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Bonfils Safari (bonfilssafari@gmail.com)
University of Rwanda

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Modelling extreme rainfall with Block Maxima and Peak-Over Threshold methods in Rwanda

Bonfils Safari[1]
[1]*School of Science, College of Science and Technology.*
University of Rwanda
Kigali, Rwanda
Email address: bonfilssafari@gmail.com
ORCID iD: 0000-0003-0145-5886

Abstract

In this study two fundamental approaches of extreme value theory (EVT) were applied on the extreme precipitation incidents over twelve synoptic stations of Rwanda: the Block Maxima (BM) and the Peak-Over Threshold (POT). Annual maximum rainfall series (AMS) and partial duration rainfall series (PDS) higher than a selected threshold were fitted respectively to the Generalized Extreme Value (GEV) distribution and the Generalized Pareto (GP) distribution at each station. Four methods were used for the estimation of the parameters of the GEV and the GP distributions: the Maximum Likelihood Estimation (MLE) method, the L-Moments Estimation (LME) method, the Bayesian Estimation (BAYE) method and the Generalized Maximum Likelihood Estimation (GMLE) method. The performances of those methods were analyzed and compared for best fitting the data based on goodness-of-fit tests. It was found that in general, those methods are suitable for the two distributions at the sites considered in Rwanda with slight differences in estimated return levels and their confidence intervals. However, the MLE and LME methods perform better than the other methods for the GEV distributions whereas for the GP distribution it is the BAYE method. Return levels of extreme rainfalls with their 95% confidence intervals were computed for return periods of 10, 20, 50, 75, 100, 150 and 200 years. It was found that using the selected parameterization methods, the GP distribution presents higher return levels than GEV distribution for all stations Those methods can therefore be recommended as best parametric methods for estimating extreme rainfall in Rwanda using EVT.

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1 Introduction

The major effects of climate change in East Africa and particularly in Rwanda include an increased occurrence of extreme rainfall causing extreme flood events, an increased duration and frequency of droughts and an increasing average temperature. These effects when combined together have destructive impacts on infrastructure, energy production, agriculture, ecosystems and communities, causing enormous financial losses for countries whose economy is already fragile and health and food insecurity (FA0, 2017). Reports from IPCC(2007, 2014) indicate that climate change and its impact are expected to become more severe over the coming decades, with intense negative effects on key economic sectors. To address the various threats posed by climate change, it is necessary to have precise information that predict sudden changes that may occur and as a result causing costly damage. Development of statistical methods that can give an appropriate prediction of extreme hydrological, meteorological and climatological events is one of the major challenges experienced by hydrologists and climatologists in providing information on the occurrence and intensity of disasters associated to in order to prevent human and infrastructure damages (Coles et al., 2003). Extreme Value Theory (EVT) first developed by Fréchet (1927), followed by Fisher and Tippet (1928), Gnedenko (1943) and Gumbel (1958).is a powerful and robust statistical framework useful for the analysis of extreme events. Discussions on the methods to use for modelling extreme events have been well developed by Coles (2001), Coles et. al. (2003), Castillo et al. (2005), Rychlik and Ryden (2006), Gomes et al. (2007), Reiss and Thomas (2007), Beirlant et al. (2004). Diverse studies in hydrology and climatology aiming to model extreme rainfall and floods have contributed to save endangered natural resources. Extreme rainfall was modelled by Ryden (2005) at Havana (Cuba), Friederichs (2010) in Germany, Benestad (2010) in Norway, Nadarajah and Choi (2007) in South Korea, Deka et al. (2011) in India, Zalina et al. (2002) and Wan Zin (2009) in Malaysia, Lazoglou and Anagnostopoulou (2017) in Mediterranean, Alam et al. (2019) in Bangladesh. Recently, some studies on modelling extreme rainfall using EVT have come out in Africa: Ngailo et al. (2016) and Rutalebwa (2017) in Tanzania, Chikobvu and Chifurira (2016) in Zimbabwe, Boudrissa et. al. (2017) in northern Algeria.

Recently, Rwanda has experienced heavy rains especially in the northern and the western province. These heavy rains in combination with natural factors like topography and loss of ecosystems services resulting from deforestation, poor agricultural practices and environmental degradation have aggravated the impacts of floods on people, agriculture, soil erosion, rock falls, landslides and physical infrastructure (roads, bridges and schools) as well as loss of human and animal lives (Uwihirwe et al., 2020). This paper provides the first application of EVT to model extreme rainfall in Rwanda using the GEV and GP distributions, with the aim of establishing an adequate forecasting model that helps meteorologists, insurers and decision-makers to understand these exceptional events and thus prevent climate risks that may occur. The performances of four commonly used methods for estimating parameters of GEV and GP distributions namely the Maximum Likelihood Estimation (MLE) method, the L-Moments Estimation (LME) method, the Bayesian Estimation (BAYE) method and the Generalized Maximum Likelihood Estimation (GMLE) method are analyzed and compared to best fit the data based on goodness-of-fit tests.

2 Study area

Rwanda is a small mountainous, landlocked country in the Great Lakes region of Central/East Africa. Bordered by the Democratic Republic of Congo (DRC) to the west, Uganda to the north, Tanzania to the east and Burundi to the south, it is located between latitudes 01°04-02°51 south and longitudes 28°53-30°53 east. The topography of Rwanda is deeply modified by the relief at a varied altitude, 900 m in south-west, 1,500 m to 2,000 m in the south and the centre, 1.800 m to 3,000 m in the highlands of the north and the west and 3,000 to 4,507 m in the regions extending along the Congo-Nile Crest and the chain of volcanoes. Rwanda is divided into two major river basins: the Nile in the east and centre, and the Congo in the west. Both are shared with neighbouring countries. **Figure 1** represents the geographic location of Rwanda as well as its topography, and the location of synoptic stations in their respective Districts.

Annual rainfall varies across the country, with drier conditions in the eastern savannah plateau regions (<900 mm) and much wetter conditions over the central plateau and high elevated north-western mountains (>1200 mm). The country experiences two rainy seasons in a year separated by two dry seasons: the period from March to May corresponding to the long rainy season with a peak in April of 150 mm on average; the period from mid-September to mid-December corresponding to the short rainy season with a peak in November of 120 mm on average; the period from mid-December to end February corresponding to the short dry season.

3 Data and method

3.1 Data

Historical time series data sets of daily rainfall at twelve synoptic stations of Rwanda were provided by the Rwanda Meteorology Agency. **Table 1** gives the descriptive statistics of the data for each station.

3.2 Method

3.2.1 Extreme value distribution functions

In the present study, two commonly approaches are used to analyzing extremes rainfall in Rwanda. The first approach commonly called "Block Maxima (BM)" consists of reducing the data considerably and analyzing maxima of long blocks of data, here annual maxima. The second approach is to analyze partial duration series (PDS) of data exceeding a high threshold commonly called "Peak Over Threshold (POT)". Both approaches can be characterized in terms of a Poisson process, which allows for simultaneously fitting of parameters concerning both the frequency and intensity of extreme events. The Generalized Extreme Value (GEV) distribution function has theoretical justification for fitting respectively to BM of data (Coles, 2001) and the Generalized Pareto (GP) distribution function has similar justification for fitting PDS of data exceeding a high threshold as it approaches the endpoint of the variable (Pickands, 1975).

BM method and the Generalized Extreme Value (GEV) distribution

Assuming $X_1, X_2, \dots, X_n$ are independent and identically random variables that follow a non degenerate distribution function distribution function . Let $M_n = max\{X_1, X_2, \dots, X_n\}$. Suppose there exists constants $a_n > 0$ and b_n such that:

$$P\left\{\frac{M_n - b_n}{a_n} \le z\right\} \to G(z) \text{ as } n \to +\infty \text{ with } -\infty < z < \infty \quad (1)$$

By asymptotic considerations, a member of GEV distribution functions family can be used for fitting to block maxima of data M_n (Coles, 2001). The family of cumulative distribution function of the GEV distribution is given by (Jenkinson, 1955) and denoted as followed:

$$G(x) = exp\left[-\left\{1 + \xi\left(\frac{x-\mu}{\sigma}\right)\right\}_+^{-\frac{1}{\xi}}\right] \quad (2)$$

where $y_+ = max(y, 0)$, $\sigma > 0$ and $-\infty < \xi, \mu < \infty$. Here, σ and ξ are called respectively the location, scale and shape parameters or the tail indexes. Equation (2) envelops three types of distribution function depending on the sign of ξ: heavy-tailed Fréchet type of distribution functions when $\xi > 0$ including fat-tailed distributions exhibiting a large skewness such as Cauchy and Pareto; upper bounded Weibull type of distribution functions when $\xi < 0$ for short tailed distributions such as uniform and beta distributions; Gumbel type of distribution functions when $\xi = 0$ including exponential tailed distributions such as exponential, normal, gamma and log-normal distributions. The Gumbel type is obtained by taking in Equation (2), the limit as $\xi \to 0$ giving:

$$G(x) = exp\left\{-\left[1 + \left(\frac{x-\mu}{\sigma}\right)\right]\right\}, \ -\infty < x < \infty, \ \sigma > 0, -\infty < \mu < \infty \tag{3}$$

The probability density function obtained from the derivation of the GEV distribution function specified in Equation (2) is given by:

$$g(x) = \begin{cases} \frac{1}{\sigma}\left[1 + \xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-\frac{1+\xi}{\xi}} exp\left\{-\left[1 + \xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\}, & \xi \neq 0 \\ \frac{1}{\sigma}\exp\left\{-\left[\frac{x-\mu}{\sigma} + exp\left(-\frac{x-\mu}{\sigma}\right)\right]\right\}, & \xi = 0 \end{cases} \tag{4}$$

POT method and the General Pareto (GP)distribution

By asymptotic considerations, a member of GP distribution functions family can be used for fitting to PDS of data exceeding a high threshold

The GP distribution function is given by:

$$H(x) = 1 - \left[1 + \xi\left(\frac{x-u}{\sigma_u}\right)\right]_+^{-\frac{1}{\xi}} \tag{5}$$

where u is a high threshold, $x > u$, the scale parameter $\sigma_u > 0$ depends on the threshold u, and the shape parameter $-\infty < \xi < \infty$. Here also, the shape parameter determines three types of distribution functions as for the GEV distribution function: heavy-tailed Pareto type of distribution functions when $\xi > 0$; upper bounded Beta type of distribution functions when $\xi < 0$ for short tailed distributions; Exponential type of distribution functions by taking in Equation (5), the limit as $\xi \to 0$ giving:

$$H(x) = 1 - exp\left(-\frac{x-u}{\sigma_u}\right) \tag{6}$$

The probability density function obtained from the derivation of the GP distribution function specified in Equation (5) is given by:

$$h(x-u) = \frac{1}{\sigma_u}\left[1 + \xi\frac{x-u}{\sigma_u}\right]^{-1(1+{}^{1}/_{\xi})} \tag{7}$$

The GEV method is the simplest and more commonly employed method as the BM series is made of annual maxima (AM), where only the greatest event for each year is considered. However, it may be subject to some difficulties in its application (Coles, 2001). The disadvantage of this method is that useful information from other extreme events during certain years may not be taken into account. In addition, some annual maximum data might be not be really extremes. Sample size could also be an issue as a small sample would give information that may contain uncertainties. The POT method allows with a low threshold level to include more extreme events information and therefore increasing the sample size and adapts better to heavy-tailed distribution (Madsen et al. 1997). However, a too low threshold may lead to errors as violation of the assumption of independence can happen if the events are too close in time (Coles, 2001; Beguería, 2005).

3.2.2 Quantiles and Return Period

Quantiles of the GEV or GP distribution function are of particular interest because of their interpretation as return levels. If the probability of observing an extreme event of a given intensity is p then the mean return period T_p is such that $T_p = 1/p$. In other words, the mean return period or simply return period $T_p = 1/p$ is the number of years we expect to wait on average before we observe another extreme event of equal or greater intensity. The return level x_p represents the value expected to be equalled or exceeded on average once every T_p years, where $1 - 1/T_p$ is the specific probability associated with the quantile.

For the GEV distribution function, the return period T_p for any given block maxima of data (here a year) is given as the solution of Equation (2).

$$G(x_p) = P\{X \leq x_p\} = 1 - 1/T_p \Longrightarrow x_p = G^{-1}(1 - 1/T_p) \tag{8}$$

Letting $y_p = -1/ln(1 - T_p)$, then the associated return level x_p is:

$$\begin{cases} \mathrm{x_p} = \mu + \frac{\sigma}{\xi}\left(y_p^{\xi} - 1\right), & \xi \neq 0 \\ \quad = \mu + \sigma ln y_p, & \xi = 0 \end{cases} \tag{9}$$

For the GP model, suppose σ_u and ξ are respectively scale and shape parameters for a probability distribution function of exceedences over a suitable high threshold u. Then, the probability of the exceedence variable Xover u can be written as:

$$P\{X > x/X > u\} = \left[1 + \xi\left(\frac{x-u}{\sigma_u}\right)\right]^{-\frac{1}{\xi}} \tag{10}$$

provided that $x > u$ and $\xi \neq 0$. If $\zeta_u = P\{X > u\}$ is the probability of the occurrence of an exceedence of a high threshold u, then

$$P\{X > x\} = \zeta_u\left[1 + \xi\left(\frac{x-u}{\sigma_u}\right)\right]^{-\frac{1}{\xi}} \tag{11}$$

Then, the level x_m that is exceeded on average once every m observations (called the m-observation return level) is the solution of the following equation:

$$\zeta_u \left[1 + \xi \left(\frac{x-u}{\sigma_u}\right)\right]^{-\frac{1}{\xi}} = \frac{1}{m} \quad (12)$$

The solution of Equation (11) is:

$$x_m = u + \frac{\sigma_u}{\xi}\left[(m\zeta_u)^{\xi} - 1\right] \quad (13)$$

for m large enough to ensure that we have excedences ($x > u$). The same procedures applied to Equation (6) for $\xi = 0$ give:

$$x_m = u + \sigma_u ln(m\zeta_u) \quad (14)$$

If $N$is a return period, expressed in years, corresponding to the return level x_N and n_y is the number of observation per year, then $m = n_y N$, then we have:

$$\begin{cases} \mathrm{x_N} = u + \frac{\sigma_u}{\xi}\left[\left(n_y N\zeta_u\right)^{\xi} - 1\right], & \xi \neq 0 \\ \quad = u + \sigma_u ln\left(n_y N\zeta_u\right), & \xi = 0 \end{cases} \quad (15)$$

3.2.3 Parameter estimation

An overview of the developments on the estimation of parameters of extreme events was presented by Gomes and Guillou (2015). In this study four most common methods used in hydrology and climatology are used for the estimation of the parameters of the GEV and the GP distributions to fit the BM and the PDS of rainfall data: the Maximum Likelihood Estimation Method (MLE) (Prescott and Walden, 1980; Prescott and Walden, 1983; Hosking et al., 1985; Smith and Naylor, 1987; Coles and Dixon, 1999; Katz et al., 2002; Boudrissa et. al. (2017), the L-moments Estimation Method also called Probability-Weighted Moments Estimation Method (only without parameter covariates) (Greenwood et al., 1979; Landwehr et al., 1979; Hosking et al., 1985; Hosking, 1990; Hosking and Wallis, 1995; Hosking and Wallis, 1997), the Bayesian Estimation Method (Smith and Naylor, 1987; Lye et al., 1993, Coles and Powell,1996; Coles and Tawn, 1996; Coles and Tawn, 2005), and the Generalized Maximum Likelihood Estimation Method (Martins and Stedinger, 2000; Martins and Stedinger, 2001; El Adlouni et al. (2007).

Maximum Likelihood Estimation (MLE) Method

Assuming $x_1, x_2, \ldots, x_m$ represent the block maxima of some sample consisting of m blocks of length n, the likelihood function denoted by $L(\mu, \sigma, \xi, x_1, x_2, \ldots, x_m$is the joint density of the variables involved, that is:

$$L(\mu, \sigma, \xi, x_1, x_2, \dots, x_m) = \prod_{i=1}^{m} f(\mu, \sigma, \xi, x_i) \quad (16)$$

The log-likelihood for the GEV distribution function is:

$$lnL = \begin{cases} l(\mu, \sigma, \xi, x_1, x_2, \dots, x_m) = \\ -nln\sigma - (1 + 1/\xi) \sum_{i=1}^{m} ln\left[1 + \xi\left(\frac{x_i - \mu}{\sigma}\right)\right]_+ - \sum_{i=1}^{m}\left[1 + \xi\left(\frac{x_i - \mu}{\sigma}\right)\right]_+^{-\frac{1}{\xi}}, \ \xi \neq 0 \end{cases} \quad (17)$$

where $y_+ = \max\{y, 0\}$ to make sure that any value x_i with parameter combination such that $1 + \xi(x_i - \mu)/\sigma \leq 0$ is finite. For$\xi = 0$, the GEV distribution function reduces to the Gumbel distribution function and the log-likelihood is given by:

$$lnL = \begin{cases} l(\mu, \sigma, \xi, x_1, x_2, \dots, x_m) = \\ -nln\sigma - \sum_{i=1}^{m}\left(\frac{x_i - \mu}{\sigma}\right) - \sum_{i=1}^{m} exp\left[-\left(\frac{x_i - \mu}{\sigma}\right)\right] \end{cases} \quad (18)$$

The maximum likelihood estimator $\hat{\mu}, \hat{\sigma}, \hat{\xi}$ is then the solution of the following score equations:

$$\frac{\partial l(\mu,\sigma,\xi)}{\partial \mu} = 0,\ \frac{\partial l(\mu,\sigma,\xi)}{\partial \sigma} = 0,\ \frac{\partial l(\mu,\sigma,\xi)}{\partial \xi} = 0 \quad (19)$$

Now, for the POT method, let $y = (y_1, y_2, \dots, y_m)$ be m exceedances of a threshold u from the GP density function (7). For $\xi \neq 0$, The likelihood function is given by:

$$L(\sigma_u, \xi, y_1, y_2, \dots, y_m) = \prod_{i=1}^{m} h(\sigma_u, \xi, y_i) = \frac{1}{\sigma_u^m} \prod_{i=1}^{m} \left\{\left[1 + \xi \frac{y_i}{\sigma_u}\right]^{-1(1+{}^{1}/_{\xi})}\right\} \quad (20)$$

The log-likelihood for the GP distribution function is then:

$$lnL = \begin{cases} l(\sigma_u, \xi, y_1, y_2, \dots, y_m) = \\ -mln\sigma_u - \left(1 + {}^{1}/_{\xi}\right) \sum_{i=1}^{m}\left(1 + \xi \frac{y_i}{\sigma_u}\right) \end{cases} \quad (21)$$

The maximum likelihood estimation of σ_u and ξ for the GP distribution of the POT method can be identified by setting to zero the partial derivative of Equation (21).

However, the MLE is not always valid and regularity conditions do not always exist. The MLE is valid for $\xi > -1$, but the asymptotically normal properties of the MLE is only valid for $\xi > -1/2$. When $\xi < -1$, MLEs generally do not exist (Smith, 1985). No explicit solution for this system is found for the estimators of parameters for both GEV and GP. Numerical methods are needed to solve the obtained system of equations. Details on the computation of the MLE of the GEV parameters are described by Prescott and Walden (1980) and Prescott and Walden (1983). As for GP parameters details are given by Hosking and Wallis (1987).

L-Moments Estimation (LME) Method or Probability Weighted Moments (PWM)

The probability-weighted moments first introduced by Greenwood et al. (1979), of a random variable X with distribution function $F(X) = P(X \leq x)$ are the quantities:

$$M_{p,r,s} = E[X^p\{F(X)\}^r\{1 - F(X)\}^s] \tag{22}$$

where p, r and s are real numbers

The most convenient way of evaluating these moments is when the inverse distribution function $x(F)$ can be written in closed form, for then we may write:

$$M_{p.r.s} = \int_0^1 \{x(F)\}^p\, F^r(1 - F)^s dF \tag{23}$$

For the GEV distribution, it has been demonstrated by Hosking et al.(1985) that with $p = 1, r = 0{,}1{,}2$ and $s = 0$, the probability-weighted moments $E[X\{F(X)\}^r]$ can be written as:

$$\beta_r = \frac{1}{r+1}\left\{\mu + \frac{\sigma}{\xi}\left[1 - (r+1)^{-\xi}\Gamma(1+\xi)\right]\right\}, \quad \xi > -1, \xi \neq 0 \tag{24}$$

From Equation (23), we have:

$$\begin{cases} \beta_0 = \mu + \frac{\sigma}{\xi}[1 - \Gamma(1+\xi)] \\ 2\beta_1 - \beta_0 = \frac{\sigma}{\mu}\Gamma(1+\xi)\left(1 - 2^{-\xi}\right) \\ (3\beta_2 - \beta_0)/(2\beta_1 - \beta_0) = \left(1 - 3^{-\xi}\right)/\left(1 - 2^{-\xi}\right) \end{cases} \tag{25}$$

The exact solution requires iterative methods. The PWM estimators $\hat{\mu}, \hat{\sigma}, \hat{\xi}$ of the parameters are the solutions of Equation (25) for μ, σ, ξ when the probability-weighted moments β_r are replaced by their estimators$\hat{\beta}_r$.

The following unbiased estimator was proposed by Landwehr et al. (1979):

$$\hat{\beta}_r = \frac{1}{n}\sum_{j=1}^{n}\prod_{l=1}^{r}\left(\frac{j-l}{n-l}\right)X_{j,n} \tag{26}$$

Where $X_{1,n}, X_{2,n}, \dots, X_{n,n}$ represents the ordered GEV distributed sample. A detailed study on the properties of the estimators $(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2)$ and $(\hat{\mu}, \hat{\sigma}, \hat{\xi})$ was developed by Hosking et al. (1985) who showed the asymptotic normality of these estimators for $-0.5 < \xi < 0.5$.

For the GP distribution, the L-Moment estimators (Hosking and Wallis,1987; Madsen et al., 1997; Martins and Stedinget, 2001) are:

$$\begin{cases} \hat{\sigma} = \hat{\lambda}_1\left(\frac{1}{\hat{\tau}_2} - 1\right) \\ \hat{\xi} = \frac{1}{\hat{\tau}_2} - 2 \end{cases} \tag{27}$$

where the L-Moment estimators $\hat{\lambda}_1$, $\hat{\lambda}_2$, $\hat{\lambda}_3$, and $\hat{\tau}_2 = \hat{\lambda}_2/\hat{\lambda}_1$are obtained by using an estimator for the first two probability weighted moments β_1 and β_2 of:

$$\beta_r = \frac{\sigma}{(r+1)(r+1+\xi)} \tag{28}$$

The same unbiased estimator given in (26) is used (Hosking and Wallis,1987; Madsen et al., 1997 and Martins and Stedinget, 2001) to compute $\lambda_1 = \beta_0$, $\lambda_2 = 2\beta_1 - \beta_0$ and $\lambda_3 = 6\beta_2 - 6\beta_1 + \beta_0$.

Generalized Maximum Likelihood (GMLE) Method

The range of the GEV is for $\xi < 0$, $\mu + \sigma/\xi \leq x \leq \mu$ and for $\xi > 0$, $\mu \leq x \leq \mu + \sigma/\xi$ and that of GP is for $\xi \leq 0$, $u \leq x < \infty$ and for $\xi > 0$, $u < x \leq u + \sigma/\xi$, and otherwise unbounded) distributions depends on their parameters, so that the regularity conditions for maximum likelihood estimation are not necessarily satisfied. The desirable asymptotic properties of efficiency and normality of MLEs do hold if $\xi < 0.5$ (Smith, 1985; Cheng and Iles, 1987; Davison and Smith, 1990). Martins and Stedinger (2000) have demonstrated that maximum likelihood estimators can yield absurd values of the GEV shape parameter ξ in small samples. For POT analysis, a GP distribution can experience similar problems (Martins and Stedinger, 2001).

The Generalized Maximum Likelihood consists of considering the true shape parameter ξ of the GEV or GP distribution as a random variable whose range is ξ_L, ξ_U with prior density $\pi(\xi)$. The following beta distribution was used by Martins and Stedinger (2000, 2001) as prior referred to here as the geophysical prior:

$$\pi(\xi) = (0.5 + \xi)^{p-1}(0.5 - \xi)^{q-1}/B(p,q), \quad \xi \epsilon [-0.5, +0.5] \tag{29}$$

with $p = 6$ and $q = 9$ where $B(p,q) = \Gamma(p)\Gamma(q)/\Gamma(p+q)$, $E[\xi] = -0.10$ and $Var[\xi] = (0.122)^2$.

It is relatively flat for ξ values of the range of interest of the probability density function $f(\mu, \sigma, \xi, x_1, x_2, \ldots, x_n)$. Once the prior $\pi(\xi)$ is chosen, the joint density (or the generalized-likelihood function) is computed as $GL(\mu, \sigma, \xi/x_i) = L(\mu, \sigma, \xi/x_i)\pi(\xi)$ which shows the relationship between the generalized-likelihood GL function and the likelihood function L. Thus $ln[GL(\mu, \sigma, \xi/x_i)]$ equals the expression in Equation (18) for GEV in BM analysis and (21) for GP in POT analysis plus $ln[\pi(\xi)]$. The generalized maximum likelihood estimator of μ, σ and ξ can be found by maximizing the generalized log-likelihood function and the generalized maximum estimators will have the desired asymptotic properties if both the likelihood and prior satisfy a few regularity condition.

Bayesian Estimation (BAYE)Method

Suppose the data $x = (x_1, x_2, \ldots, x_n)$ are realizations of a random variable with a density from the parametric family $F = \{f(x; \theta, \theta \epsilon \Theta)\}$ where$\theta = (\mu, \sigma, \xi)$. Assuming the parameter θ as random variable in the domainΘ, the Bayesian Estimation Method of θ is based on the Bayesian inference with its core Bayes' theorem expressed as follows:

$$\begin{cases} \pi(\theta/x) = \frac{\pi(\theta)\pi(x/\theta)}{\pi(x)} \\ \qquad\quad = \frac{\pi(\theta)\pi(x/\theta)}{\int_\Theta \pi(\theta)\pi(x/\theta)} \end{cases} \tag{30}$$

where $\pi(\theta)$ is the prior (marginal) distribution of the parameter set θ (it does not take into account any information contained in the observed data), $\pi(x/\theta)$ is the sampling distribution given the parameters θ and $\pi(x)$ is the marginal prior distribution or the prior predictive distribution of the data x, which indicates what x should look like, given the model, before it has been observed, and $\pi(\theta/x)$, is the joint posterior distribution of the parameters θ. The latest expresses the updated beliefs about θ after taking both prior and data into account. The prior predictive distribution $\int_\Theta \pi(\theta)\,\pi(x/\theta)d\theta$ normalizes the joint posterior distribution $\pi(\theta/x)$ by a factor equal to the probability density function $\pi(\theta)$ of the prior beliefs about the parameterθ. If a suitable prior distribution $\pi(\theta)$ can be specified in Equation (30), Bayesian procedures can conveniently be used to determine the parameterθ. Numerical computational methods are needed for the computation of the integral in the denominator of Equation (30). Markov chain Monte Carlo (MCMC) methods represent a class of such algorithms and can be found in literature (Metropolis et al., 1953; Besag et al., 1995, Hastings, 1970; Tierney, 1994; Coles and Powell, 1996; Coles and Tawn, 1996; Gamerman,1997; Brooks[a], 1998; Brooks[b], 1998; Brooks and Roberts, 1998; Gelman et al., 2004; Besag, 2004; Reis and Stedinger, 2005; Robert and Casella, 2009). The most applied MCMC algorithms are the Gibbs sampler which was used by Geman and Geman (1984) and Gelfand and Smith (1990) for models with the Gibbs distribution, and the Metropolis-Hastings algorithm introduced first by Metropolis et al. (1953), later by Hastings (1970) and developed by Tierney (1994) and Gelman et al. (2004).

The likelihood function for θ is given by:

$$\begin{cases} L(\theta,x) = L(\mu,\sigma,\xi) = f(x,\theta) = \prod_{i=1}^{n} f(x_i;\theta) \\ \qquad\quad = \prod_{i=1}^{n} \frac{1}{\sigma}\xi\left(\frac{x_i-\mu}{\sigma}\right)^{-\frac{1+\xi}{\xi}} exp\left\{-\left[1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\} \\ \qquad\quad = \frac{1}{\sigma^n} exp\left\{-\sum_{i=1}^{n}\left[1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\}\prod_{i=1}^{n}\left[1+\xi\left(\frac{x_i-\xi}{\sigma}\right)\right]^{-\frac{1+\xi}{\xi}} \end{cases} \quad (31)$$

The sampling distribution $\pi(x/\theta)$ in Equation (30) can be replaced by $L(\theta,x)$ to give:

$$\pi(\theta/x) = \frac{\pi(\theta)L(\theta,x)}{\int_{\Theta}\pi(\theta)L(\theta,x)d\theta} = \frac{\pi(\theta)L(\theta,x)}{f(x)} \quad (32)$$

Bayesian analysis is applied to the annual maximum daily rainfall data for GEV and to Partial Duration Series for GP. For specification of the prior, the parameterization $\phi = log\sigma$ is easy to work with, σ being positive. The prior density is chosen to be:

$$\pi(\theta) = \pi(\mu,\phi,\xi) = \pi_{\mu}(\mu)\pi_{\phi}(\phi)\pi_{\xi}(\xi) \quad (34)$$

where the marginal priors $\pi_{\mu}(\mu)$, $\pi_{\phi}(\phi)$, $\pi_{\xi}(\xi)$ are normally distributed with mean zero and variances v_{μ}, v_{ϕ}, v_{ξ}, respectively. The variances are chosen to be large enough so that the distributions are almost flat, corresponding to prior ignorance.

The density of interest is the posterior of the form:

$$\pi(\mu,\phi,\xi/x) \propto \pi(\mu,\phi,\xi)L(\mu,\phi,\xi/x) = \pi_{\mu}(\mu)\pi_{\phi}(\phi)\pi_{\xi}(\xi)L(\mu,\phi,\xi/x) \quad (35)$$

The full conditionals of consideration are of the form:

$$\begin{cases} \pi(\mu/\phi,\xi) = \pi_{\mu}(\mu)L(\mu,\phi,\xi/x) \\ \pi(\phi/\mu,\xi) = \pi_{\phi}(\phi)L(\mu,\phi,\xi/x) \\ \pi(\xi/\mu,\phi) = \pi_{\xi}(\xi)L(\mu,\phi,\xi/x) \end{cases} \quad (36)$$

In this study we choose:

$$\begin{cases} \pi_{\mu}(\mu) \sim N(0{,}30{,}000) \\ \pi_{\phi}(\phi) \sim N(0{,}30{,}000) \\ \pi_{\xi}(\xi) \sim N(0{,}10{,}000) \end{cases} \quad (37)$$

Since the joint posterior density is a multivariate density, a Gibbs sampler is used, and the random-walk Metropolis algorithm is used to simulate from each of the full conditionals.

3.2.4 Stationaroty test for the AMS and PDS

Before selecting parameters, stationarity tests are performed to check if the statistical properties of time series of data do not change over time. Different types of stationarity tests are found in literature (Zhijie, 2001). The following two most popular are used in this study to check the stationarity of the time series of extreme rainfall data: the Augmented Dickey Fuller (ADF)

Unit Root test (Dickey and Fuller, 1979; Said and Dickey, 1984) and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test for level or trend stationarity (Kwiatkowski et al., 1992). The null hypothesis of the ADF test, H_o, is that variable is not stationary/has a unit root, while the alterative hypothesis, H_a, is that the variable is stationary/has no unit root. The statistic Dickey-Fuller used in the test is a negative number. The more negative and lower than the critical value level it is, the stronger the rejection of the null hypothesis that there is a unit root at some level of confidence (a low p-value will indicate that the variable is stationary). The null hypothesis of the KPSS test, H_o, is that the variable has trend stationary, while the alterative hypothesis, Ha, is that the variable has not trend stationary/has a unit root (a low p-value will indicate a signal that is not trend stationary). That is, if the p-value is less than 0.05, the KPSS statistic will be greater than the 5% critical value. The KPSS statistic used in the test is a positive number. The greater than the critical value level it is, the stronger the rejection of the null hypothesis that there is trend stationary at some level of confidence

3.2.5 Threshold selection for the POT analysis

Before fitting the GP distribution function to the data, it is necessary to determine a sufficiently high threshold for which the theoretical justification is applied, in other words for the errors to be minimized. A high threshold value reduces the bias as this satisfies the convergence towards the extreme value theory but however increases the variance for the estimators of the parameters of the GP distribution function, as there will be fewer data from which to estimate parameters. A low threshold value results in the opposite i.e. a high bias but a low variance of the estimators, since there is more data with which to estimate the parameters (Coles, 2001). A compromise is therefore necessary in the selection of the threshold.

Different methods for the determination of a suitable threshold of the GP model to fit POT data series are provided in literature. The most commonly used are graphical diagnostic plots and the rule of thumb. Graphical diagnostic plots have been extensively discussed by Davison and Smith, (1990), Kratz and Resnick (1996), Behrens et al. (2004), Beguería (2005), Scarrott and MacDonald (2012), Wadsworth and Tawn (2012) and Gilleland and Katz (2016) to just mention them. They include the Mean Residual (Mean Excess) Life (MRL) Plot, the Parameter Stability Plot (PSP), Multiple-Threshold plot, the Dispersion Index (DI) plot, the Zipf plot, the Hill plot, etc. But interpreting these plots is subjective and rather challenging (Coles, 2001). Rules of thumb include the simple fixed quantile rule consisting in selecting a fixed percentile of data

(90%, 95%, 99%), the square root rule $k = \sqrt{n}$ has and the empirically driven rule $k = n^{2/3}\log(\log(n))$. The simple fixed quantile rule has been applied for example by by Papalexiou et al. (2012) comparing different models for extreme rainfall with daily data from around the world, Anagnostopoulou and Tolika(2012) for extreme rainfall in Mediterranean and Ngailo et al. (2016) studying extreme rainfall in Tanzania. The square root has been used by Ferreira et al. (2003). The empirically driven rule first proposed by Loretan and Philips (1994) was used by Omran and McKenzie (1999) and Ho and Wan (2002). Those two rules consist of defining the threshold as the k^{ith} upper order statistic X_{n-k+1} from the ordered sequence $X_1,..., X_n$ of the sample.

In this study the Parameter Stability Plot and the Mean Residual Life Plot are used for the validation of the choice of the threshold obtained by one of the three rules methods of Rule of Thumb. The Parameter Stability Plot helps to find a suitable threshold, chosen in the range from the 80% quantile to the 99% quantile, corresponding to appropriate estimates of the shape and modified scale parameters of a GPD. The parameter estimates should be constant (stable) above the selected threshold in the plot and without too high variance (Scarrot and Mac-Donald, 2012). With the Mean Residual Life Plot, the assumption is that where the plot starts showing an approximately linear behaviour, a suitable threshold can be estimated (Davison and Smith, 1990). The increasing variance for high thresholds leads to large uncertainties (confidence intervals) which may cause the plot to lose the linear behaviour.

3.2.6 **Goodness-of-Fit Test**

Goodness-of-fit test statistics are used to test if the distribution of experimental data follows a theoretical distribution. Three procedures are commonly used: i) the formal statistical tests, ii) graphical methods and iii) accuracy measure methods. Several methods exist in statistical testing procedures to compare the fit of observed cumulative distribution function with the expected theoretical cumulative distribution function for given data set and provide a measure of the discrepancy between them. Kolmogorov-Smirnov (K-S) test, Anderson-Darling (A-D) test, Shapiro Wilk (SW) test, Cramer Von Mises (CVM) test and Pearson's Chi-Square test are largely used in many statistical applications (Razali and Wah, 2011; Rahman et al., 2013; Ul Hassan, 2019). Graphical tests are used to visually compare the observed and estimated values. They include Quantile-Quantile (Q-Q) plots, Probability-Probability (P-P), Cumulative Distribution Function vs Empirical Distribution Function (CDF-ECDF) plots, Return Level (RL) plots (Coles,

2001) and L-Moment Ratio Diagram (Murray et. a, 2001). Accuracy measure methods require the computation of Skewness and Kurtosis indices, Root Mean Square Error (RMSE), Relative Root Mean Square Error (RRMSE), Mean Absolute Error (MAE), R-Square (R^2), Akaike information criterion (AIC) and Bayesian information criterion (BIC) (Chen et al., 2017).

In this study, Q-Q plots, CDF-ECDF plots and L-Moment Ratio Diagram are used to diagnose the fits of the GEV and GP distributions to respectively the BM and the PDS data. The K-S and CVM tests are applied to the data to assess the null hypothesis H_0 that the BM and the PDS of rainfall data can be described respectively by the GEV and the GP probability distributions. The alternative hypothesis H_a is that the BM and the PDS of rainfall data cannot be described by the given probability distributions. For the K-S test and the CVM test, the null hypothesis H_0 is rejected if the calculated test statistic exceeds the critical value at the 0.05 significance level. Performance of each parameterization method for each distribution at each site is assessed based on the outputs of the RMSE, MAE and R^2 and calculated K-S test statistics. The GEV or GP distribution with the lowest RMSE, and lowest MAE or the highest R^2 has the highest rank. In K-S test, the GEV or the GP distribution with the lowest statistic test (i.e. the highest p-value) has the highest rank.

4 Results and discussions

Daily rainfall data sets from Rwanda Meteorology Agency have been processed and some outliers due to data coding errors have been removed without affecting subsequent statistical analysis. **Figure 2** presents the time series of daily rainfall at the twelve synoptic stations of Rwanda.

4.1 Modelling with GEV distribution

Summary statistics of the annual maximum daily rainfall data (mm) for the twelve synoptic stations of Rwanda are presented in **Table 2.** Time series of annual maxima daily rainfall at the twelve synoptic stations of Rwanda. are respectively shown in **Figure 3**. No significant trend is observed.

GEV Parameters

The BM constituted of annual maxima daily rainfall has been fitted to the GEV distribution for each station using the four parameterization methods: MLE, BAYE. LME, GMLE. **Table 3** gives the estimated parameters of GEV distribution with their 95% confidence intervals for each method at each station. Results show that regardless of the choice of parameterization method, the confidence interval of the GEV shape parameter ξ contains zero value for the stations of Rubona, Gikongoro, Byimana, Rubengera, Nyagatare, Kayonza, Kibungo and Kanombe (except for GEV-MLE and GEV-BAYE). Therefore, the Gumbel distribution could be a more appropriate model among the GEV family of distribution to fit the annual maximum daily rainfall data set for those stations. For the stations of Byumba (except for GEV-BAYE) and Kamembe (except for GEV-BAYE), the data can be modelled using a Weibull class of distribution because ξ is significantly negative (upper bounded tail) as the confidence interval does not contain zero value. As for the stations of Kanombe (except for GEV-LME and GEV-GMLE), Gisenyi and Ruhengeri, the data can be modelled using heavy-tailed Pareto distribution type because the estimate of ξ is positive though the confidence interval does contain zero value. A further graphical support is provided by the profile likelihood. **Figure 4** shows the profile likelihood of the GEV shape parameter ξ from which a 95% confidence interval for ξ is obtained as approximately 0.014(-0.280, 0.308), 0.321(-0.031, 0.673), -0.109(-0.353, 0.135), -0.166(-0.296, -0.035) for respectively the stations of Kanombe, Gisenyi, Nyagatare and Kamembe, which is almost the same as the calculated 95% confidence interval.

4.2 Modelling with GP distribution

Stationarity tests

The ADF and KPSS tests were carried out for checking the stationarity in the POT rainfall series over the twelve stations of the study. **Table 4** presents the ADF stationary test applied to the POT rainfall data series for the period of study of each station. Statistics of the ADF tests are less than the critical values at 1%, 5% and 10% levels with significant *p*-values. The null hypothesis of no stationarity at 1%, 5% and 10% levels of significance is therefore to be rejected and the alternative hypothesis that the rainfall data are stationary and have no trend to be accepted. Results of the KPSS test shown in **Table 5** applied to same data as above indicate a high p-value (>0.1) at significance level $\alpha=0.05$ leading also to reject the null hypothesis that the data has trend stationary and accept the alterative hypothesis that the data has not trend stationary .

Threshold selection

Computed thresholds obtained using the three rules of thumb presented in section 3.2.5 are shown in **Table 6.** The thresholds obtained by the method using the rule $k = n^{2/3}\log(\log(n))$ are relatively low compared to those obtained with the rule using 99% quantile, but high compared to those obtained with rules using 90% and 95% quantiles. On the contrary, the thresholds obtained with the method using the rule $k = \sqrt{n}$, though they are also high compared to the those obtained with rules using 90% and 95% quantiles, they are smaller than but closer to those obtained by the rule using 99% quantile. **Figure 5** and **Figure 6** present respectively the Mean Residual plots and the Parameter Stability plots for the four stations (Kanombe, Gisenyi, Kamembe and Nyagatare) taken as example. A certain linearity is observed in the Mean Residual Life plots around the thresholds values corresponding to the 99% quantile of each station. The shape parameter and the modified scale parameter appear to be constant in the vicinity of the 99% quantile in the Parameter Stability plots. These two observations confirm the choice of the 99% quantile as threshold for each station.

GP Parameters

The estimates of parameters with their 95% confidence intervals of the fitted GP distribution to PDS of daily rainfall at the twelve synoptic stations of Rwanda are displayed in **Table 7**

4.3 Goodness-of-Fit Test for the GEV and GP distributions

Results from the K-S and CVM goodness-of-fit tests applied to the BM series and the PDS series of daily rainfall and the GEV and GP distributions at each station with the four methods of parameterization are presented in **Tables 8&9** and **Tables 10&11**. It is shown that the K-S statistics and the CVM statistics are less than their 5% critical value for both GEV and GP distributions, suggesting a non rejection of the null hypothesis. This allows concluding that the extreme rainfall at the twelve stations considered in this study follow GEV and GP distributions for respectively the AMS and the PDS of daily rainfall at the twelve stations. Underlined values of the statistics and p-values correspond respectively to lowest KS or CVM statistics and highest p-values, therefore to parameterization methods considered to have the best fit. Italic type values correspond respectively to highest KS or CVM statistics and lowest p-values, therefore to parameterization methods considered as having less good fit.

The L-Moment ratio diagram represented in **Figure 7**shows the ratio between the L-kurtosis and L-skewness used to determine the goodness of fit for the chosen distributions at the twelve stations of the study. It is observed that though the two distributions are acceptable, the PDS of daily rainfall are closer to the GP distribution than that corresponding to AMS of daily rainfall of GEV distribution. It can be due to PD series having more data that AMS of daily rainfall.

Figure 8 and **Figure 9** represent respectively Q-Q plots and CDF-ECDF plots for GEV and GP distributions wirh the four parameterization methods for the twelve stations of this study. A straight line of regression is observed in each Q-Q plot for each station between empirical and modelled quantiles for AMS of daily rainfall and PDS of daily rainfall independently to the parameterization method used. This indicates that the AMS of daily rainfall and the PDS of daily rainfall follow well respectively a GEV distribution and a GP distribution. However, it is observed that the GP distribution approximates the PDS daily rainfall data with good accuracy, while the GEV distribution demonstrates poor approximation for the high tail. In extreme value analysis, quantile estimation of high tail is extremely important, since the major problem consists in estimating the return period for risk prevention. As for CDF-ECDF plots, theoretical cumulative distributions of both GEV and GP for the four methods of parameterization follow well the paces of the corresponding empirical cumulative distributions although the cumulative distributions of GEV present some discrepancies.

The results of RMSE, MAE and R-Square between empirical quantiles and estimated quantiles using the four methods of parameterization are reported in **Table 12** and **Table 13** respectively for GEV and GP distributions for the twelve stations of this study. Underlined values correspond to lowest values, and italic values correspond to highest values of RMSE and MAE, the opposite is for R-Square. Lowest values of RMSE or MAE correspond to parameterization methods considered to have the best fit. Highest values of R-Square indicate best regression and therefore best fit. It is observed that errors for fitting GP distribution are of the order of ten lower than those of GEV distribution. This suggests again that the GP distribution is more suitable than the GEV distribution to represent extreme rainfall in Rwanda. Another observation that should not be overlooked is that a few stations indicate that a method presenting the best fit with one type of statistical test or one type of accuracy measure is not necessarily so for the other type. The two statistical tests (KS and CVM) and the three accuracy measures (RMSE, MAE, R^2) were used in the present study as metrics and were assigned the same weight for the selection of the suitable method to be used. The method having the highest cumulative score in total was considered as the best method. **Tables 14 & 15** present the selected parameterization methods respectively for GEV and GP distribution fits of respectively AMS and PDS of daily rainfall over the twelve synoptic stations of the study. The most predominant best methods of parameterization for the majority of stations are MLE (6 stations out of 12) and LME (5 stations out of 12) for GEV whereas it is BAYE (9 stationsout of 12) for GP.

4.4 Return Levels and Return Periods

Table 16 shows the return levels estimates with their 95% confidence intervals corresponding to different return periods for GEV distribution fit of AMS of daily rainfall data with the selected parameterization method for each station, **Table 17** shows the return levels estimates with their 95% confidence intervals corresponding to different return periods for GP distribution fit of PDS of daily rainfall data with the selected parameterization method for each station. The estimated return levels and confidence intervals increase with the increase of the return periods. For stations for which parameters of the fitted GP distribution was obtained by the BAYE method, the upper bound of the confidence interval appears to deviate much more from the estimated return level for high return periods as compared with the lower bound of the confidence interval. **Figure 10** presents the return levels estimates with their 95% confidence intervals corresponding to different return periods for GEV and GP distribution fits of AMS and PDS respectively of daily rainfall data with the selected parameterization methods for each station.

5. Conclusion

In this study, EVT is applied to simulate extreme rainfall in Rwanda. AMS and PDS of daily rainfall are fitted respectively to GEV distribution and GP distribution using four parameterization methods for twelve synoptic stations of Rwanda. Goodness-of-fit statistical tests and accuracy measures are used for assessing the performance of the parameterization methods. The most predominant best methods of parameterization for the majority of stations are MLE and LME for GEV whereas it is BAYE for GP. However, the GP distribution is found to perform better compared to GEV distribution regardless of the parameterization method used. The selected parameterization methods can be considered as suitable for the estimation of the parameters for both GEV and GP distributions of respectively AMS and PDS daily rainfall for sites considered in Rwanda Return levels of extreme rainfalls with their 95% confidence intervals were computed with the selected parameterization methods for return periods of 10, 20, 50, 75, 100, 150 and 200 years. It was found that the GP distribution presents slightly higher return levels than GEV distribution for all stations. Although the BAYE method is found to be better appropriate for GP distribution in most of the stations in this study, it should be, however, noted that we used a non-informative priors to obtain the parameters of the distributions. Taking into consideration expert priors when available, would improve the model. This will be investigated in subsequent studies as well as a thoroughly analysis of spatial variability of frequency of extreme rainfall incidences in Rwanda. Another area of further research is to analyze extreme rainfalls in Rwanda using other models.

This paper provides the first application of EVT to rainfall in Rwanda. It makes a contribution to understanding extreme rainfalls and their occurrence probability or their return periods in Rwanda. This could contribute in the formulation of strategic measures against risks of loss of life, and the destruction of property and infrastructure, caused by such natural hazard.

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Statements snd Declarations

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Competing interests

The author declares that this work has no conflict of interest.

Author Contributions

The author contributed to the study conception and design, material preparation, data collection, application of statistical, mathematical, computational techniques to analyse and synthesize study. The writing 0f the initial draft was done by the author as well the submission of the final manuscript.

Figures

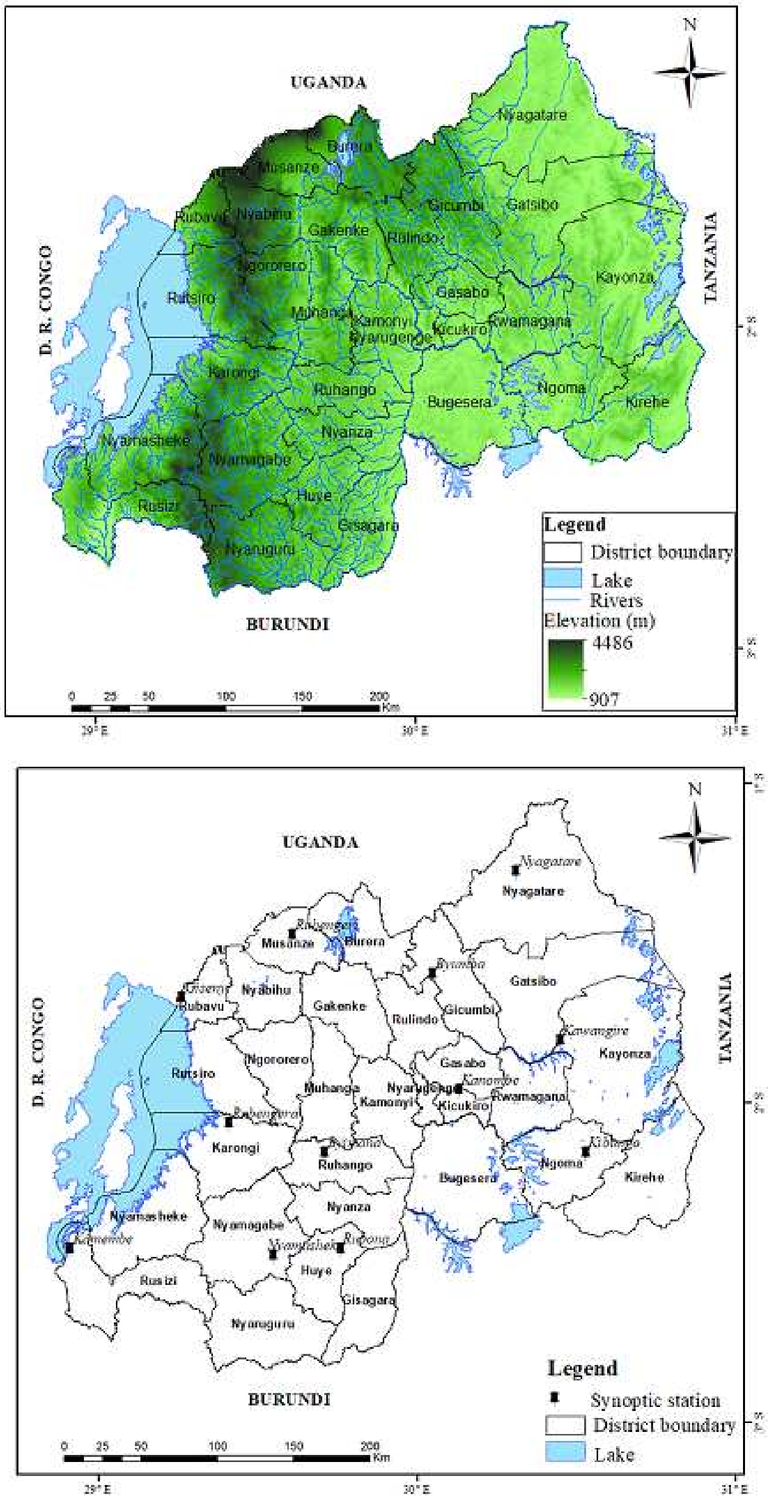

Figure 1

Geographic location and topography of Rwanda (left) and location of synoptic stations in their respective Districts (right)

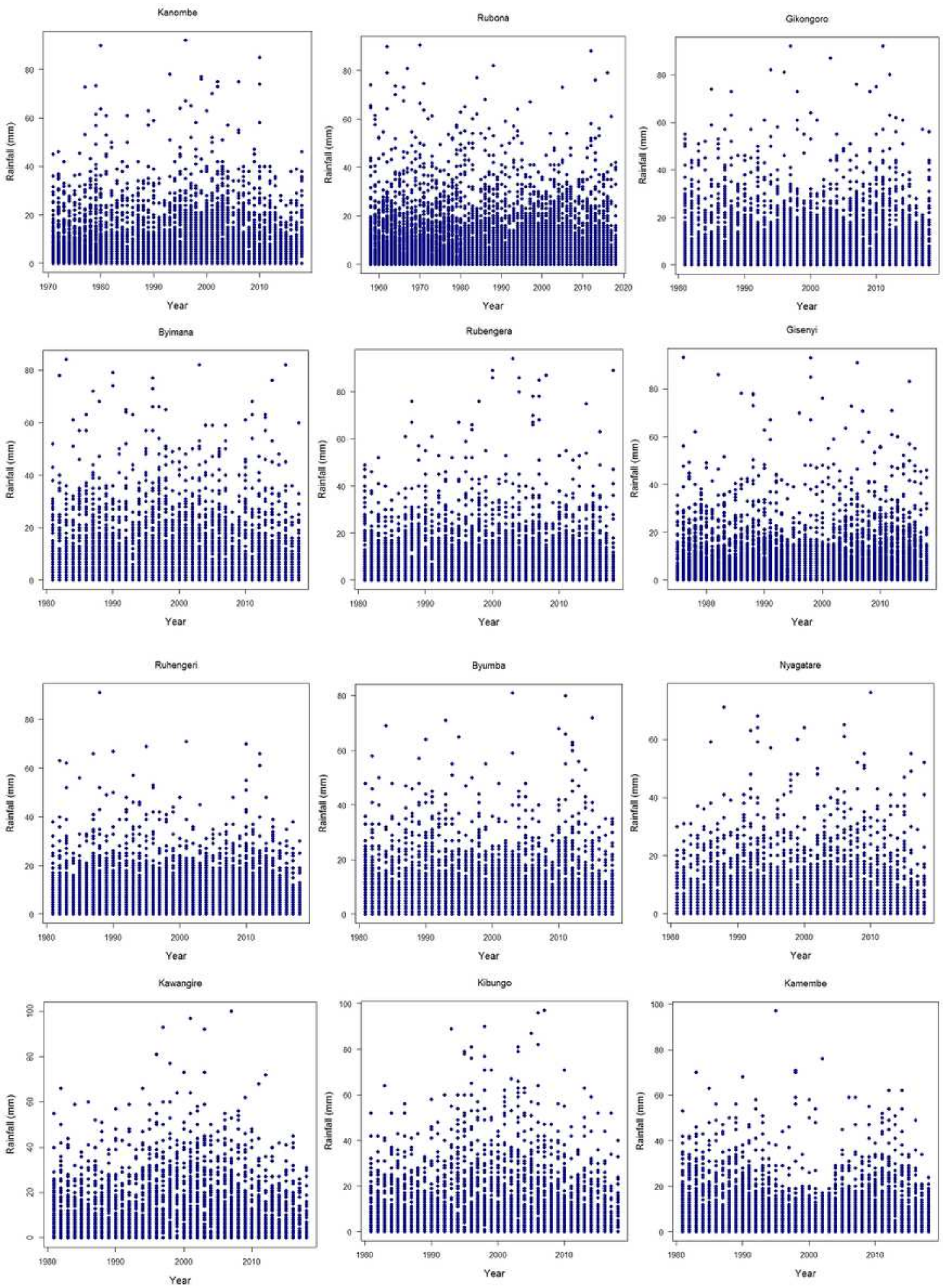

Figure 2

Time series of daily rainfall at the twelve synoptic of Rwanda

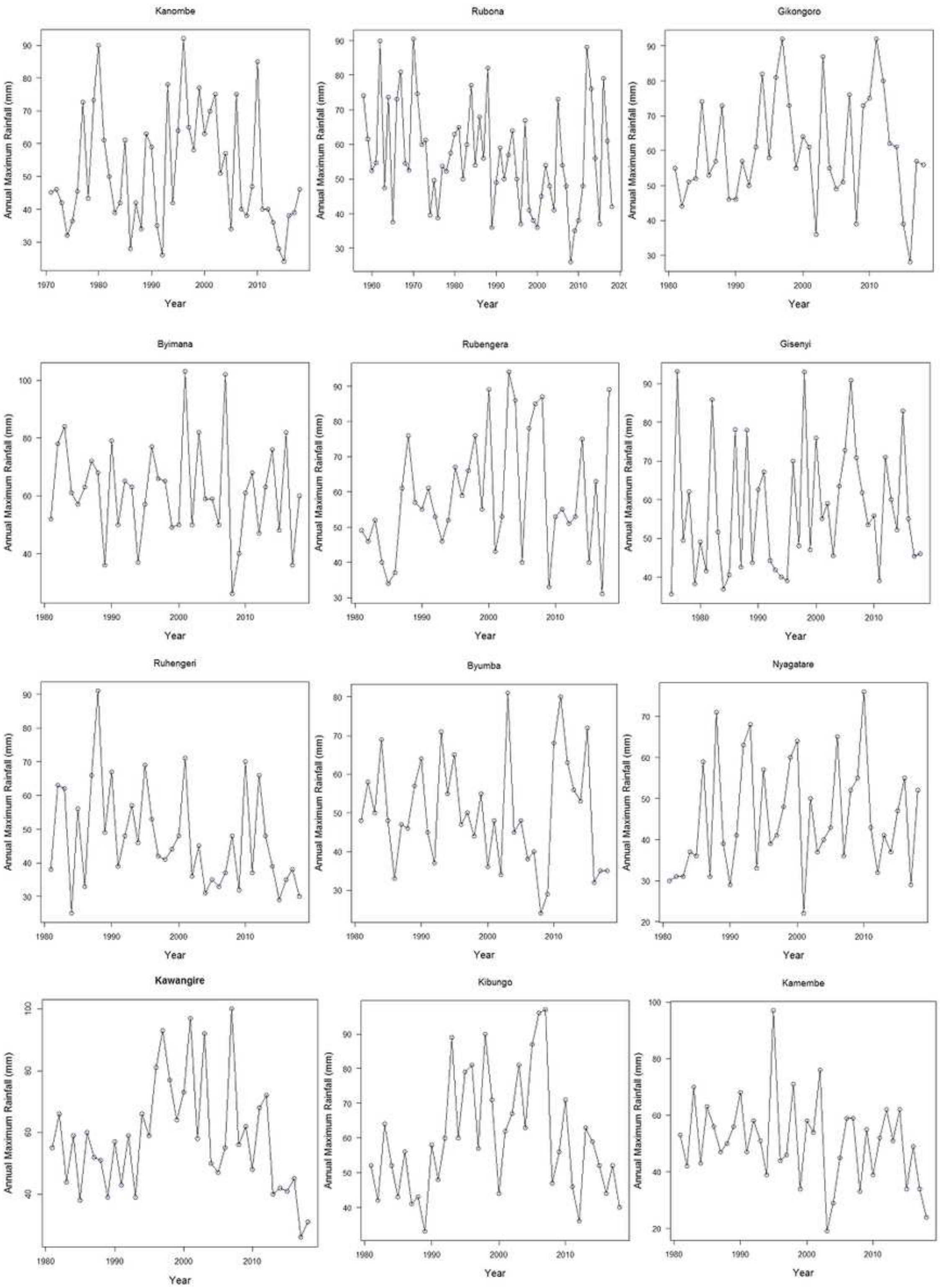

Figure 3

Time series of annual maxima daily rainfall at the twelve synoptic stations of Rwanda.

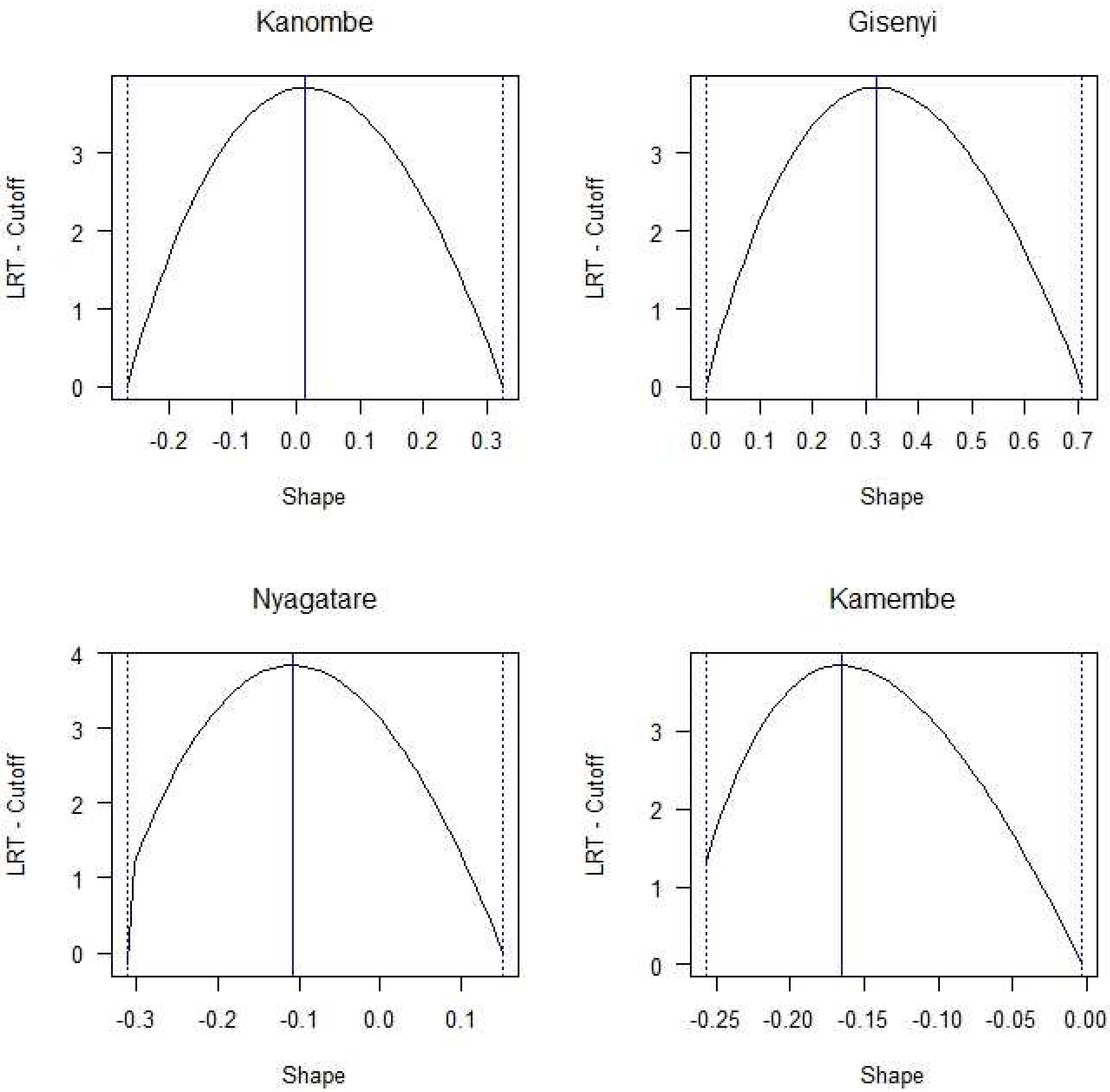

Figure 4

Profile likelihood (likelihood ratio test (LRT)-Cutoff) of the GEV shape parameter for annual maximum daily rainfall for the stations of Kanombe, Gisenyi, Nyagatare and Kamembe for their respective period of study.

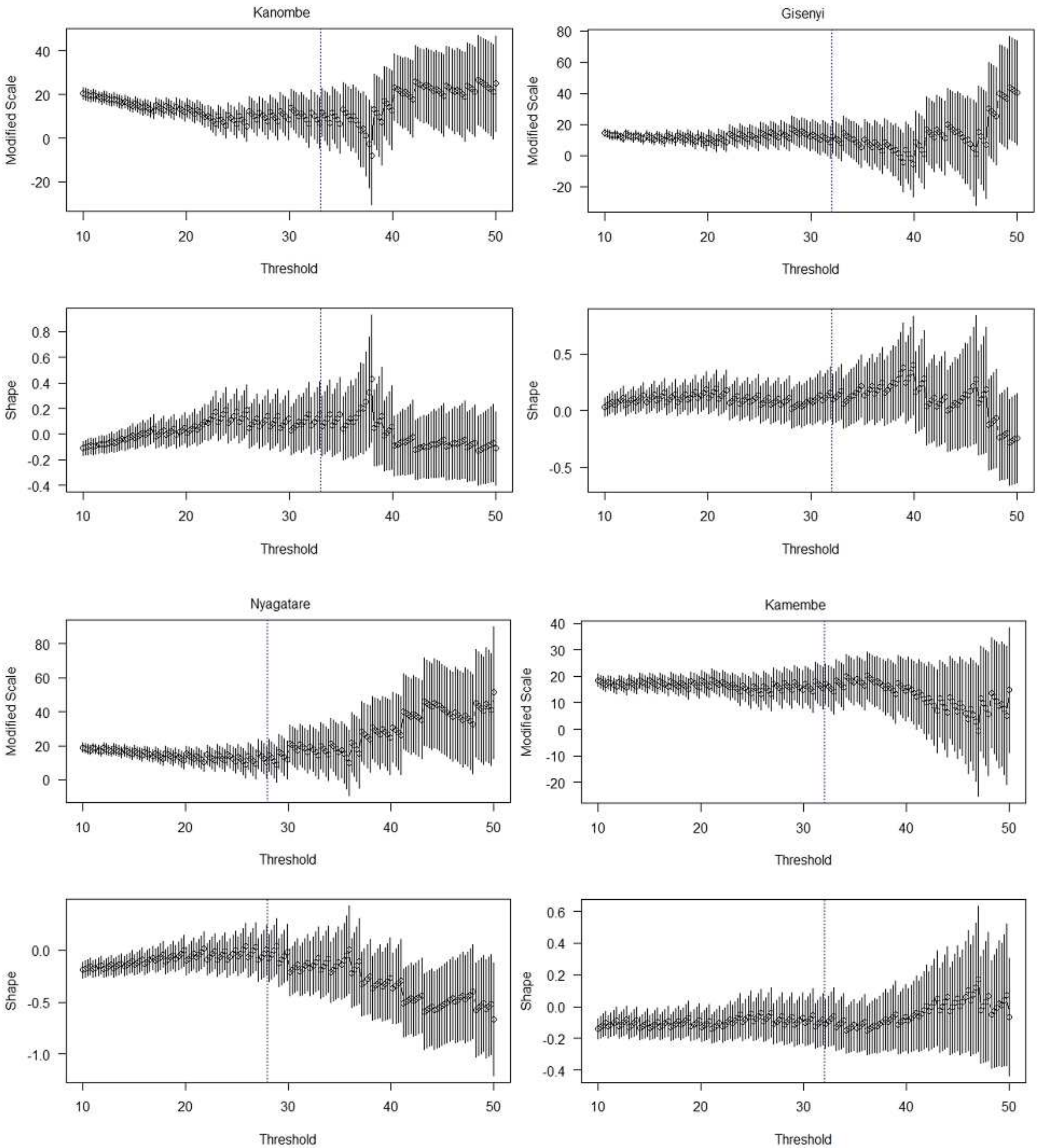

Figure 5

Mean Residual plots for the four synoptic stations (Kanombe, Gisenyi, Nyagatare and Kamembe) taken as example

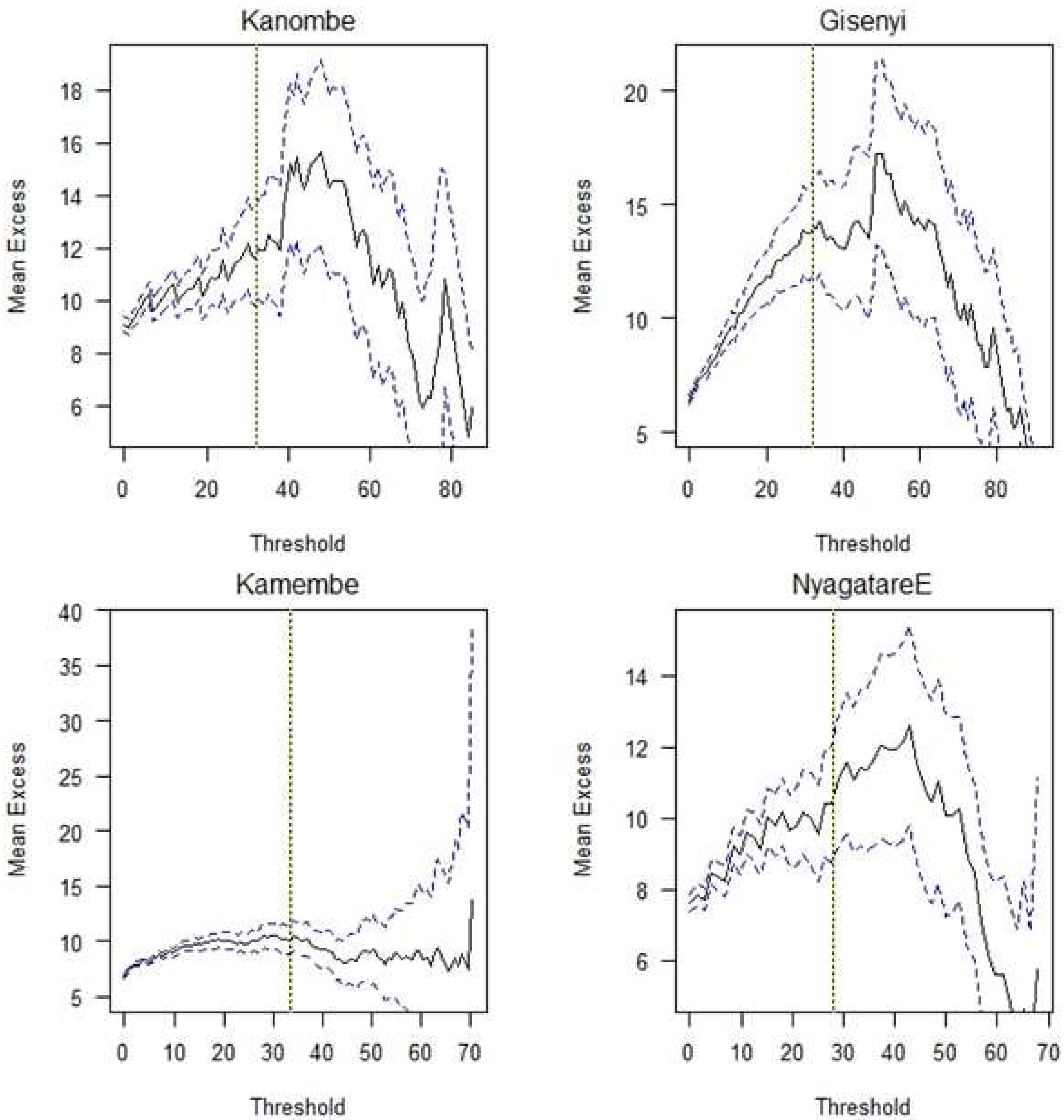

Figure 6

Parameter Stability plots for the four synoptic stations taken as example

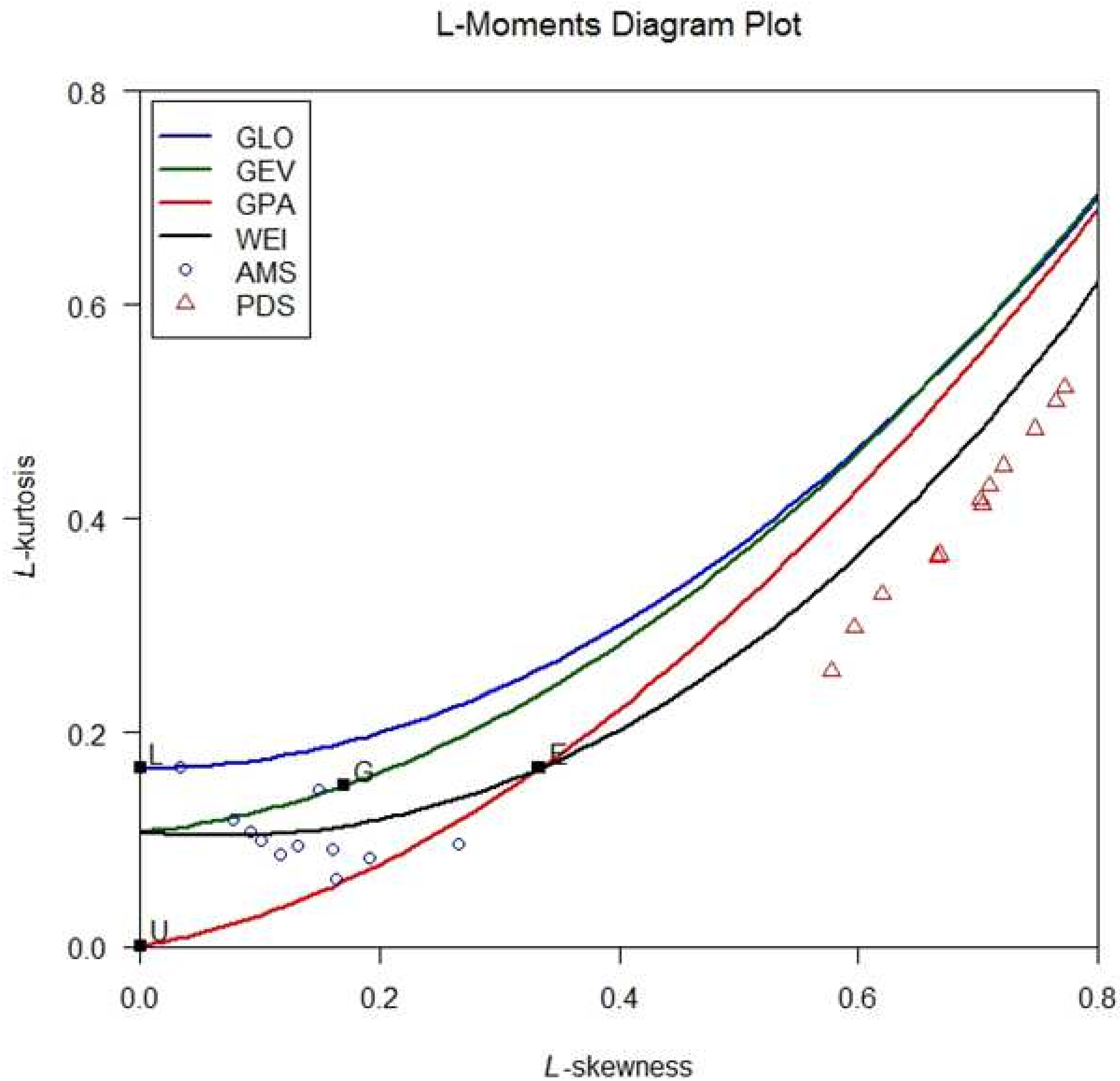

Figure 7

L-Moments Diagram plot presenting the ratio between L-kurtosis and L-skewness for goodness of fit of the chosen distributions at the twelve stations of the study. GLO: General Logistic; GEV: General Extreme Value; GPA: General Pareto; WEI: Weibul.

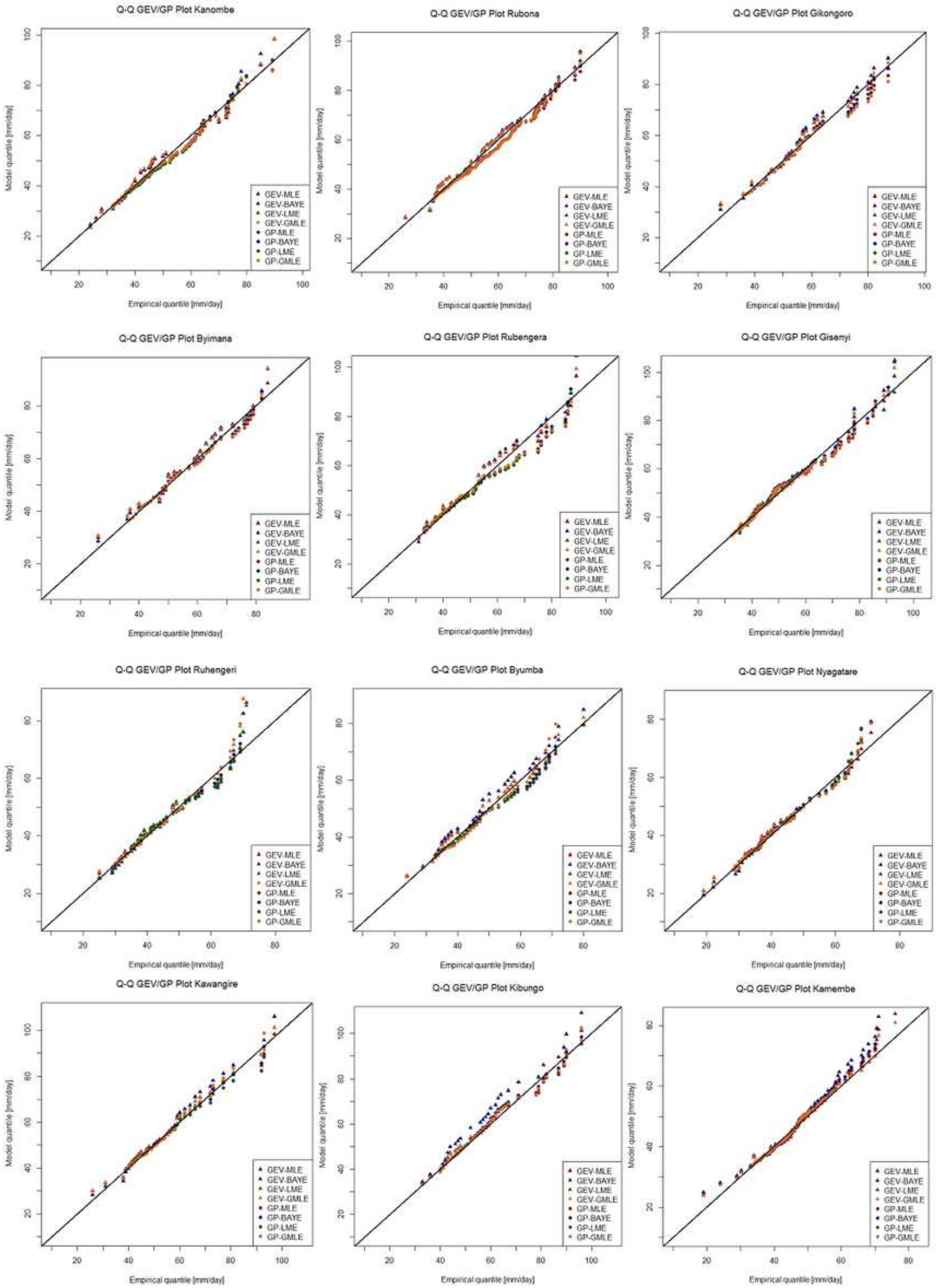

Figure 8

Q-Q plots for GEV and GP distributions with the four parameterisation methods for the twelve synoptic stations of the study.

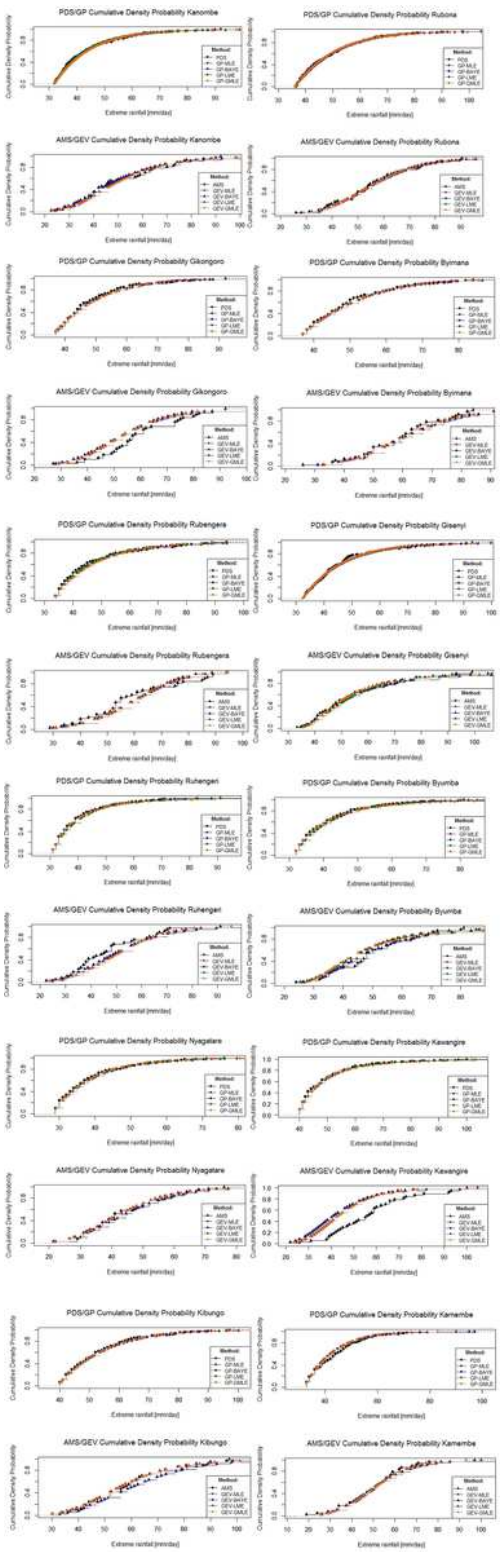

Figure 9

Cumulative Density plots for AMS, PDS, GEV and GP distributions with the four parameterisation methods for the twelve synoptic stations of the study.

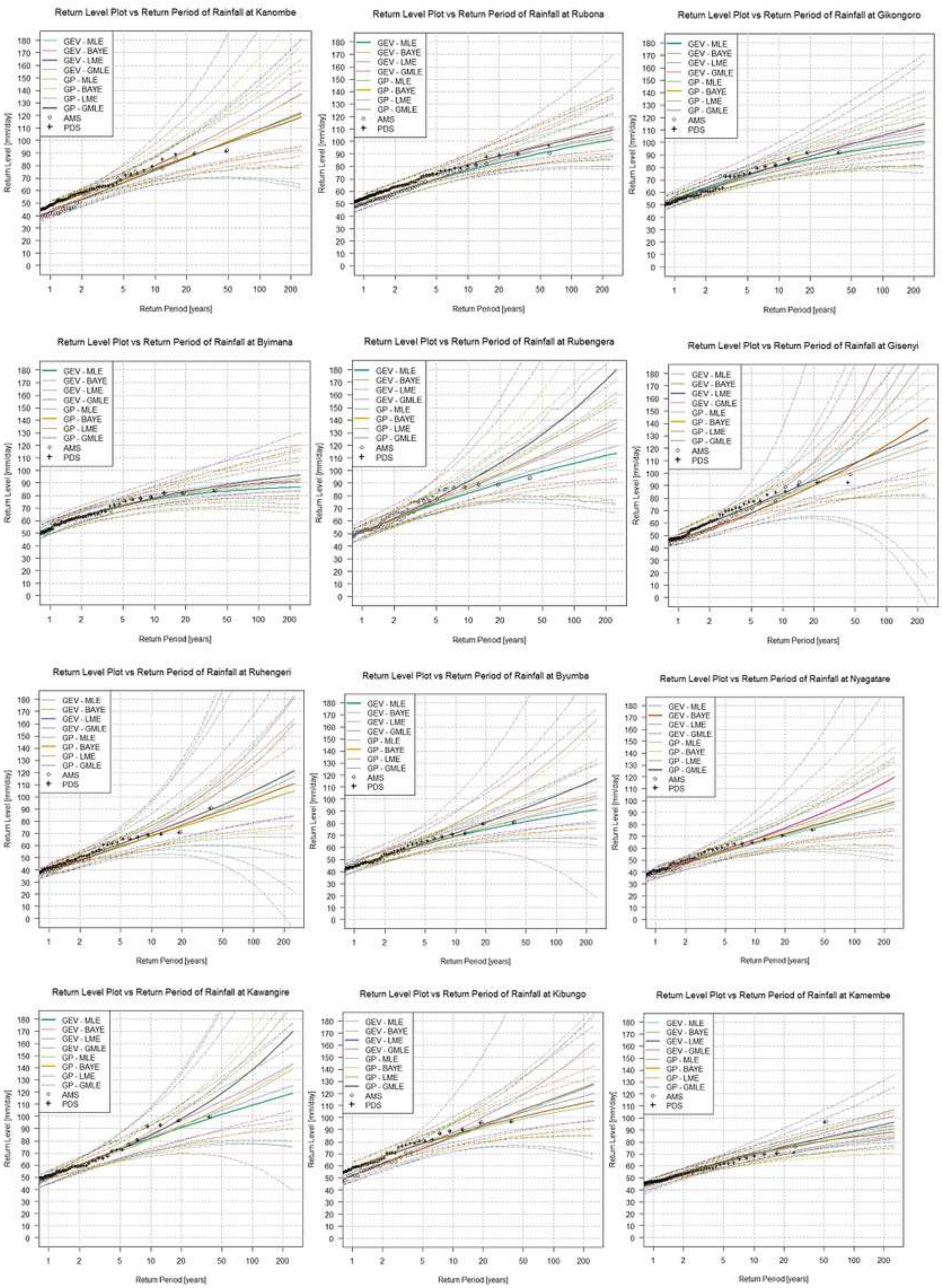

Figure 10

Return levels estimates with their 95% confidence intervals corresponding to different return periods for GEV and GP distribution fits of AMS and PDS respectively of daily rainfall data set with the selected parameterization methods (thick lines) for each station.