series of the Rössler circuit at the initial conditions are taken as ($x_2=y_2=z_2=1$) is shown in Fig. 2, where the x_2 , y_2 , and z_2 -dynamics are $[-10$ to $12]$, $[-12$ to $9]$, and $[0$ to $27]$ respectively.

The attractors of Rössler system are a strange attractors as shown in Fig.3 which takes a pair of state variables (x_2-y_2), (x_2-z_2), and (y_2-z_2) respectively.

When the variable x_1 of Lorenz system is coupled with the variable x_2 of Rössler system, the time series of Rössler system will take different values while the dynamics remain chaotic as shown in Fig.4 which illustrate the dynamics of Rössler at (x_1+x_2) for three state variables (x_2, y_2, z_2) respectively.

The attractors after coupling between the variable x_1 of Lorenz system and the variable x_2 of Rössler system remain strange (chaotic behavior), but the attractor shapes vary, as shown in Fig.5 which explain the attractor of Rössler at (x_1+x_2) which take a pair of state variables (x_2-y_2), (x_2-z_2), and (y_2-z_2) respectively. It is observed that there is an increase in the value of chaotic dynamic systems ranges as shown in table (1), this positively causes an increase in bandwidth, which plays an important role in communication applications especially confidential and secure ones. This is confirmed during the process of performing fast Fourier transforms (FFT) of chaotic dynamics before and after coupling. It is believed that the chaotic systems with higher dimensional attractors have much wider applications. The spectrum of FFT of the Lorenz and Rössler systems, is illustrated in Fig.(6), where it was observed that when comparing the two spectra, the Lorenz system has a wider spectrum of the

Rössler system with a distinct frequency at the low frequency of about 1.7(a.u). As for Figs 7, 8 and 9, it is noticed that the bandwidth of the x_2 , y_2 , and z_2 dynamics of Rössler system has increased and became wider after the mixing process, and this observation is important and the basis for the work of secure communications. The distinct unwanted frequency of Rössler system has been noticed to disappear within the new frequency range. The best bandwidth that can be used in communications is within z -dynamics due to its distinct exponential dicey distribution.

Conclusions

In this paper the coupling between two chaotic systems (Lorenz and Rössler) is investigated. The study revealed that there are variations in time series and attractor of Rössler system. This means that the behavior of Rössler system experienced a significant change after coupling, which can be considered as a good method for controlling the chaotic systems. This new system can be used in secure communications due to its chaotic behavior and its wide bandwidth, by performing fast Fourier transforms (FFT) of chaotic dynamics before and after coupling. It is possible to build an electronic circuit using the new chaotic system. Over all, the results indicate that the theoretical model has practical feasibility for implementation.

References

- [1] W. K. Chen, "The Circuits and Filters Handbook", CRC Press (2003) 396–397.
- [2] S. R. Mehrotra, "The Synthetic floating negative inductor using only two op-amps", Electronics World 111 (2005) 47.
- [3] E. Ott, "Chaos in dynamical system", Cambridge University Press (2000).
- [4] E. N. Lorenz, "Deterministic nonperiodic flow" J. Atm. Sci. 20 (1963) 130-141.
- [5] S. Vaidyanathan and C. Volos, "Analysis adaptive control of a novel 3-D conservative no-equilibrium chaotic system", Archives of control sciences 25(3) (2015) 333-353.
- [6] R. K. Jamal and D. A. Kafi, "Secure communications by chaotic carrier signal using Lorenz model", Iraqi Journal of Physics 14 (30) (2016) 51-63.
- [7] R. Banupriya, J. Deepa, and S. Suganthi, "Video steganography using LSB algorithm for security application", Int. J. Mech. Eng. Technol. 10(1) (2019) 203-211.
- [8] Z. Hua, Y. Zhou, and H. Huang, "Cosine-transform-based chaotic system for image encryption", Inf. Sci. 480(1) (2019) 403-419.
- [9] A. Uchida, S. Kinugawa, T. Matsuura, and S. Yoshimori, "Dual synchronization of chaos in one-way coupled microchip lasers", Phys. Rev. E 68 (2003).

[10] O. E. Rössler, "An Equation for Continuous Chaos", Physics Letters 57(5) (1976) 397–398.

Figure (1): x_1 -dynamics of Lorenz system where $r=28$, $\sigma=10$, $\beta=2.7$, and initial values $x_1=y_1=z_1=0.01$ origin.

Figure (2):(a) x_2 -dynamics, (b) y_2 -dynamics, (c) z_2 -dynamics of Rössler system where $a=b=0.2, c=6.3$, and initial values $x_2=y_2=z_2=1$.

Figure (3): (a) (x_2-y_2) attractor, (b) (x_2-z_2) attractor, and (c) (y_2-z_2) attractor of Rössler where $a=b=0.2, c=6.3$, and initial values $x_2=y_2=z_2=1$ origin.

Figure (4):(a) x_2 -dynamics, (b) y_2 -dynamics, (c) z_2 -dynamics of Rössler at (x_1+x_2) .

Figure(5):(a) (x_2-y_2) attractor, (b) (x_2-z_2) attractor, (c) (y_2-z_2) attractor of Rössler at x_1+x_2 .

Table (1): Ranges of Rossler dynamics before and after mixing.

Attractor before mixing	(x_2-y_2)	(x_2-z_2)	(y_2-z_2)
Range	$x_2: [-12, 8]$ $y_2: [-9, 13]$	$x_2: [0, 233]$ $z_2: [-3, 13]$	$y_2: [0, 28]$ $z_2: [-12, 9]$
Attractor after mixing	(x_2-y_2)	(x_2-z_2)	(y_2-z_2)
Range	$x_2: [-28, 13]$ $y_2: [-22, 22]$	$x_2: [0, 28]$ $z_2: [-22, 24]$	$y_2: [0, 233]$ $z_2: [-28, 13]$

Figure (6): FFT spectrum of x1: Lorenz (blue line) and x2:Rossler (black line).

Figure (7): FFT spectrum of x2:mix (blue line) and x2:Rossler (black line).

Figure (8): FFT spectrum of y2:mix (blue line) and y2:Rossler (black line).

Figure (9): FFT spectrum of z2:mix (blue line) and z2:Rossler (black line).

Figures

Figure 1

x1-dynamics of Lorenz system where $r=28$, $\sigma=10$, $\beta=2.7$, and initial values $x_1=y_1=z_1=0.01$ origin.

Figure 2

(a) x_2 -dynamics, (b) y_2 -dynamics, (c) z_2 -dynamics of Rössler system where $a=b=0.2, c=6.3$, and initial values $x_2=y_2=z_2=1$.

Figure 3

(a) (x_2-y_2) attractor, (b) (x_2-z_2) attractor, and (c) (y_2-z_2) attractor of Rössler where $a=b=0.2, c=6.3$, and initial values $x_2=y_2=z_2=1$ origin.

Figure 4

(a) x_2 -dynamics, (b) y_2 -dynamics, (c) z_2 -dynamics of Rössler at (x_1+x_2) .

Figure 5

(a) $(x_2 - y_2)$ attractor, (b) $(x_2 - z_2)$ attractor, (c) $(y_2 - z_2)$ attractor of Rössler at $x_1 + x_2$.

Figure 6

FFT spectrum of x1: Lorenz (blue line) and x2:Rossler (black line).

Figure 7

FFT spectrum of x2:mix (blue line) and x2:Rossler (black line).

Figure 8

FFT spectrum of y2:mix (blue line) and y2:Rossler (black line).

Figure 9

FFT spectrum of z2:mix (blue line) and z2:Rossler (black line).