Empirical Modeling of Optimum Tilt Angle for Flat Solar Collectors and PV Panels

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Abstract

Global historical experimental measurements regarding temporal and geographical variations of sky clearness index worldwide and their associated diffused light estimations are used to model the optimum tilt angle between a flat solar collector, or a PV solar panel and the horizontal for all latitudes within both hemispheres. Equations for the yearly, seasonal and monthly relations between optimum tilt angle and latitude angle are obtained. General good agreements between this modeling results and most of both experimental and other models predictions are found. A freely downloadable MATLAB software that output yearly, seasonal and monthly optimum tilt angle for any latitude is made available.

Introduction

Driven by global warming, economic and political concerns worldwide, renewable energies are playing an ever increasing role in providing the world energy needs. The major players in this respect are the wind and solar energies. As far as the latter is concerned, photovoltaic (PV) electricity represents the most important way to generate electric power on both industrial and household levels.

According to International Energy Agency [(IEA) (2021)] report, global PV electricity generation reached 160 gigawatts (GW) in 2021. This represents an annual growth of 17% [IEA (2022)]. Annual PV industry growth for 2020 was 23% despise the COVID-19 pandemic. Average annual PV growth in the US over the last decade reached about 33% [SEIA (2022)]. Household PV installation annual growth was about 30% for the same period.

While many industrial PV installations adopt sun tracking systems, most household installations do not involve such systems. There are several reasons for this. The first is related to cost. Solar trackers are expected to increase generated power by 20–40%. This comes at increased installation costs of about 30% [Solar Reviews (2022), Wind & Solar (2022)]. Furthermore, tracking systems have moving mechanical parts which require periodic maintenance [Lee *et al* (2009)] In addition, it is relatively much easier and cheaper to install household PV systems directly on the inclined rooftop. Consequently, it may prove more economical to install larger number of PV panels instead of spending the same amount of capital on tracking systems.

The use of fixed PV systems naturally requires the optimization of the angle θ between PV panel and the horizontal surface which is called the *tilt* angle. Such optimization allows the system to produce the maximum electric power. The optimum tilt angle θ_{opt} is defined as the angle between the horizontal and the flat solar collector or PV panel that produces maximum solar energy output. Unfortunately, every tilt angle is associated with maximum power on a particular day of the year at a particular location. In other words, the best tilt angle for a summer day is the worst one on a winter day and vasa versa. This is due to changing sun direction throughout the year. Consequently, it becomes almost meaningless to define an optimum tilt angle unless that is coupled with the definition of an associated particular period of the year, or otherwise define the optimum angle averaged for the whole year. To be precise, we need to specify the optimum tilt angle at particular latitude on daily basis. However, and as said earlier, the practical and economic considerations suggested that economic gains achieved using such daily adjustments are very small. Instead, most PV systems employ fixed directions installations. In such installations, the panel is directed south in the northern hemisphere and north in the southern hemisphere. The azimuthal tilt angle is fixed at a value that produces the maximum average possible output over a specified period of the year or over the whole year

Many works have been performed to establish the year averaged optimum tilt angle at particular locations. [Jacobson & Jadhav (2018)] used the National Renewable Energy Laboratory's *PVWatts* program [PVWatt (2022)] to produce a list of optimum tilt angles at 216 meteorological stations locations in all world countries. The software uses 30 years historical sunshine data from each station to estimate the optimum tilt angle. A review article on the subject of optimum tilt angle by [Ogundimu *et al* (2019)] listed about 25 related research works carried out at different world locations. Some works measured optimum tilt angles at different locations experimentally [Ajao et al (2013), Krishna *et al* (2020), Kamanga *et al* (2014), Memon et al (2021), Karafil et al (2015), Asowata et al (2012) & Mohammed (2019)], Other works used different modeling or computer algorithms to predict optimum tilt angles at different locations [Apeh *et al* (2021), Matius *et al* (2021), Sarr *et al* (2021). Yassir et al (2019), Kaddoura *et al* (2016), Kazem et al (2013), Nfaoui & El-Hami (2020), Mamun *et al* (2017), Hailu & Fung (2019), Uba, & Sarsah (2013), Benghanem (2011), Calabrò (2009), Calabrò (2013), Guo (2017), Dervishi & Mahdavi (2012), Krivoshein (2020) and Darhmaoui & Lahjouji (2020)]. Most theoretical, modeling and computer algorithm works are oriented towards estimating optimum tilt angles at a particular location using local weather data as model input ingredients. Some works suggest different relations between tilt angle and latitude for summer and winter months. This local confinement of optimum tilt angle modeling is dictated by the fact that while estimating the optimum tilt angle at any location using clear sky calculations is not a difficult task, sky clearness and diffused light effects complicates the situation. In short, a global model needs adequate global information regarding sky clearness index and diffusion radiation information. Consequently, and although there have been several models suggesting several relations between optimum tilt angle and latitude angle. Large differences among different model predictions are not uncommon. Large deviations between model predictions and experimental results are also registered. The main reason for the discrepancies between different results is the almost random nature of the sky clearness factor (K_c)

and the different mathematical approaches used to model diffused radiation contribution. In a review paper, [Maleki & Hizam (2017)] listed about 35 such models. As a result, most models tend to restrict themselves to specific locations where there exists adequate information regarding the variation of *Kc* throughout the year. Some other works are based on assuming constant value of K_c [Elsayed (1989)]

It is the purpose of this work to present model calculation of the global yearly, seasonal and monthly optimum tilt angles. The modeling is based on numerically maximizing the amount of solar radiation incident on a PV panel during a particular period of the year at any particular location. The model makes use of available global K_c data to generate diffusion radiation effects at all latitude angles.

Freely available MATLAB software which outputs the optimum tilt angle for time periods ranging between one day and one year at any particular latitude is written.

Optimum Tilt Angle: Calculations And Modeling

The directional angle β for sunlight on a horizontal flat solar collector or PV panel installed at a latitude location position of ϕ on a particular time during a particular hour of a particular day of the year when the Hour angle between longitude line and the sun is ω and the Declination angle between the sun direction and the equatorial plane is δ can be written as [Duffie & Beckman (2006), Messenger & Ventre (2003)].

$$\cos(\beta) = [\cos(\delta) \,.\, \cos(\phi) \,.\, \cos(\omega) + \sin(\delta) \,.\, \sin(\phi)]$$

1

At any particular location, the hour angle is $\omega = 0$ at noon, negative before noon and positive in the afternoon. It basically defines the time difference from noon time multiplied by the value of the angle spanned by earth rotation during one hour which is 15 ^{o}. Consequently,

$$\omega = 15(t_s - 12)$$

2

t_s is the solar time

The Declination angle δ between the sun direction and the equator is caused by 23.45 ^{o} angle between earth rotational axis and earth orbit plane. This angle varies between − 23.45 ^{o} and + 23.45 ^{o} and it assumes these two extreme values on 22nd of December and 21st of June respectively. For a particular day of a non-leap year, δ is given by

$$\delta_n = 23.45 sin\left[360 * \frac{(284+n)}{365}\right]$$

3

It is trivial to say that the amount of solar energy received on a particular day depends on the date during the year. Neglecting all other secondary effects, day time during winter is shorter than that during summer. On a clear day and neglecting all weather effects like clouds and dust, the amount of solar energy received by a horizontal surface during the day is proportional to the daytime length (*L*). The daytime length is the time period between sunrise and sunset for the nth day of the year starting from 1st of January. This is given by [Duffie & Beckman (2006), Messenger & Ventre (2003)].

$$L(n,\phi) = \frac{2}{15} cos^{-1}[-\tan(\phi) \times \tan(\delta_n)]$$

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Equation (4) does not take into consideration the effect of sun light refraction when passing through the atmosphere. To account for such refraction effects, Eq. (4) is modified to include a correction term *sin(a)*, with *a* being an angle having the value of -0.83 ^{o}. Eq. (4) for the energy metric on a horizontal surface is thus rewritten as [Duffie & Beckman (2006)].

$$L(n,\phi) = \frac{2}{15} cos^{-1}\left[\frac{\sin(a) - sin(\phi) * sin(\delta_n)}{\cos(\phi) * \cos(\delta_n)}\right]$$

5

Calculation shown in figure (2) for the difference produced by Eq. (5) correction compared to Eq. (4) show that this correction is highest in winter months of December and January and it increases with increasing latitude reaching about 1.8% of the daytime length at latitude angle of 40 ^{o} in winter while it does not exceed 1.2% of the day time length in summer. This corresponds to increase in calculated daytime of 6.6–7.2 minutes on the equator and about 8.7–10.2 minutes at 40 ^{o} latitude.

As stated earlier, the aim of obtaining the optimum tilt angle is to maximize the energy produced during a specified period of time during the year. Starting with a hypothetical clear sky model where the sky clearness index (K_c= $H/H0$) is equal to unity (H and H_0 are the earth surface and the extrteristial radiations respectively) and assuming that no diffused (D) or surface reflected radiation (r) are present, the daily energy (E) per unit area received will be directly proportional to the daytime length Consequently, the total energy received over a period between day n_1 and day n_2 during the year will be proportional to the sum of daytime lengths (X) during the specified period n_1 *to* n_2 days. From now on, the variable X will be used as our energy metric for convenience.

$$X\left(\phi\right)=\sum_{n=n_1}^{n=n_2}L(n,\phi)$$

6

At any variable tilt angle θ with the horizontal deviating from the optimum position, the energy incident on the solar collector or PV panel will be

$$X(\phi,\theta)=\sum_{n=n_1}^{n=n_2}L_n\cos(\phi-\delta_n-\theta)$$

7

Equation (7) suggests that any inclination deviating from optimum will have the effect identical to reducing the daytime length for each day during the specified period through multiplication by the *cosine* term associated with that particular day. One method to find the optimum tilt angle θ_{opt} is to calculate the quantity under the summation in Eq. (7) after substitution from (5) as a function of θ and finding the value of θ that gives the maximum value of the quantity

$$X(\phi,\theta)=\sum_{n=n_1}^{n=n_2}cos^{-1}\left[\frac{\sin\left(a\right)-sin\left(\phi\right)*sin\left(\delta_n\right)}{\cos\left(\phi\right)*\cos\left(\delta_n\right)}\right]\cos(\phi-\delta_n-\theta)$$

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Calculations And Modeling

Equation (8) gives the relation between the metric of energy received by the PV panel (X) and the angle $0^0<\theta<90^o$ between the plane of the panel and the horizontal at any geographical location with latitude angleϕ. Calculating X for a specified period of time between day n_1 and n_2 during the year and finding the position in θ values which gives maximum X value (X_{max}) will correspond to the optimum tilt angle θ_{opt} for the clear sky model. The average θ_{opt} for a whole year is obtained by setting n_1 = 1 and n_2 = 365 for a non-leap year.

Figure (3-a) show the results of calculations of the yearly sum of effective daylight hours X potted against tilt angles at all latitudes from the equator up to 65 ^{o} north. Needless to say, the same applies to the southern hemisphere. For more clarity purpose, results for latitude angles of 15, 30, 45 and 60^o are re-plotted in figure (3-b).

The above analysis uses the clear sky model with all days sky clearness index ($K_c(n) = 1$) and no diffused radiation ($D = 0$). However, the situation is far more complicated due to three reasons at least. These are:

1. The atmospheric weather effects caused by clouds, water vapor and dust particles reduce the sky clearness index.
2. Diffused light caused by scattering of sunlight from atmospheric clouds and dust particles.
3. Surface reflected radiation. This depends on the surrounding terrain. This effect depends on the area surrounding the solar collector and it is thus case sensitive. Even so, its effect is small compared to direct and diffused radiations, and it is more or less isotropic with limited angular effects.

As far as the first effect is concerned, it is well known that the sky clearness possesses both geographical and seasonal variations. It is trivial to say that it is impossible to precisely model these variations on global bases because they are mainly dependent on instantaneous local weather conditions at a particular moment in time. Consequently, one has to make some reasonable assumptions regarding how such weather conditions affect sky clearness. One assumption we introduce here is that the geographical and temporal effects are independent. In mathematical terms, this means that K_c is factorable into the product of two separate functions.

$$K_c\left(n,\phi\right)=C\times A\left(n\right)\times B\left(\phi\right)$$

9

C is a normalization constant required in order to avoid double downscaling by the two function *A* and *B*. To investigate these two function, global experimental data compilation published by [HOMER (2022)] related to the variations of K_c with latitude angle during each month of the year are used. The data include measured averaged monthly *Kc* values at 331 latitude angle locations. In all, the database contains 3972 data points. The temporal dependence of K_c is studied by averaging all measurements over all latitude angles for each month of the year. The results of such averaging are plotted against the day of the year (*n*) which represents dates at the middle of each month in figure (4). The data can be reasonably fitted to a second degree polynomial of the form

$$A\left(n\right)=-3.1\times 10^{-6}\times n^2+0.001\times n+0.46$$

10

In order to establish the behavior of $B\left(\phi\right)$, the experimental values of K_c are averaged over all months for each value of ϕ. The averaged data are plotted as circles in figure (5). To put these data in a usable modeling form, they are fitted to a double Gaussian equation of the form

$$B\left(\phi\right)=a_1e^{-a_2\left(\phi-a_3\right)^2}+a_4e^{-a_5\left(\phi-a_6\right)^2}+a_7$$

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The dark smooth solid line in figure (5) represent the 95% confidence level fit to Eq. (11) with best fitting parameters values of a_1 = 0.2632, a_2 = 0.0008, a_3 = 27.3063, a_4 = 0.1420, a_5 = 0.0302, a_6 = 66.7598, a_7 = 0.2982. This gives

$$K_c\left(n,\phi\right)=1.8182\times\left[-3.1\times 10^{-6}\times n^2+0.001\times n+0.46\right]\left[a_1e^{-a_2\left(\phi-a_3\right)^2}+a_4e^{-a_5\left(\phi-a_6\right)^2}+a_7\right]$$

(12)

The factor of *C = 1.8182* is the reciprocal of 0.55 which is the maximum value of both *A(n)* and $B\left(\phi\right)$ It is used here to avoid double downscaling of K_c by both functions.

In order to have the best possible representation of the K_c data, one further modification is necessary. This is because although Eq. (12) provides the best possible fit to the data, the actual data possess large fluctuations above and below the fitted smooth curve. These fluctuations are manifestations of changing weather conditions. Such fluctuations are of almost random nature in the sense that they change values and positions from one year to another. The maximum full span between the highest and lowest K_c values is 0.36. To account for such fluctuations within the model, a random number term (*R*) such that $-0.18\le R\le 0.18$ is added to Eq. (12). The equation thus becomes

$$K_c\left(n,\phi\right)=1.8182\times\left[-3.1\times 10^{-6}\times n^2+0.001\times n+0.46\right]\left[a_1e^{-a_2\left(\phi-a_3\right)^2}+a_4e^{-a_5\left(\phi-a_6\right)^2}+a_7+R\right]$$

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The addition of these random numbers makes the model more representative of the actual data. The result of such addition is shown as dotted line in figure (5).

The second consideration which needs to be included in this modeling is the effect of diffused radiation. This modification is related to the fact a significant part of the light incident on a solar collector is diffused light. This light results from scattering of direct sun light against atmospheric constituents. To model the effect of diffused radiation. There are several empirical models that relate the diffuse fraction of the radiation (K_d) to the sky clearness factor K_c [Erbs et al (1982), Orgill & Hollands (1977), Reindle (1990), Lam & Li (1996), Skartveit & Olseth (1987), Louche et al (1991), Vignola & McDaniels (1984)]. Based on conclusions by [Dervishi Mahdavi (2012)] that the model suggested by [Orgill and Hollands (1977)] was found to be more compatible with experimental results, this model is selected for use in our modeling of optimum tilt angle. The model suggests the following relation between K_d and K_c.

for $K_c < 0.35$ $K_d = 1-0.249K_c$ (14-a)

for $0.35\le K_c\le 0.75 K_d=1.577-1.84K_c$ (14-b)

for $K_c > 0.75$ $K_d = 0.177$ (14-c)

The daily energy metric on a horizontal surface according to the clear sky model is given by Eq. (5). Invoking the sky clearness index K_c modifies this daily metric to $M(n,\phi)$ given by

$$M\left(n,\phi\right)=K_c(n,\phi)\times L(n,\phi)$$

15
This quantity involves both diffused and direct parts. The diffused part is isotropic with no significant tilt angle dependence. The direct part carries the major part of the tilt angle dependence. The diffused part $D(n,\phi)$ can be written as

$$D\left(n,\phi\right)=K_d\times M\left(n,\phi\right)=K_d\times K_c(n,\phi)\times L(n,\phi)$$

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The direct part can be obtained by subtracting the diffusion radiation from the total radiation reaching the surface. We can thus write the daily tilt angle θ dependence of the total radiation T as

$$T\left(n,\phi,\theta\right)=D\left(n,\phi\right)+\left(K_c-K_d\right)\times L(n,\phi)\times\cos(\phi-\delta_n-\theta)$$

17
The total energy metric X for the energy received by a flat solar collector or a PV panel making a tilt angle θ with the horizontal during the time period between day's n_1 and n_2 of the year is given by

$$X\left(\phi,\theta\right)=\sum_{n=n_1}^{n=n_2}\left[D\left(n,\phi\right)+\left(K_c-K_d\right)\times L(n,\phi)\times\cos(\phi-\delta_n-\theta)\right]$$

18
Equation (18) is used to calculate the yearly, seasonal and monthly tilt angle dependence of the energy metric at each latitude angle. Results for the yearly calculations are shown in figure (6-a). For clarity purpose, only results for latitudes 0f 15 and 45^{o} are re-plotted in figure (6-b). The optimum tilt angle in each case is obtained by fitting the energy metric dependence for each latitude angle to a second degree polynomial and the tilt angle that gives maximum value to the fitted curve is registered as the optimum tilt angle. Plots in figure (6 - a) correspond to those in figure (3 –a), after taking into account the sky clearness and diffusion radiation. It is interesting to note here that deviating from optimum tilt angle by as much as 10^{o} does not produce significant reduction in energy received calculated on yearly bases. The reduction is about only 2% for the clear sky model and about 1% only for the modified model. This means that one should not be much concerned about optimum tilt angle positioning of PV panels if one is interested in making the most of the solar energy on yearly base. In other words, energy lost during one season due to bad angular positioning will be automatically gained during another season.

Comparing between results in figure (3) and (6) one also observes that the tilt angle dependence in figure (3) is stronger than the corresponding one in figure (6). This is further manifested in figure (7) where the relations between θ_{opt}andϕ for both models are plotted. The relation for the clear sky model has a larger slop. This is much expected because of the role of isotropic diffused radiation in diluting the overall angular dependence.

Although deviations from θ_{opt} position may not result in significant overall annual energy loss, the same cannot be said as far as seasonal and monthly energies. This means that if the PV panel is needed to be geared to produce maximum output during a particular month or even season, then more attention as far as tilt angle positioning is required. To demonstrate this the relations for the optimum angle as a function of latitude for the four seasons and for the twelve months are plotted in figures (8) and (9) respectively.

Needless to mention here that above result is for the northern hemisphere. All signs are reversed when the equations are applied to the southern hemisphere.

In spite of the limited effect of tilt angle on the yearly energy yield of a flat solar collector, seasonal variations are more significant. This means that gearing the system to produce the maximum output during a specified season or month needs more attention as far as tilt angle is concerned. To demonstrate this, the dependence of θ_{opt} on $\phi$is studied for each season and each month of the year. The results are plotted in figures (8) and (9) respectively.

From the latter two figures one notes that while the relation between θ_{opt} and ϕ for autumn and winter in figure (8) and their associated months in figure (9) tend to be almost linear, the relations for spring and summer and their associated months consist of two parts. The first is for latitude angle below the limit 20–30 ^{o}. this part is characterized by weak negative slope dependence of θ_{opt} onϕ. The second part has a positive stronger dependence θ_{opt} on ϕ above this limit.

All relations for the yearly, seasonal and monthly results are fitted to appropriate linear relations. The results are summarized in table (1). However, one important point related to these fits needs to be mentioned here. This is and because of random variation deliberately introduced in the modeling of K_c to account for random weather variation, results of repeating the same fit are not identical. This is very much expected because the computer uses different set of random numbers during each run. Consequently, the results for the slopes and intercepts in table (1) are the averaged outcome of twenty runs. The standard deviations associated with each value is also indicated on the table. The set of linear equations of the annual, seasonal and monthly relations between latitude and optimum tilt angle are programed in MATLAB, and are freely available on the MATLAB file exchange website [Azooz (2022)]

Table (1) yearly, seasonal and monthly linear fitting parameters for fits of the form $\theta_{opt} = A\,\phi + B$ fitted to

Period	$A \mp \sigma_A$	$B \mp \sigma_B$	ϕ range
Yearly	$0.78 \mp (0.05)$	$3.5 \mp (2.0)$	All
Autumn	$1.07 \mp (0.05)$	$11.1 \mp (1.8)$	All
Winter	$1.05 \mp (0.05)$	$12.3 \mp (2.0)$	All
Spring	$1.05 \mp (0.05)$ $-0.3 \mp (0.20)$	$-19.2 \mp (2.0)$ $9.4 \mp (2.6)$	> 20 ≤ 20
Summer	$1.06 \mp (0.04) {-} 0.2 \mp (0.30)$	$-19.7 \mp (2.0)$ $9.7 \mp (3.6)$	> 20 ≤ 20
January	$1.08 \mp (0.04)$	$18.8 \mp (1.3)$	All
February	$1.11 \mp (0.05)$	$9.6 \mp (1.9)$	All
March	$1.06 \mp (0.03)$	$-0.8 \mp (1.4)$	All
April	$1.1 \mp (0.05)$ $0.05 \mp (0.20)$	$-15.2 \mp (2.6)$ $3.7 \mp (2.0)$	> 20 ≤ 20
May	$1.04 \mp (0.05)$ $-0.5 \mp (0.10)$	$-22.1 \mp (3.4)$ $12.3 \mp (1.8)$	> 25 ≤ 25
June	$1.05 \mp (0.05)$ $-0.7 \mp (0.10)$	$-26.6 \mp (3.5)$ $18.7 \mp (2.5)$	> 30 ≤ 30
July	$1.07 \mp (0.05) - 0.42 \mp (0.15)$	$-25.3 \mp (3.1)$ $13.2 \mp (2.1)$	> 30 ≤ 30
August	$1.11 \mp (0.05)$ - $0.02 \mp (0.10)$	$-19.9 \mp (3.5)$ $4.1 \mp (2.5)$	> 25 ≤ 25

Period	$A \mp \sigma_A$	$B \mp \sigma_B$	ϕ range
September	$1.07 \mp (0.05)$	$-6.2 \mp (2.6)$	>10
	$0.3 \mp (0.40)$	$1.7 \mp (2.0)$	≤ 10
October	$1.08 \mp (0.04)$	$8.6 \mp (1.5)$	All
November	$1.09 \mp (0.04)$	$17.0 \mp (1.3)$	All
December	$1.09 \mp (0.05)$	$20.9 \mp (2.0)$	All

Discussions

In order to test the applicability of the suggested modeling, comparisons with experimental measurements and predictions of other models are necessary. As far as testing model predictions against experimental results is concerned, results published in some references are used. It must be pointed out however that not all experimental data are measured during periods not strictly annual, seasonal or monthly. Consequently averages over the period specified are taken. Table (2) summarizes the results of comparisons between available published experimental data and model prediction.

Table (2) Comparisons between some published experimental optimum tilt angle results and this model predictions.

Ref.	Location	Lat.	Period	Exp.	Model	Remarks
Ajao *et al* (2013)	Ilorin, Nigeria	8.5	10 days May & June	22	12.7	1- measurements are for 10 min. only 2- almost no tilt angle dependence Fig. 3 3- Fig. 4 latitude scale is not consistent. 4- Experimental optimum may be close to 12^o
Krishna *et al* (2020)	Nitte, India	13.18	January	33	33.0	Agreement between Experiment and model is very good
			February	24	24.2	
			March	13	13.2	
			April	0.0	4.4	
			December	36	35.3	
Kamanga *et al* (2014)	Zomba District, Malawi	-15.4	October to February	0	-9.4	Agreement is good for March to September. Experimental tilt angles of only 0, 15, 20, and 25 are used in the experiment
			March to September	-25	-27.5	
Memon *et al* (2021)	Sindh Pakistan	27.7	September to March	29.5	30.3	Good agreement. Model calculations averaged over all months September to March
Asowata *et al* (2012)	Vaal Triangle, South Africa	-26.4	Winter	-36.8	-40.0	Agreement is fair
Mohammed (2016)	Baghdad Iraq	33.3	Yearly	35	30	Agreement is fair for annual and good for winter
			Winter	45	47	

The comparisons presented in table (3) suggest good general agreement between experimental data and model predictions in most cases.

In spite of the fact that the number of available optimum tilt angle experimental data is limited, there are large corresponding number of locally oriented model and theoretical calculations. The most extensive such work is that of [Jacobson & Jadhav (2018)], where National Renewable Energy Laboratory's *PVWatts* program which uses 30 years historical sunshine data obtained from 216 meteorological stations round the world is used to produce a list of yearly optimum tilt angles at 216 meteorological stations locations in all world countries. The results are compared to the predictions of this model. Figures (10) shows a histogram of the distribution of differences between data presented in reference [7] and the predictions of the current model. The histogram suggests that the median of these differences is zero and over 75% of the differences fall within $\mp 5^o$. This can be considered as acceptable agreement remembering that there are as much as $\mp 15^o$ data scattering between tilt angles for different locations having same or close latitudes. This is clearly demonstrated in figure (11).

Results of a study for the monthly, seasonal and yearly optimum tilt angles at Bilecik Turkey Latitude 40.1^o are presented in reference [14]. Table (3) shows a comparison between those data and current model predictions.

Table (3) Comparison between tilt angles from [Karafil et al (2015)] and current model

time	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Aut	win	Spr	Sum	Ann.
[14]	60.9	53.4	42.4	30.1	21.2	16.9	18.9	26.7	38.0	49.8	59.0	63.1	48.9	59.1	31.4	20.9	39.98
mod	62.1	54.1	41.7	28.9	19.6	15.5	17.6	24.6	36.7	51.9	60.7	64.6	54.0	54.4	22.9	22.8	35.2

The results show good monthly agreement with mean of absolute differences of only 1.4^o. The corresponding differences for the seasonal and the yearly data are equal to 5^o.

One last comparison is between the current model's monthly relations between ϕ and θ_{opt} and the corresponding ones predicted by [Nirncorodov et al (1994)] where monthly relations based on modeling using a constant sky clearness index $K_c = 0.75$are listed. Sample comparisons for the yearly, January, October, April, and July results are presented in figure (12 - a - f) respectively. Other not shown results for February, March, November and December are much similar to those of January and October, while results for May, June, August and September are similar to those for April and July. Figures (12-e) show the average absolute values of the degree by degree differences between the two models. It is clear from these figures that the differences between the predictions of the two models are small and they fall in the range of about 2^o for most of the six months of January, February, March, October, November and December. Differences as large as 10–20^o are observed for the six months of April, May, June, July, August and September. Such large differences are not very much surprising and are to be expected. This is because while the differences between the two models are small for latitudes above *20–250*, large differences between the two models are registered below such latitudes for the months of April to September. The current model shows two behavior trends below and above these latitudes while the model in Ref [47] predict that the same linear relation will persist below this range. This leads to the latter model predicting negative optimum tilt angles for the northern hemisphere during these months for all latitudes below *20–25^o*. The current model predicts different relations for these latitudes. This difference in predictions may be related to the fact that Ref [47] model uses a constant value for K_c while the current model is governed by the actual monthly dependence of K_c suggested by Eq. (10)

Conclusions

From the above analysis and discussion one may conclude the following

1. Energy collected by a flat solar collector or a PV solar panel over an entire year period is not so sensitive to the tilt angle value. Installing solar panels with any convenient tilt angle will produce almost the same amount of energy over the entire year. This means that any loss during a particular season will be offset by gain during opposite season.
2. Optimum tilt angles for autumn and winter tend to show almost similar linear relationships to latitude angle.
3. Relations for spring and summer show a change of the nature of the linear relationship to latitude at about 20^o.
4. Optimum tilt angles for January, February, March, October, November and December tend to show linear relationships to latitude angle, while those for April, May, June, July, August and September suffer from a change of linear behavior at latitudes 20–30^o
5. Results for optimum tilt angles obtained using this empirical modeling show good agreement with most experimental data experimental and statistical uncertainties.
6. Result are consistent with predictions of other published models within uncertainties dictated by different assumptions related to sky clearness and diffused radiation.

Declarations

Ethical Approval

All authors declare that the work comply with all ethics related to scientific procedures and actions.

Consent to Participate

All authors have read the final draft of the manuscript and agreed to its contents..

Consent to Publish

All authors have given their permissions to publish this work and have authorized the corresponding author (Aasim Azooz) to carry out the necessary actions in this respect

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Competing Interests

All authors have no relevant financial or non-financial interests to disclose.

Availability of data and materials

All data, material and software are available.

Author Contributions

All authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by Sonia Issaq, Shamil Talal and Aasim Azooz. The first draft of the manuscript was written by Aasim Azooz and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript

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Figures

Figure 1

Installation of PV panels with tilt angle equals to roof inclination

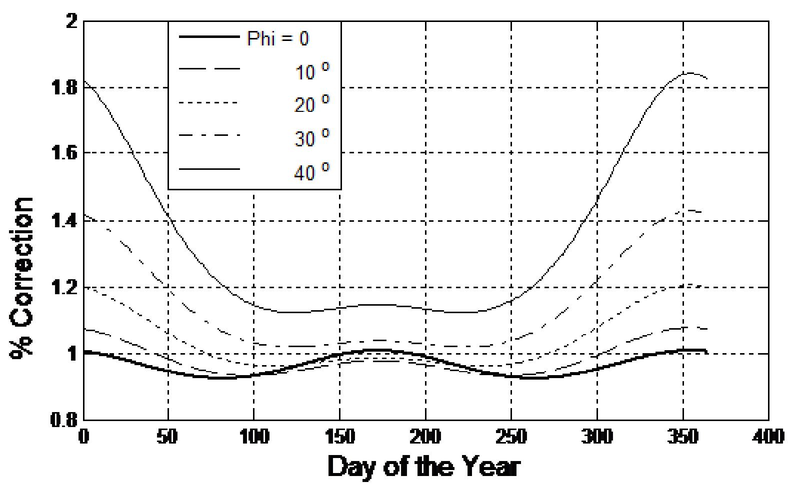

Figure 2

Percentage differences between equations (3) and (4) for the daytime correction due to atmospheric refraction.

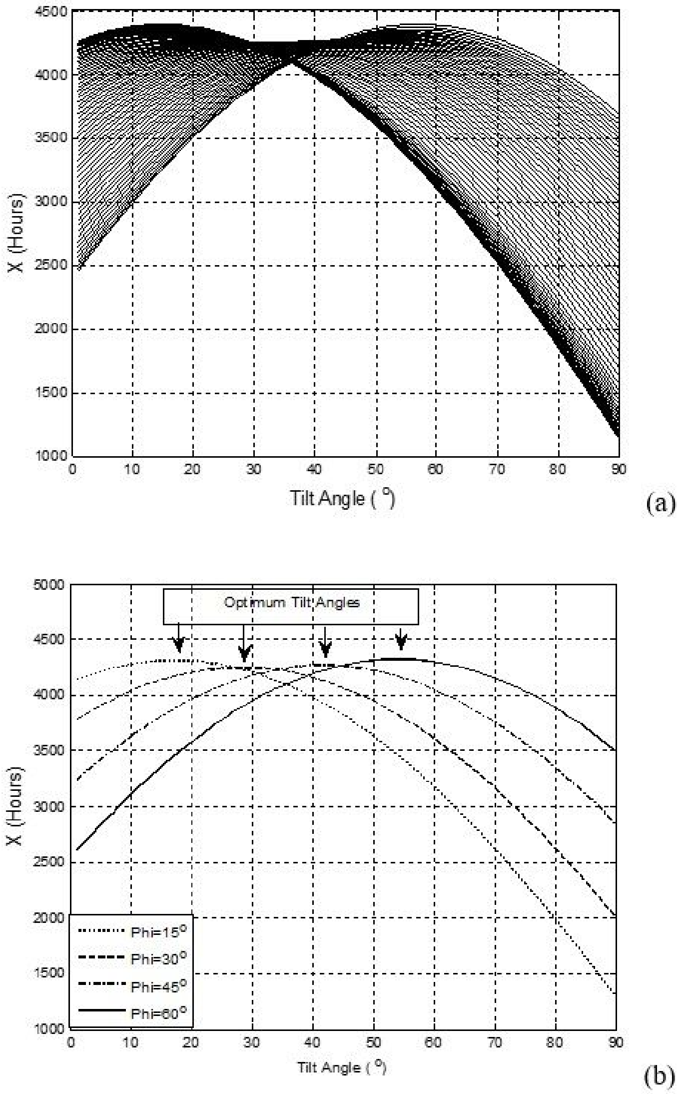

Figure 3

Yearly Energy metric against tilt angle from clear sky model

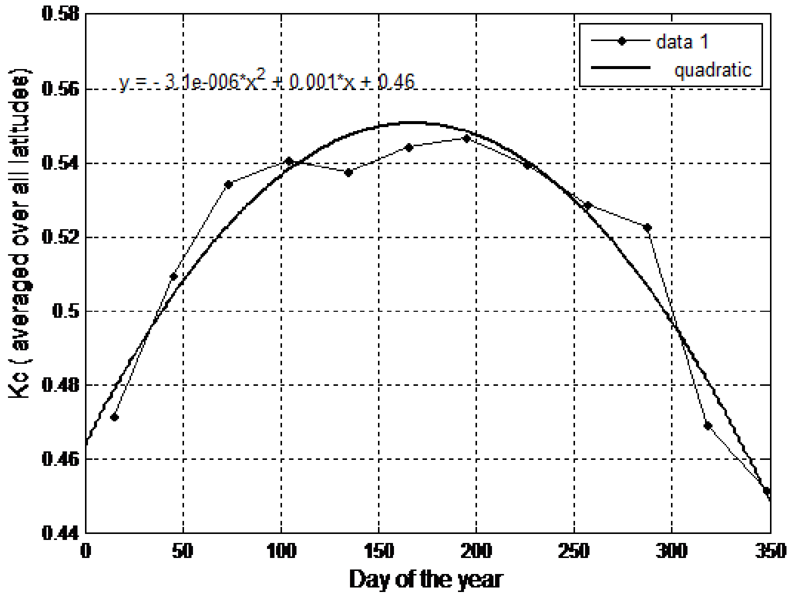

Figure 4

Monthly experimental K_c values averaged over all latitudes.

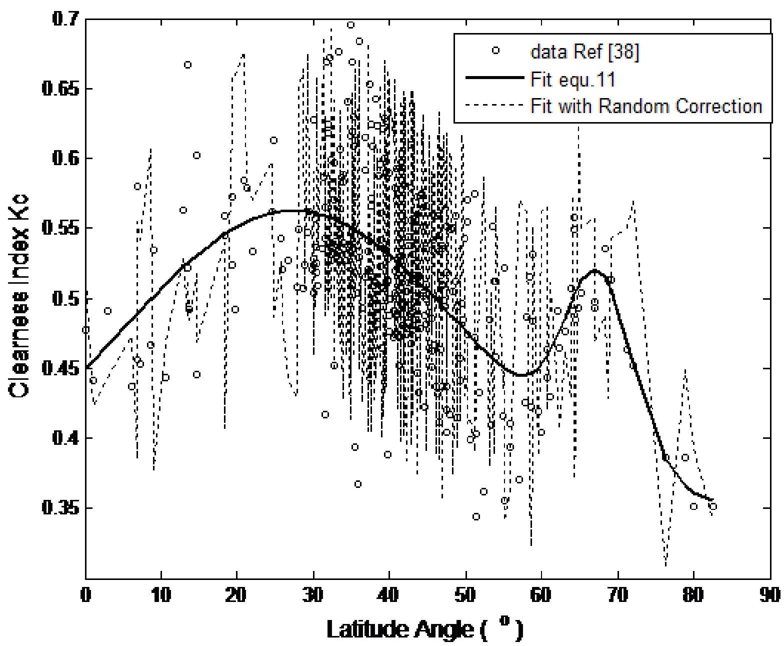

Figure 5

Experimental K_c against φ Ref [38] together with equations (12) & (13)

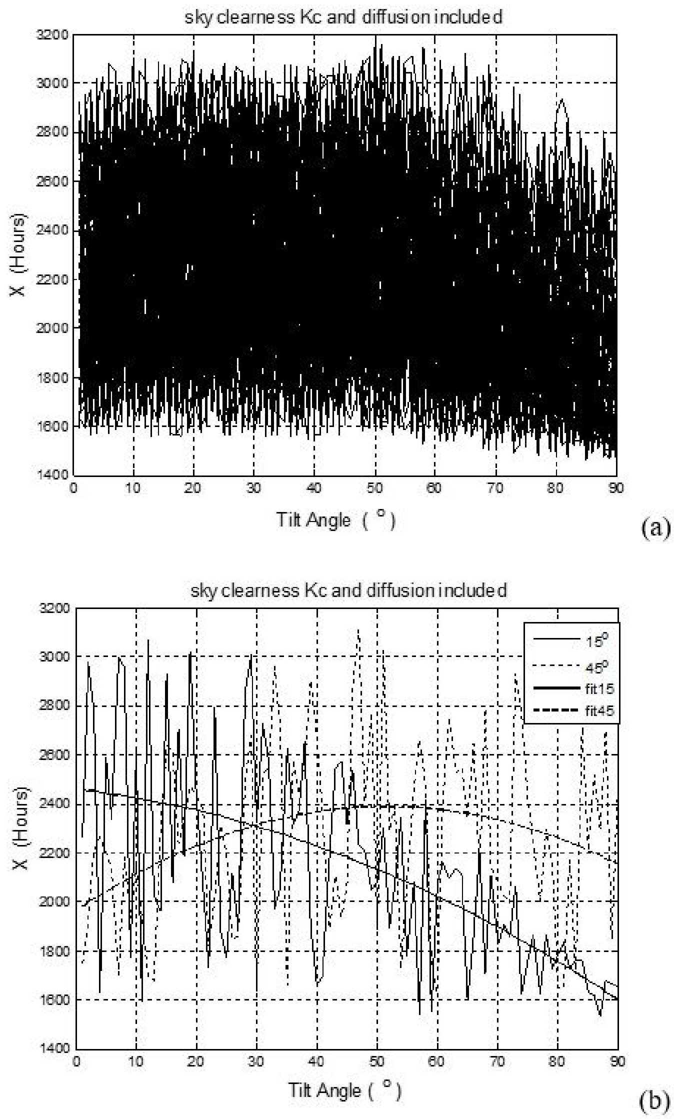

Figure 6

Yearly Energy metric *X* against tilt angle after including sky clearness factor and diffused radiation effects. (a) For all latitude angles. (b) For latitudes of 15and 45°

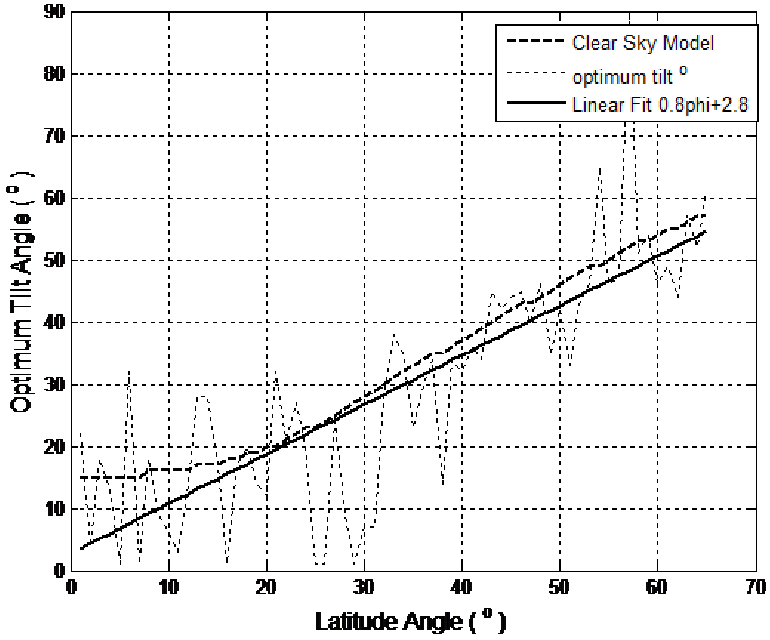

Figure 7

Relation between yearly optimum tilt angle and latitude angle deduced from clear sky model and final model in equation (18)

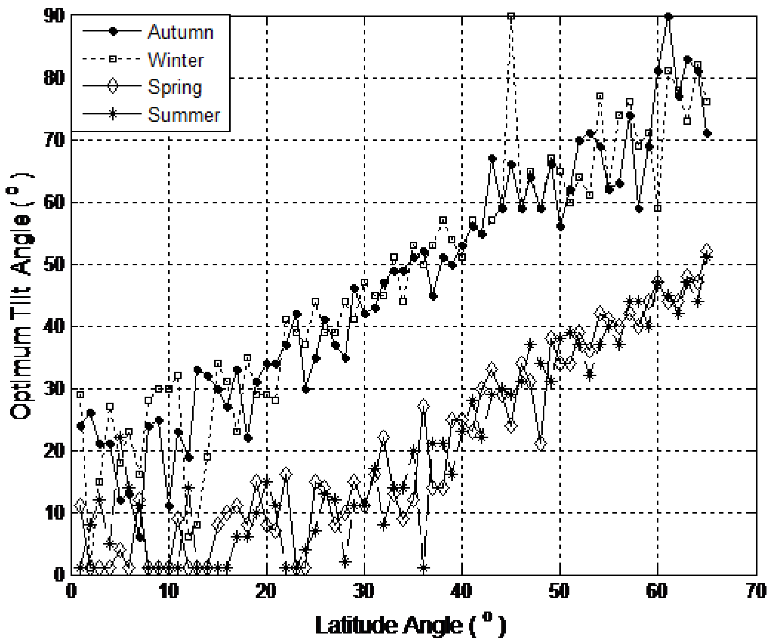

Figure 8

Relation between θ_{opt} and φ for the four seasons

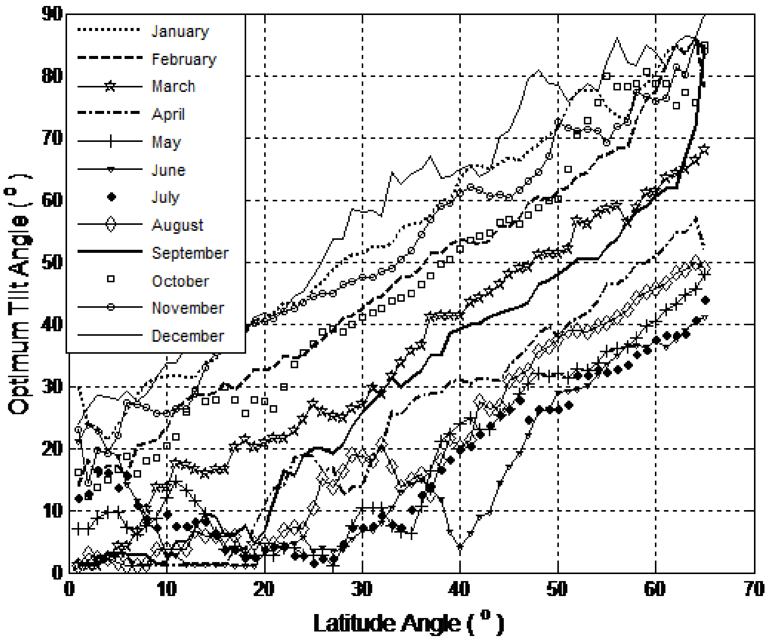

Figure 9

Relation between θ_{opt} and φ for the twelve months of the year

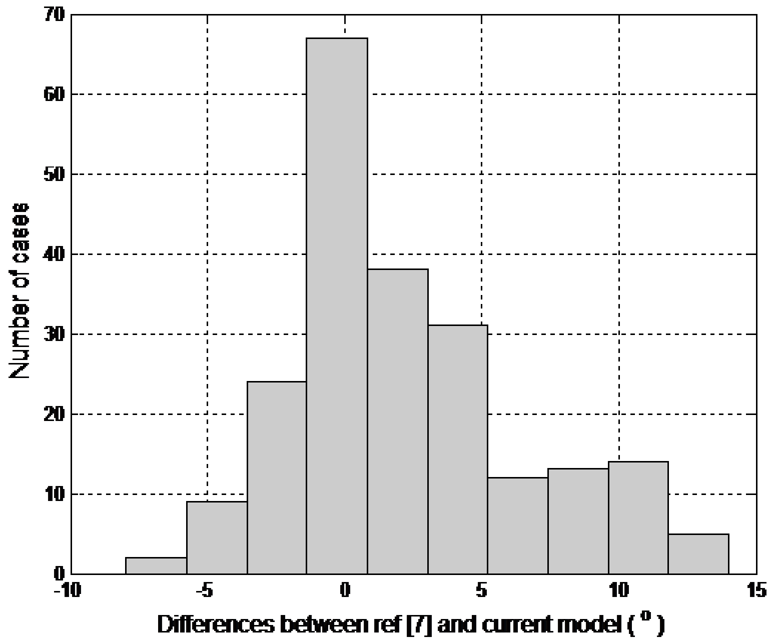

Figure 10

Differences between optimum tilt angles compiled by [Jacobson & Jadhav (2018)] and current model

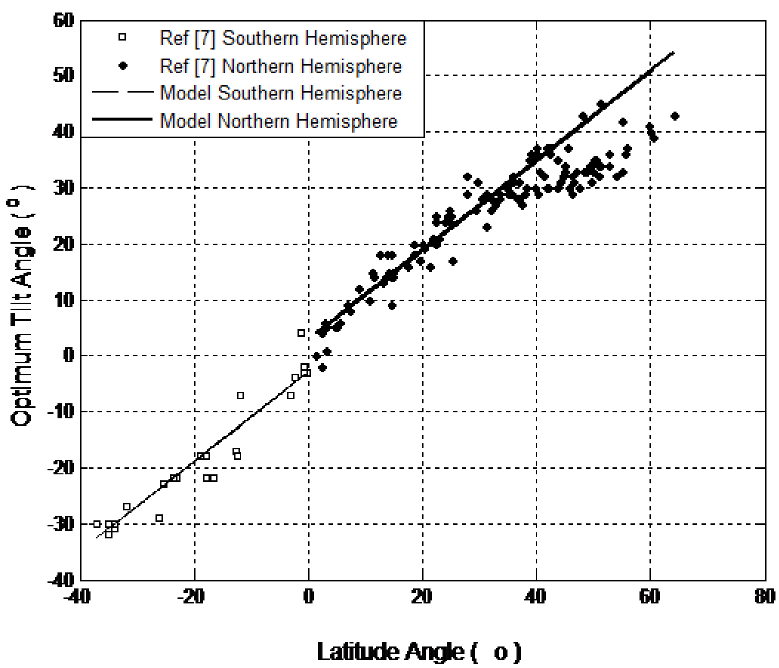

Figure 11

comparison of tilt angles compiled by [Jacobson & Jadhav (2018)] and the current model

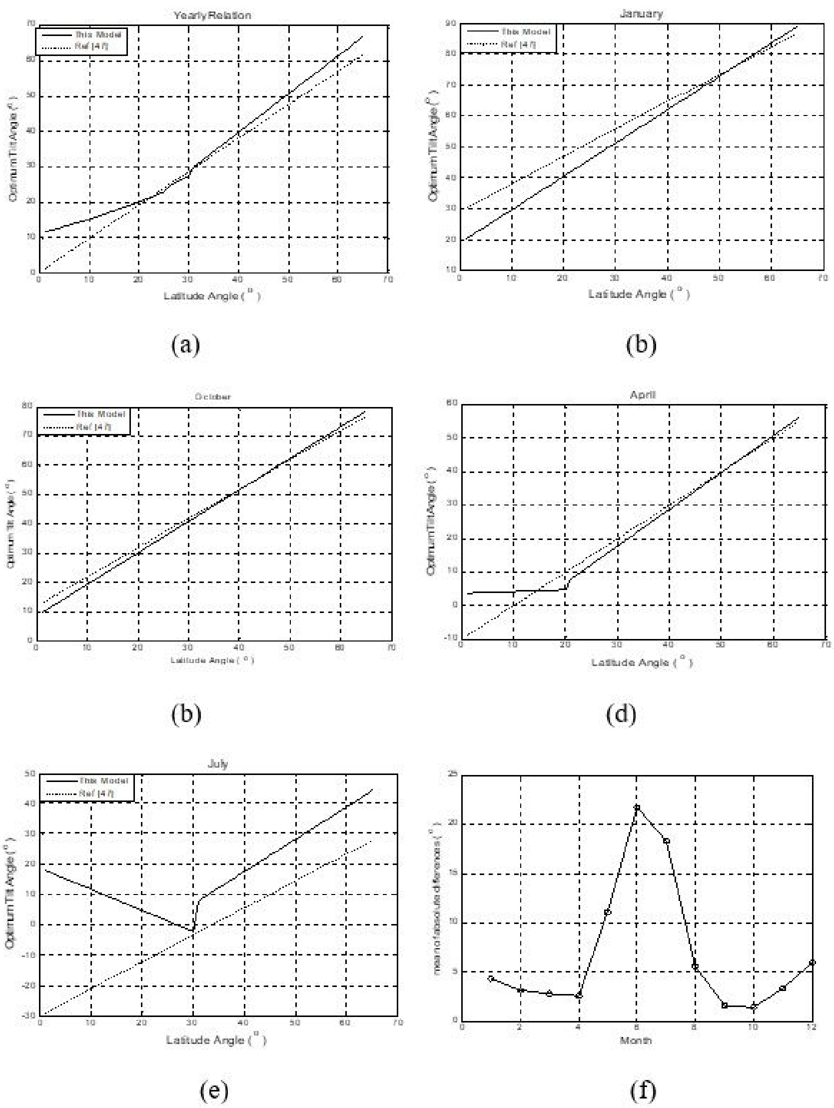

Figure 12

Comparisons between current model and Reference (47) for (a) annual, (b) January, (c) October, (d) April, (e) July, (f) monthly means of absolute differences between the two models