

Electromagnetic Radiation Under Phase Symmetry Breaking in Quantum Systems

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Electromagnetic Radiation Under Phase Symmetry Breaking in Quantum Systems

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Abstract

According to classical mechanics, electron acceleration results in electromagnetic radiation while in quantum mechanics radiation is considered to be arising out of a transition of the charged particle from a higher to a lower energy state. A different narrative is presented in quantum field theory, which considers radiation as an outcome of the perturbation of zero point energy of quantum harmonic oscillator which results in a change in density of electrons in a given state. The theoretical disconnect in the phenomenological aspect of radiation in classical and quantum mechanics remains an unresolved theoretical challenge. As a charged particle changes its energy state, its wavefunction undergoes a spatial phase change, hence, we argue that the spatial phase symmetry breaking of the wavefunction is a critical aspect of radiation in quantum mechanics. This is also observed in Josephson junction, where a static voltage induces spatial phase symmetry breaking of current resulting in emission of electromagnetic waves. As temporal symmetry breaking of the magnetic vector potential generates classical radiation and a wavefunction of a charged particle can always be associated with a specific magnetic vector potential; the concept of radiation under spatial phase symmetry breaking offers a novel perspective towards unifying the phenomenon of radiation in quantum mechanics and classical electromagnetism.

I. INTRODUCTION

In classical electromagnetism, radiation is considered to be an outcome of acceleration of a charged particle [1, 2], while in quantum mechanics, the transition of a charged particle from a higher energy state to a lower energy state, generates quantized radiation [3]. Schrödinger equation, which defines the basic mathematical structure of quantum mechanics, explains stimulated emission of radiation where a photon is emitted under the effect of an external electromagnetic field, but it cannot explain spontaneous emission where photon emission does not require the action of an external field[3]. These problems are addressed in quantum field theory (QFT), which considers electromagnetic field as a set of harmonic oscillators, comprising of non-zero minimum energy states in vacuum, called the zero-point energy and spontaneous emission is described using perturbation theory. The fluctuation of the electromagnetic field stimulates spontaneous emission of radiation by atoms which is associated with a change in charge density and particle number in a given state [4]. There is a general theoretical disconnect in our current understanding of radiation in classical electromagnetism and quantum mechanics. The phenomenological aspects of radiation in QFT, still remains detached from that of classical electromagnetism. The key objective of the current work is to provide a theoretical framework, which could provide a phenomenological link between radiation in classical electrodynamics and quantum mechanics.

Despite the differences in the nature of mathematical formulations and their physical interpretations in classical and quantum systems, global conservation of charge, expressed through gauge invariance, is the central theme, which is encountered in both these fields [5]. A key role in this context is played by magnetic potential, \mathbf{A} , where a change in its value, by a scalar function, ψ , i.e. $\mathbf{A} \rightarrow \mathbf{A} + \nabla\psi$, implies that the electric potential is reduced, i.e. $V \rightarrow V - \partial\psi/\partial t$, which leads to invariance of the electric field, $\mathbf{E} = -\nabla V - \partial\mathbf{A}/\partial t$ [6].

Hermann Weyl established the connection between gauge symmetry and global conservation of charges in electromagnetism [7]. It had a profound impact in quantum mechanics and QFT [8]. However, Gauge invariance is looked upon from the perspective of a theoretical construct as a redundant degree of freedom to understand symmetry of the electromagnetic field. For example, Nathan Sieberg argues that gauge symmetry is not a symmetry, it is gauge redundancy [9]. He points that gauge symmetry is not intrinsic and hence gauge symmetries are not fundamental. It is often convenient to use them to make the descrip-

tion manifestly Lorentz invariant, unitary and local. But there can be different such (dual) descriptions. Sieberg further argues that as gauge symmetry is not a symmetry and so it is never broken [9]. Greiter believes that Gauge symmetry is not broken in superconductors, it is because gauge symmetries do not relate to different states, but they relate to the same state [10].

Despite the concerns of some theorists, gauge invariance is empirically evident in an electrodynamic system like an antenna radiating a constant power driven by a finite potential source, where for every change in the value of magnetic vector potential, its invariance is associated with a corresponding drop in the potential values resulting in global invariance of the associated Lagrangian [12]. The magnetic vector potential is expressed by, $\mathbf{A} = \mu_0/(4\pi r \int Idl)$, where r is distance from a finite section of length dl with current I , μ_0 is magnetic permeability [11]. A change in the value of current I , leads to an increase in the value of magnetic vector potential \mathbf{A} , and a corresponding reduction in the voltage V as the source of electric potential is finite and it can drive a finite amount of current in a load. Thus, during the process of radiation, gauge invariance ensures a constant value of the electric field \mathbf{E} [12]. For radiation in free space, the static electric potential is zero and the electric field is $\mathbf{E} = -\partial\mathbf{A}/\partial t = j\omega\mathbf{A}$. The frequency is expressed as, $\omega = d\phi(t)/dt$, where $\phi(t)$, is phase at given instance of time [11]. Thus, temporal symmetry breaking of the phase of current is an integral feature of classical radiation, which may provide a link between classical and quantum mechanics. The theoretical challenge is to establish a connection between the varying phase of charge motion in classical systems with the phase of electron wavefunction in quantum mechanics.

II. PHASE SYMMETRY BREAKING AND RADIATION

The Schrödinger wave equation for an electron is given by [13],

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi \quad (1)$$

where, ψ is the particle wavefunction, m is its mass, \hbar is the reduced Planck's constant, t is time, V is the potential and $i = \sqrt{-1}$. Its solution is expressed as, $\psi(r, t) = \psi_0 e^{i(\mathbf{p}\cdot\mathbf{r})/\hbar} e^{i(-Et)/\hbar}$, where \mathbf{p} is momentum, \mathbf{r} is position vector and E is energy. The coefficient, $e^{i\mathbf{p}\cdot\mathbf{r}}$ describes the phase of the wavefunction in geometric space and can be referred

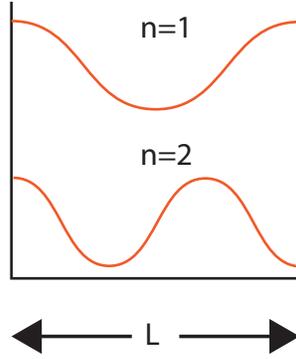


FIG. 1. Wavefunction of an electron in a quantum well.

to as the spatial phase of the wavefunction. Under electron acceleration, there is a change in momentum expressed as, $d\mathbf{p}/dt$ which influences the spatial phase of the wavefunction. The wavefunction at a time t' can be written as, $\psi(r, t') = \psi_0 e^{\frac{i}{\hbar} \int \frac{d\mathbf{p} \cdot \mathbf{r}}{dt'} dt'} e^{-iEt'/\hbar}$, which has a different spatial phase in comparison to the wavefunction at time t . It is important to establish this aspect of spatial phase change in the context of transition of an electron in a quantum well and radiation.

Solving the Schrödinger wave equation for one dimensional case of a particle in a potential well of magnitude V and length L , along a given dimension z for a constant value of energy (Fig. 1), we get [13],

$$E_n u = -\frac{\hbar^2}{2m} \frac{d^2 u_n}{dz^2} + V u_n \quad |z| < L/2 \quad (2)$$

where n is an integer, E_n is energy eigenfunction and u_n is eigenfunction associated with the n th state of the electron, expressed by, $\psi_n = u_n e^{-iEt/\hbar}$. We assume that the wavefunction is entirely confined within the potential well. Applying the boundary condition, $u_z(L/2) = u_z(-L/2) = 0$, we get the following expression,

$$u_n(z) = \sqrt{\frac{2}{L}} \sin \left[n \frac{\pi}{L} \left(z + \frac{L}{2} \right) \right] \quad (3)$$

The energy eigenstates are given by,

$$E_n = -\frac{\hbar^2}{2m} \left[\frac{n\pi}{L} \right]^2 \quad (4)$$

As the electron makes a transition between different energy states, E_2 and E_1 , a photon is emitted with the energy, $\Delta E = E_f - E_i = \hbar\omega$, where \hbar is reduced Planck's constant and ω

is angular frequency of radiation. The eigenfunction of an electron corresponding to these two electron states has a net spatial phase difference, $\Delta\phi = n\pi/L(z + L/2)$. As the electron transition takes place over a finite period of time, we can state that spatial phase symmetry of the electron eigenfunction has undergone a change which is associated with radiation.

According to Fermi's golden rule, the transition rate of an electron from one energy eigenstate to another eigenstate under perturbation is proportional to the coupling strength between the two states[13]. Its value in a direct band gap semiconductor, where, an electron with a wave number, k_α in the valence band, with an initial eigenstate $|\alpha\rangle = \psi_{v,\mathbf{k}_\alpha}(\mathbf{r})$ with a Hamiltonian H excited by a photon, moves to the conduction band with an eigenstate $|\beta\rangle = \psi_{c,\mathbf{k}_\beta}(\mathbf{r})$ with a wavenumber k_β under a perturbation H' is,

$$\Gamma_{\alpha\beta} = \frac{2\pi}{\hbar} |\langle\alpha| H' |\beta\rangle|^2 \delta(E_\beta - E_\alpha \pm \hbar\omega) \quad (5)$$

where, $H' = e\mathbf{A}\boldsymbol{\epsilon}\cdot\mathbf{p}/m$ and $\boldsymbol{\epsilon}$ is the light polarization vector. It also represents the transition rate when a photon is emitted as the electron combines with the hole. The wavefunction for electrons corresponding to the two states are expressed as,

$$\psi_{v,\mathbf{k}_\alpha}(\mathbf{r}) = \frac{1}{\sqrt{V_L}} u_{n_v,\mathbf{k}_\alpha}(\mathbf{r}) e^{i\mathbf{k}_\alpha\cdot\mathbf{r}} \quad (6)$$

$$\psi_{c,\mathbf{k}_\beta}(\mathbf{r}) = \frac{1}{\sqrt{V_L}} u_{n_c,\mathbf{k}_\beta}(\mathbf{r}) e^{i\mathbf{k}_\beta\cdot\mathbf{r}} \quad (7)$$

where, V_L is a volume and $u_{n_v,\mathbf{k}_\alpha}(\mathbf{r})$ and $u_{n_c,\mathbf{k}_\beta}(\mathbf{r})$ are the maximum values of amplitudes of the electron wavefunction in states k_α and k_β respectively. The spatial phase difference is also evident here as the wave number changes from k_α to k_β .

The observables in quantum mechanics remain invariant under a global change in phase λ of the wavefunction expressed as, $\psi(\mathbf{r}, t) \rightarrow e^{i\lambda(\mathbf{r}, t)}\psi(\mathbf{r}, t)$. This is called U(1) symmetry which is associated with charge conservation. The wavefunction remains invariant under transformation of the magnetic vector potential expressed as, $\mathbf{A} \rightarrow \mathbf{A} - \nabla f(\mathbf{r}, t)$ and electric potential, $V \rightarrow \phi + \partial f(\mathbf{r}, t)/\partial t$, where $f(\mathbf{r}, t)$ is the potential and is related to the phase via the relationship, $f(\mathbf{r}, t) = (\hbar/q)\lambda(\mathbf{r}, t)$, where q is charge of an electron. The function $f(\mathbf{r}, t)$ is related to \mathbf{A} using the relationship, $\int_{\mathbf{r}_0}^{\mathbf{r}} d\mathbf{r} \cdot \mathbf{A} = \int_{\mathbf{r}_0}^{\mathbf{r}} d\mathbf{r} \cdot \nabla f = f(\mathbf{r}) - f(\mathbf{r}_0)$, where \mathbf{r} is the initial position vector of the electron. Thus, the wavefunction is expressed as, $\psi(\mathbf{r}, t) \rightarrow e^{-i\frac{e}{\hbar}f(\mathbf{r}, t)}\psi(\mathbf{r}, t)$. Under radiation, the condition of this gauge symmetry between the two quantum wavefunctions breaks down as the initial and final wavefunctions have

different phases as defined through Eq. 6 and Eq. 7. Thus spatial phase symmetry breaking is an integral aspect of radiation.

III. RADIATION FROM JOSEPHSON JUNCTION

Josephson effect is an instance of spontaneous symmetry breaking of the phase of the electronic wavefunction in a superconductor [16]. In a normal metal, the Fermi states have random phases which are invariant with respect to translational or temporal variance. The symmetry of the phase is spontaneously broken in a superconductor. The phase symmetry breaking can be measured by introducing a junction in a superconductor. In other words, it is the spontaneous symmetry breaking of the Ginzburg–Landau complex order parameter. In the current work, our focus is on variations in existing spatial phase of the wavefunctions of electrons under radiation along the temporal dimension, which we are referring to as spatial phase symmetry breaking.

We can also look at radiation from the Josephson junction under a specific voltage bias, from the perspective of spatial phase symmetry breaking, leading to generation of electromagnetic radiation. A superconductor is characterized by an order parameter which is its wavefunction representing the collective quantum state of the cooper pairs [14, 15]. In a Josephson junction, the wavefunction along the two sides of insulating barriers, separating the superconducting ring can be represented by, $\psi_1 = \sqrt{\rho_1}e^{i\delta_1}$ and $\psi_2 = \sqrt{\rho_2}e^{i\delta_2}$, where ρ_1 and ρ_2 are cooper pair densities, δ_1 and δ_2 are phases [17]. The Schrödinger wave equation for each of the wavefunctions is represented as, $i\hbar\frac{\partial\psi_1}{\partial t} + U_1\psi_1 = K\psi_2$ and $i\hbar\frac{\partial\psi_2}{\partial t} + U_2\psi_2 = K\psi_1$, where, K is a constant which is a function of junction width, its composition and temperature.

When a voltage is applied across a Josephson junction, the net difference in energy across the junction is, $U_2 - U_1 = 2eV_0$, where, V_0 represents the voltage drop associated with the junction, $2e$ is the Cooper pair charge. The weak link current is expressed as, $I(t) = I_C \sin \delta$, where, I_C is the value of critical current below which superconductivity is maintained and, $\delta = \delta_1 - \delta_2$, is the difference in phases of the current across the junction [17]. The net current, I , which tunnels through the junction is directly proportional to the rate of variation of density of Cooper pairs, $d\rho_1/dt = -d\rho_2/dt$. In order to solve these equations, the equations which are needed are, $I = I_C \sin \delta$ and $d\delta/dt = 2eV(t)/\hbar$. It is worth mentioning that the

phase difference between currents across the two sections of Josephson junction is directly linked to the spatial phase of the wavefunction [19].

The current in the weak link in Josephson junction, has a constant phase, δ , when there is no voltage across the junction. When a static voltage, V_0 is applied at a junction, the instantaneous current is expressed by, $I = I_C \sin(\delta + \frac{2e}{\hbar}V_0t)$ [20]. As \hbar present in the denominator of the phase of current is very small, the phase of the sine function becomes extremely large, with a time average value of zero, which makes the value of tunneling supercurrent to be zero for $V \neq 0$. A measurable current, $I = I_C \sin \delta$ appears only for $V = 0$. As a static voltage is applied at the junction, the energy difference associated with the Cooper pair which crosses the junction is equal to $2eV_0$. It results in emission of a photon with a frequency of $\omega = 2eV/\hbar$. The change of phase of Josephson current specifically highlights the fact that temporal symmetry breaking of the phase is associated with the voltage across the weak link.

When a fixed voltage V_{DC} is applied to a Josephson junction, the phase varies with time and an alternating current is set up in the system with an amplitude I and frequency $2eV_{DC}/\hbar$ [18]. The current can be expressed as, $I = Cdv/dt + I \sin \phi + V/R$. The phase is expressed by, $\phi = \phi_0 + n\omega t + a \sin \omega t$, the voltage is $U(t) = (\frac{\hbar}{2}e)\omega(n + a \cos \omega t)$ and current is $I(t) = I_C \sum J_m(a) \sin(\phi_0 + (n + m)\omega t)$. Thus, it also transforms time varying frequency to voltage, where the phase evolution is dependent on the voltage $V(t)$ across the junction according to the following equation,

$$\frac{\partial \phi}{\partial t} = \frac{2eV}{\hbar} \quad (8)$$

Here, $\hbar/2e$ is magnetic flux constant. Under such a condition, the spatial phase change of current in the two sections of the superconducting ring is observed, which is associated with electromagnetic radiation.

During the voltage sweep, there is a step-wise increase in current given by, $\Delta I = \frac{V}{2V_0}I_C$ corresponding to the condition of resonance, $\omega = 2eV_0/\hbar$. The Josephson constant is expressed as, $2e/\hbar = 483.5979 \text{ MHz}/\mu\text{V}$, which offers a frequency to voltage conversion framework based on AC Josephson effect [20]. When the junction is irradiated by radio frequency radiation, it induces phase changes in the junction current, expressed as, $I = I_C \sin(\delta + \frac{2e}{\hbar}V_0t + \frac{2eV}{\hbar\omega} \sin \omega t) = n\frac{v}{2V_0}I_C$, where, $\omega = 2neV_0 \sin \delta/\hbar$ for $n = 1, 2, 3, \dots$ [21]. The high frequency components have been neglected. Thus, the input electromagnetic wave

imparts a specific spatial phase to the Josephson current.

Josephson effect offers an interesting theoretical avenue to explore the role of spatial phase changes in radiation, which has remained unexplored until now. For example, in a related experiment, current in a superconducting ring was excited by a pulse of laser light from an external source which resulted in GHz wide band radiation in the GHz range. The radiation could be tuned by varying the pattern of laser illumination. It could be argued that laser excitation resulted in spatial phase symmetry breaking of the cooper pairs, which resulted in radiation[22]. Such observations can be used to design further experiments to investigate related physical effects in radiation from Josephson junctions.

IV. AHARONOV-BOHM EFFECT

The magnetic vector potential \mathbf{A} , is related to magnetic flux ϕ_B , by the relation, $\phi_B = \int_C \mathbf{A}dr$ along a closed path, C . In quantum mechanics, any change in magnetic vector potential induces a change in phase of the wavefunction [23]. Considering the symmetry principle in quantum mechanics, its reverse should also be true, which means that we can assign a finite value of magnetic vector potential to a change in spatial phase of wavefunction of a charged particle.

An ideal solenoid comprises of a long and uniform current distribution along a cylindrical surface enclosing a magnetic field, \mathbf{B} (Fig. 2). The magnetic field outside the cylindrical surface is zero. Hence, the Lorentz force on a charged body traversing the region outside the cylinder is zero. But the magnetic vector potential, \mathbf{A} in the given region is finite. Thus, the presence of current in the solenoid can modulate the phase of the charged particle leading to a change in the interference fringe. Turning on or off of current in the solenoid can affect the spatial phase of the wavefunction. The spatial phase change in the wavefunction of a charged particle with electric charge q traveling along some path P , in a region with zero magnetic field \mathbf{B} , but a non-zero value of \mathbf{A} is expressed as [23–25],

$$\varphi = \frac{q}{\hbar} \int \mathbf{A}dx \tag{9}$$

Particles starting from the same start point and finishing at the same end point, which travel along two different paths, will develop a geometric phase difference, $\Delta\varphi$ dependent on the magnetic flux density, Φ_B passing the area enclosed between the paths (via Stokes' theorem

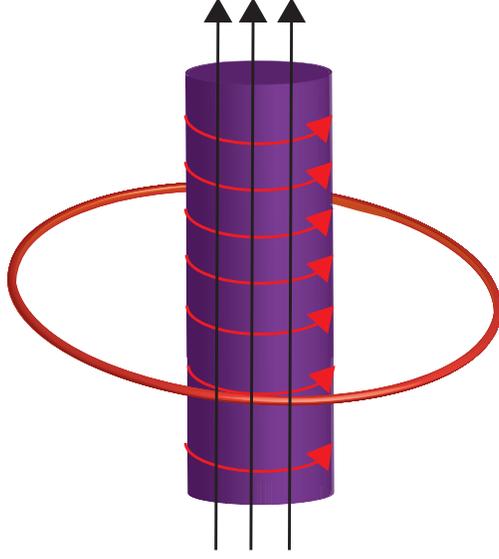


FIG. 2. A graphical illustration of magnetic vector potential outside a long solenoid.

and $\nabla \times \mathbf{A} = vB$. The phase difference acquired is,

$$\Delta\varphi = \varphi_d = \int_1 A(x, t) dx - \int_2 A(x, t) dx = \frac{q\varphi_B}{\hbar} \quad (10)$$

Taking the derivative of Eq. 10, we get,

$$\frac{\partial\varphi_d}{\partial t} = \int_1 \frac{\partial\mathbf{A}}{\partial t} dx - \int_2 \frac{\partial\mathbf{A}}{\partial t} dx = \frac{q}{\hbar} V \quad (11)$$

As $\partial\mathbf{A}/\partial t$ is associated with radiation, it proves that symmetry breaking of the spatial phase can be associated with radiation. Applying Gauge transformation, $\mathbf{A}(x, t) \rightarrow \mathbf{A}(x, t) + \nabla\Lambda(x, t)$, where, $\Lambda(x, t)$ is an arbitrary, scalar function defined between two distinct spatial points, x_1 and x_2 as, $\Lambda(x, t) = \int_{x_1}^{x_2} A(x, t) dx$, and $V(x, t) \rightarrow V(x, t) - \partial\Lambda(x, t)/\partial t$, the wavefunction associated with it, $\psi'(x, t)$, undergoes a phase change expressed as,

$$\psi(x, t) = \psi'(x, t) e^{-\frac{iq}{\hbar}\Lambda(x, t)} \quad (12)$$

Thus, the scalar function, $\Lambda(x, t)$ changes the spatial phase of the wavefunction. The wavefunction associated with the two paths are, $\psi_1 = \psi_{10} e^{-\frac{iq}{\hbar} \int_1 A(x, t) dx}$, $\psi_2 = \psi_{20} e^{\frac{iq}{\hbar} \int_2 A(x, t) dx}$.

It is worth mentioning that gauge symmetry defines the global conservation of charges and it applies to radiating systems. In classical electromagnetism, from a physical perspective the invariance of the electric field, with a decrease in electric potential and an increase in

magnetic vector potential, essentially, indicates that some voltage has been transformed into current which is driving the radiation and in the process, the global charge is being conserved. At a quantum mechanical level, it results in a change in spatial phase of the wavefunction. The magnetic vector potential undergoes a change along temporal dimension, which will also introduce a change in the spatial phase of the wavefunction. Thus, a temporal change in the value of magnetic vector potential can be correlated to a change in spatial phase and radiation. The effect is also quite evident in classical electromagnetism, where, we can always associate a radiation field with time rate of change of magnetic vector potential, $\partial\mathbf{A}/\partial t$.

In Aharonov-Bohm effect, the phase of wavefunction of an electron is modulated by magnetic vector potential, \mathbf{A} in a region where the value of magnetic flux density and electric fields are zero. The wavefunction of a charged particle present around a long solenoid experiences a phase shift. The magnetic Aharonov-Bohm effect can be seen as a result of the requirement that quantum physics be invariant with respect to the gauge choice for the electromagnetic potential, of which the magnetic vector potential \mathbf{A} , forms an integral part. Thus, it is evident that a spatial change in phase of an electronic wave function results in electromagnetic radiation.

V. CONNECTIONS TO BERRY PHASE

During the adiabatic evolution of a quantum system, where the external parameter varies slowly, eventually forming a loop in space, the initial eigenstate is regained, but there is a geometric phase difference, which is called the Berry Phase. The Hamiltonian is expressed by, $H(\mathbf{R})$, where \mathbf{R} is a physical quantity, which parametrizes the cyclic adiabatic process. The system is in an eigenstate, $|n(\mathbf{R}(0))\rangle$ at time, $t = 0$. The eigenstate at time, t is $|n(\mathbf{R}(t))\rangle$ and the wavefunction is expressed as, $|\Psi_n(t)\rangle = e^{i\gamma_n(t)} e^{-\frac{i}{\hbar} \int_0^t dt' \epsilon_n(\mathbf{R}(t'))} |n(\mathbf{R}(t))\rangle$. If the system evolves adiabatically such that, $\mathbf{R}(t) = \mathbf{R}(0)$, the Berry phase is expressed as, $\gamma_n = \int_C d\mathbf{R} \cdot \mathcal{A}_n(\mathbf{R})$, where, $\mathcal{A}_n(\mathbf{R}) = i \langle n(\mathbf{R}) | \nabla_{\mathbf{R}} | n(\mathbf{R}) \rangle$, is called the Berry potential or the Berry connection [27, 28].

The Gauge Transformation of the wavefunction expressed as, $|\tilde{n}(\mathbf{R})\rangle = e^{-i\beta\mathbf{R}} |n(\mathbf{R})\rangle$, is associated with, $\tilde{\mathcal{A}}_n(\mathbf{R}) = \mathcal{A}_n(\mathbf{R}) - \nabla_{\mathbf{R}}\beta(\mathbf{R})$. It is worth noting that, if an electron has been subjected to a cyclic process, by using a magnetic vector potential, any change in phase

would naturally be associated with radiation, as the application of a finite value of magnetic vector potential to an electron, will always be associated with a finite value of $\partial\mathbf{A}/\partial t$ in the inertial frame of the electron. Hence, generation of Berry phase, must be accompanied by radiation. However, if the process is adiabatic, then, in the ideal scenario, the process lasts in a slow manner, such that, the electron has been in equilibrium at all times of the process. Thus, the process of acquiring Berry's phase, under an adiabatic process, can also be considered to be a special example, where radiation is explicitly neglected, because of the physical assumptions interleaved with the adiabatic nature of the process. A slow adiabatic process related to acquiring Berry phase is an idealized perspective. The point finds support from Bohm et al.[29], who write, "However, it is easy to show that for a Hamiltonian with changing eigenvectors the adiabatic approximation of the dynamics of a cyclic evolution cannot be exact. Therefore Berry's phase could only be an approximation of the true quantum geometric phase....Therefore, exact adiabatic cyclic evolutions do not exist. The adiabatic equality can only be an approximation".

VI. RADIATION IN QUANTUM FIELD THEORY

The contradictions in the mechanism of radiation in classical electromagnetism and quantum mechanics is addressed using quantum optics, where a transition from an excited state towards the ground state results in the emission of photons in a Fock state. It is assumed that fluctuations in the quantized light fields act as a perturbation on the excited state, which places the system into a coherent superposition state between the excited state and the ground state. For this superposition state, a second derivative of the spatial probability distribution couples to the probability amplitude and probability for photon emission. This superposition state then collapses to some eigenstate of the joint system when a measurement is performed, e.g. by detecting a photon. This appears to directly yield some connection to classical electromagnetism, where a second derivative of charge density is coupled to radiation. This point can be used to establish a possible connection between quantum mechanics and classical electrodynamics. However, radiation as a consequence of a change in density of states does not show a direct link between classical electromagnetism and quantum mechanics as the issue of electron acceleration is not addressed in quantum optics.

In quantum optics, the electric field is expressed as[30],

$$\mathbf{E}(r, t) = i \sum_{k, \mu} \sqrt{\frac{\hbar \omega}{2V_L \epsilon_0}} \left(e^{(\mu)} a^{(\mu)}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}} - e^{-(\mu)} a_{\mathbf{k}}^{\dagger(\mu)}(\mathbf{k}) e^{-i\mathbf{k} \cdot \mathbf{r}} \right) \quad (13)$$

where, \hbar is reduced Planck's constant, V_L is volume, ω is angular frequency and \mathbf{k} is wave number and \mathbf{r} is position vector, $e^{(\mu)}$ is polarization vector, $a^{\dagger(\mu)}(\mathbf{k})$ is the creation operator which excites from an n fold excited state to an $n + 1$ excited state, $a^{\dagger(\mu)}(\mathbf{k}) |n\rangle = |n + 1\rangle \sqrt{n + 1}$ it excites from an n fold excited state to an $n + 1$ fold excited state. The annihilation operator, $a^{(\mu)}(\mathbf{k})$, which destroys one photon from vacuum, $a^{(\mu)}(\mathbf{k}) |n\rangle = |n - 1\rangle \sqrt{n}$. They satisfy a set of specific relations, $[a^{(\mu)}(\mathbf{k}), a^{(\mu')}(\mathbf{k})] = 0$, $[a^{\dagger(\mu)}(\mathbf{k}), a^{\dagger(\mu')}(\mathbf{k})] = 0$ and $[a^{(\mu)}(\mathbf{k}), a^{\dagger(\mu')}(\mathbf{k})] = \delta_{k, k'}, \delta_{\mu, \mu'}$

The vector magnetic potential is expressed as,

$$\mathbf{A}(r, t) = \sum_{k, \mu} \sqrt{\frac{\hbar}{2\omega V_L \epsilon_0}} \left(e^{(\mu)} a^{(\mu)}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}} + e^{-(\mu)} a_{\mathbf{k}}^{\dagger(\mu)}(\mathbf{k}) e^{-i\mathbf{k} \cdot \mathbf{r}} \right) \quad (14)$$

The Hamiltonian for the electromagnetic field is,

$$\frac{1}{2} \sum_{k, \mu=\pm 1} \hbar \omega [a^{\dagger(\mu)}(\mathbf{k}) a^{(\mu)}(\mathbf{k}) + a^{(\mu)}(\mathbf{k}) a^{\dagger(\mu)}(\mathbf{k})] = \sum_{k, \mu} \hbar \omega \left(N^{\mu}(\mathbf{k}) + \frac{1}{2} \right) \quad (15)$$

where, $N^{(\mu)}(\mathbf{k}) = a^{\dagger(\mu)}(\mathbf{k}) a^{(\mu)}(\mathbf{k})$ is the number operator which acting on a quantum mechanical photon number state, returns the number of photons in mode (k, μ) .

The concept of radiation under phase symmetry breaking do not appear to have a counterpart in QFT. In quantum mechanics, the observables or the operators act on the wavefunction and in QFT, the operator valued field acts on the space of states. The wavefunction is transformed into observables through operator valued quantum fields, which is called second quantization. States in quantum mechanics are associated with a spatio-temporal meaning expressed through probability measurements while in QFT, states are expressed through quantum field operators having spatio-temporal interpretations.

The concept of particle phases do not occur in QFT, where the second derivative of the spatial probability distribution of the charged particle couples to the probability amplitude and probability for photon emission. However, a change in density ρ_p of a given field, is linked to a corresponding change in its current density \mathbf{J}_p , which can be written as, $\partial \rho_p / \partial t = \nabla \cdot \mathbf{J}_p$. A second derivative of the density can be associated with a change in phase of current, i.e. $\partial^2 \rho_p / \partial t^2 = \partial(\nabla \cdot \mathbf{J}_p) / \partial t$. Thus, despite apparent differences in terminology, QFT implicitly

incorporates the role of phase symmetry breaking in radiation. Some of the existing gaps in the physical mechanism of radiation could be addressed if we consider the spatial phases of particles, however, it would entail some newer formulations, which could be taken up in future work.

VII. CONCLUSION

We have proved that the spatial phase symmetry breaking along the temporal dimension of an electron wavefunction is associated with quantum mechanical radiation, which has a classical counterpart in the form of temporal symmetry breaking of the magnetic vector potential. Analysis of radiation from the perspective of geometric phase symmetry breaking of a wavefunction also implies that geometric phase changes in Aharonov Bohm effect must be accompanied by finite value of electromagnetic radiation, although, if the process is adiabatic, radiation can be explicitly neglected. The perspective offers a unifying framework of radiation in classical electromagnetism and quantum mechanics.

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Figures

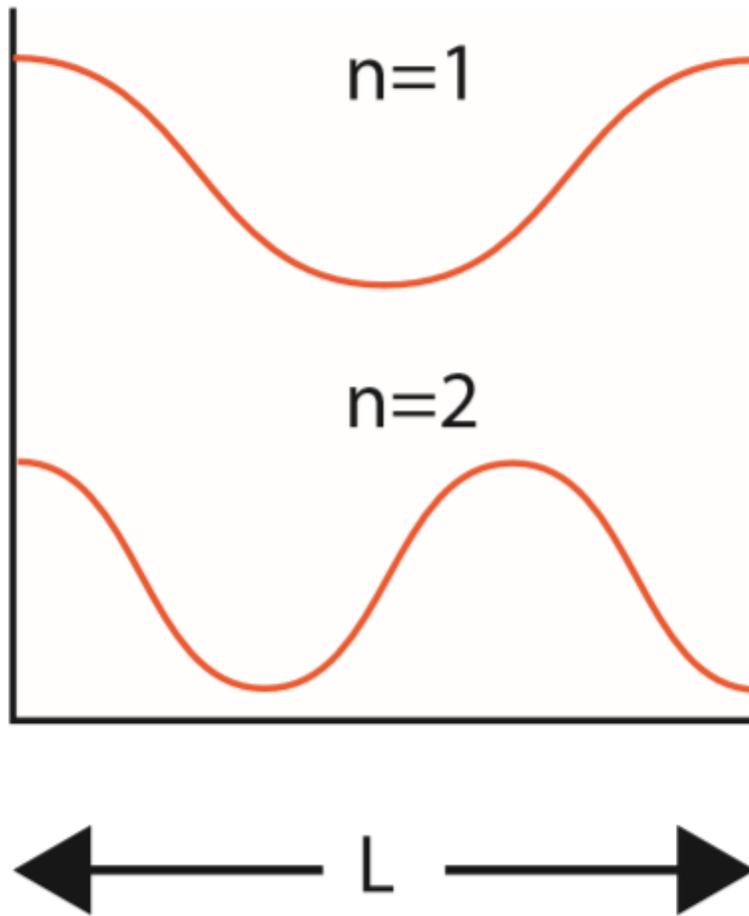


Figure 1

Wavefunction of an electron in a quantum well.

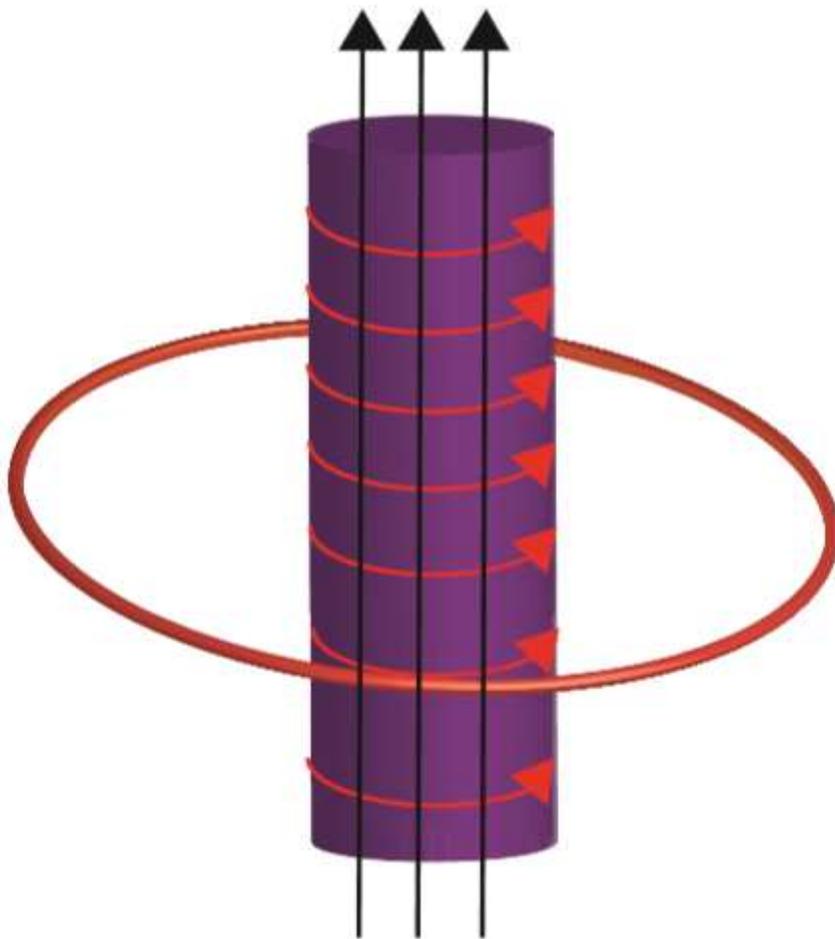


Figure 2

A graphical illustration of magnetic vector potential outside a long solenoid.